CSL003P1M: Probability and Statistics
Semester-1: 2024-2025
Instructor: Nitin Kumar
Last date of submission: **22 September 2023**

भारतीय प्रौद्योगिकी संस्थान जम्मू
INDIAN INSTITUTE OF TECHNOLOGY JAMMU
विद्याधनं सर्वधन प्रधानम्

Assignment-2

Useful Data: $\Phi(1/3) = 0.6293, \Phi(5/6) = 0.7967, \Phi(1) = 0.8413, \Phi(4/3) = 0.918$

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1. The m.g.f. of a random variable X is given by $M_X(t) = \frac{1}{2}e^{-5t} + \frac{1}{6}e^{4t} + \frac{1}{8}e^{5t} + \frac{5}{24}e^{25t}$. Find the distribution function of the random variable.

2. Let X be a random variable with $E(X) = 3$ and $E(X^2) = 13$, determine a lower bound for $P(-2 < X < 8)$.

3. Let X be a random variable with p.m.f. $P(X = x) = \begin{cases} 1/8, & x = -1,\ 1 \\ 6/8, & x = 0 \\ 0, & \text{otherwise} \end{cases}$. Using the p.m.f., show that the bound for Chebychev's inequality cannot be improved.

4. A machine contains two belts of different lengths. These have times to failure which are exponentially distributed, with means α and 2α. The machine will stop if either belt fails. The failures of the belts are assumed to be independent. What is the probability that the system performs after time α from the start?

5. Let X be a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$. Compute (a) $P(X > 5)$, (b) $P(4 < X < 16)$, (c) $P(X < 8)$.

6. Let $X \sim N(\mu, \sigma^2)$. If $P(X \le 0) = 0.5$ and $P(-1.96 \le X \le 1.96) = 0.95$, find μ and σ^2.

7. It is assumed that the lifetime of computer chips produced by a certain semiconductor manufacturer are normally distributed with parameters $\mu = 1.4 \times 10^6$ and $\sigma = 3 \times 10^5$ hours. What is the approximate probability that a batch of 10 chips will contain at least 2 chips whose lifetime are less than 1.8×10^6 hours?

8. Let $X \sim P(\lambda)$, then (i) Find $E((2 + X)^{-1})$; (ii) Find the p.m.f. of $Y = X^2 - 5$.

9. Consider the discrete random variable X with the probability mass function $P(X = -2) = \frac{1}{5}$, $P(X = -1) = \frac{1}{6}$, $P(X = 0) = \frac{1}{5}$, $P(X = 1) = \frac{1}{15}$, $P(X = 2) = \frac{1}{3}$, $P(X = 3) = \frac{1}{30}$. Find the probability mass function of $Y = X^2$.

10. Find the p.m.f. of $Y = \dfrac{X}{X+1}$ where the p.m.f. of the random variable X is given by
$$P(X = x) = \begin{cases} \frac{1}{3}\left(\frac{2}{3}\right)^x, & x = 0, 1, 2, \ldots \\ 0, & \text{otherwise} \end{cases}$$

11. Let $X \sim U(0, 1)$. Find the distribution of the following functions of X:
(a) $Y = \sqrt{X}$; (b) $Y = X^2$; (c) $Y = 2X + 3$; (d) $Y = -\lambda \log X; \lambda > 0$.

12. Let $X \sim U(0, \theta),\ \theta > 0$. Find the distribution of $Y = \min(X, \theta/2)$.

13. The probability density function of X is given by $f_X(x) = \begin{cases} \frac{1}{2}, & -\frac{1}{2} < x < \frac{3}{2} \\ 0, & \text{otherwise} \end{cases}$. Find the distribution of $Y = X^2$.

14. Let $X \sim U(0,1)$. Find the distribution function of $Y = \min(X, 1-X)$ and the p.d.f. of $Z = \frac{1-Y}{Y}$.

15. Let $X \sim N(\mu, \sigma^2)$. Find the distribution of $Y = 2X - 6$.

Following questions are just for the practice purpose.

1. Find the moment generating function (m.g.f.) of the following distributions (a) $Bin(n,p)$, (b) $NB(r,p)$, (c) $P(\lambda)$, (d) $G(\alpha, \beta)$ and (e) $N(\mu, \sigma^2)$. Find mean and variance from the m.g.f.s.

2. Let $X \sim B(n,p)$ and $Y \sim NB(r,p)$; $0 < p < 1$ and $r \in \{1, 2, \ldots, n\}$. Prove that $P(X \geq r) = P(Y \leq n - r)$.

3. Let X be a continuous random variable on (a,b) with p.d.f f and c.d.f. F. Find the p.d.f. of $Z = -\log(F(X))$.

4. Let $X \sim B(n,p)$. Find the probability mass function of $Y = n - X$.

*** All the best ***