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1

1. All winning moves given (1, 3, 5) as a Nim position

Nim sum of 1, 3, 5

$1 :- 0 0 0 1_2$

$3 :- 0 0 1 1_2$

$5 :- 0 1 0 1_2$

Nim sum :- $0 1 1 1_2$

As it is not '0' this is an N-position, so we have to find a move to P-position, i.e to a position with even number of 1's in each column.

if we do XOR 1 & nim sum :-

$$\begin{array}{r} 0001_2 \\ 0101_2 \\ \hline 0110 \end{array} \rightarrow 6$$

as we cannot remove 6 piles from 1

if we do XOR 3 & nimsum :-

$$\begin{array}{r} 0001 \\ 0111 \\ \hline 0100 \end{array} \rightarrow 4$$

we can not remove 4 from 3

if we do XOR 5 & nimsum :

$$\begin{array}{r} 0101 \\ 0111 \\ \hline 0010 \end{array} \rightarrow 2$$

we can remove minimum 2 from 5 so

the winning move would be x. moving 3 piles from 5

So
$$\begin{array}{cccccc} 1 & - & 0 & 0 & 0 & 1 \\ 3 & - & 0 & 0 & 1 & 1 \\ 2 & - & 0 & 0 & 1 & 0 \\ \hline & & 0 & 0 & 0 & 0 \end{array}$$

So the winning move would be $(1,3,5) \to (1,3,2)$

(2) By assuming $x = \{2,3\}$, $Y = \{2,3\}$ the payoff matrix is given by

$$\begin{array}{ccc} & & \text{Player II} \\ & 2 & 3 \\ \text{player I} \quad 2 & \left[\begin{array}{cc} -4 & +6 \\ +6 & -9 \end{array}\right. & \\ 3 & & \end{array}$$

$$\begin{array}{cc} & \text{Player II} \\ & \begin{array}{cc} 2 & 3 \end{array} \\ \text{player I} \begin{array}{c} 2 \\ 3 \end{array} & \begin{bmatrix} -4 & +6 \\ +6 & -9 \end{bmatrix} \end{array}$$

let x be proportion of time player I calls '2'.

The optimal strategy is

Then P I should select $x \Rightarrow -4p + 6(1-p) = 6p - 9(1-p)$

$$-10p + 6 = 15p - 9$$

$$p = 3/5$$

if I call '2' with probability $3/5$ and '3' with $2/5$

on an average player I wins $-4 \times 3/5 + 6 \times 2/5 = 0$

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the 2nd average loss is $6(3/5) - 9(2/5) = 0$

so the game is fair

value of game is 0

(3)

		Player II	
		C	D
Player I	A	(6,-10)	(0,10)
	B	(4,1)	(1,0)

player I plays (A,B) with probability (P,1-P)

player II plays (C,D) with probability (q, 1-q)

$$EP_c = P(-10) + (1-P)(1) = 1 - 11P$$

$$EP_d = P(10) + (1-P)(0) = 10P$$

$$EU_c = EU_d$$

$$1 - 11p = 10p \Rightarrow 1 = 21P \Rightarrow P = 1/21$$

for player II Expected payoffs 10p is $10/21$

$$EP_A = q(6) + (1-q)(0) = 6q$$

$$EP_B = (4)q + (1-q)(1) = 1 + 3q$$

$$EP_A = EP_B$$

$$6q = 1 + 3q \Rightarrow q = 1/3$$

Expected payoffs for player I is $6q = 6 \times 1/3 = 2$

unique mixed strategy nash equilibrium is $p = 1/21$ & $q = 1/2$

with payoffs 2 for player I

$10/21$ for player II

$\Rightarrow$ if we assume $P = 3/21 = 1/7$ slightly greater than $1/21$

a) payoffs for player I would be $6q = 6 \times 1/3 = \underline{\underline{2}}$

when she maintains her payoff as expected

payoff for Player I would be $1 + 3q - P + 3Pq$

$$1 + \cancel{3} \times 1/\cancel{3} - 1/7 + \cancel{3} \times 1/7 \times 1/\cancel{3}$$

