Home Work - 6

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7.15

Given the System is with Capacity $C=2$

$\mu_A = 10$ min $\Rightarrow \lambda_A = 1/10$ min^{-1}

$\mu_S = 15$ min $\Rightarrow \lambda_S = 1/15$ min^{-1}

given $\Delta = 3$ min

$P_A = \lambda_A \Delta = \frac{1}{10} \times 3 = 0.3$

$P_S = \lambda_S \Delta = \frac{1}{15} \times 3 = 0.2$

Transition prob for X

$P_{00} = 1 - P_A = 1 - 0.3 = 0.7$

$P_{01} = P_A = 0.3$

$P_{10} = (1-P_A)P_S = 0.7 \times 0.2 = 0.14$

$P_{12} = (1-P_S)P_A = (1-0.2) \times 0.3 = 0.24$

$P_{11} = 1 - P_{10} - P_{12} = 1 - 0.14 - 0.24 = 0.62$

$P_{20} = 0$

$P_{21} = (1-P_A)P_S = 0.14$

$P_{22} = 1 - 0.14 = 0.86$

markov chain $X(t)$ has 3 states $X=0, X=1, \& X=2$

Transition table.

$$P = \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0.14 & 0.62 & 0.24 \\ 0 & 0.14 & 0.86 \end{bmatrix}$$

Steady State Eq.

$$\pi P = \pi$$

$$\pi = [\pi_0, \pi_1, \pi_2]$$

$$0.7\pi_0 + 0.14\pi_1 = \pi_0 \longrightarrow (1)$$

$$0.3\pi_0 + 0.62\pi_1 + 0.24 = \pi_1 \longrightarrow (2)$$

$$0.24\pi_1 + 0.86\pi_2 = \pi_2 \longrightarrow (3)$$

with $\pi_0 + \pi_1 + \pi_2 = 1$

Solving (1) (2) & (3)

$$\pi = [0.1467 \quad 0.3144 \quad 0.5389]$$

7.11 Given

Sol) arrival Rate $\lambda_A = 10/25 = 2/5$

Service Rate $\lambda_S = 1/2$

Interarrive Ration $(\rho) = \frac{\lambda_A}{\lambda_S} = \frac{2}{5/1/2} = 4/5$

(a) Average No. of Customer's in System:-

$$L_S = \frac{\lambda}{\mu - \lambda} = \frac{\rho}{1-\rho} = \frac{\frac{4}{5}}{1-4/5} = \frac{\frac{4}{5}}{\frac{1}{4}} = 4 \text{ Customer's}$$

(a) Average No. of Customers in waiting:-

$$W_S = \left(\frac{\lambda}{\mu}\right)\frac{\lambda}{\mu-\lambda} = \frac{\rho^2}{1-\rho} = \frac{4}{(5)}(4) = \frac{16}{5} = 3.2 = 3 \text{ Custom}$$

(c) The fraction of time when teller is busy and atleast 5 other Custerms are waiting in line

$= P(n \geq 6) = p^n = (0.8)^6$

$= 0.2621$

b) The fraction of time when Teller is =

$= p$ (at least one customer)

$= p(n \geq 1)$

$= p = $ probability $=$ Interarartiv Ration : $\rho = 4/5 = (0.8)$.

$$p = \frac{E(T)}{T} = \rho = \boxed{0.8}$$

7.12

Given

$\lambda_A = 60/5 = 12 / \text{hour}$

$\lambda_S = 60/3 = 20 / \text{hour}$

$\rho = \frac{\lambda_A}{\lambda_S} = \frac{12}{20} = \frac{6}{10} = 0.6$

a) Response Time $= \frac{1/20}{1-0.6} = \frac{0.05}{0.4} = 0.125 = 7.5$ mins

b) fraction of Time there are n customer in the System $= (1-\rho)(\rho)^n$

p(less than 2 Customers in System) $= (1-0.6)(1)$
$+ (1-0.6)(0.6)$
$= 0.4 + 0.24$
$= 0.64$

c) probability of Having at least one Customer in the System (wait Before Service Starts) $= 0.6$.

1.17

Sol) little's law States the relation B/W Expect

Jobs, response time and Arrival time.

$$E(X) = \lambda_A E(n)$$

Queuing Syst (M/M/1)

$$E(n) = \frac{U_s}{1-\gamma}$$

$$E(x) = \frac{\gamma}{1-\gamma}$$

$$\lambda_A * E(n) = \lambda_A * \frac{U_s}{1-\gamma}$$

$$= \frac{\gamma}{1-\gamma}$$

$$= E(x)$$

$E(x) = E(S) + E(W)$ Wait & Servic time Relation

$$E(W) = \frac{U_s}{1-\gamma} - U_s$$

$$= \frac{\gamma}{1-\gamma} E(S)$$

$$E(W) = E(x) * E(S)$$