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17.1-3 For the sequence of 'n' No. of operations the total powers of 2 will be present $\lfloor \log_2 n \rfloor + 1$ which are like $1, 2, 4, \ldots, 2^{\lfloor \log_2 n \rfloor}$

The On doing their geometric sum

$$\text{Geometric Sum} = \frac{(a^n - 1)a_1}{(a-1)}$$ where a = Common Ratio, a_1 = first term.

Then On summing up we will uptain.

$$\sum_{i=0}^{\lfloor \log_2 n \rfloor} 2^i = 2^{\lfloor \log_2 n \rfloor + 1} - 1 \leq 2^{\log_2 n + 1} = 2n$$

Also The remaining Operation's are small & "1" Complexity & for total 'n' Such operation's the Complexity $T(n) \leq 2n + n$

$\leq 3n = O(n)$

* It means that $O(1)$ is amortized Complexity per Operation's.

* To find the average, we divide by n, and the amortized Cost per Operation's is $O(1)$.

17-5:

a) Since the Heuristic is picked in advance, given any sequence of Requests given so far, we can simulate what ordering the Heuristic will call for then, we will pick our next Request to be whatever Element will of been in the last position of the list. Continuing until all the Request have been made, we have that the Cost of this Sequence is accesses is $= mn$.

b) The Cost of finding an Element is $Rank_L(x)$ and since it needs to be swapped with all the Elements before it, of which there are $Rank_L(x)-1$, the total cost is $2 \cdot rank_L(x) - 1$.

c) Regardless of the heuristic used, we first need to locate the Element, which is left where ever it was after the previous Step, so, needs $Rank_{L_{i-1}}(x)$ After that, By definition, there are (t_i) transpositions made so $c_i^* = Rank_{L_{i-1}}(x) + t_i$

D) if we perform a transposition of Elements y and z where y is towards the left. Then there are two Cases. The first is that the final Ordering of the list in L_i^* is with y in front of z in which case we have just increased the no. of inversions By 1, So the potential increases By 2. The Second is that in $L_2^* z$ occurs Before y, in which Case, we have just reduced the Number of inversions by One, reducing the potential by 2.

In Both Cases, whether or not there is an inversion between y (or) z and Other Elements has not Changed, Since the transpositions Only Changed the Relative Ordering of those two Elements.

e) By definition A and B are the Only two of the four Categories to place Elements that precede x in L_{i-1}, Since there are $|A|+|B|$ Elements preceding it, it's Rank in L_{i-1} is $|A|+|B|+1$. Similarly, the two Categories in which an Element Can be if it precedes x in L_{i-1}^* are A and C, So, in L_{i-1}^*, x has Rank $|A|+|C|+1$.

f) We have from part d that the potential increases By (2) if we transpose two Elements that are Being Swapped So that their Relatives Order in the final Ordering is being Screwed up, and decreased by two if they are being placed into their Correct Order in L_i^*

In particular, they increased it By at most 2, Since we are keeping track of the No. of inversions that may not be the direct Effect of the transpositions that Heuristic "H" made, we see which Ones the move to front Heuristic may of added.

In particular, Since the move to front heuristic Only Changed the Relative Order of x with Respect to the Other Elements, moving it in front of the Elements the preceded it in L_{i-1}, We Only Care about Sets A and B. For an Element in A, moving it to be behind x Created an inversion, Since that Element preceded x in L_i^*.

However, if the Element were in B, we are removing an inversion by placing x in front of it.

G.)

$$\hat{c}_i \le 2(|A|+|B|+1)-1+2(|A|-|B|+t_i^*)$$

$$= 4|A|+1+2t_i^*$$

$$\le 4(|A|+|C|+1+t_i^*)$$

$$= 4c_i^*$$

H) We Showed that the amortized Cost of each operation under the move to front Heuristic was at most four times the Cost of the Operation using any Other Heuristic. Since the amortized Cost added up over all these Operation is at most the total (real) Cost, So we have that the total Cost with movetofront is at most four times the total Cost with an arbitrary Other Heuristic.