(1) Bidder – $b_1 \rightarrow s_1$
$b_2 \rightarrow s_2$

Prize amount maximum to win :– $s_1 + s_2$
One bidder quits and Other bidder pays p & Receives
$s_1 + s_2$

* The Equilibrium is the case where both players has to bid double price they have (individual signal)

Winner payment (p) < maximum be should pay

* A Bidder who wins at a prince p knows the actual value is

$$v = s_1 + P/2 > p$$

if $p < 2s_i$, so i is ~~plea~~ pleased to be a winner. at any price up to $2s_i$ but would lose money if he "won" the auction at any higher price. this is unique Symmetric Equilibrium.

* if b_1 wins at P, then $v = s_1 + \frac{9P}{10} > p$

$$\Leftrightarrow \quad p < 10 s_1$$

* if b_2 wins at P, then $v = s_2 + P/10 > p$

$$\Leftrightarrow \quad p < \frac{10}{9} s_2 \,;$$

② Analyze the allocation algorithm for downward sloping valuation for following $v_i(k)$, values buyer 1 i)

$$v_i(k) = v_{i1} + v_{i2} + \cdots + v_{ik}$$

					k				
$V_i(k)$	0	1	2	3	4	5	6	7	8
1	0	50	100	143	182	219	244	259	269
		50	50	43	39	37	25	15	10
2	0	70	135	188	223	257	287	313	323
		70	65	53	35	34	30	26	10
3	0	60	115	160	200	236	236	287	304

$n = 3$, $m = 8$

formula:

$$v_i(m_i) - v_i(m_i - 1) \geq p \geq v_i(m_i + 1) - v_i(m_i)$$

if $\Sigma m_i > m$ then p is too low

$\Sigma m_i < m$ then p is too high

Otherwise we have to found the right value of "p".

Clearing price in $v[0, v]$ is p.

p	m_1	m_2	m_3	Σ
10	8	8	8	24
20	6	7	7	20
30	5	6	5	16
40	3	3	4	10
50	2	3	2	7
60	0	2	1	3
70	0	1	0	1

We calculate the m_1, m_2, m_3 by

If $P = 10$ $\quad i = 1$

$(50-0) \geq 10 \geq (100-50)$ ×

$(143-100) \geq 10 \geq (182-143)$ ×

$(219-182) \geq 10 \geq (244-219)$ ×

$(269-259) \geq 10 \geq (259-244)$ ×

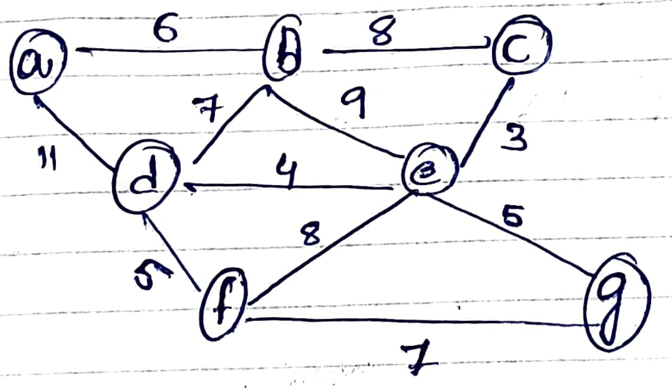
$(143-100) < 10 < (100-50)$ ×

$(219-182) < 10 < (182-143)$ ×

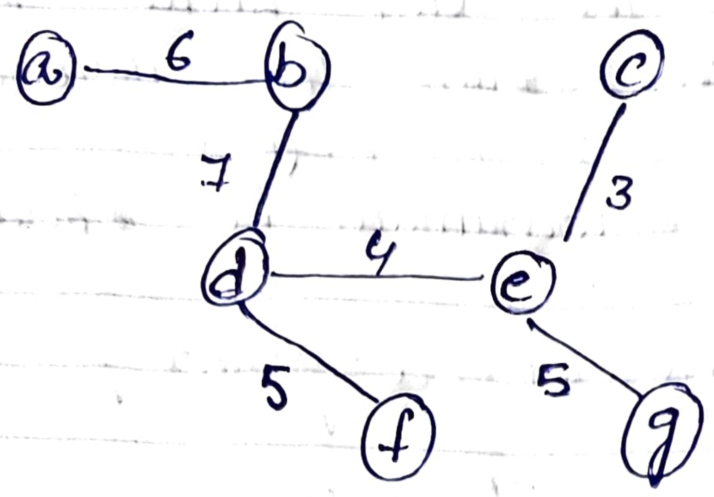
$(259-244) < 10 < (244-219)$ ✓

So it's $\underline{\underline{8}}$

③ Given graph



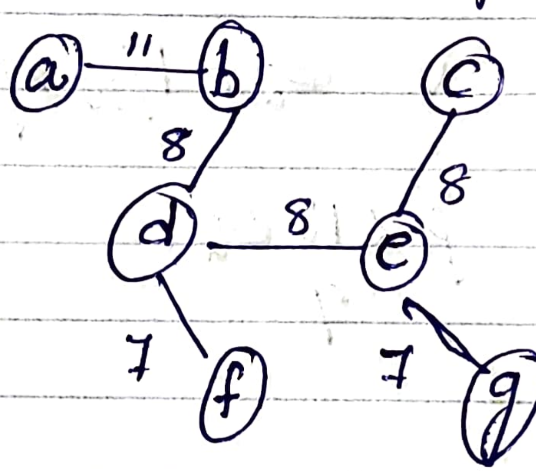
The minimum Spanning tree with Shortest distance is the optimal (mst) will be



The VCG Calculate payments

AB = 35 − 24 = 11
BD = 31 − 23 = 8
CE = 35 − 27 = 8
DE = 34 − 26 = 8
DF = 32 − 25 = 7
EG = 32 − 25 = 7

* The final minimum Spanning tree is



(4) Given bids for copies of a digital good

10 10 10 9 9 8 8 8 7 7 6 6 5 5 4 4 4 4 4 4

$n \rightarrow$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$i \rightarrow$	10	10	10	9	9	8	8	8	7	7	6	6	5	5	4	4	4	4	4	4
Revenue	10	20	30	36	45	48	56	64	63	70	66	72	65	70	60	64	68	72	76	80

Notice the highest value is 80

The optimal fixed price would be 80.