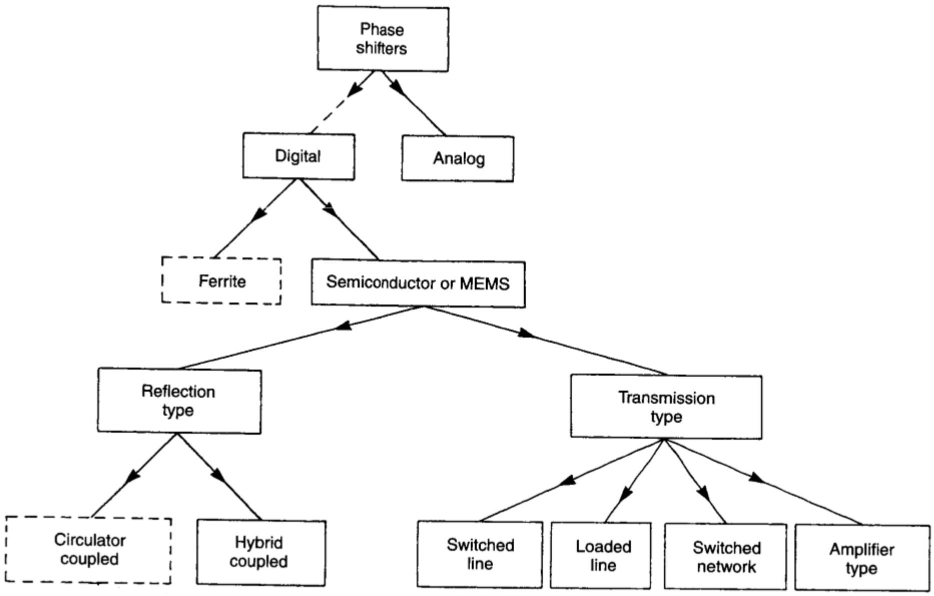
Phase Shifter Design

- A phase shifter is a two-port network with a provision that the phase difference between the output and the input signals may be controlled by a control signal (DC bias).

- Phase shifters are called DIGITAL when the differential phase shift can be changed by only a few predetermined discrete values, such as 180°, 90°, 45°, 22.5°, and 11.25°. On the other hand, in ANALOG phase shifters, the differential phase shift can be varied in a continuous manner by a corresponding continuous variation of the control signal.



Switched-Line Phase Shifters

Two SPDT switches are used to route the signal via one of the two alternative transmission path lengths l_1 or l_2.

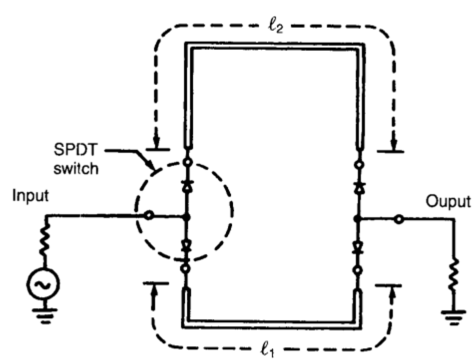
When the signal passes through the longer path, it goes through an additional phase delay given by

$$\Delta\phi = \beta(l_2 - l_1) = \frac{2\pi f}{v_p}(l_2 - l_1)$$

The differential phase shift $\Delta\phi$ is directly proportional to frequency. Because of this feature, switched-line phase shifters are also called switchable time delay networks.

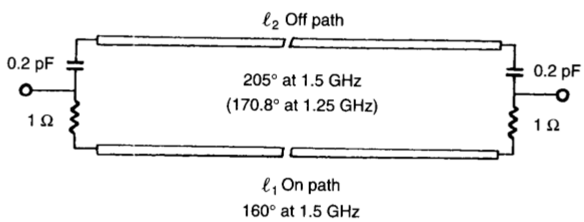
The time delay τ_d is given by:

