CSL003P1M: Probability and Statistics
Semester-1: 2024-2025
Instructor: Nitin Kumar
Last date of submission: **No need to submit**

भारतीय प्रौद्योगिकी संस्थान जम्मू
INDIAN INSTITUTE OF TECHNOLOGY JAMMU
विद्याधनं सर्वधन प्रधानम्

Practice Set-3

---

1. Let $X \sim Poisson(1)$ and $Y \sim Poisson(2)$ be independent random variables. Find the conditional distribution of X given $X+Y=t$, $t \in \{0,1,2...\}$.

2. Let X_1, X_2, X_3 and X_4 be four mutually independent random variables each having p.d.f. $f(x) = \begin{cases} 3(1-x)^2, & 0<x<1 \\ 0, & \text{otherwise.} \end{cases}$ Find the p.d.f. of $Y = \min(X_1, X_2, X_3, X_4)$ and $Z = \max(X_1, X_2, X_3, X_4)$.

3. Let X_1 and X_2 be i.i.d $U(0,1)$. Define two new random variables as $Y_1 = X_1 + X_2$ and $Y_2 = X_2 - X_1$. Find the joint probability density function of Y_1 and Y_2 and also marginal probability density of Y_1 and Y_2.

4. Let X and Y be i.i.d $N(0,1)$. Find the probability density function of $Z = \dfrac{X}{Y}$.

5. Let X and Y be i.i.d random variables with common probability density function $f(x) = \begin{cases} \frac{c}{1+x^4}, & -\infty < x < \infty \\ 0, & \text{Otherwise.} \end{cases}$ where, c is a normalizing constant. Find the p.d.f. of $Z = \dfrac{X}{Y}$

6. Let X and Y be i.i.d $N(0,1)$. Define the random variables R and θ by $X = Rcos(\theta)$, $Y = Rsin(\theta)$

   (a) Show that R and θ are independent with $\dfrac{R^2}{2} \sim Exp(1)$ and $\theta \sim U(0, 2\pi)$.

   (b) Show that X^2+Y^2 and $\dfrac{X}{Y}$ are independently distributed.

7. Let U_1 and U_2 be i.i.d $U(0,1)$ random variables. Show that $X_1 = \sqrt{-2\ln U_1}\cos(2\pi U_2)$ and $X_2 = \sqrt{-2\ln U_1}\sin(2\pi U_2)$ are i.i.d $N(0,1)$ random variables.

8. Let X_1, X_2 and X_3 be i.i.d with p.d.f. $f(x) = \begin{cases} e^{-x}, & x>0 \\ 0, & \text{otherwise.} \end{cases}$ Find the p.d.f. of Y_1, Y_2, Y_3; where $Y_1 = \dfrac{X_1}{X_1+X_2}, Y_2 = \dfrac{X_1+X_2}{X_1+X_2+X_3}, Y_3 = X_1+X_2+X_3$.

9. X and Y are i.i.d. random variables each with p.m.f $P(X=x) = \begin{cases} (1-p)^x p, & x = 0,1,\ldots \\ 0, & \text{Otherwise.} \end{cases}$ Identify the distribution of $\frac{X}{X+Y}$. Further find the p.m.f of $Z = \min(X,Y)$.

*** All the best ***