CSE-5319- SPECIAL TOPICS THEORY ALGORITHMS

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1.) List three winning moves given (10, 18, 24, 46, 53) as a NIM position

ans Step-1 Lets convert the positions in binary so we can perform XOR

	number	binary
n_1	10	→ 001 010
n_2	18	010 010
n_3	24	011 000
n_4	46	101 110
n_5	53	110 101
	XOR	011 011

→ to achieve 0 we can do it by

making 53 ie 110 101 to 101 110 ie → 46

So. binary XOR is →

	number	binary
n_1	10	001 010
n_2	18	010 010
n_3	24	011 000
n_4	46	101 110
n_5	46	101 110
		000 000

→ is 0

So 46 is ~~wrong~~ win move ie remove 6 from 53.

(ii) Lets change ~~46~~ ~~48~~ 18 to 010010 to 001001 ie 9

So here XOR is

x_1	10	001 010
x_2	9	001 001
x_3	24	011 000
x_4	46	101 110
x_5	53	110 101
$x_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_5$	XOR	000 000

→ is 0 so here it is a winning move

18 − 9 is remove 9 from 18

(iii) Let's change 24 011 000 to 000 011 → 3

ie remove 21 in 24

the XOR is

10	001 010
18	010 010
3	000 011
46	101 110
53	110 101
	000 000

is 0

So removing 21 in 24 is a winning move

2.). A = {2, 3} for player ①
B = {2, 3} for player ②

pay off matrix is for player ① →
$$\begin{array}{c|cc} & 2 & 3 \\ \hline 2 & -4 & +6 \\ 3 & +6 & -9 \end{array}$$

as for player ① where (2, 2) → A loses so = −4
if (2, 3) → A win and = +6
(3, 2) → A win and = +6
(3, 3) → A lose and = −9

for player ② when (2, 2) → B win = +4
(2, 3) → B lose = −6
(3, 2) → B lose = −6
(3, 3) → A lose = +9

pay off matrix → for player ②
$$\begin{array}{c|cc} & 2 & 3 \\ \hline 2 & 4 & -6 \\ 3 & -6 & 9 \end{array}$$

refer coneq and coarsecorreq for files. and for .dat file refer gambit for files. → Q2.gbt and Q2-2pusoBsum .dat

3.). ~~Ex~~ matrix is →

		player II C	D
player I	A	(6, −10)	(0, 10)
	B	(4, 1)	(1, 0)

So lete p is probability of Player 1
1−p is other chance of Player 1

Let q be probability of Player II
$1-q$ be prob of player II

	(q)	$(1-q)$
p	$(6,-10)$	$(0,10)$
$1-p$	$(4,1)$	$(1,0)$

Expected pay off from 'A' is

$$E_1(A) \Rightarrow 6 \times q + 0 \times (1-q)$$
$$= 6q + 0$$
$$= 6q$$

$$E_1(A) = 6q$$

Expected pay off from 'B' is

$$E_1(B) = 4 \times q + 1(1-q)$$
$$= 4q + 1 - q$$
$$= 3q + 1$$

$E_1(A)$ and $E_1(B)$ are same as expected payoff are same

$$E_1(A) = E_1(B)$$

$$6q = 3q + 1$$
$$\boxed{q = 1/3}$$

So player II when playing 'C'

$$E_2(C) = -10(P) + 1(1-P)$$
$$= -10 + 1 - P$$
$$= -11p + 1$$

If when player II playing 'D'

$$= 10p + 0(1-p)$$
$$E_2(D) = 10p$$

as $E_2(C)$ and $E_2(D)$ are same so $E_2(C) = E_2(D)$

$$\Rightarrow 10p = -11p + 1$$
$$21p = 1$$
$$\boxed{p = 1/21}$$

so by looking at $q = 1/3$.

$p = 1/21$ we can confirm that game has unique nash equilibrium

refer files for correlated and coarse correlated equilibrium
Q3_2nd.dat file to calculate
correq and coarse correq.

3.

~~coreq~~ Correlated equilibrium

	1	2
1	0.015873	0.031746
2	0.31746	0.634921

{Q6.coreq} in files

Coarse correlated equilibrium

	1	2
1	0.015873	0.031746
2	0.31746	0.634921

{Q6.cocoreq} in files

4). Refer Q4.gbt for gambit file and for equilibrium

refer Q4_a_equilibrium.png for files. for a)
refer Q4_b for . b)
refer Q4_c for c).

