

HOMEWORK

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2.4) Given Here employees of firm

* people with c/ett known is Represents with $C = 70\% = 0.7$

* people with fortan known is $F = 60\% = 0.6$

* And people with both known fortan + c/ett is

Sol) $(C \cap F) = 50\% = 0.5$

a) Does not know fortan?

Sol) So we know people How know fortan:

$$P(F) = 0.6 \text{ we want } P(\bar{F}) = 1 - P(F)$$

$$= 1 - 0.6 = 0.4$$

So people How ~~know~~ "don't know" = 0.4

b) Does not know fortan and c/ett

$$\text{So we want } P(\bar{F} \cap \bar{C}) = 1 - P(F \cup C)$$

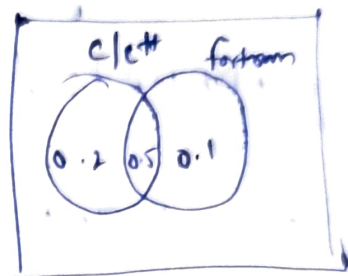
$$= 1 - [P(F) + P(C) - P(F \cap C)]$$

(c) know's C/C++ but not fortran?

Sol) $p(\bar{c}) = P(c) - p(c \cap f)$
 $= 0.7 - 0.5 = 0.2$

(d) know's fortran, but not C/C++

Sol) $p(\bar{f}) = p(f) - p(c \cap f)$
 $= 0.6 - 0.5 = 0.1$



(e) If Someone knows fortran, what is the probability that he/she knows C/C++ too?

Sol) total fortran known = 0.6
How knows Both = 0.5

So fortran + C/C++ $p(f|c) = \frac{0.5}{0.6} = 0.833$

(f) If Someone knows C/C++ what is the probability that he/she knows fortran too?

Sol) total C/C++ known = 0.7
How knows Both = 0.5

So the $p(c|f) = \frac{0.5}{0.7} = 0.714$

2.5) Given Here $p(A) = 0.2$: Error found by test 1
 $p(B) = 0.3$: Error found by test 2
 $p(C) = 0.5$: Error found by test 3

* And also given 3 independent test so

$$p(A \cap B \cap C) = 0 \text{ So } p(A \cap B \cap C) = p(A) \times p(B) \times p(C)$$

* We want Not found probabilities so

$$p(\bar{A}) = 1 - 0.2 = 0.8$$

$$p(\bar{B}) = 1 - 0.3 = 0.7$$

$$p(\bar{C}) = 1 - 0.5 = 0.5$$

* Required is found by at least one test

$$\text{So: } 1 - (p(\bar{A}) \times p(\bar{B}) \times p(\bar{C}))$$

$$= 1 - (0.8 \times 0.7 \times 0.5)$$

$$= 1 - (0.28) = \underline{\underline{0.82}}$$

2.8) Given that a Shuttle launch on any day
 of fail independent Event. $p(A) = 0.01$ tailed Event
 $p(B) = 0.02$

$$p(\bar{c}) = 1 - 0.02 = 0.98$$

So probability that all three do not fail is

$$= 0.99 \times 0.98 \times 0.98$$

$$= \underline{0.95}$$

2.11) Given that are 5 independent tests and probability of Error are

$$p(A) = 0.1 \Rightarrow p(\bar{A}) = 1 - 0.1 = 0.9$$

$$p(B) = 0.2 \Rightarrow p(\bar{B}) = 1 - 0.2 = 0.8$$

$$p(C) = 0.3 \Rightarrow p(\bar{C}) = 1 - 0.3 = 0.7$$

$$p(D) = 0.4 \Rightarrow p(\bar{D}) = 1 - 0.4 = 0.6$$

$$p(E) = 0.5 \Rightarrow p(\bar{E}) = 1 - 0.5 = 0.5$$

So (a) by at least one test means from total probability of - Success Case

$$\Rightarrow 1 - (0.9 \times 0.8 \times 0.7 \times 0.6 \times 0.5)$$

$$= \underline{0.848}$$

(b) by at least two means

$$= 1 - [0.9 \times 0.8 \times 0.7 \times 0.6 \times 0.5] - [0.1 \times 0.8 \times 0.7 \times 0.6 \times 0.5] - [0.9 \times 0.2 \times 0.7 \times 0.6 \times 0.5] - [0.9 \times 0.8 \times 0.3 \times 0.6 \times 0.5] - [0.9 \times 0.8 \times 0.7 \times 0.4 \times 0.5] - [0.9 \times 0.8 \times 0.7 \times 0.6 \times 0.5]$$

$$= 0.8488 - 0.0168 - 0.0648 - 0.1008 - 0.1512$$

$$= \underline{0.4774}$$

© By all test means all are failed

condition So

$$0.1 \times 0.2 \times 0.3 \times 0.4 \times 0.5$$

$$= 0.0012$$

2.14) Given No. of attempts: 10,00,000

(a) 6 different lower-case letters

So) let (26) alphabet So

if one letter is used we shouldn't use

$$\text{So } \underline{26} \quad \underline{25} \quad \underline{24} \quad \underline{23} \quad \underline{22} \quad \underline{21}$$

$$\text{So) } = 26 \times 25 \times 24 \times 23 \times 22 \times 21 = \text{Total Attempts}$$

$$\text{that is } \frac{10,00,000}{26 \times 25 \times 24 \times 23 \times 22 \times 21} = \frac{10^6}{165 \times 18} = \frac{1}{165} = 0.006$$

