

Crack the Code: An Interactive Journey Through Optimization

Rohit Kannan

Assistant Professor

Grado Department of Industrial and Systems Engineering, Virginia Tech

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Let's do a quick round of introductions!

I'll start...

- Hometown: Chennai, India.
- Hobbies: Board Games and Puzzles.
- Education/Training:



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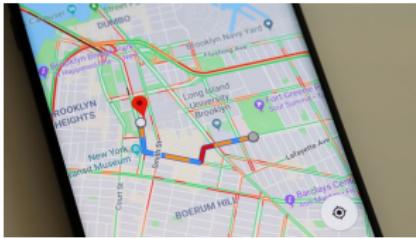
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Who are you? What are your interests?

Where Do You See Math Here?



ChatGPT

What decisions are optimized in each application?

What is an Optimization Problem?

Three Key Ingredients

- ① **Decisions:** What choices can we make?
- ② **Constraints:** What limits or rules must we follow?
- ③ **Goal:** What are we trying to maximize or minimize?

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Pizza Topping Example

You have a budget of \$10, can choose up to 4 toppings, and want to get the *most delicious* pizza possible.

Example : ten toppings 1 2 ... 10 costs: c_1, c_2, \dots, c_{10}

Decisions x_1, x_2, \dots, x_{10}

(choose the topping? YES/NO)

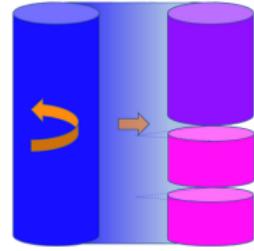
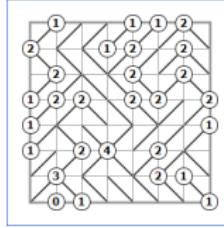
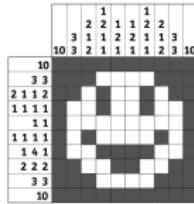
Constraints : $x_1 + x_2 + \dots + x_{10} \leq 4$
 $c_1 x_1 + c_2 x_2 + \dots + c_{10} x_{10} \leq 10$

Objective :
max. deliciousness
of pizza ?

Real-World Applications and Careers Using Optimization

- **Amazon/UPS:** Plan millions of delivery routes every day.
- **Costco/Walmart:** Optimize store layouts & shelf restocking.
- **Lyft/Uber:** Match drivers to riders in real time.
- **Google Maps:** Re-route you around traffic jams.
- **Airlines:** Schedule crews and aircraft efficiently.
- **Netflix/TikTok/YouTube:** Personalize recommendations to keep you engaged.
- **Tesla, IKEA:** Reduce waste and optimize factory workflows.
- **Airbnb/Expedia:** Set prices dynamically based on demand.
- **Finance:** Build investment portfolios balancing risk vs return.
- **Power Grid Operators:** Which generators to run each hour.
- **Chemical Plants:** Mix and heat chemicals efficiently, safely, and with minimal waste.
- **ChatGPT/Gemini:** Adjust billions of settings to generate better answers and fewer mistakes.

Line-Up of Hands-On Activities



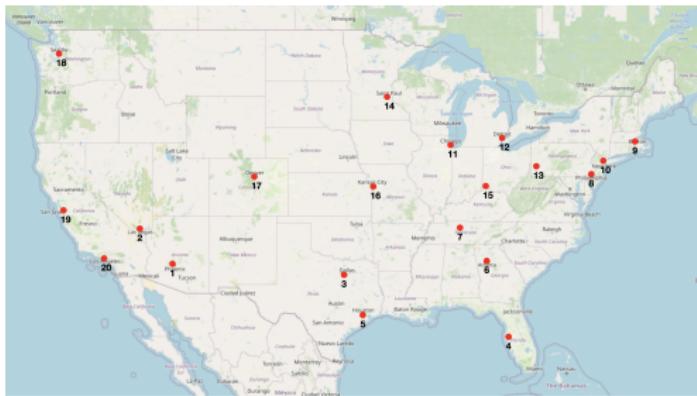
Today We Will...

- ① Help Taylor Swift plan her next tour (the smart way).
- ② Solve a puzzle where logic meets art.
- ③ Play a game called Slant — it's trickier than it looks!
- ④ Explore the science of cutting things without wasting material.

