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HOMEWORK - 5

1. By finding the maximum weight bipartite matching via ascending auction is (on auction.2.dat) is

5

100	200	300	400	500
201	202	203	204	205
301	321	341	361	381
401	431	461	481	491
462	463	464	465	466

Total weight is 1986

Matching:

0	4
1	0
2	2
3	3
4	1

M^v be a maximum weight matching and let $\|M^v\|$ be its weight; that is $\|M^v\| = \sum_i v_i M^v(i)$. write v_{-i} for the matrix obtained by replacing row i of v by 0.

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2nd row :- Total weight : 1785

Matching:

0	4
1	0
2	2
3	3
4	1

Total weight : 1647

Matching

0	4
1	1
2	0
3	3
4	2

Total weight : 1527

Matching:

0	4
1	1
2	3
3	0
4	2

Total weight : 1524

Matching:

0	4
1	1
2	2
3	3
4	0

The result will be:

0	0
1	1
2	2
3	22
4	42

$= \underline{\underline{67}}$

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lowest envy-free price vector p for v and the corresponding utility vector u are

$$M^v(i) = j \Rightarrow P_j = \|M^v - i\| - (\|M^v\| - v_{ij})$$

$$U_i = \|M^v\| - \|M^v - i\|$$

for 1st row

Total weight = 1528

Matching:

0	0
1	1
2	4
3	3
4	2

$P_1 = 1986 - 1528 = 458$

Similarly

$P_2 = 1985 - 1528 = 201$

$P_3 = 1985 - 1647 = 339$

$P_4 = 1985 - 1527 = 459$

$P_5 = 1985 - 1524 = 462$

The lowest envy-free price vector is

	P_j
0	458
1	201
2	339
3	459
4	462

The lowest total weight is 67

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2. Highest ~~lowest~~ envy-free rent $\bar{R} = \Sigma_j \bar{P_j}$,

Highest envy free price vector P for V and the corresponding utility vector U are given by

$$M^V(i) = j \Rightarrow U_i = \|M\|^{V-j} - (\|M^V\| - v_{ij})$$

$$P_j = \|M^V\| - \|M^{V-j}\| \quad \forall j$$

where V^{-j} is the matrix obtained by replacing column j of V by 0.

by replacing each column by 0 the vectors could be

1st

Total weight 1785

Matching :

0	4
1	0
2	2
3	3
4	1

Total weight : 1784

Matching :

0	4
1	1
2	2
3	3
4	0

Total weight : 1764

Matching :

0	4
1	2
2	1
3	3
4	0

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Total weight : 1744

Matching :

0	4
1	3
2	1
3	2
4	0

Total weight : 1644

Matching :

0	3
1	4
2	1
3	2
4	0

$P_1 = 1986 + 1785 = 201$

$P_2 = 1986 - 1784 = 202$

$P_3 = 1986 - 1764 = 222$

$P_4 = 1986 - 1744 = 242$

$P_5 = 1986 - 1644 = 342$

The highest envy-free price vector

0	201
1	202
2	222
3	242
4	342

The highest weight is 1209

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3. Given 5-room apartment with monthly rent of \$1000

The results of problem 1 and 2 are

lowest		highest	
0	458	0	201
1	201	1	202
2	339	2	222
3	459	3	242
4	462	4	342

$R = \alpha \underline{R} + (1-\alpha)\overline{R}$ (highest envy free rent)

$\underline{R}$ → lowest envy free rent

$0 < \alpha < 1$

$$P_j = \alpha \underline{P_j} + (1-\alpha)\overline{P_j}$$

by calculating the resulting vector will be

0	158
1	0 223
2	119
3	239
4	261

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by Assuming that atleast one matching has weights whose sum is no less than the sum for the lowest envy-free rent vector and no more than the sum for the highest envyfree rent vector envy-free rent may be achieved.

The lowest envy free rent would be 67

The highest envy free rent would be 1209

~~As in the~~ By calculating the maximum match via ascending auction we get Total weight 1986

Matching

0	4
1	0
2	2
3	3
4	1

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4. In Roughgarden Theorem

λ is a positive integer

Φ is a potential function which can take on only polynomially many distinct values. The potential function can take on exponentially many different values and best response dynamics can decrease the potential function very slowly, requiring an exponential number of iteration to converge.

k is used to know number of strategies

α - bounded jump conditions, cost functions satisfy ~~the~~ satisfies this.

The Max Gain variant of ϵ-best-response dynamics is used: in every iteration, among players with an ϵ-move available, the player who can obtain the biggest absolute cost decrease moves to its minimum-cost deviation.

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Then an ϵ-PNE is reached in $\left(\frac{k\alpha}{\epsilon} \log \frac{\phi(s^0)}{\phi_{min}}\right)$ iterations.

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