5). 4 buyers with indicated budgets : a $ 300, b/$400, c/$200.
d/$100
4 goods with indicated 0/20, 1/10, 2/30, 3/40

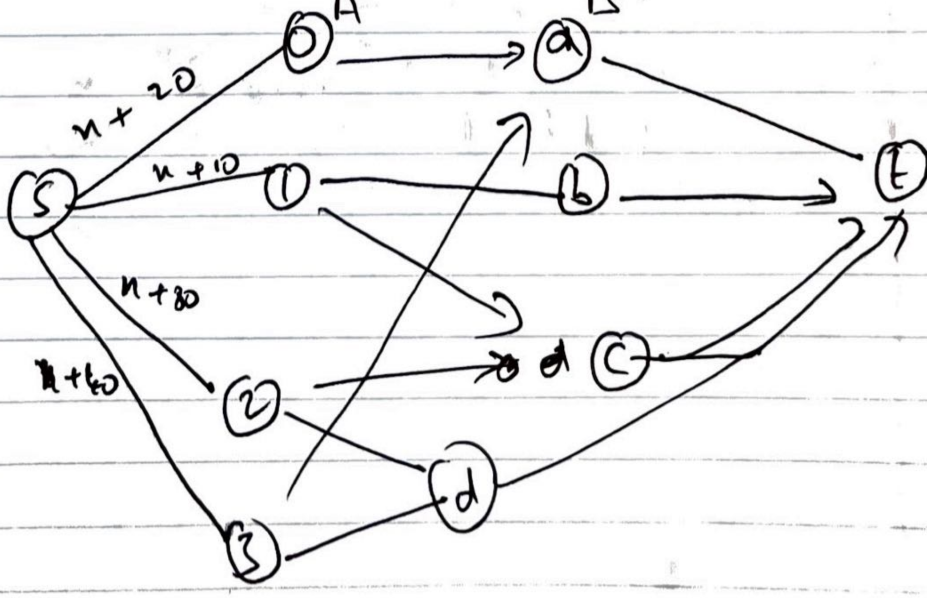
· Access to goods?

a) 0, 3 b: 0, 1 c: 1, 2 d: 2, 3

ans · Binary search (between 0 & $\left(\frac{300+400+200+100}{20+10+30+40}\right) = 10$

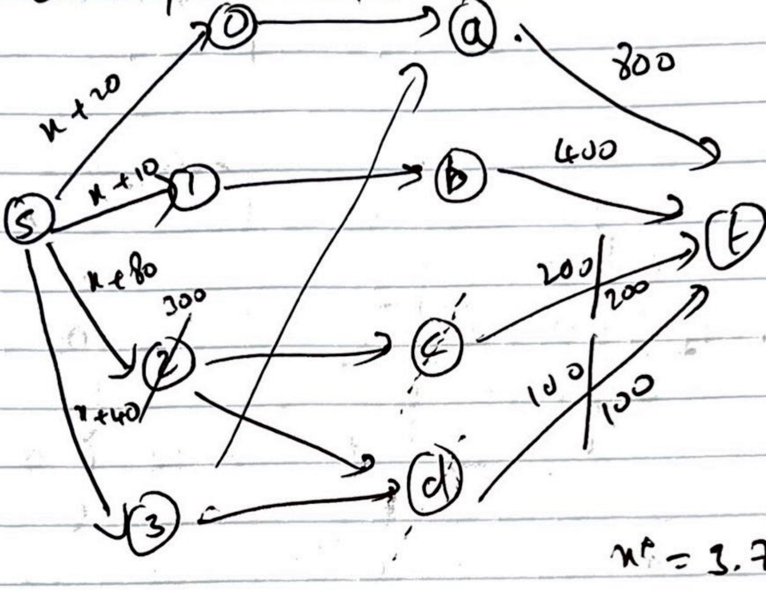
So $x^* = 10$ market clearing price (per unit)

So, value is b/w 0 & 10



It does not fit as it does not saturate at anything interpolation again b/w 0 & 5
$x^* = 2.5$