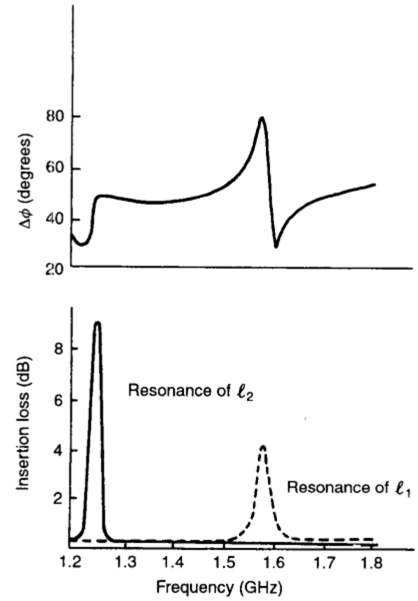
$$\tau_d = \frac{l_2 - l_1}{v_p}$$



[Single-bit switched-line phase shifter]

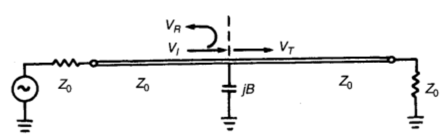
Problems in Switched-Line Phase Shifters
(Off-path Resonances)





Loaded-Line Phase Shifters

A very common design for 45° and 22.5° phase bits is known as the loaded-line phase shifter. The mechanism of phase shift in this circuit is based on the loading of a uniform transmission line by a small reactance as shown in Fig.



It can be shown that the transmitted wave undergoes a phase shift $\Delta\phi$ that depends upon the normalized susceptance $b = B/Y$. The reflection caused by b is given by

$$\Gamma = \frac{1-(1+jb)}{1+(1+jb)} = \frac{-jb}{2+jb}$$

The voltage associated with the transmitted wave is $V_T = V_I + V_R$, where V_I and V_R are voltages of the incoming wave and the reflected wave, respectively.

The transmission coefficient T can therefore be written as

$$T = \frac{V_T}{V_I} = \frac{V_I + V_R}{V_I} = 1 + \Gamma = \frac{2}{2+jb}$$

$$V_T = TV_I = V_I \frac{2}{2+jb} = V_I \left(\frac{4}{4+b^2}\right)^{1/2} \exp\left[-j \tan^{-1}\left(\tfrac{1}{2}b\right)\right]$$

The phase difference introduced (phase of V_I – phase of V_T) may therefore be written as

$$\Delta\phi = \tan^{-1}\left(\tfrac{1}{2}b\right)$$

If the normalized susceptance $b = 0.2$ (i.e., capacitive), $\Delta\phi = \tan^{-1}(0.1) = 0.1$ rad $= 5.7°$. If, on the other hand, we load the line with $b = -0.2$ (a shunt inductor), the phase delay $\Delta\phi$ is negative, that is, $\Delta\phi = \tan^{-1}(-0.1) \simeq -5.7°$.

That is, the transmitted wave advances in phase as compared with the transmitted wave for $b = 0$.

The insertion loss is given by:

$$\frac{V_T}{V_I} = \left(\frac{4}{4+b^2}\right)^{1/2}$$

As $|b|$ is equal in two states (only its sign is changed), insertion loss has the same value in two states. For $b = 0.2$, $V_T/V_I = 0.995$, which corresponds to a loss of 0.04 dB. Although, in the present case, the value of insertion loss is not significant, the presence of this loss is an unfavorable feature of the circuit

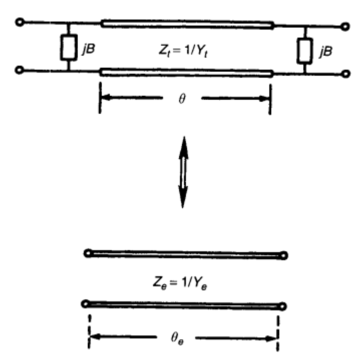
This drawback of reflection from the susceptance producing the phase shift can be overcome by using two identical susceptances separated by a quarter-wave line length. Such an arrangement is shown in Fig.

If we consider partial reflections caused by the two susceptances, these reflected waves are almost equal in magnitude and out-of-phase (180° phase difference) when looking into the input terminals. Thus these undesirable reflections cancel each other.

A quantitative assessment of the situation may be obtained by writing a uniform transmission line equivalent circuit as shown in Fig.

Equivalent Y_e and θ_e may be obtained by comparing *ABCD* matrices of the two networks.

