Digital Signal Processing (ECN-312)

Lecture 4 (Quantization)

Dheeraj Kumar

dheeraj.kumar@ece.iitr.ac.in

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From discrete-time to digital signals

- So far, our discussions of the representation of continuous-time signals by discrete-time sequence have focused on:
  - Idealized models of periodic sampling
  - Bandlimited interpolation
- But discrete-time systems have infinite precision
  - $x[n]$ can take any real values (infinite options)
- To convert a discrete-time signal into a digital signal, one more step is needed
  - Limit the range of values $x[n]$ can take
  - Quantization
    - Like sampling along the y-axis

Discrete-time vs digital signal processing

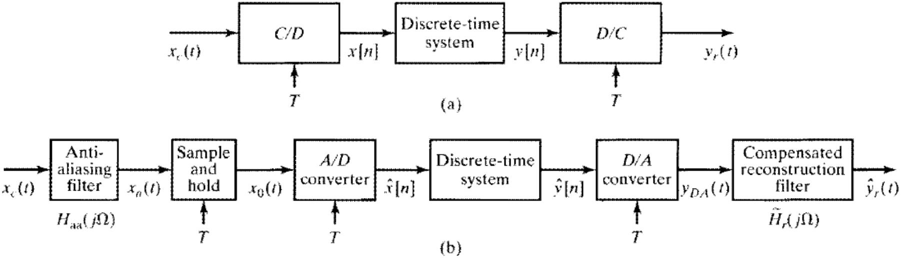

Figure 4.41 (a) Discrete-time filtering of continuous-time signals. (b) Digital processing of analog signals.

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Introduction

- An ideal C/D converter converts a continuous-time signal into a discrete-time signal
  - Each sample is known with infinite precision
- For *digital signal processing* (DSP), we need to converts a continuous-time (analog) signal into a digital signal
  - A sequence of finite-precision or quantized samples

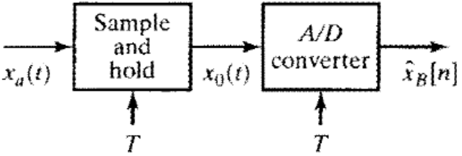


Figure 4.45 Physical configuration for analog-to-digital conversion.

Sample and hold

- A physical device that converts a voltage or current amplitude at its input into a binary code
  - A quantized amplitude value closest to the amplitude of the input
  - Start and complete an A/D conversion every T seconds (not instantaneous)
- High-performance A/D system typically includes a sample-and-hold
  - $x_0(t) = h_0(t) * \sum_{n=-\infty}^{\infty} x_a(nT)\delta(t-nT) = \sum_{n=-\infty}^{\infty} x[n]h_0(t-nT)$
  - $x[n] = x_a(nT)$ are the ideal samples of $x_a(t)$
  - $h_0(t)$ is the impulse response of the zero-order-hold system
    - $h_0(t) = \begin{cases} 1, & 0 < t < T, \\ 0, & otherwise \end{cases}$

Sample and hold

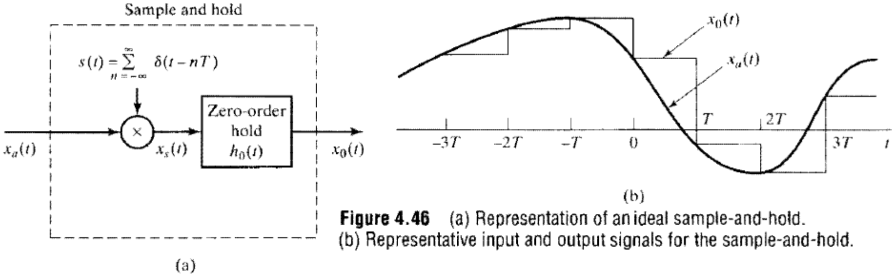
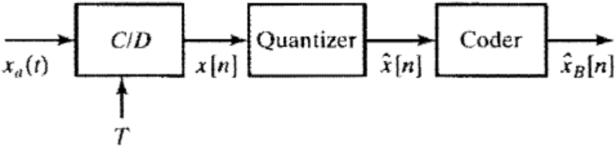


Figure 4.46 (a) Representation of an ideal sample-and-hold. (b) Representative input and output signals for the sample-and-hold.

- Output of the zero-order hold ($x_0(t)$) is a staircase waveform where the sample values are held constant during the sampling period of T seconds

A/D converter

- Physical configuration for analog-to-digital converter can be shown as:

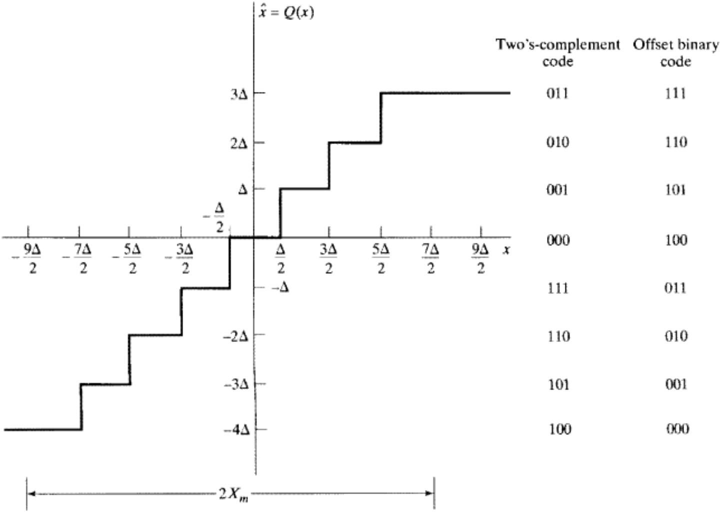


- Ideal C/D converter represents the sampling performed by the sample-and-hold
- The quantizer and coder together represent the operation of the A/D converter

Quantizer and coder

- A quantizer is a nonlinear system whose purpose is to transform the input sample $x[n]$ into one of a finite set of prescribed values
  - $\hat{x}[n] = Q(x[n])$
  - Can be defined with either uniformly or non-uniformly spaced quantization levels
    - Leads to “uniform” and “non-uniform” quantizers respectively
- A coder is a *linear* mapping from quantized output ($\hat{x}[n]$) to a sequence of binary numbers ($\hat{x}_B[n]$)

Typical quantizer for A/D conversion (Uniform quantization)



Typical quantizer for A/D conversion

- This quantizer is appropriate for a signal whose samples are both positive and negative (bipolar)
  - If input samples are always positive (or negative), then a different distribution of the quantization levels would be appropriate
- In general, 2^{B+1} quantization levels can be coded with a $B+1$-bit binary code
  - In this example, 8 quantization levels $\rightarrow$ 3-bit binary code
- Many binary coding schemes exist, each with its own advantages and disadvantages, depending on the application
  - Offset binary coding: symbols are assigned in numeric order, starting with the most negative quantization level
  - Two's complement coding: Most significant bit is considered as the sign bit, and the remaining bits as binary integers or fractions

Step size

- Relationship between the code words and the quantized signal levels depends on the parameter X_m
  - Full-scale level of the A/D converter
  - Typical values are 10,5, or 1 volt
- Step size of the quantizer: $\Delta = \frac{2X_m}{2^{B+1}} = \frac{X_m}{2^B}$
  - Smallest quantization levels ($\pm\Delta$) correspond to the least significant bit of the binary code word

Quantization and coding example

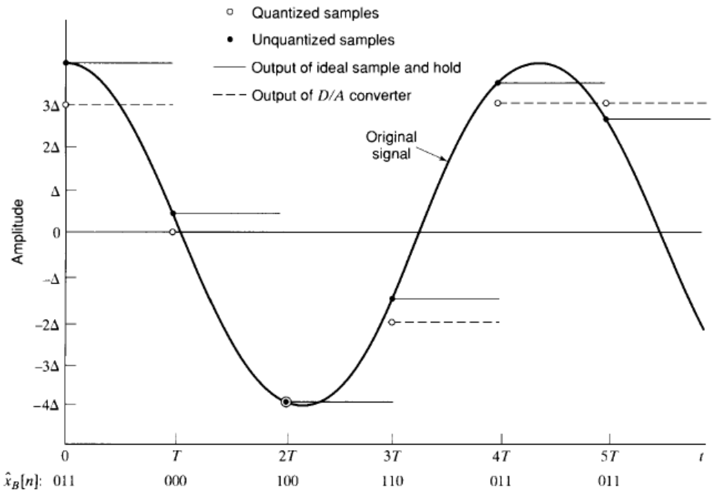


Figure 4.49 Sampling, quantization, coding, and D/A conversion with a 3-bit quantizer.

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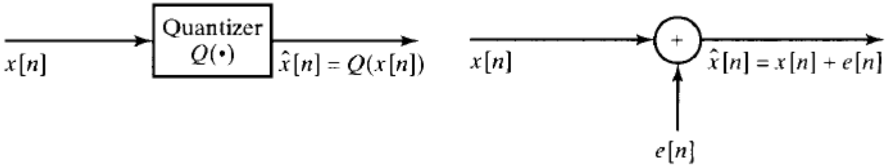
Quantization error

- Quantized sample $\hat{x}[n]$ will generally be different from the true sample value $x[n]$
  - Difference between them is the quantization error: $e[n] = \hat{x}[n] - x[n]$
- For a $(B+1)$-bit quantizer with step size Δ
  - If $x[n]$ lies between two quantization levels, i.e., $-X_m - \frac{\Delta}{2} < x[n] \leq X_m - \frac{\Delta}{2}$
  - Then $-\frac{\Delta}{2} < e[n] \leq \frac{\Delta}{2}$
  - If $x[n]$ is outside this range, (e.g., sample at $t = 0$), $|e[n]| > \frac{\Delta}{2}$
    - Such samples are said to be clipped

Additive noise model for quantizer

- A simplified, but useful model of the quantizer considers quantization error as an additive noise signal



- Model is exactly equivalent to the quantizer if we know $e[n]$
- However, $e[n]$ is usually not known, and a statistical model is useful in representing the effects of quantization

Thanks.