Outline

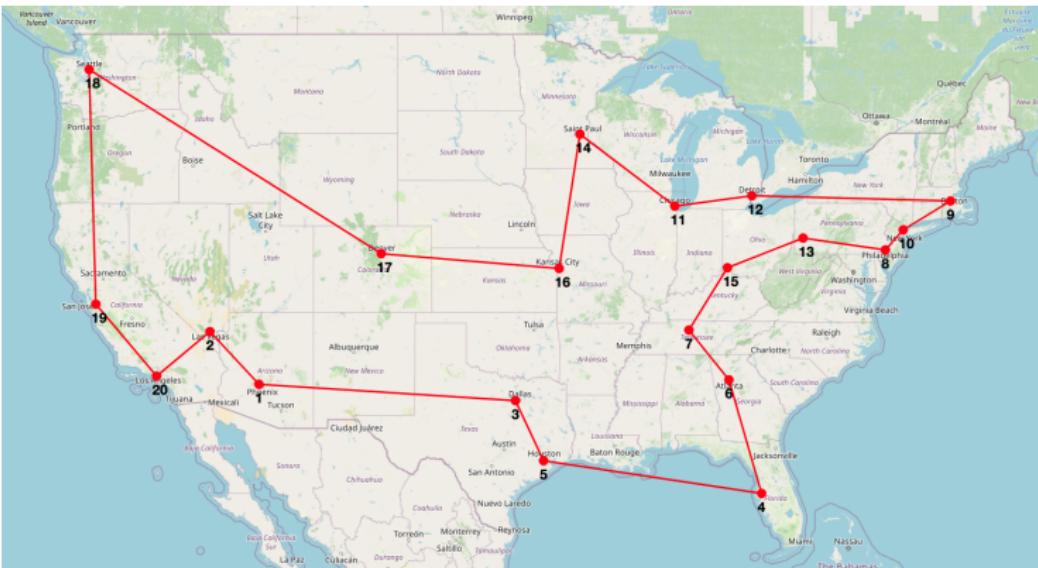
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Help Taylor Swift Optimize Her Next Tour!



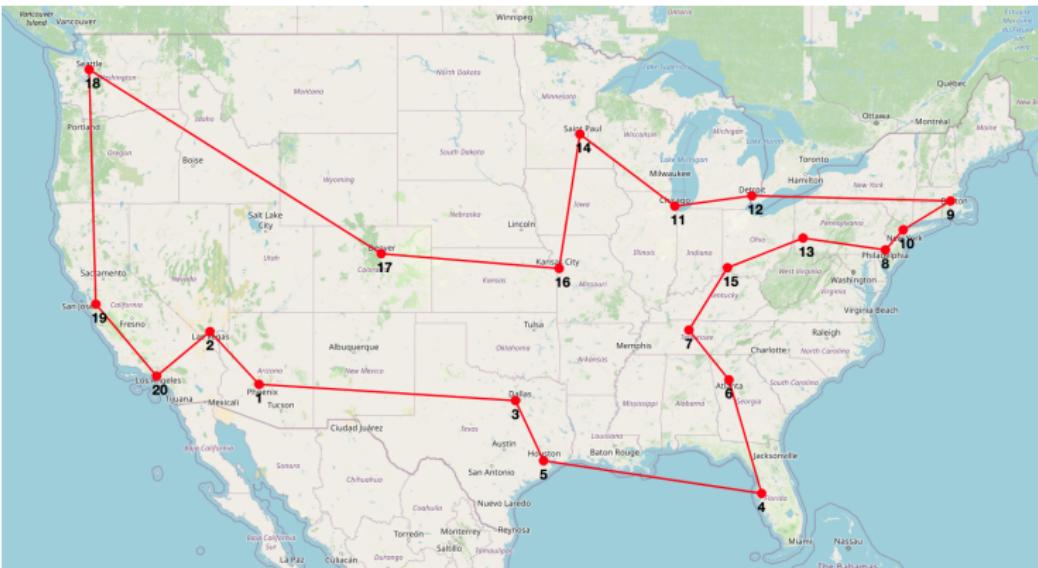
- **Scenario:** Taylor needs your help planning her next tour!
- She wants to start in Phoenix (#1), visit each venue exactly once, and return to Phoenix.
- Work in teams of two to help Taylor minimize the total distance traveled.
- No sub-tours allowed!

Help Taylor Swift Optimize Her Next Tour!