$$= 2$$

b) Expected payoff for player II $10P + q - 21Pq$

$$= 10 \times 1/7 + 1/3 - 21 \times 1/3 \times 1/7$$

$$= 10/7 + 1/3 - 1$$

$$= \frac{30+7}{21} - 1 = \frac{37-21}{21} = \frac{16}{21}$$

which is greater than $10/21$

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$\Rightarrow$ player II can commit to playing strategy c with probability $> 1/3$

by assuming $q = 2/3$

Expected payoffs for player II $10P + q - 21Pq$

$$10 \times 1/21 + 2/3 - 21 \times 1/21 \times 2/3$$

$$= \frac{10}{21}$$

maintains his payoffs

Expected payoffs for player I $1 + 3q - P + 3Pq$

$$1 + 3 \times \frac{2}{3} - 1/21 + 3 \times 1/21 \times 2/3$$

$$= 3 + \frac{2}{21} - 1/21 = 3 + 1/21$$

$$\approx 3.05 > 2$$

Player I benefits by obtaining a greater pay than she did in nash equilibrium.

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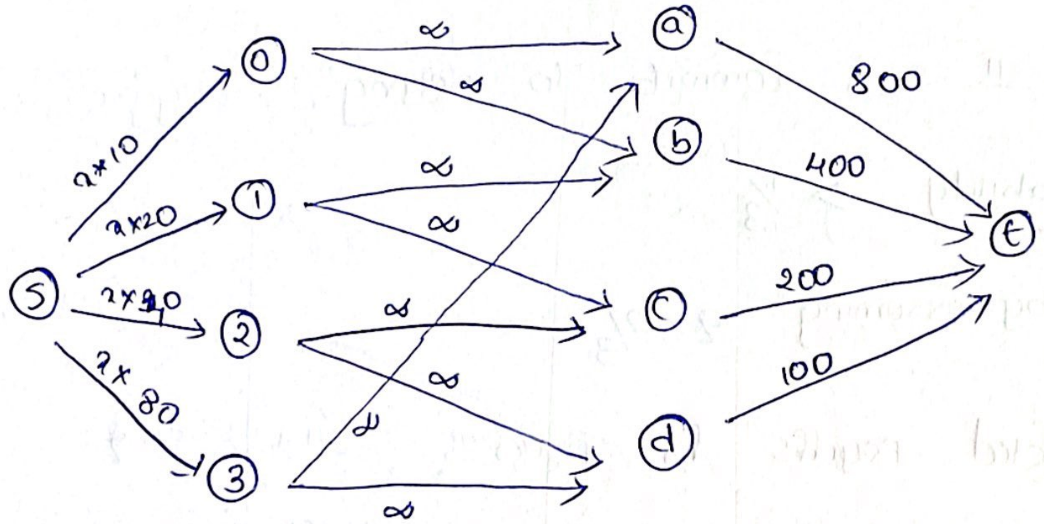
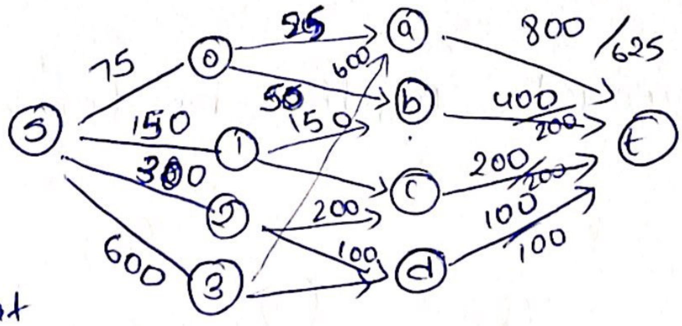
5.



