EXAM 2- ADDENDUM

Sample Statistics (Used as estimators for Population Properties)

Mean

DEFINITION 8.3

Sample mean $\bar{X}$ is the arithmetic average,

$$\bar{X} = \frac{X_1 + \ldots + X_n}{n}$$

Variance and Std. Deviation

DEFINITION 8.8

For a sample $(X_1, X_2, \ldots, X_n)$, a **sample variance** is defined as

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2. \qquad (8.4)$$

It measures variability among observations and estimates the population variance $\sigma^2 = \mathrm{Var}(X)$.

Sample standard deviation is a square root of a sample variance,

$$s = \sqrt{s^2}.$$

It measures variability in the same units as X and estimates the population standard deviation $\sigma = \mathrm{Std}(X)$.

Alt formula for Variance: $$s^2 = \frac{\sum_{i=1}^{n} X_i^2 - n\bar{X}^2}{n-1}.$$

Quantiles, percentiles, and quartiles

DEFINITION 8.7

A p-**quantile** of a population is such a number x that solves equations

$$\begin{cases} \boldsymbol{P}\{X < x\} & \leq & p \\ \boldsymbol{P}\{X > x\} & \leq & 1-p \end{cases}$$

A **sample** p-**quantile** is any number that exceeds at most $100p\%$ of the sample, and is exceeded by at most $100(1-p)\%$ of the sample.

A γ-**percentile** is (0.01γ)-quantile.

First, second, and third **quartiles** are the 25th, 50th, and 75th percentiles. They split a population or a sample into four equal parts.

A **median** is at the same time a 0.5-quantile, 50th percentile, and 2nd quartile.

Shape of a distribution (comparing mean and median)

Symmetric distribution	$\Rightarrow$	$M = \mu$
Right-skewed distribution	$\Rightarrow$	$M < \mu$
Left-skewed distribution	$\Rightarrow$	$M > \mu$

IQR and outliers.

DEFINITION 8.10

An **interquartile range** is defined as the difference between the first and the third quartiles,

$$IQR = Q_3 - Q_1.$$

It measures variability of data. Not much affected by outliers, it is often used to detect them. IQR is estimated by the *sample interquartile range*

$$\widehat{IQR} = \hat{Q}_3 - \hat{Q}_1.$$

Any samples that are less than $Q_1 - 1.5(IQR)$ or more than $Q_3 + 1.5(IQR)$ can be treated as potential outliers.

Standard error of any estimator is its std deviation.

$$\begin{array}{lcl} \sigma(\hat{\theta}) & = & \text{standard error of estimator } \hat{\theta} \text{ of parameter } \theta \\ s(\hat{\theta}) & = & \text{estimated standard error } = \hat{\sigma}(\hat{\theta}) \end{array}$$

Parameter Estimation

Method of moments Estimation: Equate the population and sample moments to estimate the parameters (number of parameters = number of moments)

Population moment and Population Central Moment:

$$\mu_i = E(X^i)$$
$$\mu_i' = E(X - \mu_1)^i$$

Sample moment and Sample Central Moment:

$$m_i = \frac{\sum_n X^i}{n}$$
$$m_i' = \frac{\sum_n (X - m_1)^i}{n}$$

Method of Max Likelihood: Find the value for parameter that maximizes likelihood or log-likelihood by equating its derivative (w.r.t to each parameter) to 0

$$\frac{\partial}{\partial\theta}\big(L(X_1, X_2, \dots X_n)\big) = \frac{\partial}{\partial\theta}\big((P(X_1)P(X_2)\dots P(X_n))\big) = 0$$

or

$$\frac{\partial}{\partial\theta}(\ln L(X_1, X_2, \dots X_n)) = \frac{\partial}{\partial\theta}(\ln P(X_1) + \ln P(X_2) + \cdots + \ln P(X_n)) = 0$$

Solve the family equations that result (number of equations will be same as number of parameters)

Confidence Intervals:

Confidence interval, Normal distribution

If parameter θ has an unbiased, Normally distributed estimator $\hat{\theta}$, then

$$\hat{\theta} \pm z_{\alpha/2} \cdot \sigma(\hat{\theta}) = \left[\hat{\theta} - z_{\alpha/2} \cdot \sigma(\hat{\theta}),\ \hat{\theta} + z_{\alpha/2} \cdot \sigma(\hat{\theta})\right]$$

is a $(1-\alpha)100\%$ confidence interval for θ.

If the distribution of $\hat{\theta}$ is *approximately* Normal, we get an *approximately* $(1-\alpha)100\%$ confidence interval.