Binary search b/w 3.75 & 10

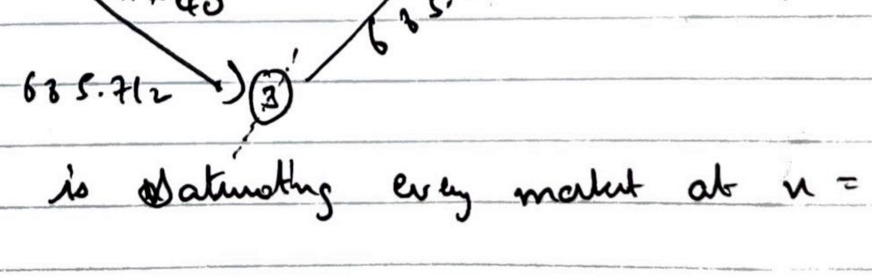


$x^* = 3.75$

· So as it saturates 2nd c & d · $x^* = 3.75$

Now, we do binary search b/w (3.75 & (300+400+60+20+10))
= 3.75 & 17.1428)

So $x^* = 17.1428$



It is saturating every market at $x = 17.1428$

A	Price	B	Units
0	17.1428	a	6.67 of 0
1	17.1428	b	11.33 of 0, 10 of 1
2	3.75	c	53.33 of 2
		d	26.666 of 2
3		a	40 of 3.

6.) correq equilibrium

(1 1 0)
(1 2 0)
(1 3 0)
(2 1 0)
(2 2 1)
(2 3 0)

	1	2	3
1	0	0	0
2	0	1	0

Coarse correq equilibrium

(1 1 0.5)
(1 2 0)
(1 3 0)
(2 1 0)
(2 2 0.5)
(2 3 0)

	1	2	3
1	0.5	0	0
2	0	0.5	0

CSE-5319 SPEC TOPS THEORY / ALGORITHMS

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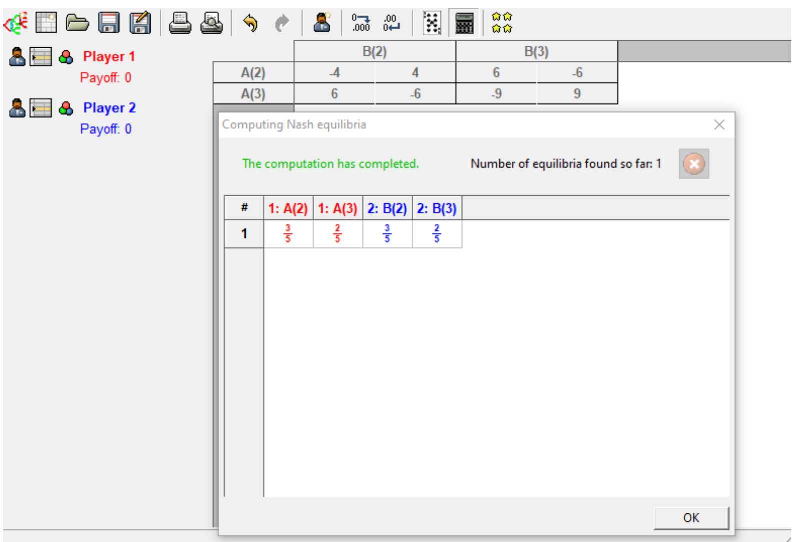
Q2.Gambit equilibrium
run glpsol --model CorrEq.mod --data Q2_2per0sum.dat
run glpsol --model coarseCorrEq.mod --data Q2_2per0sum.dat
in cmd