- **Original Tour Distance:** 11,016 miles
(based on the order of cities in the original tour)
- **Optimized Tour Distance:** 8,221 miles (**25% shorter!**)

Help Taylor Swift Optimize Her Next Tour!



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- **Optimized Tour Distance:** 8,221 miles (**25% shorter!**)
- **But what's wrong with this solution?**

Traveling Salesperson Problem



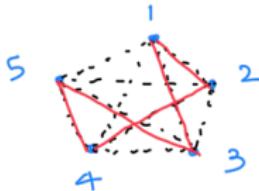
Optimal tour through 50 landmarks in the U.S., computed using data from Google Maps.

<https://www.math.uwaterloo.ca/tsp/usa50/index.html>

Check out <https://www.math.uwaterloo.ca/tsp/> for a history of the TSP, cool applications, and an interactive TSP solver

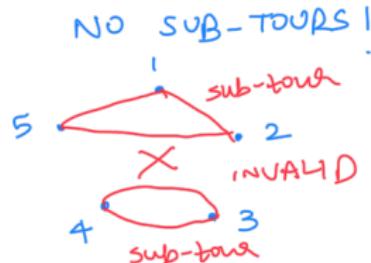
Real-World Applications: Amazon, UPS, Uber, Google Maps, ...

Example with five locations



Decisions : which arcs to traverse

(example tour highlighted)

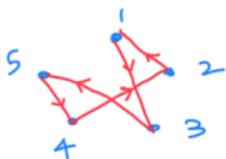


$$x_{ij} = 1 \text{ if we traverse arc } (i,j) \\ = 0 \text{ otherwise}$$

Traveling Salesperson Problem

Optimization Model

- *Variables:* $x_{ij} = 1$ if we travel directly from location $i \rightarrow j$.
- *Visit Each Location Once:* Each city must be entered and exited exactly once.
- *No Sub-tours:* Prevent smaller loops that don't include all cities.
- *Objective:* Minimize total distance $\sum_{i,j} d_{ij}x_{ij}$.



visit each location once : e.g. $x_{21} + x_{31} + x_{41} + x_{51} = 1$
 $x_{12} + x_{13} + x_{14} + x_{15} = 1$

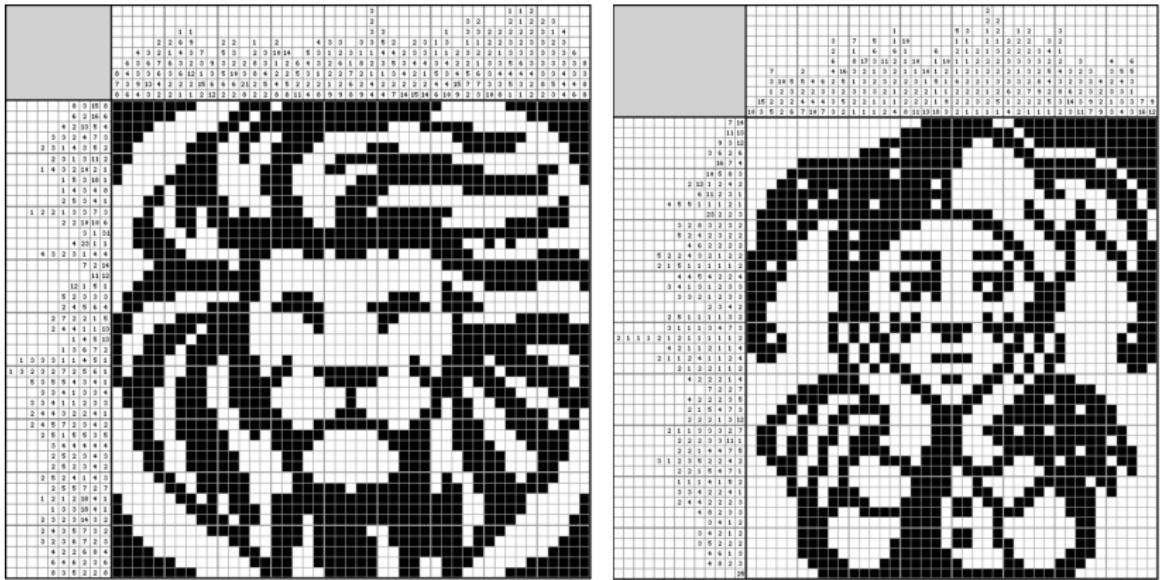
No sub-tours : exponentially many constraints in the number of locations !

