8.3 Hw.3 (1)

$P(X \geq 127.78) = P(X - 48.23 \geq 79.54)$

$\leq P\left(\frac{|X - 4.23|}{26.51} \geq \frac{79.54}{26.51}\right)$

$= P\left\{\frac{|X-\mu|}{\delta} \geq 3\right\} \leq 1/9$

$P(X < 127.78) \geq 8/9$

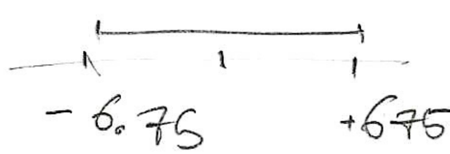
---

8.4

X within 1.5 IQR

Standard Normal quantile $= \pm 0.675$

Stand Normal IQR $= 2(0.675) = 1.35$




$X \sim N(\mu, \delta)$ and quantil $= \mu \pm 0.675\delta$

$P(Q_1 - 1.5(IQR) \leq X \leq Q_3 + 1.5(IQR)\}$

$= P\left\{-0.675 - 1.5(1.35) \leq \frac{X-\mu}{\delta} \leq 0.675 + 1.5(1.35)\right\}$

$= P\{-2.70 \leq Z \leq 2.70\} = \Phi(2.7) - \Phi(-2.7)$

$= 0.99 - 0.0035 = \boxed{0.99}$

8.8

check diagram.

---

9.2 a) Given Geometric sample:

$\mu_1 = 1/p$ and $\bar{X} = 4 \Rightarrow \frac{1}{p} = 4$

$\hat{p} = 0.25$

b) Pmf ?

$P(X) = \prod_{i=1}^{5} p(1-p)^{x_i - 1} = p^5 (1-p)^{\sum x_i - 5} = p^5 (1-p)^{15}$

$\Rightarrow P(x) = p^5 (1-p)^{15} \Rightarrow$

$\operatorname{Ln} P(X) = 5 \operatorname{Ln}(p) + 15 \operatorname{Ln}(1-p)$

MLE: Find by calculating the deretive with respect p and equal to zero

$\frac{\partial}{\partial p} \operatorname{Ln} P(X) = 5/p - \cancel{15}/1-p = 0 \Rightarrow$

$5 - 5p = 15p \Rightarrow \hat{p} = \frac{5}{20} = 0.25$

9.4 First moment: by using method of moment 

$$m_1 = \mu_1 = E(X) = \int_0^1 x f(x)\,dx = \int_0^1 \theta x^{\theta}\,dx$$

$$= \frac{\theta x^{\theta+1}}{\theta+1}\Big]_{x=0}^{x=1} = \frac{\theta}{\theta+1}$$

$$m_1 = \bar{X} = \frac{0.7+0.9+0.4}{3} = 2/3$$

$$\frac{\theta}{\theta+1} = 2/3 \Rightarrow \boxed{\theta = 2}$$

Method of maximum likelihood.

Joint density is

$$f(x) = \prod_{i=1}^{3} \theta x_i^{\theta-1}$$

$$\ln f(x) = \sum_{i=1}^{3} (\ln\theta + (\theta-1)\ln x_i)$$

$$= 3\ln\theta + (\theta-1)\sum_{i=1}^{3} \ln x_i$$

$$\frac{\partial \ln f(x)}{\partial\theta} = \frac{\partial\left(3\ln\theta + (\theta-1)\sum_{i=1}^{3}\ln x_i\right)}{\partial\theta} = 0$$

$$\Rightarrow \theta = -\frac{3}{\sum_i^3 \ln x_i} = \frac{-3}{\ln(0.4)+\ln(0.7)+\ln(0.9)} = 2.1799$$

9.8 ④

a) $\bar{X} \pm Z_{0.025} \frac{\delta}{\sqrt{n}} = 42 \pm (1.96) \frac{5}{\sqrt{64}} =$

$42 \pm 1.225 \Rightarrow \boxed{[40.775, 43.225]}$

b) $P\{40.775 \leq X \leq 43.225\} =$

$P\left\{ \frac{40.775 - \mu}{\delta} \leq Z \leq \frac{43.225 - \mu}{\delta} \right\}$

$= P\left\{ \frac{40.775 - 40}{5} \leq Z \leq \frac{43.40}{5} \right\}$

$= \Phi(0.645) - \Phi(0.155) = .740 - 0.561$
$= 0.179$

doesn't belong 95% CI

---

9.9 Given $\delta$ unkown using $\bar{X} \pm t_{\alpha/2} \frac{S}{\sqrt{n}}$ ①

a) $\alpha = 1 - .9 = 0.1$, $n = 3$, $\rightarrow df = n-1 = 2$
$\hookrightarrow t_{\alpha/2} = t_{0.05} = 2.920$

