**CSE 5301 – Fall 2019**
**Exam 1, Variant $\alpha$, Tuesday 10/01/2019**

**Name:** ______________________________

**Student ID:** ______________________________

**Row:** ______________________________

*(Not providing this information: -10 Points)*
*(ID missing in Individual pages: -5 Points)*

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**Total Exam Points: 100**

**Score**

| Question | Points | Max Points |
|---|---|---|
| 1 | | 8 |
| 2 | | 12 |
| 3 | | 12 |
| 4 | | 12 |
| 5 | | 4 |
| 6 | | 8 |
| 7 | | 12 |
| 8 | | 8 |
| 9 | | 12 |
| 10 | | 12 |
| **Total** | | **100** |

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**Question 1 – 8 points**

Consider a collection 15 people. 10 are men and 5 are women. We are selecting groups of people.

(a) What is probability that a group of 4 will contain at least 2 women.
(b) What is probability that a group of 3 will contain at most 2 men.

(a)

P(atleast 2 women)

$= 1 - P(\text{less tha } 2 \text{ women})$

$= 1 - [P(1 \text{ women}) + P(0 \text{ women})]$

$$= 1 - \left[\frac{10C_3 \; 5C_1}{15C_4} + \frac{10C_4 \; 5C_0}{15C_4}\right]$$

$= 1 - [0.4396 + 0.1538] = 0.4066$

(b) $P(\text{atmost } 2 \text{ men}) = P(2 \text{ men}) + P(1 \text{ men}) + P(0 \text{ men})$

$$= \frac{10C_2 \; 5C_1}{15C_3} + \frac{10C_1 \; 5C_2}{15C_3} + \frac{10C_0 \; 5C_3}{15C_3}$$

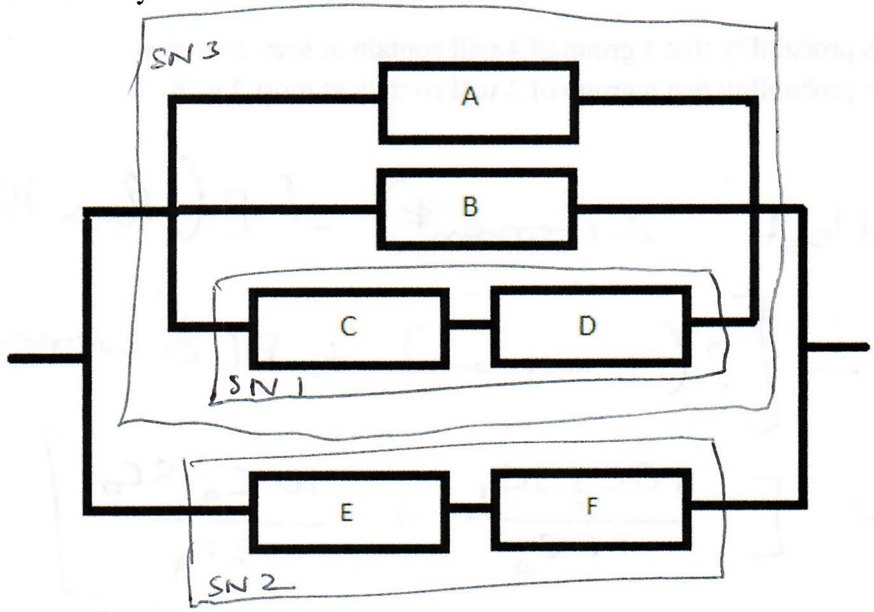
$$= \frac{45 \times 5}{455} + \frac{10 \times 10}{455} + \frac{1 \times 10}{455}$$

$= 0.4945 + 0.2198 + 0.0220$

$= 0.7363$

**Question 2 – 12 points**

In the following figure, the components are independent of each other. Components A, B, C have probability of failure 0.2. Components D, E, F have probability of failure 0.3. What is probability of failure of the entire system?




$P(A) = P(B) = P(C) = 0.8$

$P(D) = P(E) = P(F) = 0.7$

$P(SN1) = P(C) \cdot P(D)$

$= 0.56$

$P(SN2) = P(E) \cdot P(F) = 0.49$

$P(SN3) = 1 - P(\overline{SN3})$

$= 1 - P(\overline{A}) \cdot P(\overline{B}) \cdot P(\overline{SN1})$

$= 1 - 0.2 \times 0.2 \times 0.44$

$= 0.9824$

$$P(\overline{\text{Network}}) = P(\overline{SN3}) \cdot P(\overline{SN2})$$

$$= 0.0176 \times 0.51$$

$$= \underline{0.008976}$$

**Question 3 – 12 points**

You play a game with a six-sided dice where you win 10 times the number rolled on the dice.