Research Question: How can we design algorithms that still work when there are thousands of locations?

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Nonogram: Logic Meets Art

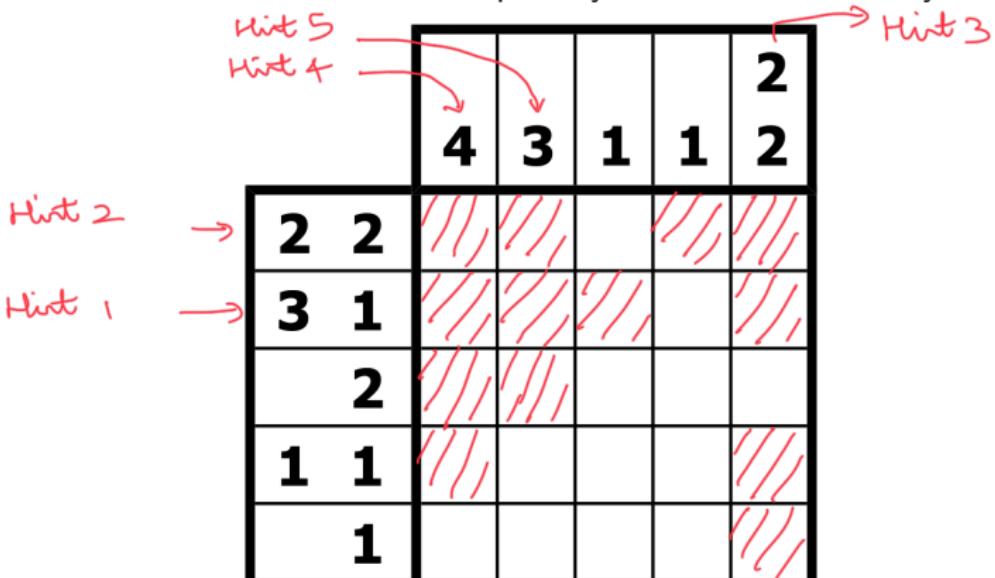


- Fill squares to satisfy row/column clues.
- Hidden pixel picture appears!
- Interested? Try solving more at:
 - <https://www.nonograms.org/>
 - <https://www.puzzle-nonograms.com/>

Let's Solve a Simple Nonogram Together

You have a grid of squares. Your goal is to shade the correct squares.

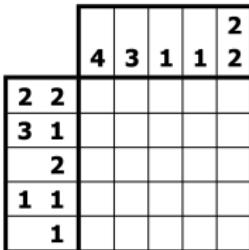
- Numbers next to each row and column tell you how many groups of shaded squares are in that line and how long each group is. For example, "3 1" means a group of 3 shaded squares, then a group of 1, with at least one blank square in between.
- Use logic to figure out which squares to shade or leave blank.
- Use a small X or dot to mark squares you are sure should stay blank.



Can You Solve This Larger Nonogram?

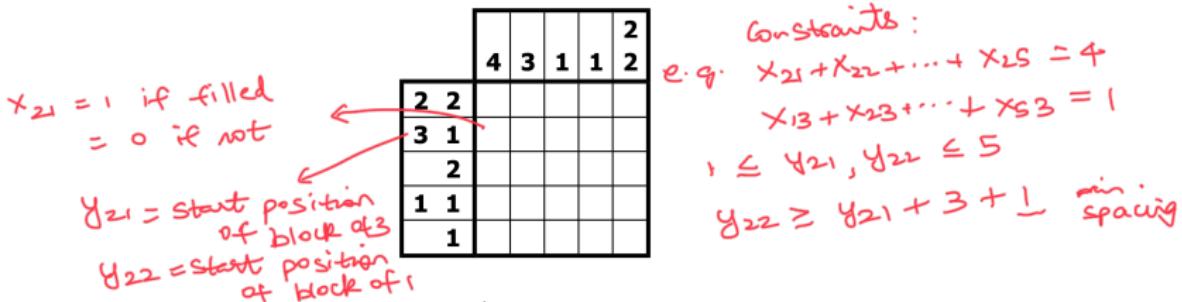
	4	1	1	1	3	3	5	3	9	8	6	Hint1	Hint2	Hint3
Hint4	6	x	x	x	x	/ /	/ /	/ /	/ /	/ /	/ /			
2	1	3	x	/ /	/ /	x	x	/ /	x	/ /	/ /			
Hint10	1	3	x	x	x	x	x	/ /	x	/ /	/ /			
2	3	/ /	/ /	x	x	x	x	x	/ /	/ /	/ /			
Hint9	1	3	/ /	x	x	x	x	x	x	/ /	/ /			
Hint6	1	5	/ /	x	x	x	x	/ /	/ /	/ /	/ /			
1	4	/ /	x	x	x	x	/ /	/ /	/ /	/ /	/ /	x		
Hint5	7	x	x	/ /	/ /	/ /	/ /	/ /	/ /	/ /	/ /	x		
3	1	x	x	x	/ /	/ /	/ /	/ /	x	/ /	x	x		
3	x	x	x	/ /	/ /	/ /	/ /	x	x	x	x			