An *ABCD* matrix of the transmission line of length θ, shunt-loaded by susceptances at either end, may be obtained by multiplying *ABCD* matrices of the shunt susceptance, of the line section, and of the second susceptance at the other end. This yields



Circuit configuration for loaded-line phase shifter and equivalent representation.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jB & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & jZ_t \sin\theta \\ jY_t \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ jB & 1 \end{bmatrix}$$
$$= \begin{bmatrix} (\cos\theta - BZ_t \sin\theta), & j(Z_t \sin\theta) \\ j(2B\cos\theta + Y_t \sin\theta - B^2 Z_t \sin\theta), & (\cos\theta - BZ_t \sin\theta) \end{bmatrix}$$

ABCD of the equivalent transmission line may be written as:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta_e & jZ_e \sin\theta_e \\ jY_e \sin\theta_e & \cos\theta_e \end{bmatrix}$$

By comparing the *A* terms of the above two equations, we can find the length of the equivalent uniform line, as

$$\cos\theta_e = \cos\theta - BZ_t \sin\theta$$

The equivalent admittance Y_e is obtained by comparing the ratios C/B, as

$$Y_e = Y_t[1 - (BZ_t)^2 + 2BZ_t \cot\theta]^{1/2}$$

When $\theta = 90^\circ$, the above two relations may be written as:

$$\cos\theta_e = -BZ_t$$

or

$$\theta_e = \tfrac{1}{2}\pi + BZ_t + \tfrac{1}{6}(BZ_t)^3$$
$$Y_e = Y_t[1 - (BZ_t)^2]^{1/2}$$

$Y_e < Y_t$ and its magnitude is independent of the sign of B.

Thus, if only the sign of B is changed (and its magnitude kept unchanged), the circuit can remain matched in both states of the phase shifter. For example, if $b_1 = 0.2$ and $b_2 = -0.2$, we have $\theta_{e1} = 101.54^\circ$ and $\theta_{e2} = 78.46^\circ$, resulting in a differential phase shift $\Delta\phi = \theta_{e2} - \theta_{e1} = 23.08^\circ \simeq 0.4$ rad.

Various Configurations for Loaded-Line Phase Shifters

Various designs for loaded-line phase shifters differ to the extent that the susceptances B_1 and B_2 for two states of the phase shifter are realized by different circuit configurations.

The basic design equations are obtained by combining

$$\cos\theta_e = \cos\theta - BZ_t \sin\theta \qquad Y_e = Y_t[1 - (BZ_t)^2 + 2BZ_t \cot\theta]^{1/2}$$

with

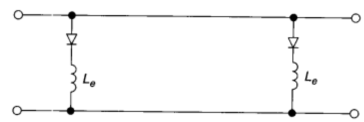
$$\Delta\phi = \tan^{-1}\left(\tfrac{1}{2}b\right)$$

And may be written as:

$$Y_T = Y_0 \sec(\tfrac{1}{2}\Delta\phi) \sin\theta$$
$$B_{1,2} = Y_0\left[\sec(\tfrac{1}{2}\Delta\phi)\cos\theta \pm \tan(\tfrac{1}{2}\Delta\phi)\right]$$

1. Main-Line Mounted Circuit:

In this case, the switching devices are mounted directly across the main line. The idea is to use the high-impedance state capacitance C_j and the low-impedance state inductance L_s directly for B_1 and B_2, respectively. However, it becomes necessary to add an external inductance L_e in series with the device as shown in Fig.



$$B_1 = \frac{\omega C_j}{1-\omega^2 L C_j} \qquad B_2 = \frac{-1}{\omega L}$$

where $L = L_s + L_e$.

Using the above equations, the following relationship can be derived:

$$Y_0 = \omega C_j \frac{\sin\Delta\phi}{\sin^2(\Delta\phi/2) - \cos^2\theta}$$

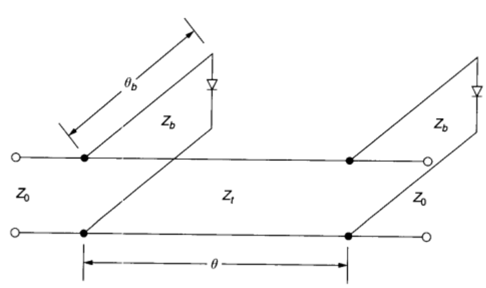
Usually one would like to select θ so as to obtain the maximum bandwidth from the phase shifter circuit, which occurs at $\theta = 90°$. For this case

$$Y_0 = 2\omega C_j \cot(\tfrac{1}{2}\Delta\phi)$$

This puts restrictions on the input–output line impedance unless C_j can be chosen arbitrarily, which becomes difficult because of the limited availability of switching device capacitances. Because of these difficulties, this design configuration has not become very popular.