Confidence interval for the mean; σ is known

$$\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Confidence interval for the difference of means; known standard deviations

$$\bar{X} - \bar{Y} \pm z_{\alpha/2} \sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}$$

If we do not know the population std. dev. but we know the n is large, then σ(θ) can be replaced by s(θ)

Confidence interval for a population proportion

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Confidence interval for the difference of proportions

$$\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Common Z values:

$$z_{0.10} = 1.282, \qquad z_{0.05} = 1.645, \qquad z_{0.025} = 1.960$$
$$z_{0.01} = 2.326, \qquad z_{0.005} = 2.576.$$

Can also obtained from Z-table or from T-table with v = ∞

If n is small,

Confidence interval for the mean; σ is unknown

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

where $t_{\alpha/2}$ is a critical value from T-distribution with $n-1$ degrees of freedom

Confidence interval for the difference of means; equal, unknown standard deviations

$$\bar{X} - \bar{Y} \pm t_{\alpha/2}\, s_p \sqrt{\frac{1}{n} + \frac{1}{m}}$$

where s_p is the *pooled standard deviation*, a root of the pooled variance in (9.11)

and $t_{\alpha/2}$ is a critical value from T-distribution with $(n+m-2)$ degrees of freedom

Pooled Std. Deviation:

$$s_p^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2 + \sum_{i=1}^{m}(Y_i - \bar{Y})^2}{n+m-2} = \frac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2}. \quad (9.11)$$

Confidence interval for the difference of means; unequal, unknown standard deviations	$\bar{X} - \bar{Y} \pm t_{\alpha/2}\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}$ where $t_{\alpha/2}$ is a critical value from T-distribution with ν degrees of freedom given by formula (9.12)

Statterthwaite approximation of degrees of freedom (9.12) [Round to nearest integer]:

$$\nu = \frac{\left(\frac{s_X^2}{n} + \frac{s_Y^2}{m}\right)^2}{\frac{s_X^4}{n^2(n-1)} + \frac{s_Y^4}{m^2(m-1)}}$$

Hypothesis Testing (Z-tests):	**Hypothesis Testing (t-tests):**
Right Tailed Test (**H_A: θ > $θ_0$**): $\begin{cases} \text{reject } H_0 & \text{if} \quad Z \geq z_\alpha \\ \text{accept } H_0 & \text{if} \quad Z < z_\alpha \end{cases}$	For a **right-tail alternative**, $\begin{cases} \text{reject } H_0 & \text{if} \quad t \geq t_\alpha \\ \text{accept } H_0 & \text{if} \quad t < t_\alpha \end{cases}$
Left Tailed Test (**H_A: θ < $θ_0$**): $\begin{cases} \text{reject } H_0 & \text{if} \quad Z \leq -z_\alpha \\ \text{accept } H_0 & \text{if} \quad Z > -z_\alpha \end{cases}$	For a **left-tail alternative**, $\begin{cases} \text{reject } H_0 & \text{if} \quad t \leq -t_\alpha \\ \text{accept } H_0 & \text{if} \quad t > -t_\alpha \end{cases}$
Two Tailed Test (**H_A: θ ≠ $θ_0$**): $\begin{cases} \text{reject } H_0 & \text{if} \quad \lvert Z\rvert \geq z_{\alpha/2} \\ \text{accept } H_0 & \text{if} \quad \lvert Z\rvert < z_{\alpha/2} \end{cases}$	For a **two-sided alternative**, $\begin{cases} \text{reject } H_0 & \text{if} \quad \lvert t\rvert \geq t_{\alpha/2} \\ \text{accept } H_0 & \text{if} \quad \lvert t\rvert < t_{\alpha/2} \end{cases}$

Summary of t-tests

Hypothesis H_0	Conditions	Test statistic t	Degrees of freedom
$\mu = \mu_0$	Sample size n; unknown σ	$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$	$n - 1$
$\mu_X - \mu_Y = D$	Sample sizes n, m; unknown but equal standard deviations, $\sigma_X = \sigma_Y$	$t = \frac{\bar{X} - \bar{Y} - D}{s_p\sqrt{\frac{1}{n} + \frac{1}{m}}}$	$n + m - 2$
$\mu_X - \mu_Y = D$	Sample sizes n, m; unknown, unequal standard deviations, $\sigma_X \neq \sigma_Y$	$t = \frac{\bar{X} - \bar{Y} - D}{\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}}$	Satterthwaite approximation, formula (9.12)