Coarse and Coarse correlated equilibria

```
Generating zprob...
Generating aconstraint...
Generating bconstraint...
Generating computeasum...
Generating computebsum...
Generating oconstraint...
Generating obj...
Model has been successfully generated
GLPK Simplex Optimizer, v4.65
9 rows, 7 columns, 26 non-zeros
Preprocessing...
5 rows, 4 columns, 12 non-zeros
Scaling...
 A: min|aij| =  1.000e+00  max|aij| =  1.500e+01  ratio =  1.500e+01
GM: min|aij| =  8.589e-01  max|aij| =  1.164e+00  ratio =  1.355e+00
EQ: min|aij| =  7.567e-01  max|aij| =  1.000e+00  ratio =  1.321e+00
Constructing initial basis...
Size of triangular part is 5
      0: obj =  -0.000000000e+00 inf =   1.135e+00 (1)
      3: obj =  -0.000000000e+00 inf =   0.000e+00 (0)
OPTIMAL LP SOLUTION FOUND
Time used:   0.0 secs
Memory used: 0.1 Mb (126549 bytes)

a payoff          0 b payoff          0 objective is          0
z distribution is:
 (1 1       0.36)
 (1 2       0.24)
 (2 1       0.24)
 (2 2       0.16)
Model has been successfully processed
```

```
Model has been successfully generated
GLPK Simplex Optimizer, v4.65
13 rows, 7 columns, 26 non-zeros
Preprocessing...
5 rows, 4 columns, 12 non-zeros
Scaling...
 A: min|aij| =  1.000e+00  max|aij| =  1.500e+01  ratio =  1.500e+01
GM: min|aij| =  8.589e-01  max|aij| =  1.164e+00  ratio =  1.355e+00
EQ: min|aij| =  7.567e-01  max|aij| =  1.000e+00  ratio =  1.321e+00
Constructing initial basis...
Size of triangular part is 5
      0: obj =  -0.000000000e+00 inf =   1.135e+00 (1)
      3: obj =  -0.000000000e+00 inf =   0.000e+00 (0)
OPTIMAL LP SOLUTION FOUND
Time used:   0.1 secs
Memory used: 0.1 Mb (126575 bytes)

a payoff          0 b payoff          0 objective is         0
z distribution is:
 (1 1       0.36)
 (1 2       0.24)
 (2 1       0.24)
 (2 2       0.16)
Model has been successfully processed
```

Q3

Obtained by

run glpsol --model CorrEq.mod --data Q3_2nd.dat

run glpsol --model coarseCorrEq.mod --data Q3_2nd.dat

in cmd

Coarse and Coarse correlated equilibria

```
a payoff          2 b payoff    0.47619 objective is    2.47619
z distribution is:
 (1 1   0.015873)
 (1 2   0.031746)
 (2 1    0.31746)
 (2 2   0.634921)
Model has been successfully processed
```

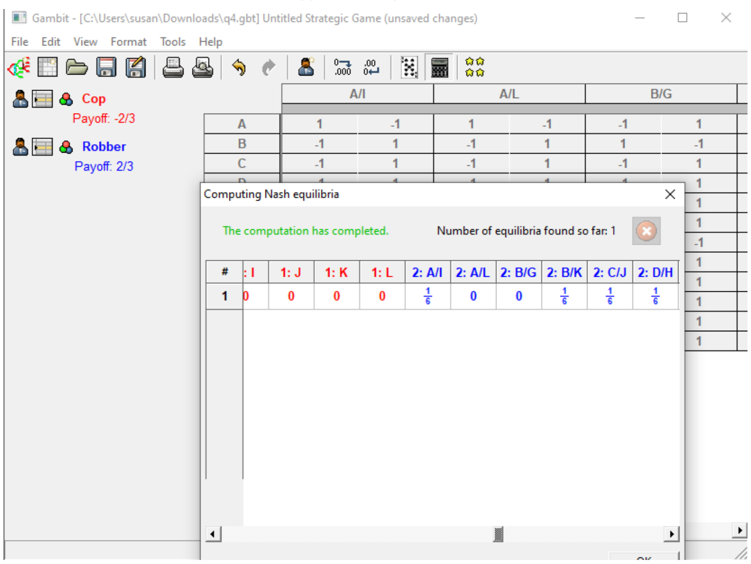
```
a payoff          2 b payoff    0.47619 objective is    2.47619
z distribution is:
 (1 1   0.015873)
 (1 2   0.031746)
 (2 1    0.31746)
 (2 2   0.634921)
Model has been successfully processed
```

Q4

a) # run this as glpsol --model 2per0sum.mod --Q4_a.dat

```
V is      -0.6
X distribution is: (1       0) (2       0) (3       0) (4       0) (5     0.2) (6       0) (7     0.2) (8       0) (9     0.2) (10      0) (11    0.2) (12    0.2)
Y distribution is: (1     0.2) (2       0) (3       0) (4       0) (5     0.2) (6     0.2) (7       0) (8     0.2) (9       0) (10      0) (11    0.2)
Model has been successfully processed
```

One equilibrium found (gambit)