Binary search (between 0 and $\frac{(800+400+200+100)}{(10+20+40+80)} = 10$

assume $\lambda^* = 5$ it can increased assume 7.5

if $\lambda^* = 7.5$

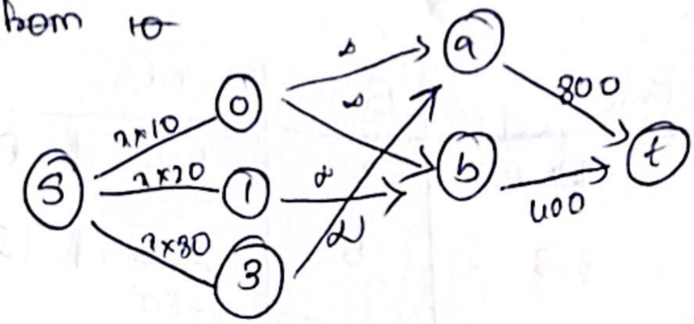


~~as~~ ~~maxd~~ ~~Cleaning~~

(largest λ value that

saturates all edges leaving s)

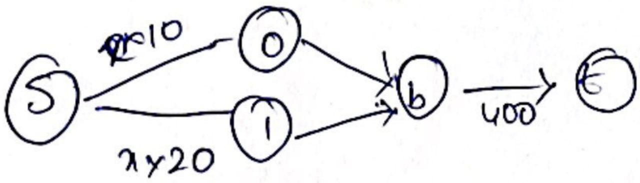
by starting from 10



Binary search (between 7.5 and $\frac{800+400}{10+20+80} = \frac{1200}{110} \approx 10.9$

$x^* = 10$ (largest x value that saturates all edges leaving s)

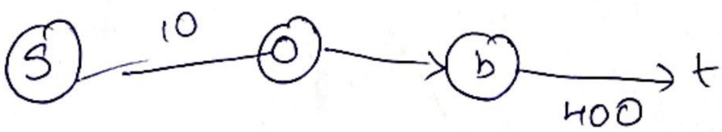
$\omega = \{0, 1, b\}$



Binary search (between 10 and $\frac{400}{10+20} = \frac{40}{3} = 13.33$

$x^* = 13.3$ (largest x value that saturates all edges leaving s)

A Price B ~~Units~~



$x^* = 40$

A	Price	B	Units
0	40	b	400 of 0
1	13.3	b	260 of 1
3	10	b	800 of 3
2	7.5	d	100 of 3
2		c	300 of 2

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Payoff: -2/3

Robber
Payoff: 2/3

	1st St /3rd Ave		1st St / 6th Ave		2nd St / 1st Ave		2nd St / 5th Ave		3rd St / 4th Ave		4th St / 2nd Ave		4th St/ 5th Ave		5th/ 2nd Ave
1st St	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
2nd St	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1
3rd St	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1	-1
4th St	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	-1
5th St	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1
6th St	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
1st Ave	-1	1	-1	1	1	-1	-1	1	-1	1	-1	1	-1	1	-1
2nd Ave	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	1
3rd Ave	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
4th Ave	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1	-1
5th Ave	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1	1	-1	-1
6th Ave	-1	1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1

Profiles All equilibria by enumeration of mixed strategies in strategic game

#	1: 1st St	1: 2nd St	1: 3rd St	1: 4th St	1: 5th St	1: 6th St	1: 1st Ave	1: 2nd Ave	1: 3rd Ave	1: 4th Ave	1: 5th Ave	1: 6th Ave
1	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0
2	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0	0	$\frac{1}{6}$	0
3	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	0
4	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	0
5	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	0	0	0
6	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	0	0
7	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0	0	0	0
8	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	0	0
9	0	0	0	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
10	$\frac{1}{6}$	0	0	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

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Payoff: -2/3

Robber
Payoff: 2/3

	1st St / 3rd Ave		2nd St / 5th Ave		3rd St / 4th Ave		4th St/ 2nd Ave		5th St/ 6th Ave		6th St/ 1st Ave	
1st St	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1
2nd St	-1	1	1	-1	-1	1	-1	1	-1	1	-1	1
3rd St	-1	1	-1	1	1	-1	-1	1	-1	1	-1	1
4th St	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1
5th St	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1
6th St	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1

Profiles All equilibria by enumeration of mixed strategies in strategic game

#	1: 1st St	1: 2nd St	1: 3rd St	1: 4th St	1: 5th St	1: 6th St	2: 1st St / 3rd Ave	2: 2nd St / 5th Ave	2: 3rd St / 4th Ave	2: 4th St/ 2nd Ave
1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Computing Nash equilibria

The computation has completed. Number of equilibria found so far: 1

#	1: 1st St	1: 2nd St	1: 3rd St	1: 4th St
1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$