$\bar{X} = \frac{30+50+70}{3} = 50 \Rightarrow S = \sqrt{\frac{(30-50)^2 + (50-50) + (70-50)^2}{n-1}}$

$\boxed{\delta = 20}$ Plug in ① $50 \pm 2.920 \frac{20}{\sqrt{3}} = 50 \pm 3.77$
$[16.3, 83.7]$

9.9 b) Hypo: $H_0: \mu = 80$

$H_A: \mu \neq 80$

10% level

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{50 - 80}{\frac{20}{\sqrt{3}}} = -2.598$$

$\Rightarrow$ Acceptance region $[-2.920, 2.920]$

No significant evidence against $H_0$

(c) 90% CI for $\sigma$

$$\left[ \sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}} \right]$$

$$= \left[ \sqrt{\frac{(2)(400)}{5.99}}, \sqrt{\frac{(2)(100)}{0.10}} \right]$$

$\Rightarrow [11.6, 89.4]$ \$

9.10 (a)

$\hat{P} = \frac{24}{200} = 0.12$

$\hat{P} \pm Z_{0.02} \sqrt{\frac{\hat{P}(1-\hat{P})}{n}} = 0.12 \pm (2.054) \sqrt{\frac{0.12(1-0.12)}{200}}$

$\alpha = 1 - .96 = 0.04$

$= 0.12 \pm 0.047$

$Z_{\alpha/2} = Z_{0.02} = 2.054$

$[0.073, 0.167]$

(b)

$H_0 : P \leq 0.1$

$H_A \quad P > 0.1$

$$Z = \frac{\hat{P} - P_0}{\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}} = \frac{.12 - 0.1}{\sqrt{\frac{0.12(1-0.12)}{200}}} = 0.870$$

P-value : $P(Z > 0.870) = 1 - \Phi(0.870)$

$= 1 - 0.8070 = 0.192$

P-value bigger 0.04, 0.15

do not have significance evidence to reject.

9.11

$H_0 : P_1 = P_2$

$H_A : P_1 > P_2$

$\hat{p}_1 = 0.12 , n_1 = 200$

$\hat{p}_2 = \frac{13}{150} , n_2 = 150$

$$\hat{p} \text{ (pooled)} = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{24+13}{200+150} = 0.1057$$

$$Z = \frac{0.12 - 0.0867}{\sqrt{(0.1057)(1-0.1057)\left(\frac{1}{200}+\frac{1}{150}\right)}} = 1.0027$$

P_value: $P(Z > 1.0027) = 1 - 0.84 = 0.158$

No Significance evidence that quality of item by new is higher.

9.14

(a) $H_0 = \mu_A = \mu_B$

$H_A = \mu_A \neq \mu_B$

(b) $p = 2P\{t > |-2.7603|\}$ between $[0.01, 0.02]$

Not significant. to reject.

(C) $H_0 \quad \mu_A = \mu_B$

$H_A \quad \mu_A < \mu_B$

$P = P\{t < -2.7603\}$ between $[0.005, 0.001]$

~~A~~ reject $H_0$

9.17

$\hat{P}_A = .45$

$$Z_{0.025}\sqrt{\frac{\hat{P}_A(1-\hat{P}_A)}{n_A}} = 1.96\sqrt{\frac{.45 \times .55}{900}}$$

$$= \underline{0.0325}$$

$$\hat{P}_B = .35 \Rightarrow Z_{0.025}\sqrt{\frac{\hat{P}_B(1-\hat{P}_B)}{n_B}} = 1.96\sqrt{\frac{.35 \times .65}{900}} =$$

$$= 0.0312$$

$\hat{P}_A - \hat{P}_B = .1$ and Margin err:

$$Z_{0.025}\sqrt{\frac{\hat{P}_A(1-\hat{P}_A)}{n_A} + \frac{\hat{P}_B(1-\hat{P}_B)}{n_B}} = 1.96\sqrt{\frac{.45 \times .55}{900} + \frac{0.35 \times .65}{900}}$$

$$\boxed{= 0.0450}$$

---

9.20 $H_0 = \quad \delta = 5$

$H_A: \quad \delta \neq 5$

$$\chi^2_{obs} = \frac{(n-1)S^2}{\delta_0^2} = \frac{39 \times (6.2)^2}{5^2}$$

$$= 60$$

P-value $= 2\min\left(P(\chi^2 \geq \chi^2_{obs}), P(\chi^2 \leq \chi^2_{obs})\right\}$

$= 2\min\left\{F(\chi^2_{obs}, 1 - F(\chi^2_{obs})\right\}$

$\nu = n-1 = 39$ dif.