2. Stub-Mounted-Type Circuit:

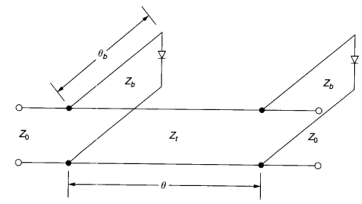
In the circuit arrangement shown in Figure, switching devices are mounted at the ends of two shunt-connected stubs separated by a line length θ. This allows greater design flexibility.



Stub impedance Z_b and length θ_b are determined from device reactances X_f and X_r (in low and high impedance states, respectively) as follows:

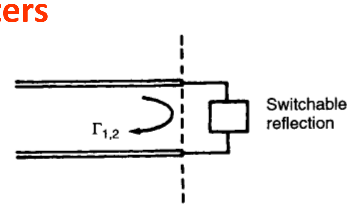
$$Z_b = \left(\frac{X_f - X_r - X_f X_r (B_1 - B_2)}{B_1 - B_2 - B_1 B_2 (X_f - X_r)} \right)^{1/2}$$

$$\tan \theta_b = \frac{Z_b (1 - X_f B_1)}{X_f - B_1 Z_b^2}$$



where Z_t and θ may be selected for wide bandwidth. It has been reported [12] that the best bandwidths are obtained for $\theta \sim 90°$. These design methods make use of device reactances only and assume the device resistances to be negligible.

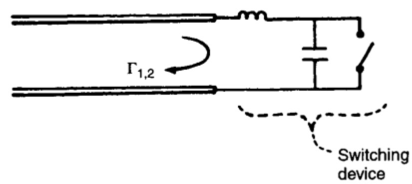
Reflection-Type Phase Shifters



[Basic concept of reflection-type phase shifter]

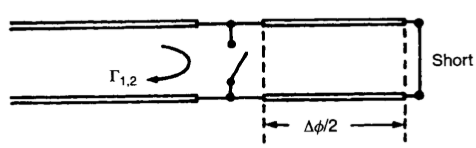
the reflection coefficient is switched from $\Gamma_1 = |\Gamma_1| \angle \phi_1$ to $\Gamma_2 = |\Gamma_2| \angle \phi_2$, the reflected signal undergoes a differential phase shift $\Delta\phi = \phi_1 - \phi_2$.

the ratio of the reflected power to the incident power is given by $|\Gamma|^2$. Ideally $|\Gamma|$ should be unity so that there is no loss associated with the phase shifting operation.



[Switchable-reactance-type reflection phase shifter]

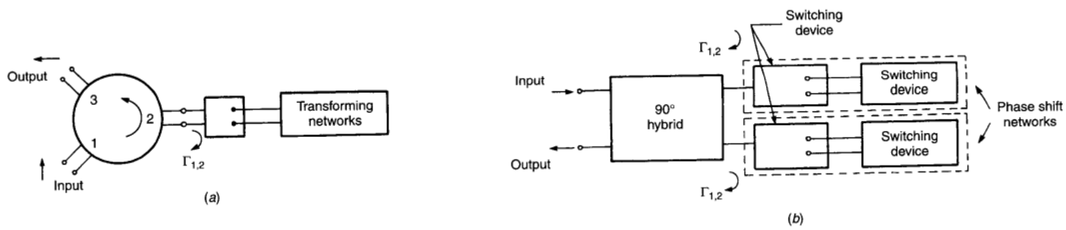
Subnetworks providing the switchable reflection coefficients may be of two different types. In the first group, the reactance terminating the line is changed (say from inductive to capacitive) as shown in Fig. Phase shifters using these subnetworks are called switchable reactance types and constitute the more commonly used variety of reflection phase shifters.



