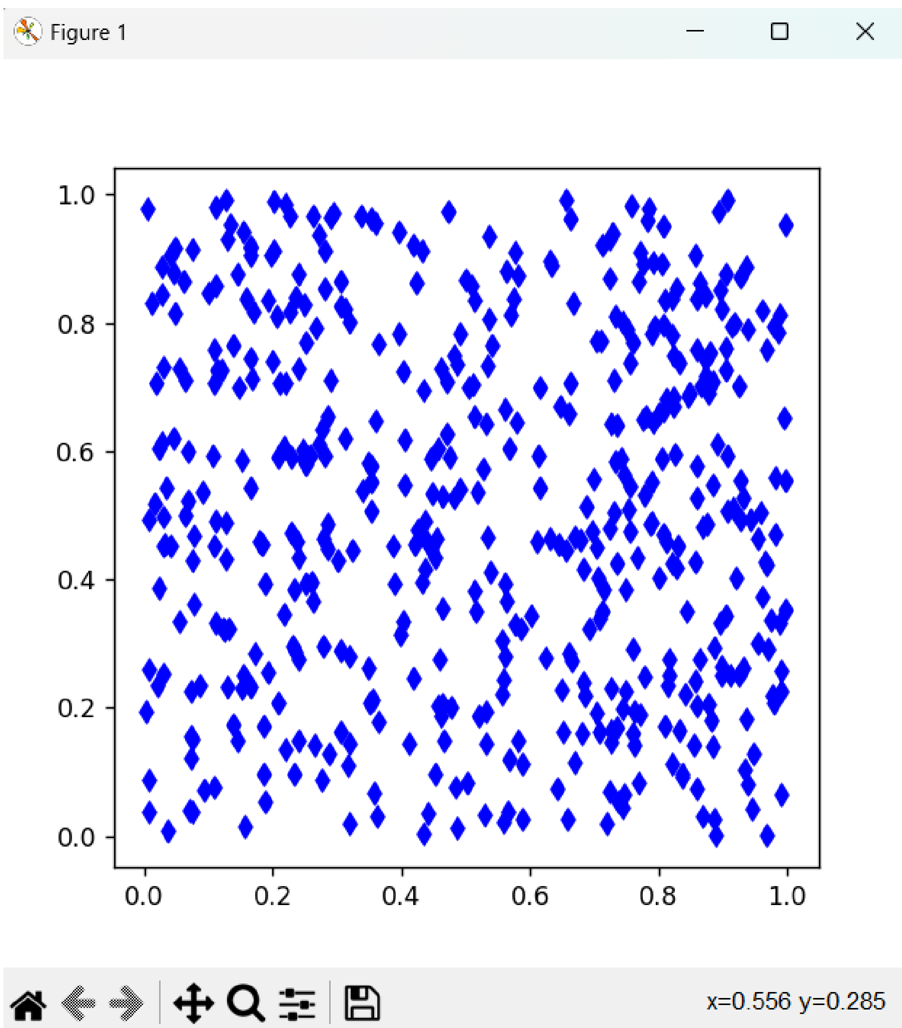
1Q)

Pesudo Random Number Generator

Code:

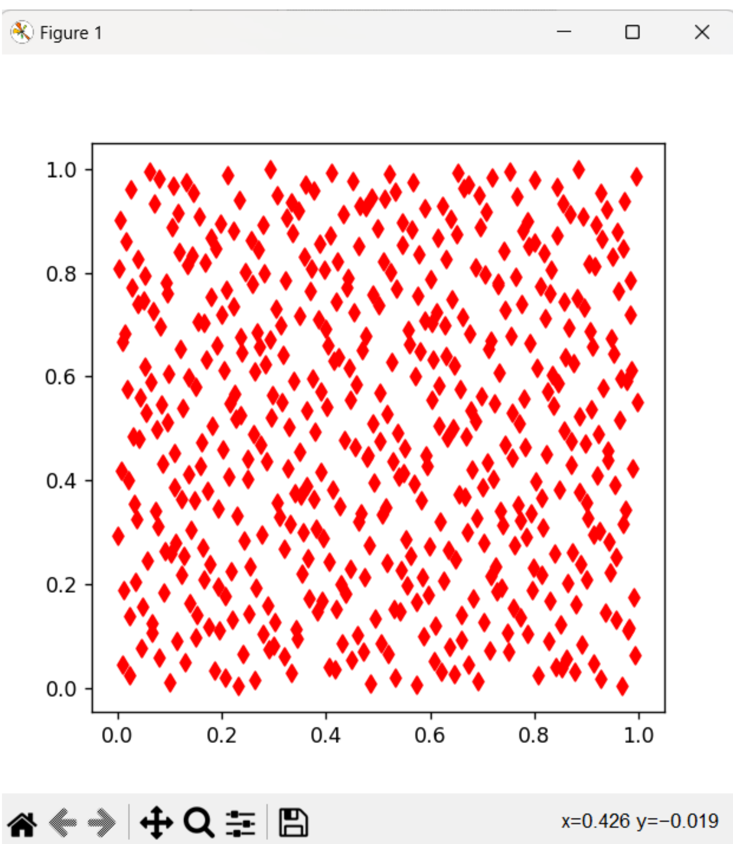
```
from scipy.stats import qmc, stats
import matplotlib.pyplot as plt
import numpy as np

# Pesudo Random Number Generator
num = 1e3/2
x1 = np.random.uniform(0,1,int(num))
y1 = np.random.uniform(0,1,int(num))

plt.rcParams["figure.figsize"] = (5, 5)
plt.scatter(x1,y1,marker='d', c= 'b')
plt.show()
```



