CSE 5301 – Spring 2018
Exam 1, Variant β, Tuesday 02/19/2018

Name: SOLUTION

Student ID:

Row:

(Not providing this information: -10 Points)
(ID missing in Individual pages: -5 Points)

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Total Exam Points: 100

Score

Question	**Points**	**Max Points**
1		8
2		12
3		10
4		12
5		4
6		8
7		8
8		10
9		8
10		10
11		10
Total		**100**

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Question 1 – 8 points

Consider a collection 10 people. 6 are men and 4 are women. We are selecting groups of people.

(a) What is probability that a group of 3 will contain at most 2 women.
(b) What is probability that a group of 4 will contain at least 2 men.

(a) P(Gp has at most 2 women)

= P(Gp has 0 women) + P(Gp has 1 women) + P(Gp has 2 women)

$$= \frac{6C_3\, 4C_0}{10C_3} + \frac{6C_2\, 4C_1}{10C_3} + \frac{6C_1\, 4C_2}{10C_3}$$

$$= \frac{\frac{6\times5\times4}{3\times2\times1}\times 1 + \frac{6\times5}{2\times1}\overset{2}{\cancel{4}} + 6\times\frac{\overset{2}{\cancel{4}}\times3}{2\times1}}{10C_3}$$

$$= \frac{20 + 60 + 36}{120} = \frac{116}{120} = 0.9667$$

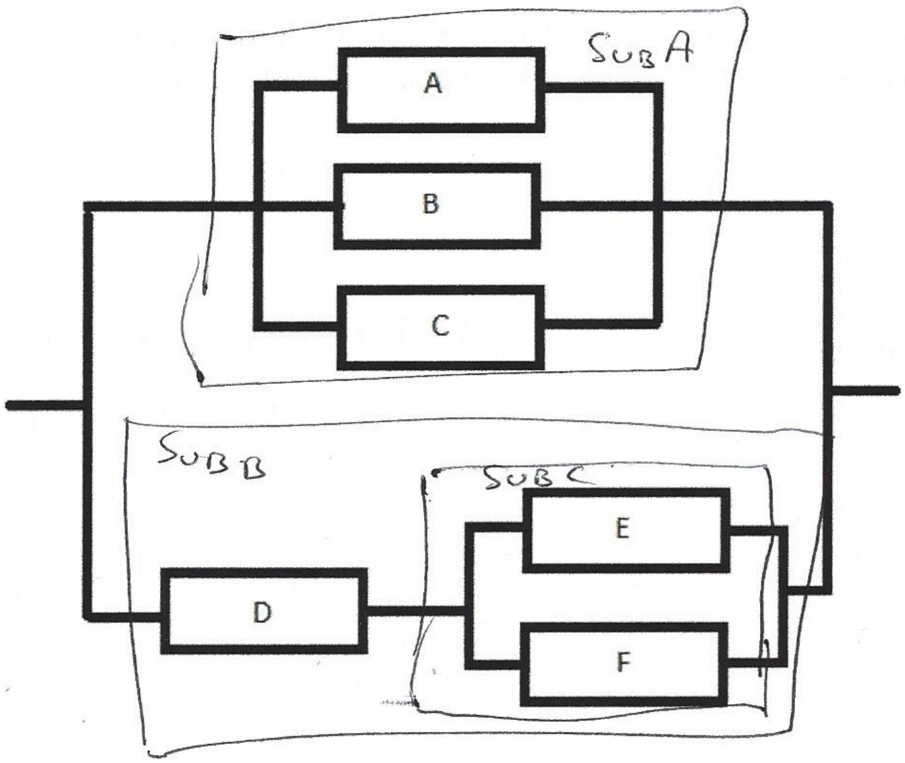
(b) P(Gp has atleast 2 men) =

$$= \frac{6C_2\, 4C_2}{10C_4} + \frac{6C_3\, 4C_1}{10C_4} + \frac{6C_4\, 4C_0}{10C_4}$$

$$= \frac{90 + 80 + \frac{\overset{3}{\cancel{6}}\times5}{2\times1}}{210} = \underline{0.8809}$$

Question 2 – 12 points

In the following figure, each component has a probability of failure of 0.2 independent of all others. What is the reliability (probability of successful operation) of the entire system?