[Switchable-length-type reflection phase shifter]

In the second group of these phase shifter circuits, an additional line length is added at the reflection plane by using an SPST switch. The concept is similar to that used in switched-line phase shifters. Here, as shown in Fig. the signal travels a longer path when the switch is open. The differential phase shift, in this case, is twice the electrical length of the shorted line switched in by the SPST switch.

Transformation of a reflection phase bit into a two-port network:



[(a) Circulator-coupled reflection-type phase shifter, (b) Hybrid-coupled reflection-type phase shifter]

It may be noted that hybrid-coupled arrangements require two identical phase bits (and hence twice the number of devices). However, this circuit is preferred to the circulator-coupled phase shifters because of the following two features: (1) 90° hybrids are a lot more integrable in MICs and MMICs than the circulators, and (2) the use of two switching devices in hybrid-coupled phase shifters increases the power-handling capability by a factor of 2, which is needed in several systems.

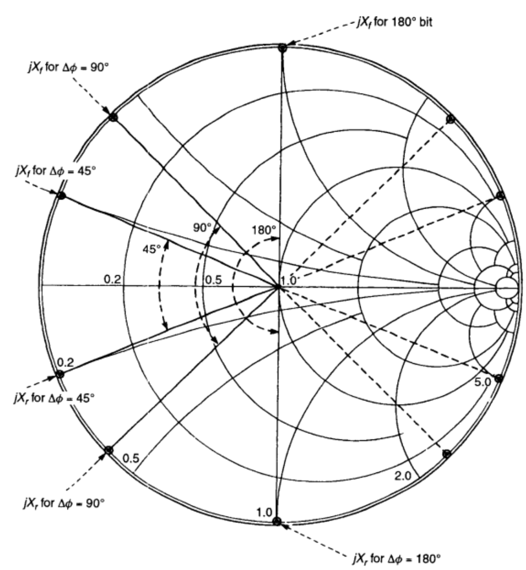
Design of reflection-type phase shifters:

Various designs for hybrid-coupled reflection phase shifters use different configurations for the two transforming networks.

In order to obtain a phase shift $\Delta\phi$, the reflection coefficient values (as seen at two coupled ports of the hybrid) should differ in phase by $\Delta\phi$. Ideally their magnitudes should be unity (no loss). Thus for a 180° bit, values of the reflection coefficients in two states would lie on the outer circle ($|\Gamma| = 1$) of the Smith chart and their locations would be diametrically opposite.

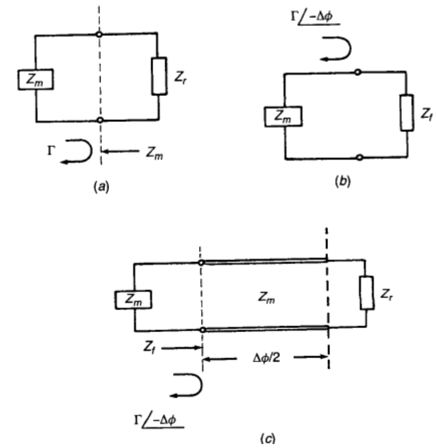
Although an infinite number of such combinations are possible, it can be shown that for a reflection phase shifter using a two-branch branch-line hybrid, the maximum phase shifter bandwidth is obtained when the two values are located at points corresponding to normalized reactances equal to $+j$ and $-j$ on the Smith chart (i.e., when located symmetrically with respect to the $X = 0$ axis).

Similarly for other values of $\Delta\phi$, Γ_1 and Γ_2 in two states should also be located symmetrically with respect to the $X = 0$ axis.



Based on the preceding discussion, we can divide the function of the transforming network into two parts.

1. It arranges Γ_f and Γ_r so that the desired phase shift $\Delta\phi$ is obtained.
2. It locates Γ_f and Γ_r symmetrically about the $X = 0$ axis on the Smith chart so that the condition for maximum bandwidth is satisfied.



It may be noted that the second part of the function just mentioned is implemented by connecting a suitable line length between the transforming networks designed for part 1 and the ports of the hybrid. This line length will rotate Γ_f and Γ_r points around the Smith chart without disturbing the phase relationship between them.