(a) What is the expectation and standard deviation of the amount of money you win if the dice is unbiased?

(b) What is the expectation and standard deviation of the amount if you used a loaded dice whose probability mass function is given by

| x | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| P(x) | 1/9 | 1/9 | 1/9 | 2/9 | 2/9 | 2/9 |

$y = 10x$

(a)

$E(y) = 10\,E(x)$

$= 10\left[\frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}\right]$

$= 10\,[3.5] = \$35$

$Var(y) = 100\;Var(x)$

$= 100\left[\frac{1^2}{6} + \frac{2^2}{6} + \frac{3^2}{6} + \frac{4^2}{6} + \frac{5^2}{6} + \frac{6^2}{6} - 3.5^2\right]$

$= 100\,[2.91667]$

$= 291.6667$

$\text{Std Dev}(y) = \$\,17.0783$

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(b) $y = 10x$

$E(y)$

$$= 10\left[\frac{1}{9} + \frac{2}{9} + \frac{3}{9} + \frac{8}{9} + \frac{10}{9} + \frac{12}{9}\right]$$

$$= \$40$$

$$Var(y) = 100 \; Var(x)$$

$$= 100\left[\left[\frac{1}{9} + \frac{4}{9} + \frac{9}{9} + \frac{32}{9} + \frac{50}{9} + \frac{72}{9}\right] - 4^2\right]$$

$$= 100\left[2.6667\right]$$

$$= 266.6667$$

$$Std\ Dev(y) = \$16.329$$

**Question 4 – 12 points**

You are checking the quality of components that you are using for an experiment. You know that every component has a 5 percent chance of being defective.

(a) What is probability that 5 out of 100 components are defective?
(b) If you check the components sequentially, what is the probability that the 5[th] component you check is the first defective one.
(c) What is probability that you need to check 10 components before finding 5 defective ones.

$p = 0.05$

Poisson(5)

OR

(a) $X \approx \text{Binomial}(100, 0.05) \approx \text{Normal}(5, 2.1794)$

$P(X=5) = P(X \le 5) - P(X \le 4)$

$= P(Z \le 0) - P\left(Z \le \frac{4-5}{2.1794}\right)$

$= P(Z \le 0) - P(Z \le -0.46)$

$= \phi(0) - \phi(-0.46)$

$= 0.5 - 0.3228 = 0.1772$

$P(X=5) \approx \frac{e^{-5} 5^5}{5!}$

$= 0.1754$

(b) $X = \text{Geometric}(0.05)$

$P(X=5) = (1-0.05)^4 (0.05)$

$= 0.0407$

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(c) $X = \text{Neg-Binomial}(5, 0.05)$

$P(X = 10) = P(y = 5-1) * 0.05$

$(\text{When } y = \text{Binomial}(9, 0.05))$

$P(X=10) = [P(y \le 4) - P(y \le 3)] * 0.05$

$= (1 - 0.999) * 0.05$

$= 0.00005$

**Question 5 – 4 points**

On average it takes 1 minute to download a file with a standard deviation of 10 seconds. What can you tell me about the probability of spending more than 1.5 minutes to download the file?

$\mu = 60 \quad \sigma = 10$

$P\{x > 90\} = P\{X - 60 > 30\}$

$= P\{|X-60| > 30\}$

$\le \left(\frac{10}{30}\right)^2$ by Chebychev's Ineq.

$P\{X > 90\} \le 0.1111$

**Question 6 – 8 points**

The TTF (Time to Fail) for a component follows a Gamma distribution with parameters $\alpha = 2$ and $\lambda = 3$ years$^{-1}$. What is the probability that it fails after 4 months?

$$T = \text{Gamma}(2, 3)$$

$$P\{T > 4/12\} = P\{X < 2\}$$

By Gama-Poisson

where $X = \text{Poisson}(1)$

$$P\{X < 2\} = P\{X \leq 1\}$$

$$= 0.736$$

$$P\{T > 4/12\} = 0.736$$

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**Question 7 – 12 points**

For the following datasets, give sample mean, variance and the five-point summary. Also give the stem and leaf plot.