(b) 6 different letters, upper, some may be upper case and it is Case-Sensitive

$$\Rightarrow \underline{52} \quad \underline{51} \quad \underline{50} \quad \underline{49} \quad \underline{48} \quad \underline{47} \quad (26+26) = \text{Total } (52)$$

$$= \frac{10^6}{\binom{52}{6}} = \frac{625}{9161334}$$

c) Any 6 letters, upper- or lower-case and it is case-sensitive.

$$\underline{52} \quad \underline{52} \quad \underline{52} \quad \underline{52} \quad \underline{52} \quad \underline{52}$$

So)

* Here we have to repeat the numbers again.

$$\text{So)} \quad \frac{10^6}{(52)^6}$$

d) Any 6 letters including letter and digits and case sensitive.

So).

$$\underline{62} \quad \underline{62} \quad \underline{62} \quad \underline{62} \quad \underline{62} \quad \underline{62}$$

26 (lower case)

26 (upper case)

10 (digits 0-9).

$$26 + 10 = 62$$

* We are 1 million

$$= \frac{10^6}{(62)^6}$$

$$(62)^6$$

$$\begin{array}{l|l} \text{2.16) Given: } p(S_1) = 0.5 & p(D|S_1) = 0.05 \\ p(S_2) = 0.2 & p(D|S_2) = 0.03 \\ p(S_3) = 0.3 & p(D|S_3) = 0.06 \end{array}$$

We need to find $p(S_1|D)$

a) Bay the law of total probability.

$$\begin{aligned} p(D) &= p(D|S_1)p(S_1) + p(D|S_2)p(S_2) + p(D|S_3)p(S_3) \\ &= (0.05)(0.5) + (0.2)(0.03) + (0.3)(0.06) \\ &= 0.049 \end{aligned}$$

(b) Bayes Rule:

$$p(S_1|D) = \frac{p(D|S_1)p(S_1)}{p(D)} = \frac{(0.5)(0.05)}{0.49} = \frac{0.025}{0.49} = 0.051$$

2.18) Given $p(\bar{G}) = 0.75$

$$p(C|\bar{G}) = 0.9$$

$$p(C|G) = 1/4 = 0.25$$

$$\text{Also } p(G) = 1 - 0.75 = 0.25$$

Then By the Bayes Rule,

$$p(G|C) = \frac{p(C|G)p(G)}{p(C|G)p(G) + p(C|\bar{G})p(\bar{G})}$$

$$= \frac{(0.25)(0.25)}{(0.25)(0.25) + (0.9)(0.75)}$$

$$= \frac{0.0625}{0.0625 + 0.675} = \frac{0.0625}{0.7375} = 0.0847$$

2.23) Given $p(A) = 0.9$ | $p(E) = 0.5$
 $p(B) = 0.8$
 $p(C) = 0.7$
 $p(D) = 0.6$

So Now A and B are in series. So, Reliability

$$= P(A \cap B) = P(A) P(B) \\ = 0.9 \times 0.8 = 0.72$$

2.20) Given $t = 9$ be total numbers of databases, of which $m = 4$ have no and $K = 5$

let $c = 4$ databases chosen and s database

The Question is asking to find $S_{X,2}$ i.e.

$$S = 2, 3, 4$$

$$* P(S_{X,2}) = P(S=2) + P(S=3) + P(S=4)$$

$$= \frac{\binom{5}{2} \binom{4}{2}}{\binom{9}{4}} + \frac{\binom{5}{3} \binom{4}{1}}{\binom{9}{4}} + \frac{\binom{5}{4} \binom{4}{0}}{\binom{9}{4}}$$

$$= \frac{60 + 40 + 5}{126} = \frac{35}{42} \approx \underline{\underline{0.83}}$$

2.30) All outcomes are listed in the table below. According to the problem, they are equally likely

②

Outcome	The older child	The younger child	Who's more
1	girl	girl	the older girl
2	girl	girl	The younger
3	girl	boy	the girl
4	girl	boy	the girl
5	boy	girl	The girl
6	boy	girl	The boy
7	boy	boy	The older
8	boy	boy	The younger