Let a network with the equivalent Thévenin impedance Z_m terminated by an impedance Z_r (of the high-impedance state of the switching device) produce a reflection coefficient Γ, as shown in Fig. *a*. When the same network is terminated with an impedance Z_f (of the low-impedance state of the device), we want a reflection coefficient of $\Gamma\underline{/-\Delta\phi}$ (Fig. *b*). A reflection coefficient of $\Gamma\underline{/-\Delta\phi}$ will also be produced when a transmission-line section of length $\Delta\phi/2$ and impedance Z_m is connected between the network and Z_r, as shown in Fig. *c*. Thus networks shown in Fig. *b* and *c* should be equivalent.

For this equivalence to hold good, the following relationship should be satisfied:

$$Z_f = Z_m \frac{Z_r + jZ_m \tan(\Delta\phi/2)}{Z_m + jZ_r \tan(\Delta\phi/2)}$$

For low-loss switching devices, Z_f and Z_r may be approximated by jX_f and jX_r, respectively. In this case, ( ) may be rewritten to express Z_m explicitly in terms of X_f, X_r, and $\Delta\phi$. That is, given device reactances, the desired phase shift $\Delta\phi$ may be obtained by choosing Z_m given by

$$Z_m = \frac{X_f - X_r}{2\tan(\Delta\phi/2)} \pm \sqrt{\left(\frac{X_f - X_r}{2\tan(\Delta\phi/2)}\right)^2 - X_f X_r}$$

For a 180° bit, Z_m becomes

$$Z_m = \sqrt{-X_f X_r}$$

and for a 90° bit, we should have

$$Z_m = \frac{X_f - X_r}{2} \pm \sqrt{\left(\frac{X_f - X_r}{2}\right)^2 - X_f X_r}$$

It should be remembered that the reactance X_r has a negative value (it is capacitive). This design approach holds good even when the switching device is lossy.

Phase-shifter using a λ/4 transforming network:

The design is simple. A quarter-wave line of impedance Z_t is used to transform the hybrid impedance Z_0 to a value Z_m, which the device should look into in order to provide the desired phase shift $\Delta\phi$. So

$$Z_t = \sqrt{Z_0 Z_m}$$

The line length θ_t is selected so that the impedances Z_b (in two bias states) are located symmetrically with respect to the $X = 0$ axis on the Smith chart. For a 180° phase bit, Z_b in the low-impedance state should be $-jZ_0$ (since a 90° line length has already been added). This yields

$$-jZ_0 = Z_b = jZ_0 \frac{-Z_t^2/X_f + Z_0 \tan\theta_t}{Z_0 + (Z_t^2/X_f)\tan\theta_t}$$

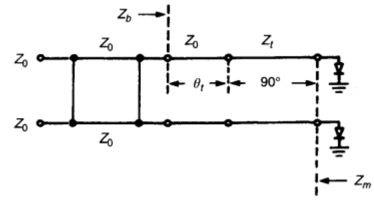
which gives

$$\theta_t|_{\Delta\phi=180^\circ} = \tan^{-1}\left(\frac{Z_t^2 - Z_0 X_f}{Z_t^2 + Z_0 X_f}\right)$$

Similarly, for a 90° bit, we should have $Z_b = -j2.4142 Z_0$

which yields

$$\theta_t|_{\Delta\phi=90^\circ} = \tan^{-1}\left(\frac{Z_t^2 - 2.4142 X_f Z_0}{Z_0 X_f + 2.4142 Z_t^2}\right)$$



Switched-Network Phase Shifters

When the input signal, originally passing through network 1, is switched to pass through network 2, we get a differential phase shift $(\phi_2 - \phi_1)$. The switched-line phase shifter is a special case of a switched-network phase shifter .

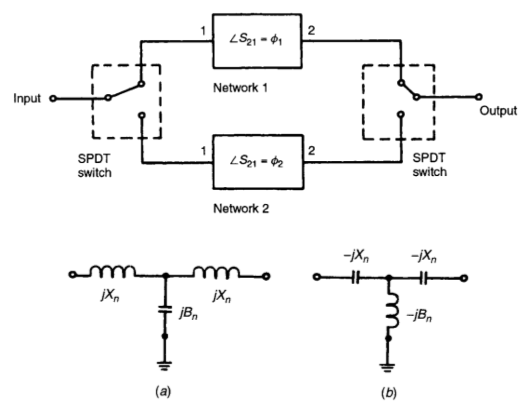
The main advantage of generalizing a switched-line phase shifter into a switched-network configuration is that one can design the variations of ϕ_1 and ϕ_2 with frequency appropriately and obtain a wider bandwidth or a desired frequency response of the phase shifter.