How Can We Solve Nonograms Using Optimization?



- Fill squares to satisfy row/column clues.

How Can We Solve Nonograms Using Optimization?



- Fill squares to satisfy row/column clues.

Optimization Model

- Variables:* $x_{ij} = 1$ if square (i, j) is filled; include additional variables to track where each block starts in a row or column.
- Constraints:* Ensure at least one blank square between consecutive blocks; satisfy the given clues.
- No objective — want *any* solution that satisfies all constraints.

How Can We Solve Nonograms Using Optimization?

					2
4	3	1	1	2	
2	2				
3	1				
2					
1	1				
1					

- Fill squares to satisfy row/column clues.

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- Variables:* $x_{ij} = 1$ if square (i, j) is filled; include additional variables to track where each block starts in a row or column.
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Related Real-World Applications: circuit board design, classroom scheduling, image segmentation.

Outline

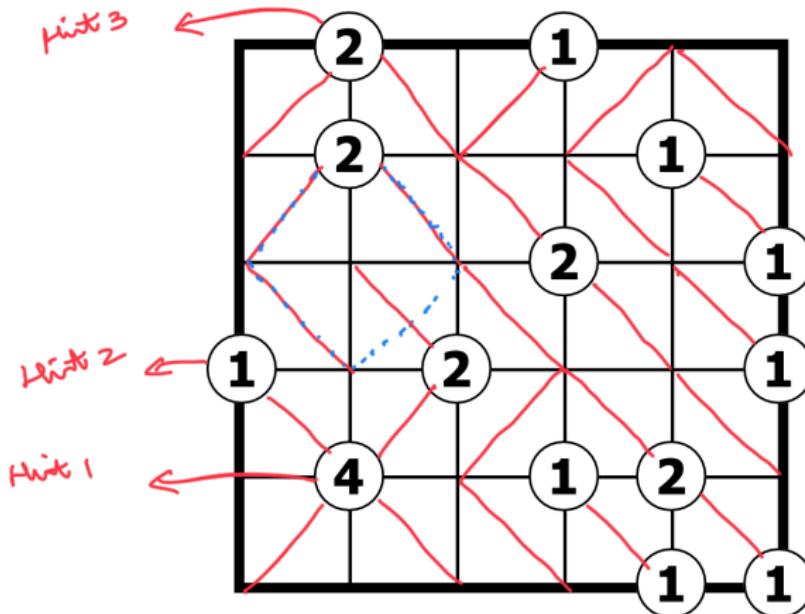
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Let's Play a Game of Slant

Place a diagonal line (either / or \) in every cell of the grid

- Numbers at the corners show how many lines meet at that point (from the surrounding one, two, or four cells).
- The lines must not form a closed loop anywhere in the grid.