Summary of Z - Tests

Null hypothesis H_0	Parameter, estimator $\theta, \hat{\theta}$	If H_0 is true: $\mathbf{E}(\hat{\theta})$	If H_0 is true: $\mathrm{Var}(\hat{\theta})$	Test statistic $Z = \frac{\hat{\theta} - \theta_0}{\sqrt{\mathrm{Var}(\hat{\theta})}}$
One-sample Z-tests for means and proportions, based on a sample of size n				
$\mu = \mu_0$	μ, $\bar{X}$	μ_0	$\frac{\sigma^2}{n}$	$\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$
$p = p_0$	p, $\hat{p}$	p_0	$\frac{p_0(1-p_0)}{n}$	$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
Two-sample Z-tests comparing means and proportions of two populations, based on independent samples of size n and m				
$\mu_X - \mu_Y = D$	$\mu_X - \mu_Y$, $\bar{X} - \bar{Y}$	D	$\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}$	$\frac{\bar{X} - \bar{Y} - D}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}}$
$p_1 - p_2 = D$	$p_1 - p_2$, $\hat{p}_1 - \hat{p}_2$	D	$\frac{p_1(1-p_1)}{n} + \frac{p_2(1-p_2)}{m}$	$\frac{\hat{p}_1 - \hat{p}_2 - D}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n} + \frac{\hat{p}_2(1-\hat{p}_2)}{m}}}$
$p_1 = p_2$	$p_1 - p_2$, $\hat{p}_1 - \hat{p}_2$	0	$p(1-p)\left(\frac{1}{n} + \frac{1}{m}\right)$, where $p = p_1 = p_2$	$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n} + \frac{1}{m}\right)}}$ where $\hat{p} = \frac{n\hat{p}_1 + m\hat{p}_2}{n+m}$

P Values (Reject H_0 if $P < 0.01$, Accept H_0 if $P > 0.1$, Not enough evidence otherwise):

P values for Z tests:

Hypothesis H_0	Alternative H_A	P-value	Computation
$\theta = \theta_0$	right-tail $\theta > \theta_0$	$\boldsymbol{P}\{Z \geq Z_{\text{obs}}\}$	$1 - \Phi(Z_{\text{obs}})$
	left-tail $\theta < \theta_0$	$\boldsymbol{P}\{Z \leq Z_{\text{obs}}\}$	$\Phi(Z_{\text{obs}})$
	two-sided $\theta \neq \theta_0$	$\boldsymbol{P}\{\lvert Z\rvert \geq \lvert Z_{\text{obs}}\rvert\}$	$2(1 - \Phi(\lvert Z_{\text{obs}}\rvert))$

P values for t tests:

Hypothesis H_0	Alternative H_A	P-value	Computation
$\theta = \theta_0$	right-tail $\theta > \theta_0$	$\boldsymbol{P}\{t \geq t_{\text{obs}}\}$	$1 - F_\nu(t_{\text{obs}})$
	left-tail $\theta < \theta_0$	$\boldsymbol{P}\{t \leq t_{\text{obs}}\}$	$F_\nu(t_{\text{obs}})$
	two-sided $\theta \neq \theta_0$	$\boldsymbol{P}\{\lvert t\rvert \geq \lvert t_{\text{obs}}\rvert\}$	$2(1 - F_\nu(\lvert t_{\text{obs}}\rvert))$

Confidence Intervals (variance):

Confidence interval for the variance	$$\left[\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right]$$
Confidence interval for the standard deviation	$$\left[\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}}\right]$$
Confidence interval for the ratio of variances	$$\left[\frac{s_X^2}{s_Y^2 F_{\alpha/2}(n-1,m-1)}, \frac{s_X^2 F_{\alpha/2}(m-1,n-1)}{s_Y^2}\right]$$

Hypothesis tests for variance (can also be used for Std Dev by conv question to variance):

Null Hypothesis	Alternative Hypothesis	Test statistic	Rejection region	P-value
$\sigma^2 = \sigma_0^2$	$\sigma^2 > \sigma_0^2$	$\frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2_{\text{obs}} \geq \chi^2_\alpha$	$\boldsymbol{P}\left\{\chi^2 \geq \chi^2_{\text{obs}}\right\}$
	$\sigma^2 < \sigma_0^2$		$\chi^2_{\text{obs}} \leq \chi^2_{1-\alpha}$	$\boldsymbol{P}\left\{\chi^2 \leq \chi^2_{\text{obs}}\right\}$
	$\sigma^2 \neq \sigma_0^2$		$\chi^2_{\text{obs}} \geq \chi^2_{\alpha/2}$ or $\chi^2_{\text{obs}} \leq \chi^2_{1-\alpha/2}$	$2\min\left(\boldsymbol{P}\left\{\chi^2 \geq \chi^2_{\text{obs}}\right\}, \boldsymbol{P}\left\{\chi^2 \leq \chi^2_{\text{obs}}\right\}\right)$