The most commonly used networks in switched-network phase shifters are the low-pass and high-pass filter configurations shown in Fig.

The normalized $ABCD$ matrix of the network in Fig. a may be written as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_n = \begin{bmatrix} A & B/Z_0 \\ CZ_0 & D \end{bmatrix} = \begin{bmatrix} 1 & jX_n \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ jB_n & 1 \end{bmatrix}\begin{bmatrix} 1 & jX_n \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 - B_n X_n & j(2X_n - B_n X_n^2) \\ jB_n & 1 - B_n X_n \end{bmatrix}$$

where X_n and B_n are the reactance and susceptance shown in Fig. normalized with respect to transmission-line impedance Z_0 and admittance Y_0, respectively.



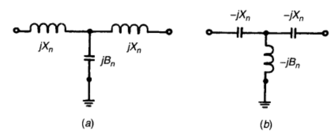
[(a) Low-pass and (b) high-pass networks used in switched network phase shifters]

The transmission coefficient S_{21} is given, in terms of the normalized $ABCD$ matrix, by

$$S_{21} = \frac{2}{A+B+C+D} = \frac{2}{2(1-B_nX_n)+j(B_n+2X_n-B_nX_n^2)}$$

The transmission phase ϕ is given by

$$\phi = \tan^{-1}\left(-\frac{B_n+2X_n-B_nX_n^2}{2(1-B_nX_n)}\right)$$



When both B_n and X_n change signs (as shown in Fig. b), the phase ϕ retains the same magnitude but changes sign. Amplitude of S_{21} does not change. Thus the phase shift $\Delta\phi$ caused by switching between low-pass and high-pass networks is given by

$$\Delta\phi = 2\tan^{-1}\left(-\frac{B_n+2X_n-B_nX_n^2}{2(1-B_nX_n)}\right)$$

For the phase shifter to be matched we need

$$|S_{11}| = 0$$

Since we are considering a lossless case,

$$|S_{11}| = \sqrt{1-|S_{21}|^2}$$

This leads to the following relationship between B_n and X_n:

$$B_n = \frac{2X_n}{X_n^2+1}$$

Thus the phase shift $\Delta\phi$ can be expressed in terms of X_n alone as

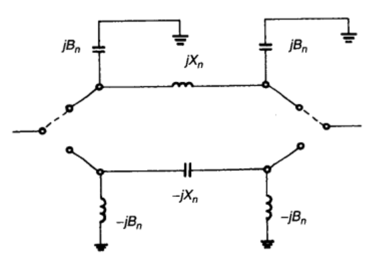
$$\Delta\phi = 2\tan^{-1}\left(\frac{2X_n}{X_n^2-1}\right)$$

which yields X_n in terms of $\Delta\phi$ as

$$X_n = \tan\left(\tfrac{1}{4}\Delta\phi\right)$$

Substituting X_n in ( ) yields

$$B_n = \sin\left(\tfrac{1}{2}\Delta\phi\right)$$



The π-section filter shown in Fig. , may also be used in place of the T configuration .

For this case

$$B_n = \tan\left(\tfrac{1}{4}\Delta\phi\right)$$
$$X_n = \sin\left(\tfrac{1}{2}\Delta\phi\right)$$

Bandwidth of these phase shifters may be discussed by considering variations of ϕ_1 and ϕ_2 with frequency. For a T-configuration low-pass filter, phase delay ϕ_1 increases with frequency; while for a high-pass filter, phase advance ϕ_2 decreases with frequency. As ϕ_1 is phase delay and ϕ_2 is phase advance, they have opposite signs and $\Delta\phi = \phi_1 - \phi_2 = |\phi_1| + |\phi_2|$. Thus the two variations tend to compensate for each other and $\Delta\phi$ stays relatively constant.