⑥ $p(RB) = p(2, 1) = 1/4$

$p(BG) = p(5, 6) = 1/4$

$p(BB) = p(3, 4) = 1/4$

⑦ $p(2, 1) = 1/2$

$p(1/2) = 1/4$

$p(4/2) = 1/4$

⑧ $p(4, 6 | \text{sim})$

$= 1/2$

3.7) Given

x = no. of Home runs in one game

$y = n \times x$ = no. of home runs in n game.

n is some positive whole number.

if $n=2$ then

$$y = n \times x$$

$$y = 2x$$

$\Rightarrow y = a + bx$ then,

$$E(y) = E(a + bx)$$

$$E(y) = E(a) + E(bx)$$

$$= E(a) + b \times E(x)$$

* new column is $0 + 0.4 + 0.4 = \underline{0.8}$

$$E(y) = a + b \times E(x)$$

$$= 2(0.8)$$

$$= \underline{1.6}$$

$$* \text{Var}(x) = E(x^2) - [E(x)]^2$$

$$\text{Var}(y) = b^2 \text{Var}(x)$$

$$= 2^2 \text{Var}(x)$$

$$= 4(0.56)$$

$$= \underline{2.24}$$

x	$p(x)$	$x \times p(x)$
0	0.4	$0 \times 0.4 = 0$
1	0.4	$1 \times 0.4 = 0.4$
2	0.2	$2 \times 0.2 = 0.4$

3.9) Given to solve in Chebyshev's Inequality

$$P(|x - \mu| > k) \leq (\sigma^2/k^2)$$

$\sigma^2 = 5$ (Standard deviation)

$$\mu = 40$$

$$k = 1$$

$$\therefore \sigma^2/k^2 = (5)/1 = 5, \underline{25\%}$$

3.18) Given for $E(x)$

$$P_A(1) = P_B(1) = 0.5, \quad P_B(-1) = 0$$

$$E_A(x) = E_B(x) = (1)(0.5) + (-1)(0.5) = 0$$

To obtain Variance

$$E_A(x^2) = E_B(x^2) = (1^2)(0.5) + (-1)^2(0.5)$$

$$= 0.5 + 0.5$$

$$= 1$$

a) 100 Share's * \$10 * $x/10$

$$E(A) = E(100x) = 100 E(x) = 100 \times 0 = 0$$

$$\text{Var}(A) = 100^2 (\text{Var}(x)) = 10,000 \times 1 = \underline{10,000}$$

b) 50 Shares $\times 10 \times X/10 + 10 \text{ Shares}(50) + Y/50$
 Spent) $= 50X + 10Y$

$$E(X) = E(50X) + E(10Y) = 50(0) + 10(0) = 0$$

$$\begin{aligned} \text{Var}(X) &= (50)^2 (\text{Var}(X)) + (10)^2 (\text{Var}(Y)) \\ &= 50^2 + 10^2 = 2500 + 100 = 2600 \end{aligned}$$

c) 40 Shares $(10)(X/10) + 12(50) + Y/50$

Spent) $= 40X + 12Y$

$$E(X) = E(40X) + E(12Y) = 40(0) + 12(0) = 0$$

$$\begin{aligned} \text{Var}(X) &= 40(2) (\text{Var}(X)) + (12)^2 (\text{Var}(Y)) \\ &= 1600 + 144 = \underline{\underline{1744}} \end{aligned}$$

3.25) Given, The proportion of users who do not close windows properly $= 0.10$

So: The proportion of users who close windows properly $= 1 - 0.10 = 0.90$,

Assume, the No. of users $= 25$

a) The Expected users who have closed windows properly $= 25(0.90) = \underline{\underline{22.5}}$

b) The Expected users who do not close windows properly $= 25(0.10)$
 $= \underline{\underline{2.5}}$

3.27) Given $p(x, 5) = 1 - p(x \leq 4)$

a)

$$= 1 - 10.55)$$

$$= 0.945$$

b)

Given $p(x=5) = \frac{e^{-\lambda} \lambda^x}{x!}$

$$= \frac{e^{-9} (9)^5}{5!} = 0.602$$

3.28) Given assume that X has a poisson distribution

$$pr(X \leq \lambda/2) \leq 4/5$$

Chebyshev's inequality

$$P(|X - E(X)| \geq a) \leq \frac{Var(X)}{a^2}$$

* The Event $\{X \leq \lambda/2\} \subset \{X \leq \lambda/2\} \cup \{X \geq 3\lambda/2\}$

$$= \{ |X - \lambda| \geq \lambda/2 \}$$

3.35) Given let T = Thunderstorm

X = poisson

$$P(T/X=7) = \frac{P(T \cap X=7)}{P(X=7)} = \frac{P(X=7/T) P(T)}{P(X=7/T) P(T) + P(X=7/\text{not } T) P(\text{not } T)}$$

$$= \frac{(e^{-10} 10^7) 70\%}{0.6}$$

$$(e^{-10} 10^7) 70\% \cdot 0.6 + (e^{-4} 147) 70\% \cdot 0.4 = \underline{\underline{0.6941}}$$