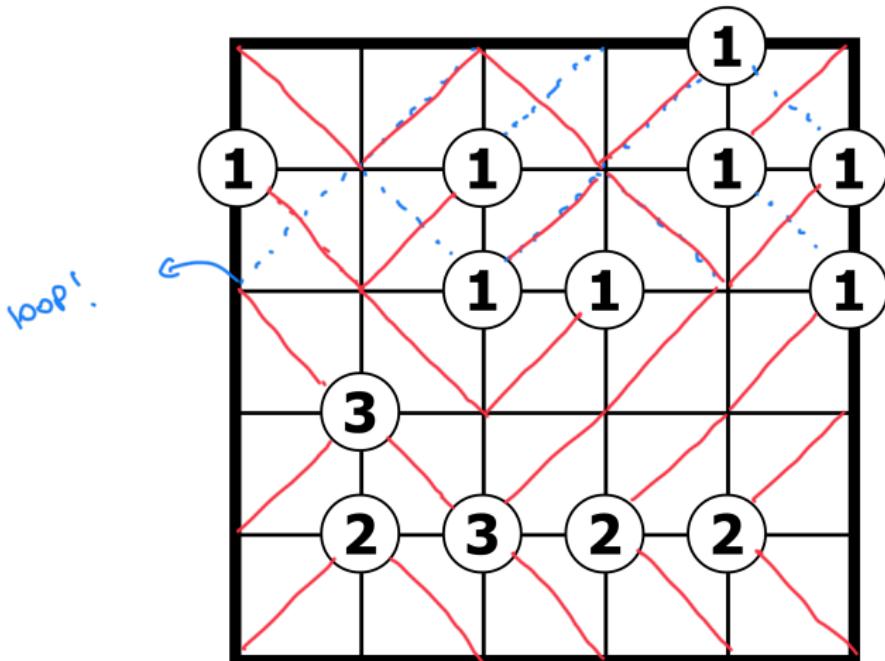
All you need is logic and a pencil!



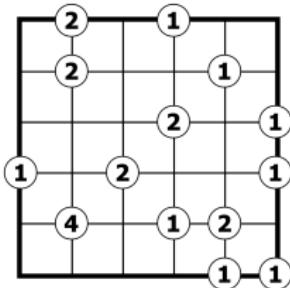
Can You Solve This Harder Puzzle?

Place a diagonal line (either / or \) in every cell of the grid

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- The lines must not form a closed loop anywhere in the grid.



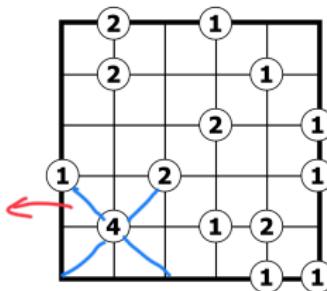
How Can We Solve Slant Using Optimization?



Curious? Try solving more at: <https://www.puzzle-slant.com/>

How Can We Solve Slant Using Optimization?

$$\begin{aligned}x_{41} &= 1 \text{ if } / \\&= 0 \text{ if } \backslash \\y_{41} &= 0 \text{ if } / \\&= 1 \text{ if } \backslash\end{aligned}$$



$$\text{Constraints: } x_{41} + y_{41} = 1$$

To satisfy ④:

$$y_{41} + x_{42} + x_{51} + y_{52} = 4$$

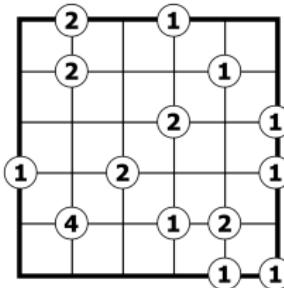
How to formulate
no loops?

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Optimization Model

- *Variables:* $x_{ij} = 1$ if square (i,j) contains a $/$; $y_{ij} = 1$ if it contains a \backslash .
- *Constraints:* $x_{ij} + y_{ij} = 1$; satisfy the numbered clues at the corners; avoid forming any loops.
- No objective — want any solution that satisfies all constraints.

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Optimization Model

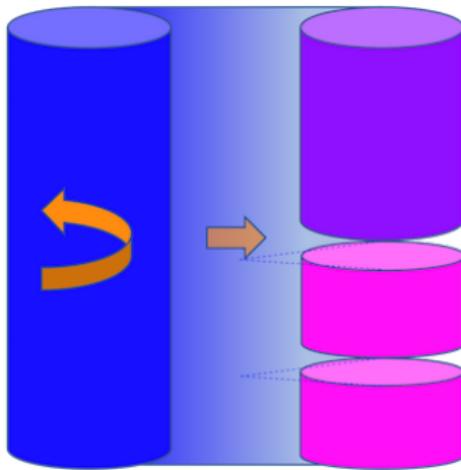
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Real-World Applications: designing loop-free telecommunication networks, routing wires on chips to avoid short circuits.

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Cutting Stock: The Science of Not Wasting Material



- Start with a large roll or sheet.
- Cut it into smaller sizes people ask for.
- Try to waste as little material as possible.
- **Real-World Applications:** Nike (cutting fabric), steel mills, and furniture makers.

Cutting Stock Problem: Craft Sticks Challenge!