Sub A : $P(\text{Fail}_A) = P(\bar{A} \cap \bar{B} \cap \bar{C}) = (0.2)^3 = 0.008$

Sub C : $P(\text{Fail}_C) = P(\bar{E} \cap \bar{F}) = (0.2)^2 = 0.04$

Sub B : $P(\text{Fail}_B) = 1 - P(\text{Succ}_B)$

$$= 1 - [P(D) \cdot P(\text{Succ}_C)]$$

$$= 1 - [(1 - P(\bar{D})) \cdot (1 - P(\text{Fail}_C))]$$

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$$= 1 - \left[(1 - 0.2)\ (1 - 0.04)\right]$$

$$= 1 - \left[0.8 \times 0.96\right] = 0.232$$

$$P(\text{Sys Succ}) = 1 - P(\text{Sys Failure})$$

$$= 1 - \left[P(\text{Fail}_A) \cdot P(\text{Fail}_B)\right]$$

$$= 1 - \left[0.008 \times 0.232\right]$$

$$= 1 - 0.001856$$

$$= 0.998144$$

Question 3 – 10 points

You play a game with a six-sided dice where you win 10 times the number rolled on the dice.

(a) What is the expectation and standard deviation of the amount of money you win?
(b) What is the expectation and standard deviation of the amount if you used a loaded dice whose probability mass function is given by

Amt
$y = 10x$

X	1	2	3	4	5	6
P(x)	1/9	1/9	2/9	2/9	2/9	1/9

(a)

SAME AS VARIANT α

(b)

$$E(y) = 10\left[1\times\frac{1}{9} + 2\times\frac{1}{9} + 3\times\frac{2}{9} + 4\times\frac{2}{9} + 5\times\frac{2}{9} + 6\times\frac{1}{9}\right]$$

$$= 36.6667$$

$$Var(y) = 10^2 \, Var(x)$$

$$= 100\left[\frac{1^2 + 2^2 + 2\cdot 3^2 + 2\cdot 4^2 + 2\cdot 5^2 + 1\cdot 6^2}{9} - (3.6667)^2\right]$$

$$= 100\left[15.6667 - 13.4444\right]$$

$$= 222.2222.$$

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std(y) = 14.9071

Question 4 – 12 points

You are checking the quality of components that you are using for an experiment. You know that every component has a 3 percent chance of being defective.

(a) What is probability that 5 out of 100 components are defective?
(b) If you check the components sequentially, what is the probability that the 5^{th} component you check is defective.
(c) What is probability that you need to check 10 components before finding 5 defective ones.

Same as Variant α

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Question 5 – 4 points

On average it takes 1 minute to download a file with a standard deviation of 10 seconds. What can you tell me about the probability of spending more than 1.5 minutes to download the file?

Same as Variant α

Question 6 – 8 points

The TTF (Time to Fail) for a component follows a Gamma distribution with parameters $\alpha = 2$ and $\lambda = 3$ years^{-1}. What is the probability that it fails in 4 months or less?

Let T be TTF

T follows Gamma $(2, 3)$

$$P(T \leq 4/12) = P(X \geq 2)$$

Where X follows Poisson$(3 \cdot \frac{4}{12})$

Poisson(1)

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - P(X \leq 1)$$

$$= 1 - 0.736$$

$$= 0.264$$

$$P(T \leq 4/12) = 0.264$$

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Question 7 – 8 points

Let X be a variable with a normal distribution. You know that it has mean 5 and variance 4. Find the following.