Testing ratio of Variances (can also be used for Std Dev by conv question to variance):

Null Hypothesis $H_0 : \frac{\sigma_X^2}{\sigma_Y^2} = \theta_0$		Test statistic $F_{\text{obs}} = \frac{s_X^2}{s_Y^2}/\theta_0$
Alternative Hypothesis	Rejection region	P-value Use F$(n-1, m-1)$ distribution
$\frac{\sigma_X^2}{\sigma_Y^2} > \theta_0$	$F_{\text{obs}} \geq F_\alpha(n-1, m-1)$	$\boldsymbol{P}\left\{F \geq F_{\text{obs}}\right\}$
$\frac{\sigma_X^2}{\sigma_Y^2} < \theta_0$	$F_{\text{obs}} \leq 1/F_\alpha(m-1, n-1)$	$\boldsymbol{P}\left\{F \leq F_{\text{obs}}\right\}$
$\frac{\sigma_X^2}{\sigma_Y^2} \neq \theta_0$	$F_{\text{obs}} \geq F_{\alpha/2}(n-1, m-1)$ or $F_{\text{obs}} \leq 1/F_{\alpha/2}(m-1, n-1)$	$2\min\left(\boldsymbol{P}\left\{F \geq F_{\text{obs}}\right\}, \boldsymbol{P}\left\{F \leq F_{\text{obs}}\right\}\right)$

Bayesian Statistics

Given a prior distribution $\pi(\theta)$ and a model for some observations $f(x|\theta) = f(x_1, x_2, x_3,\dots x_n|\theta)$ the posterior distributions $\pi(\theta|x)$ is given by

Posterior distribution

$$\boxed{\pi(\theta|\boldsymbol{x}) = \pi(\theta|\boldsymbol{X}=\boldsymbol{x}) = \frac{f(\boldsymbol{x}|\theta)\pi(\theta)}{m(\boldsymbol{x})}.}$$

Marginal distribution of data

$$m(\boldsymbol{x}) = \sum_{\theta} f(x|\theta)\pi(\theta)$$

for discrete prior distributions π

$$m(\boldsymbol{x}) = \int_{\theta} f(x|\theta)\pi(\theta)d\theta$$

for continuous prior distributions π

(for pmfs of pdfs)

Conjugate families for Bayesian statistics

Model $f(\boldsymbol{x}\|\theta)$	Prior $\pi(\theta)$	Posterior $\pi(\theta\|\boldsymbol{x})$
Poisson(θ)	Gamma(α, λ)	Gamma$(\alpha + n\bar{X}, \lambda + n)$
Binomial(k, θ)	Beta(α, β)	Beta$(\alpha + n\bar{X}, \beta + n(k - \bar{X}))$
Normal(θ, σ)	Normal(μ, τ)	Normal$\left(\frac{n\bar{X}/\sigma^2 + \mu/\tau^2}{n/\sigma^2 + 1/\tau^2}, \frac{1}{\sqrt{n/\sigma^2 + 1/\tau^2}}\right)$

Bayesian Estimate

$$\hat{\theta}_{\mathrm{B}} = \mathbf{E}\{\theta|\boldsymbol{X}=\boldsymbol{x}\} = \begin{cases} \sum_{\theta} \theta\pi(\theta|\boldsymbol{x}) \\ \int_{\theta} \theta\pi(\theta|\boldsymbol{x})d\theta \end{cases}$$

depending on discrete or continuous posterior

The variance gives posterior risk $\rho(\hat{\theta}) = \mathrm{Var}\{\theta|\boldsymbol{x}\}$

Bayesian Credible set

DEFINITION 10.5

Set C is a $(1-\alpha)100\%$ **credible set** for the parameter θ if the posterior probability for θ to belong to C equals $(1-\alpha)$. That is,

$$\boldsymbol{P}\{\theta \in C \mid \boldsymbol{X}=\boldsymbol{x}\} = \int_C \pi(\theta|\boldsymbol{x})d\theta = 1-\alpha.$$

If the posterior $\pi(\theta|x)$ is Normal (or can be approximated as Normal), This is given by.

$$\mu_x \pm z_{\alpha/2}\tau_x = [\mu_x - z_{\alpha/2}\tau_x, \mu_x + z_{\alpha/2}\tau_x]$$

Bayesian Inference

- Calculate Posterior distribution $\pi(\theta|x)$
- Identify H_0 and H_A
- If $P\{H_0\}$ is greater than $P\{H_A\}$ according to $\pi(\theta|x)$ then accept H_0. Else, reject H_0.

