HW4

WRITTEN PART

10.31.

The Model is gien by Poisson(θ)

So $$f(x/\theta) = \frac{e^{-\theta}\,\theta^{x}}{x!}$$

The Prior follows. Gama (5, 1).

So
$$\pi(\theta) = \frac{1}{\Gamma(5)}\,\theta^{4} e^{-\theta}.$$

Gama is Conjgate to Poison.

So

Posterior follow Gama $(5+n\bar{X},\ 1+n)$

$n=1$ $\bar{X}=4$. So Gama $(9, 2.)$.

Bayes estimator is.

$$\hat{\theta}_B = \frac{9}{2} = 4.5$$

Risk is.

$$\rho(\hat{\theta}_B) = \frac{9}{2^2} = 2.25.$$

10.36

(a) To find the prior distribution we have to satisfy

$$P(5.0 < \mu < 6.0) = 0.95$$

So Assume $\mu \pm Z_{0.025} \tau = [5.0, 6.0]$

$$\mu = \frac{5.0 + 6.0}{2} = 5.5 \qquad \tau = \frac{6.0 - 5.0}{2 \times Z_{0.025}} = \frac{1}{2 \times 1.96} = 0.255$$

So $\pi(\theta) = \text{Normal}(5.5, 0.255)$

(b)

We previsly calculated.

$$\bar{X} = 6.5 \quad n = 6 \quad \sigma = 2.2$$

And the observations follow Normal distribution.

Normal is conjugate to normal.

So Posterior follows Normal (μ_x, τ_x)

$$\mu_x = \frac{\frac{n\bar{X}}{\sigma^2} + \frac{\mu}{\tau^2}}{\frac{n}{\sigma^2} + \frac{1}{\tau^2}}$$

$$= \frac{\frac{6 \times 6.5}{2.2^2} + \frac{5.5}{0.255^2}}{\frac{6}{2.2^2} + \frac{1}{0.255^2}} = \underline{5.575}$$

$$\tau_x^2 = \frac{1}{\frac{n}{\sigma^2} + \frac{1}{\tau^2}} = \frac{1}{\frac{6}{2.2^2} + \frac{1}{0.255^2}} = 0.060$$

$$\boxed{\tau_x = 0.245}$$

So posterior is.

$$\text{Normal}\ (5.575,\ 0.245)$$

Bayes estimator.

$$\hat{\theta}_B = 5.575$$

Risk.

$$\rho(\hat{\theta}_B) = \tau_x^2 = 0.060$$

c)

95% HPD credible set

$$\mu_x \pm Z_{0.025}\,\tau_x$$

$$= 5.575 \pm (1.96)\,(0.245)$$

$$= 5.575 \pm 0.480$$

$$= [5.095 \quad 6.055]$$

10.37

Model for 10 heads in a row is

$$f(x/\theta) = \theta^{10}.$$

Prior distribution for θ is given by

$\pi(\theta)$	θ
0.99	0.5
0.01.	Uniform(0,1)

$$m(x) = (0.5)^{10}(0.99) + \left[\int_0^1 \theta^{10} d\theta\right](0.01)$$

$$= (0.5)^{10}(0.99) + \frac{1}{11}(0.01)$$

$$\pi(\theta = 0.5/x) = \frac{f(x/\theta=0.5)\,\pi(0.5)}{m(x)}$$

$$= \frac{(0.5)^{10}(0.99)}{(0.5)^{10}(0.99) + \frac{1}{11}(0.01)} = \underline{0.5154}$$