(a) {-20, 15, 11, 32, 54, 22, -10, -2, 6}

$\bar{x} = 12$ $\quad s^2 = 501.75$

$\{-20, -10, -2, 6, 11, 15, 22, 32, 54\}$

$\hat{q}_1 \hat{q}_1 \quad \hat{q}_2 \hat{q}_2 \quad \hat{q}_3 \hat{q}_3$

$\langle -20, -2, 11, 22, 54 \rangle$

| | |
|---|---|
| -20 | 0 |
| -10 | 0 |
| 0 | -2, 6 |
| 10 | 1, 5 |
| 20 | 2 |
| 30 | 2 |
| 40 | |
| 50 | 4. |

(b) {44, 5, -42, 10, 22, 33, 19, -4, 24, 9}

$\bar{X} = 12$

$$s^2 = \frac{6432 - 10 \times 144}{9}$$

$$= 554.6667$$

$\{-42, -4, 5, 9, 10, 19, 22, 24, 33, 44\}$

$\hat{Q}_1 \hat{Q}_1 \quad \hat{Q}_2 \quad \hat{Q}_2 \quad \hat{Q}_3 \hat{Q}_3$

$\hat{Q}_1 = 5, \quad \hat{Q}_2 = \frac{10 + 19}{2} = 14.5$

$\hat{Q}_3 = 24.$

$< -42, 5, 14.5, 24, 44 >$

| | |
|---|---|
| -40 | 2 |
| -30 | |
| -20 | |
| -10 | |
| 0 | -4, 5, 9 |
| 10 | 0, 9. |
| 20 | 2, 4. |
| 30 | 3 |
| 40 | 4 |

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**Question 8 – 8 points**

You have 50 messages being sent sequentially from a computer. Transmission time for each message follows an exponential distribution with parameter $\lambda = 5\ \text{min}^{-1}$. Find the probability that all messages are transmitted after more than 10 minutes.

Fore each message.

$t_i = Exp(5) \quad \mu_t = \frac{1}{5} \quad \sigma_t = \sqrt{\frac{1}{5^2}}$

$\mu_t = 0.2 \quad \sigma_t = 0.2.$

$T = t_1 + t_2 + t_3 + \cdots + t_{50}$

By central limit theorm

$T = \text{Normal}(50\mu_t, \sigma_t\sqrt{50})$

$= \text{Normal}(10, 1.414)$

$P(T > 10) = 1 - P(T \leq 10)$

$= 1 - P(Z \leq 0)$

$= 1 - \phi(0)$

$= 0.5$

**Question 9 – 12 points**

The execution time of a program follows an exponential distribution with some unknown parameter λ. Sample execution times for 10 runs of the program are given by 3, 40, 2, 15, 1, 4, 87, 3, 13, 8. Find out the parameter using Method of Maximum Likelihood estimation.

$$P(x_1 \ldots x_{10}) = P(x_1)P(x_2)P(x_3)\ldots P(x_{10})$$

$$= \lambda e^{-\lambda x_1} \cdot \lambda e^{-\lambda x_2} \cdots \lambda e^{-\lambda x_{10}}$$

$$= \lambda^{10}\left[e^{-\lambda x_1} \cdot e^{-\lambda x_2} \cdots e^{-\lambda x_{10}}\right]$$

$$= \lambda^{10}\left[e^{-\lambda(x_1+x_2 \cdots x_{10})}\right]$$

$$\ln P(x_1 \ldots x_{10}) = 10 \ln\lambda - \lambda(x_1+x_2+\cdots x_{10})$$

$$= 10\ln\lambda - \lambda(176)$$

$$\frac{\partial}{\partial\lambda}\left[\ln P(x_1 \ldots x_{10})\right] = \frac{10}{\lambda} - 176 = 0$$

$$\hat{\lambda} = \frac{10}{176} = 0.05682$$

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**Question 10 – 12 points**

A security engineer is aware that the number of attacks on a system is uniformly distributed. She observes the number of attacks the server is subjected two over 10 days. The numbers are 35, 45, 44, 22, 37, 32, 36, 54, 35, 31. Find the parameters of the distribution using Method of Moments estimation.

$$m_1 = \frac{371}{10} = 37.1$$

$$m_2' = \frac{(-2.1)^2 + 7.9^2 + 6.9^2 + (-15.1)^2 + (-0.1)^2 + (-5.1)^2 + (-1.1)^2 + (16.9)^2 + (-2.1)^2 + (-6.1)^2}{10}$$

$$\approx 69.69$$

$$\mu_1 = \frac{a+b}{2} \qquad \mu_1 = m_1$$

$$\mu_2' = \frac{(b-a)^2}{12} \qquad \mu_2' = m_2'$$

$$\frac{a+b}{2} = 37.1 \qquad a+b = 74.2$$

$$\frac{(b-a)^2}{12} = 69.69 \qquad b-a = 28.9185$$

$$\hat{a} = 22.64075$$

$$\hat{b} = 51.55925$$

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**SCRATCH - I**

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**SCRATCH - II**