$P \in (0.02, 0.05) \Rightarrow \alpha \geq 0.05$

$\boxed{\text{reject Ho}}$

9.23

(a)

$\bar{X} : 85 \quad S_X = 12.76$

$\bar{Y} : 80 \quad S_Y = 3.22$

$m = n = 6$

$H_0 \quad \mu_X = \mu_X$

$H_A \quad \mu_X > \mu_Y$

$H_0 \quad \delta_X = \delta_Y$

$H_A \quad \delta_X \neq \delta_Y$

$$F_{obs} = \frac{S_X^2}{S_Y^2} = \frac{12.76}{3.22} = 15.65$$

$$P = 2\min\left(P\{F \geq F_{obs}\}, P\{F \leq F_{obs}\}\right) = [0.002, 0.01]$$

reject $H_0$

To Test (a)

$$t_{obs} = \frac{85 - 80}{\sqrt{\frac{(12.76)^2}{6} + \frac{(3.22)^2}{6}}} = 0.93$$

$$v = \frac{\left(\frac{S_X^2}{m} + \cancel{S_Y^2}/m\right)^2}{\frac{S_X^2}{n^2(n-1)} + \frac{S_Y^2}{m^2(m-1)}} = \frac{\left(\frac{(12.76)^2}{6} + \frac{(3.22)^2}{6}\right)^2}{\frac{(12.76)^4}{180} + \frac{(3.22)^4}{180}} = 5.64$$

P-value $= P(t > t_{obs}) > 0.1$

~~Fail to~~ Fail To reject.

9.23 (b) $H_0 \quad \delta_X = \delta_Y$

$H_A \quad \delta_X > \delta_Y$

$F_{obs} = 15.65$

d.f: 5

$P(F \geq F_{obs}) \in [0.001, 0.005]$

$\boxed{\text{accept } H_A}$

---

9.24 (a) $\bar{X} \pm t_{\alpha/2} \frac{S_X}{\sqrt{n}} = 85 \pm (2.015) \frac{12.76}{\sqrt{6}}$

$= 85 \pm 10.50$ or $[74.50, 95.50]$

90% CI

$\bar{Y} \pm t_{\alpha/2} \frac{S_Y}{\sqrt{m}} = 80 \pm (2.015) \frac{3.22}{\sqrt{6}} =$

$= 80 \pm 2.65 \rightarrow [77.35, 82.65]$

(b) with 90% CI: $\bar{X} - \bar{Y} \pm t_{\alpha/2} \sqrt{\frac{S_X^2}{m} + \frac{S_Y^2}{m}} =$

$5 \pm (1.97) \sqrt{\frac{12.76^2}{6} + \frac{3.22^2}{6}} =$

$5 \pm 10.58 \rightarrow [-5.58, 15.58]$

Accept $H_0$

9.24

c

$$\left[\frac{(n-1)S_X^2}{\chi^2_{\alpha/2}}, \frac{(n-1)S_X^2}{\chi^2_{1-\alpha/2}}\right] = \left[\frac{5\times 12.76^2}{11.1}, \frac{5.12.76^2}{1.15}\right]$$

$$= [73.33, 707.8]$$

$$\left[\frac{(n-1)S_Y^2}{\chi^2_{\alpha/2}}, \frac{(n-1)S_Y^2}{\chi^2_{1-\alpha/2}}\right] = \left[\frac{5\times 3.22^2}{11.1}, \frac{5.3.22^2}{1.15}\right]$$

$$= [4.68, 45.22]$$

d with 90% CI, $\delta_X^2 / \delta_Y^2 = [3.10, 79.05]$

which support $\delta_X^2 \neq \delta_Y^2$

$$\left[\frac{S_X^2/S_Y^2}{F_{\alpha/2}(n-1, m-1)}, \frac{S_X^2/S_Y^2}{F_{1-\alpha/2}(n-1, m-1)}\right]$$

2/ Drug trial

Q1 : answer is slide

$H_0 \quad \mu_1 \leq \mu_2$

$H_A \quad \mu_1 > \mu_2$

$$t_c = \frac{(\bar{X}_1 - \bar{X}_2) - \delta_0}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(8-4)-0}{\sqrt{20.4933\left(\frac{1}{18} + \frac{1}{11}\right)}} = 2.31$$

$t_\alpha = 1.703 \quad df = n_1 + n_2 - 2 = 27$

$2.31 > 1.703$ reject $H_0$

3

| | |
|---|---|
| $n_1 = 174$ | $n_2 = 355$ |
| $\bar{x}_1 = 3.51$ | $\bar{x}_2 = 3.24$ |
| $s_1 = 0.51$ | $s_2 = 0.52$ |

$$\bar{x}_1 - \bar{x}_2 = 3.51 - 3.24 = 0.27$$

$$\alpha = 1 - 0.99 = 0.01$$

$$z_{\alpha/2} = z_{0.005} = 2.576$$

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 0.27 \pm 2.576\sqrt{\frac{0.51^2}{174} + \frac{0.52^2}{355}}$$

$$= 0.27 \pm 0.12$$

$$[0.15, 0.39]$$

Satisfaction for Com18 is $[0.15, 0.39]$