HOME WORK - 2

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4.4) Given Continuous random Number prob

Sol)

$$f(x) = \begin{cases} k - x/50 & \text{for } 0 < x < 10 \text{ years} \\ 0 & \text{for all other } x \end{cases}$$

Need to find ① k, ② failure in 5 years ③ Expected lifetime

a) ① => $\int f(x)dx = 1$

$$\int f(x)\,dx = \int_0^{10} (k - x/50)\,dx = kx - \frac{x^2}{2.5}\Big|_0^{10}$$

$\Rightarrow 10k - 1 = 1$

$\Rightarrow 10k = 2$

$k = 1/5 = 0.2$

$k = 0.2$

b) ② $P\{x < 5\} = \int_0^5 (0.2 - x/50)\,dx = 0.2x - \frac{x^2}{2.5}\Big|_0^5$

$= 1 - 0.25 = 0.75$

(c) (3) $E(x) = \int_0^{10} x(0.2 - x/50)\,dx = \frac{0.2x^2}{2} - \frac{x^3}{150}\Big|_0^{10}$

$= 10 - \frac{20}{3} = 3\frac{1}{3}$ (or) 3.33 years

4.7) Given the Expectation of Exponential.

Sol) Random Varible = 12 Seconds

When send to point at 10.00 But point 10.01

$= (60 \text{ Seconds})$

$E(x) = 1/\lambda = 12$

$\lambda = 1/12$

$P(X \geq a) = e^{-\lambda a}$

$P(x < 60)$

$= 1 - P(X \geq 60)$

$= 1 - e^{-60/12}$

$= 1 - e^{-5}$

$= 1 - 1/e^5 = 1 - 1/148.4$

$= 1 - 10.0$

$= 1 - (0.024)$

$= 0.9976$

4.30) let X = Connection time of the line 2

Sol) Then $X \sim \text{Gamma}(3, 2)$

$\Rightarrow f(x) = \frac{2^3 x^{3-1} e^{-2x}}{\Gamma 3} = 4x^2 e^{-2x}, \quad x \geq 0$

let y = Connection time of the line II

Then $y \sim$ uniform $(30, 50)$

$\Rightarrow f(y) = \frac{1}{50-30} = \frac{1}{20}, \; y \in [30, 50]$

P(line ① take more than 30 seconds)

$\Rightarrow P(x > 30/60) = P(x > 0.5)$

$$= \int_{0.5}^{\infty} 4x^2 e^{-2x} \, dx = 0.91978$$

P(line ② take more than 30 seconds)

$$= P(y > 30) = \int_{30}^{50} 1/20 \, dy = 1$$

Required probability

$= P$(line ① connected) $P(x > 0.5) +$
P(line ② is also connected) $P(x > 30$

$= 0.8 \times 0.91978 + 0.2 \times 1$

$= 0.935$

4.31) Given

Sol) The population mean time to install one file $\boxed{\mu = 15 sec}$

The population Variance $\sigma^2 = 11 sec^2$

The Sample Size $n = 68$ files

a) The probability that the whole package is upgraded in less than 12 min $= \frac{\cancel{12}^{3} \times 60}{\cancel{68}_{17}}$

$= 10.59$ sec

By using Z testing: $Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

$= \frac{10.59 - 15}{\sqrt{11/68}} = \frac{-4.41}{0.4022}$

$= p[Z \leq -2.20] = 0.0143$

b) Given Sample Size $= N$

* 10 mins to upload whole package $= \frac{10 \times 60}{N}$

$= \frac{600}{N}$

* The probability that the whole package is upgraded in less than 10 min's: 0.95

* The value of Z. Score at $0.95 = 1.645$

(By using Standard Normal deviation)

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\Rightarrow 1.645 = N \Big/ \sqrt{11/N}$$

$$\Rightarrow \frac{5.456}{\sqrt{N}} = \frac{600}{N} - 15$$

* multiplying by $N\sqrt{N}$ both sides

$$5.456\, N = 600\sqrt{N} - 15N\sqrt{N}$$

$$= 15(40\sqrt{N} - N\sqrt{N})$$

$$5.456\, N = -15\sqrt{N}\,(N - 40)$$

* Squaring on Both Sides: we get

$$N = 37.765$$

$$N \simeq 37$$

* Hence the required Sample size = 37.

① Entropy:

$P(x,y)$	x:	2	4	6
y1		0.		
Sensor 1		0.22	0.17	0.03
Sensor 2		0.15	0.1	0.11
Sensor 3		0.06	0.16	0

y/x	2	4	6	p(y)
2	0.22	0.17	0.03	0.42
4	0.15	0.1	0.11	0.36
6	0.06	0.16	0	0.22
p(x)	0.43	0.42	0.14	1

a) H(x) = p(x)

x	2	4	6
p(x)	0.43	0.43	0.16

* Suming of all twos, tours, and six's

b) H(y) = p(y)

y	2	4	6
p(y)	0.42	0.36	0.22

c) P(x || y) For x = 2

x	2	4	6
p(y)	0.22	0.15	0.06

for x = 4

x	2	4	6
p(y)	0.17	0.1	0.16

for x = 6

x	2	4	6
p(y)	0.03	0.11	0

$D(y \| x)$

for $y = 2$

x	2	4	6
$p(y)$	0.22	0.17	0.03

for $y = 4$

x	2	4	6
$p(y)$	0.15	0.1	0.11

for $y = 6$

x	2	4	6
$p(y)$	0.08	0.16	0

f) $H(x, y) = H(x) + H(y)$

$\Rightarrow H(x) + H(y)$

$\Rightarrow x$	2	4	6
	0.43	0.43	0.16
y	0.42	0.36	0.22

$H(x) + H(y) = 0.85 \quad 0.79 \quad 0.38$

$= 0.85 + 0.79 + 0.38 = 2.8265$

9. $H(y) - H(y/x)$

$H(y) = p(y)$

	2	4	6
$p(y)$	0.43	0.43	0.16
$H(y/x)$	0.43	0.43	0.14

$H(y) - H(y/x) = 0.43 + 0.16 + 0.14 - 0.42$

$= 0.15$

(2) The entropy p.d = (5.8) bit's

Sol)

one Hartley = $\log_2 10$ bit = 3.322 bit's

One Hartely = 2.303 nat's

Given

Entropy = 5.8 bit's

This information is enough to Calcul hartly

$$\text{Entropy} = \frac{5.8}{3.322} \text{ Hartley's}$$

$$= 1.746 \text{ Hartley's}$$

In Nat's

Entropy = 1.746 × 2.303

Entropy = 4.021 nat-s

(3)

(3.A) Given (QAM) => QAM16 Can encode 16

Sol) * The Original alphabet is 8 bits long with values between 0 and 255 (or byte

3a) The maximum Expected code length of QAM16 Symboles is 4.

This is because there are (16) possible values that can be encoded by a QAM16 Symbol So each Symbole can be send as con-
-taing 4 bits of the given data. The Original is given is 8 bits long..

* There form the above Statement the Expected code length is (4 QAM16) Symbole.

3.b)

* The maximum Expected Code length of QAM16 = 4

* and given the proablity distribution given is

$n = 255, \; p = 0.19$

* We also know that probability of the geometric distribution is dirived from the Binomial distribution, and given from (3.a) we know that maximum Expected code length of 4 Symbole.