4)b finding all equilibrium

Cop
Payoff: -2/3

Robber
Payoff: 2/3

	A/I		A/L		B/G		B/K		C/J		D/H		D/K		E/H		E/J		E/L	
A	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
B	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
C	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1
D	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1
E	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	1	-1
F	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
G	-1	1	-1	1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
H	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	1	-1	1
I	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
J	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1
K	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	-1	1	-1	1
L	-1	1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1

Profiles 5 All equilibria by enumeration of mixed strategies in strategic game

#	1: A	1: B	1: C	1: D	1: E	1: F	1: G	1: H	1: I	1: J	1: K	1: L	2: A/I	2: A/L	2: B/G	2: B/K	2: C/J	2: D/H	2: D/K	2: E/H	2: E/J	2: E/L	2: F/G
1	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$
2	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$
3	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$
4	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$
5	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	0	0	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$
6	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$
7	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0	0	0	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$
8	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$
9	0	0	0	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$
10	$\frac{1}{6}$	0	0	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$

Computing Nash equilibria

The computation has completed. Number of equilibria found so far: 10

#	1: A	1: B	1: C	1: D	1: E	1: F	1: G	1: H	1: I
1	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0
2	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0
3	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0
4	$\frac{1}{6}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0
5	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0
6	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	0	0
7	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0
8	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	0
9	0	0	0	0	0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

OK

4)c minimum line-cover (player 1) and a maximum matching (player 2)

Cop
Payoff: -2/3

Robber
Payoff: 2/3

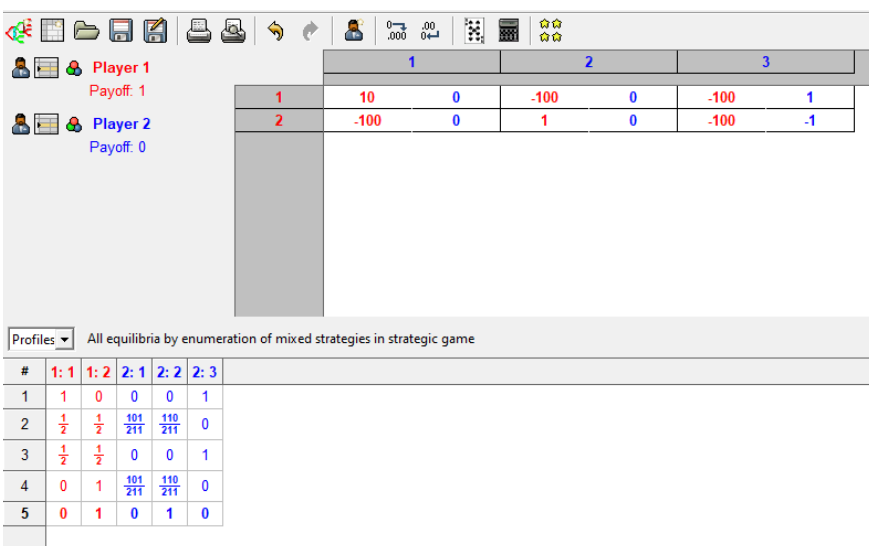
	A/I		B/K		C/J		D/H		E/L		F/G	
A	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1
D	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1
E	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1
G	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1
H	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1
J	-1	1	-1	1	1	-1	-1	1	-1	1	-1	1
K	-1	1	1	-1	-1	1	-1	1	-1	1	-1	1

6)
run glpsol --model CorrEq.mod --data Q6.dat
run glpsol --model coarseCorrEq.mod --data Q6.dat
in cmd
Coarse and Coarse correlated equilibria

```
a payoff          1 b payoff          0 objective is          1
z distribution is:
 (1 1           0)
 (1 2           0)
 (1 3           0)
 (2 1           0)
 (2 2           1)
 (2 3           0)
Model has been successfully processed
```

```
a payoff          1 b payoff          0 objective is          1
z distribution is:
 (1 1           0)
 (1 2           0)
 (1 3           0)
 (2 1           0)
 (2 2           1)
 (2 3           0)
Model has been successfully processed
```

Gambit



	1		2		3	
1	10	0	-100	0	-100	1
2	-100	0	1	0	-100	-1

#	1: 1	1: 2	2: 1	2: 2	2: 3
1	1	0	0	0	1
2	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{101}{211}$	$\frac{110}{211}$	0
3	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1
4	0	1	$\frac{101}{211}$	$\frac{110}{211}$	0
5	0	1	0	1	0