(a) $P(X \geq 2.5)$
(b) $P(|X| \leq 2.5)$

$Z = \frac{X-5}{2}$ follows std. normal.

(a) $P(X \geq 2.5) = 1 - P(X < 2.5)$
$= 1 - P(X \leq 2.5)$
$= 1 - \Phi\left(\frac{2.5-5}{2}\right)$
$= 1 - \Phi(-1.25) = 1 - 0.1056$
$= 0.8944$

(b) $P(|X| \leq 2.5) = P(-2.5 \leq X \leq 2.5)$
$= P(X \leq 2.5) - P(X \leq -2.5)$
$= \Phi\left(\frac{2.5-5}{2}\right) - \Phi\left(\frac{-2.5-5}{2}\right)$
$= 0.1056 - 0.0001$
$= 0.1055$

Question 8 – 10 points

You have 50 messages being sent sequentially from a computer. Transmission time for each message follows an exponential distribution with parameter $\lambda = 5\ \text{min}^{-1}$. Find the probability that all messages are transmitted in less than 10 minutes.

Same as Variant α.

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Question 9 – 8 points

For the following dataset, give sample mean, variance and the five-point summary.

{-20, 15, 11, 32, 54, 22, -10, -2, 6}

$$\bar{X} = \frac{108}{9} = 12$$

$$s^2 = \frac{5310 - 9(12)^2}{8} = \frac{4014}{8} = 501.75$$

Sorting the values,

$\{-20, -10, -2, 6, 11, 15, 22, 32, 54\}$

$\hat{Q}_1 = -2$ $\hat{M} = 11$

$\hat{Q}_3 = 22$

5 pt summary

$\langle -20, -2, 11, 22, 54 \rangle$

Question 10 – 10 points

A security engineer is aware that the number of attacks on a system is uniformly distributed. She observes the number of attacks the server is subjected two over 10 days. The numbers are 35, 45, 47, 22, 37, 38, 32, 54, 35, 31. Find the parameters of the distribution.

First sample moment, $m_1 = \bar{X}$

$$m_1 = \frac{376}{10} = 37.6$$

Second central sample moment: m_2'

$$m_2' = \frac{744.4}{10} = 74.44$$

For uniform Distribution $U(a, b)$

First population moment $\mu_1 = \frac{a+b}{2}$

Second central population moment $\mu_2' = \frac{(b-a)^2}{12}$

So $\frac{\hat{a} + \hat{b}}{2} = 37.6 \quad \frac{(\hat{b} - \hat{a})^2}{12} = 74.44$

$\hat{a} + \hat{b} = 75.2 \qquad \hat{b} - \hat{a} = 29.8878$

$\hat{a} = 22.6561 \qquad \hat{b} = 52.5439$

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Question 11 – 10 points

Consider the following game of chance where based on the outcome of a coin toss (of a biased coin) you win either \$5 (on getting heads) or \$10 (on getting tails). The outcomes for 8 games are as follows: \$5, \$10, \$10, \$10, \$10, \$5, \$10, \$10. From this find the probability of getting tails.

$$X_i = \begin{cases} 10 & \text{with Prob } p \\ 5 & \text{with Prob } 1-p \end{cases}$$

$$P(X) = (1-p)\cdot p\cdot p\cdot p\cdot p\cdot(1-p)\cdot p\cdot p$$

$$= p^6 (1-p)^2$$

$$\ln P(X) = 6\ln p + 2\ln(1-p)$$

$$\frac{\partial}{\partial p} P(X) = \frac{6}{p} - \frac{2}{1-p}$$

For max lg likelihood, $\frac{\partial}{\partial p} P(X) = 0$

$$\frac{6}{\hat{p}} - \frac{2}{1-\hat{p}} = 0$$

$$6 - 6\hat{p} = 2\hat{p} \qquad \hat{p} = \frac{6}{8} = 0.75$$

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SCRATCH - I

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SCRATCH - II