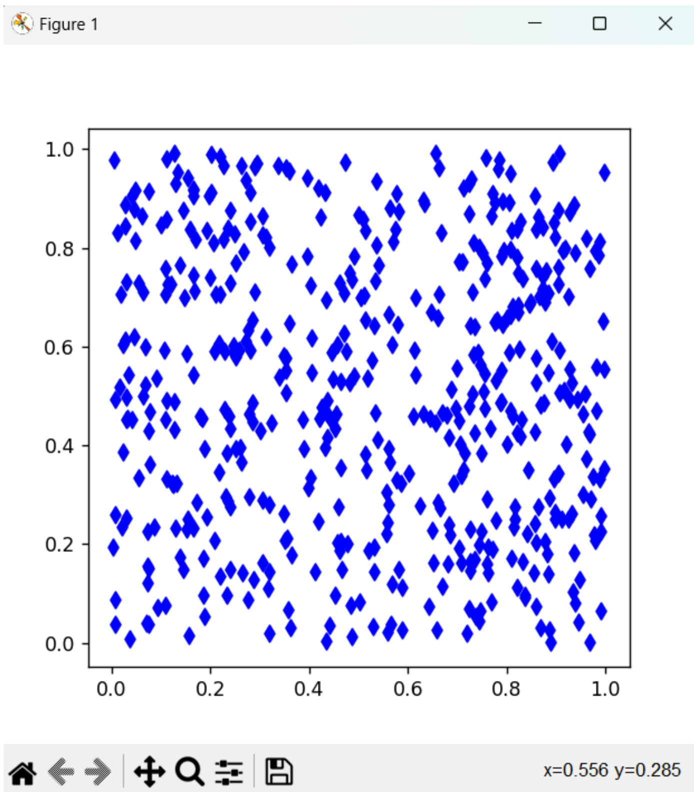
Quasi Random Number Generator

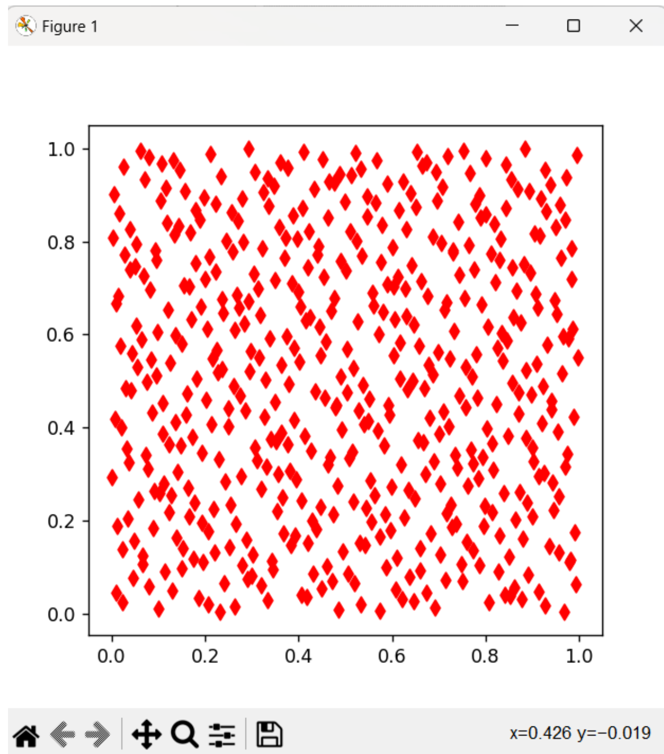
Code:

```
from scipy.stats import qmc, stats
import matplotlib.pyplot as plt
import numpy as np


# Quasi Random Number Generator
def quasi(n, d=1):
   sampler = qmc.Halton(d, scramble=True)
   return sampler.random(n)

def quasi_norm(n, d=1):
   sampler = qmc.Halton(d, scramble=True)
   x_halton = sampler. random(n)
   return stats.norm.ppf(x_halton)

x = quasi(n=500, d=2).T

plt.rcParams["figure.figsize"] = (5, 5)
plt.scatter(x[0],x[1], marker='d', c='r')
plt.show()
```