Table A5. Table of Student's T-distribution

t_α; critical values, such that $\boldsymbol{P}\{t > t_\alpha\} = \alpha$

ν	α, the right-tail probability									
(d.f.)	.10	.05	.025	.02	.01	.005	.0025	.001	.0005	.0001
1	3.078	6.314	12.706	15.89	31.82	63.66	127.3	318.3	636.6	3185
2	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60	70.71
3	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92	22.20
4	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610	13.04
5	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.894	6.869	9.676
6	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959	8.023
7	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408	7.064
8	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041	6.442
9	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781	6.009
10	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587	5.694
11	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437	5.453
12	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318	5.263
13	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221	5.111
14	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140	4.985
15	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073	4.880
16	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015	4.790
17	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965	4.715
18	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922	4.648
19	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883	4.590
20	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850	4.539
21	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819	4.492
22	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792	4.452
23	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768	4.416
24	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745	4.382
25	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725	4.352
26	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707	4.324
27	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.689	4.299
28	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674	4.276
29	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.660	4.254
30	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646	4.234
32	1.309	1.694	2.037	2.141	2.449	2.738	3.015	3.365	3.622	4.198
34	1.307	1.691	2.032	2.136	2.441	2.728	3.002	3.348	3.601	4.168
36	1.306	1.688	2.028	2.131	2.434	2.719	2.990	3.333	3.582	4.140
38	1.304	1.686	2.024	2.127	2.429	2.712	2.980	3.319	3.566	4.115
40	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551	4.094
45	1.301	1.679	2.014	2.115	2.412	2.690	2.952	3.281	3.520	4.049
50	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496	4.014
55	1.297	1.673	2.004	2.104	2.396	2.668	2.925	3.245	3.476	3.985
60	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460	3.962
70	1.294	1.667	1.994	2.093	2.381	2.648	2.899	3.211	3.435	3.926
80	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416	3.899
90	1.291	1.662	1.987	2.084	2.368	2.632	2.878	3.183	3.402	3.878
100	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390	3.861
200	1.286	1.653	1.972	2.067	2.345	2.601	2.838	3.131	3.340	3.789
∞	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.290	3.719

Binomial Distribution

$$F(x) = \boldsymbol{P}\{X \leq x\} = \sum_{k=0}^{x} \binom{n}{k} p^k (1-p)^{n-k}$$

n	x	p																		
		.050	.100	.150	.200	.250	.300	.350	.400	.450	.500	.550	.600	.650	.700	.750	.800	.850	.900	.950
20	1	.736	.392	.176	.069	.024	.008	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.925	.677	.405	.206	.091	.035	.012	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
y	3	.984	.867	.648	.411	.225	.107	.044	.016	.005	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
	4	.997	.957	.830	.630	.415	.238	.118	.051	.019	.006	.002	.000	.000	.000	.000	.000	.000	.000	.000
	5	1.0	.989	.933	.804	.617	.416	.245	.126	.055	.021	.006	.002	.000	.000	.000	.000	.000	.000	.000
	6	1.0	.998	.978	.913	.786	.608	.417	.250	.130	.058	.021	.006	.002	.000	.000	.000	.000	.000	.000
	7	1.0	1.0	.994	.968	.898	.772	.601	.416	.252	.132	.058	.021	.006	.001	.000	.000	.000	.000	.000
	8	1.0	1.0	.999	.990	.959	.887	.762	.596	.414	.252	.131	.057	.020	.005	.001	.000	.000	.000	.000
	9	1.0	1.0	1.0	.997	.986	.952	.878	.755	.591	.412	.249	.128	.053	.017	.004	.001	.000	.000	.000
	10	1.0	1.0	1.0	.999	.996	.983	.947	.872	.751	.588	.409	.245	.122	.048	.014	.003	.000	.000	.000
	11	1.0	1.0	1.0	1.0	.999	.995	.980	.943	.869	.748	.586	.404	.238	.113	.041	.010	.001	.000	.000
	12	1.0	1.0	1.0	1.0	1.0	.999	.994	.979	.942	.868	.748	.584	.399	.228	.102	.032	.006	.000	.000
	13	1.0	1.0	1.0	1.0	1.0	1.0	.998	.994	.979	.942	.870	.750	.583	.392	.214	.087	.022	.002	.000
	14	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.998	.994	.979	.945	.874	.755	.584	.383	.196	.067	.011	.000
	15	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.998	.994	.981	.949	.882	.762	.585	.370	.170	.043	.003
	16	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.995	.984	.956	.893	.775	.589	.352	.133	.016