- You have craft skill sticks of fixed length (about 11.3 cm).
- Your task: Use a pencil and ruler to mark how you would cut each stick to fulfill this customer order:
 - 6 pieces of 3 cm
 - 7 pieces of 4 cm
 - 5 pieces of 5 cm
- **Goal:** Use the fewest number of full sticks possible.
- Work in teams of two to plan your cuts and mark the sticks.

Cutting Stock Problem: Craft Sticks Challenge!

Solution using 7 sticks

- Stick 1: $4 \text{ cm} + 4 \text{ cm} + 3 \text{ cm} \implies 11 \text{ cm used, } 0.3 \text{ cm leftover}$
- Stick 2: $4 \text{ cm} + 4 \text{ cm} + 3 \text{ cm} \implies 11 \text{ cm used, } 0.3 \text{ cm leftover}$
- Stick 3: $4 \text{ cm} + 4 \text{ cm} + 3 \text{ cm} \implies 11 \text{ cm used, } 0.3 \text{ cm leftover}$
- Stick 4: $5 \text{ cm} + 3 \text{ cm} + 3 \text{ cm} \implies 11 \text{ cm used, } 0.3 \text{ cm leftover}$
- Stick 5: $5 \text{ cm} + 5 \text{ cm} \implies 10 \text{ cm used, } 1.3 \text{ cm leftover}$
- Stick 6: $5 \text{ cm} + 4 \text{ cm} \implies 9 \text{ cm used, } 2.3 \text{ cm leftover}$
- Stick 7: $5 \text{ cm} + 3 \text{ cm} \implies 8 \text{ cm used, } 3.3 \text{ cm leftover}$

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Can you prove that we need at least 7 sticks to fulfill the order?

Length of material required to fulfill orders : $6 \times 3 \text{ cm} + 7 \times 4 \text{ cm} + 5 \times 5 \text{ cm}$

$$\Rightarrow \text{Min. # sticks required} = \lceil \frac{71}{11.3} \rceil = 7 \quad = 71 \text{ cm}$$

Cutting Stock: The Science of Not Wasting Material

Eg. Patterns

$$11 = 4+4+3$$

$$11 = 5+3+3$$

$$11 = 3+3+3 + \text{waste}$$

$$11 = 5+5 + \text{waste}$$

$$11 = 4+4 + \text{waste}$$

$$11 = 5+4 + \text{waste}$$

$$11 = 5+3 + \text{waste}$$

Variables

y_1

y_2

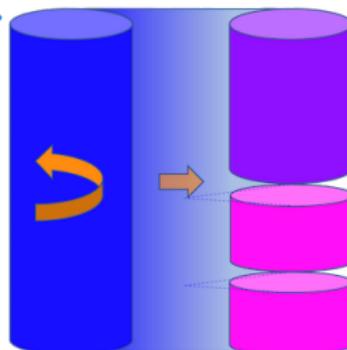
y_3

y_4

y_5

y_6

y_7



In example solution

$$y_1 = 3, y_2 = 1, y_3 = 0, \\ y_4 = 1, y_5 = 0, y_6 = 1, y_7 = 1$$

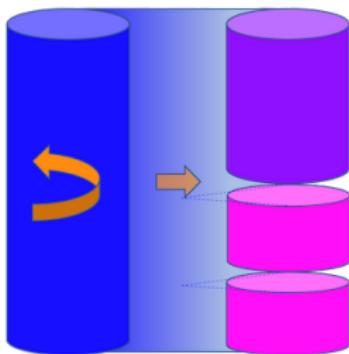
Objective: min. $\sum y_i$

If we use pattern 1,
we satisfy demand of
two 4's and one 3

Optimization Model

- *Patterns j*: different ways to cut up a sheet.
- *Variables y_j* : how many sheets we cut using pattern j .
- *Demand rule*: make enough of each piece to fill all the orders.
- *Goal*: use as few big sheets as possible.

Cutting Stock: The Science of Not Wasting Material



Optimization Model

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There can be billions of possible cutting patterns in practice, so we need smart algorithms to find the best solution fast!

Key Take-Aways

- The same recipe applies to every problem:
Optimization = Decisions + Constraints + Objective.
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 - Computer Science.
 - Mathematics.
 - Other engineering fields.

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 - Other engineering fields.
- Careers in Amazon, Apple, Capital One, Disney, Doctors Without Borders, ExxonMobil, Google, Microsoft, NASA, Procter and Gamble, Southwest, Uber, World Bank/IMF, and many more organizations!

Questions?

Thanks for playing — and optimizing!