Comparing the Pesudo and Quasi Random Numbers





6.3) a. The transaction probability matrix

$$P: \begin{array}{c|cc} & b_1 & b_2 \\ \hline b_1 & 0.6 & 0.4 \\ b_2 & 0.2 & 0.8 \end{array}$$

b. Here is brow so, (0, 1) is the initial distribution

$$p(\text{child}) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times0.6 + 1\times0.2 + 0\times0.4 + 1\times0.8 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2, 0.8 \end{bmatrix}$$

$$p(\text{grandchild}) = \begin{bmatrix} 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2\times0.6 + 0.8\times0.2 + 0.2\times0.4 + 0.8\times0.8 \end{bmatrix}$$

$$= \begin{bmatrix} 0.28 & 0.72 \end{bmatrix}$$

6.5) April 1 next year is far and may have lot of c

we use the following equations to determine i

$$0.8P_1 + 0.4P_2 = P_1$$

$$0.2P_1 + 0.6P_2 = P_2$$

$$P_1 + P_2 = 1 \longrightarrow (1)$$

$\Rightarrow P_1 = 2P_2$

Substitute in eq(1)

$\Rightarrow 3P_2 = 1$

$P_2 = 1/3$

as $P_1 + P_2 = 1 \Rightarrow P_1 + 1/3 = 1$

$P_2 = 2/3$

The probability that it is rainy is $P_2 = 1/3$

6.7) a) The sum of each row should be equal to 1.

$$\Rightarrow \begin{bmatrix} 0.3 & - & 0 \\ 0 & 0 & - \\ 1 & - & - \end{bmatrix}$$

$$\begin{bmatrix} 0.3 & 0.7 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

b) It is Regular if all the values are positive after finite number of Steps ③

$$P^2 = P \cdot P = \begin{pmatrix} 0.3 & 0.7 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0.3 & 0.7 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0.09 & 0.21 & 0.7 \\ 1 & 0 & 0 \\ 0.3 & 0.7 & 0 \end{pmatrix}$$

$$P^2 \cdot P = \begin{pmatrix} 0.09 & 0.21 & 0.7 \\ 1 & 0 & 0 \\ 0.3 & 0.7 & 0 \end{pmatrix} \begin{pmatrix} 0.3 & 0.7 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0.727 & 0.063 & 0.21 \\ 0.3 & 0.7 & 0 \\ 0.09 & 0.21 & 0.7 \end{pmatrix}$$

$$P^3 P = \begin{pmatrix} 0.727 & 0.063 & 0.21 \\ 0.3 & 0.7 & 0 \\ 0.09 & 0.21 & 0.7 \end{pmatrix} \begin{pmatrix} 0.3 & 0.7 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0.4281 & 0.5089 & 0.063 \\ 0.09 & 0.21 & 0.7 \\ 0.727 & 0.063 & 0.21 \end{pmatrix}$$

c) let $\pi = (\pi_1, \pi_2, \pi_3)$ be the Steady State and

$$\sum_{i=1}^{3} \pi_i = 1$$

$$\pi_j = \sum_{i=1}^{3} \pi_i P_{ij} \quad \forall\ j = 1,2,3$$

$$(\pi_1, \pi_2\ \pi_3) = (\pi_1, \pi_2\ \pi_3) \begin{pmatrix} 0.3 & 0.7 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= (0.3\pi_1 + \pi_3,\ 0.7\pi_1,\ \pi_2)$$

$$\pi_1 = 0.3\pi_1 + \pi_3 \Rightarrow 0.7\pi_1 = \pi_3$$

$$\pi_2 = 0.7\pi_1$$

$$\pi_2 = \pi_3$$

Since $\pi_1 + \pi_2 + \pi_3 = 1$

$$\Rightarrow 0.7\pi_1 + 0.7\pi_2 + \pi_1 = 1$$

$$\Rightarrow 2.4\pi_1 = 1$$

$$\Rightarrow \pi_1 = 1/2.4$$

$$\Rightarrow \pi_1 = 0.42$$

$$\pi_2 = \pi_3 = 0.7\pi_1 = (0.7 \times 0.42) = 0.29$$

$$\Rightarrow (\pi_1, \pi_2, \pi_3) = (0.42, 0.29, 0.29)$$

a) avarage = 2 jobs

2/60 jobs per sec

for a frame lengths of 3 second

=> $3 \times 2/60$

=> 0.1

b) probability of one job = 1/30

No. of jobs sent in 1 hr = $3600 \times 1/30$

= 120 jobs

Standard deviation = $\sqrt{120 \times 0.9}$ = 10.39 jobs

6.23)

a) poission distribution with $\lambda = 5$

The avarage rate per 2 minutes = $\lambda t = 10$

$$p(X=0) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}$$

$$= \frac{e^{-10}(10)^0}{0!}$$

$$= e^{-10}$$

$$\simeq 0.000045$$

b) Expectation if first time offer. (6)

$E[T] = 1/\lambda$

$= 1/5$

$= 0.20$

Variance of first $= 1/\lambda^2$

$= 1/25$

$= 0.04$