Assignment - 4

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Q1 Acc. to convergence in probability :- $X_n \xrightarrow{p} X$ as $n \to \infty$ if

$P(|X_n - X| \geq \varepsilon) = 0$ as $n \to \infty$ $\forall \varepsilon > 0$

we know $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$

where each X_i is iid $U(a, b)$.

$\therefore E(X_i) = \frac{a+b}{2}$ & $V(X_i) = \frac{(b-a)^2}{12}$

Now, Acc. to chebyshev's Inequality.

$P(|\bar{X}_n - E(\bar{X}_n)| \geq \varepsilon) \leq \frac{V(\bar{X}_n)}{\varepsilon^2}$

$\Rightarrow P(|\bar{X}_n - \frac{a+b}{2}| \geq \varepsilon) \leq \frac{(b-a)^2}{12 n \varepsilon^2}$

$\because E(\bar{X}_n) = E\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right]$

$= \frac{1}{n}\sum_{i=1}^{n} E[X_i] = \frac{1}{n} \cdot n\left(\frac{a+b}{2}\right)$

Also, $V(\bar{X}_n) = \frac{Var(X)}{n}$.

Now, as $n \to \infty \Rightarrow \frac{(b-a)^2}{12 n \varepsilon^2} = 0$

$\Rightarrow P(|\bar{X}_n - \frac{a+b}{2}| \geq \varepsilon) \leq 0$

So, as $P() \geq 0$

$\Rightarrow P(|\bar{X}_n - \frac{a+b}{2}| \geq \varepsilon) = 0$

$\therefore$ Acc to convergence in probability :- $\bar{X}_n \xrightarrow{p} \frac{a+b}{2}$ as $n \to \infty$

$\therefore$ The ~~se~~ required value of $c = \frac{a+b}{2}$.

Q.2 Given $\bar{X}_n \xrightarrow{p} 0 \Rightarrow E(X) = 0$

as X_i is iid $U(a, b) \Rightarrow E(X_i) = \frac{a+b}{2}$

$\Rightarrow \frac{a+b}{2} = 0 \Rightarrow \boxed{a = -b}$

Q.3: Given $\overline{X}_n \xrightarrow{p} 0 \Rightarrow E(X) = 0$

as X_i is iid $U(-2, b) \Rightarrow E(X_i) = \frac{-2+b}{2}$

$\Rightarrow \frac{-2+b}{2} = 0 \Rightarrow \boxed{b = +2}$

Q.4 Given $\overline{X}_n \xrightarrow{p} 1 \Rightarrow E(X) = 1$

as X_i is iid $U(-2, b) \Rightarrow E(X_i) = \frac{-2+b}{2}$

$\Rightarrow \frac{-2+b}{2} = 1 \Rightarrow \boxed{b = 4}$

Q.5: Given $\overline{X}_n \xrightarrow{p} 0$ as $n \to \infty \Rightarrow E(X) = 0$

as X_i is iid $U(a, 1) \Rightarrow E(X) = \frac{a+1}{2}$.

$\Rightarrow \frac{a+1}{2} = 0 \Rightarrow \boxed{a = -1}$

Q.6 Given X_is are iid $N(0,1) \Rightarrow E(X_i) = 0$ & $V(X_i) = 1$

$\Rightarrow E(\overline{X}_n) = E\left(\frac{1}{n} \sum_{i=1}^{n} X_i\right) = \frac{1}{n} \sum_{i=1}^{n} E(X_i) = \frac{1}{n} \cdot n \cdot (0) = 0$

& $V(\overline{X}_n) = V(X)/n = 1/n$.

Acc. to chebyshev's Inequality

$P(|\overline{X}_n - E(\overline{X}_n)| \geq \varepsilon) \leq \frac{V(\overline{X}_n)}{\varepsilon^2}$

$\Rightarrow P(|\overline{X}_n - 0| \geq \varepsilon) \leq \frac{1}{n\varepsilon^2}$

as $n \to \infty$

$\Rightarrow P(|\overline{X}_n - 0| \geq \varepsilon) = 0$.

$\therefore$ Acc. to convergence in prob

$\overline{X}_n \xrightarrow{p} 0 \quad \therefore \boxed{c = 0}$

Q.7 We know K-WLLN,

$\overline{X}_n \xrightarrow{P} E(\overline{X}_n)$ as $n \to \infty$

& $E(\overline{X}_n) = \mu$ which is in $N(\mu, \sigma^2)$

$\because E(\overline{X}_n) = E\left(\frac{1}{n} \sum_{i=1}^{n} X_i\right) = \frac{1}{n} \sum_{i=1}^{n} E(X_i) = \frac{1}{n} n E(X_i)$

& as each X_i is iid $N(\mu, \sigma^2)$

$\Rightarrow E(\bar{X}_n) = \frac{1}{n} n \mu = \mu$

Given $\bar{X}_n \xrightarrow{P} 0$ as $n \to \infty$

$\therefore E(\bar{X}_n) = \boxed{\mu = 0}$

<u>Q.8</u> As, $\bar{X}_n \xrightarrow{P} 5$ as $n \to \infty$ & from WLLN, & we know

$\bar{X}_n \xrightarrow{P} E(\bar{X}_n)$ as $n \to \infty$

$E(\bar{X}_n) = E(X) = \mu$

$\therefore E(\bar{X}_n) = \boxed{\mu = 5}$

<u>Q.9</u> Given X_i iid $B(2, 1/2)$

$\Rightarrow E(X_i) = np = 2 \times 1/2 = 1$

& $V(X_i) = np(1-p) = 2 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$.

<u>Also</u> $E(\bar{X}_n) = E\left(\frac{1}{n} \sum_{i=1}^{n} X_i\right) = \frac{1}{n} \sum_{i=1}^{n} E(X_i) = \frac{1}{n} \cdot n \cdot 1 = 1$

& $V(\bar{X}_n) = V\left(\frac{1}{n} \sum_{i=1}^{n} X_i\right) = \frac{1}{n^2} \sum_{i=1}^{n} V(X_i) = \frac{1}{n^2} n \, V(X_i)$

$= \frac{V(X_i)}{n} = \frac{1}{2n}$.

Acc. to chebyshev's Inequality,

$P(|\bar{X}_n - E(\bar{X}_n)| \geq \varepsilon) \leq \frac{V(\bar{X}_n)}{\varepsilon^2}$

$\Rightarrow P(|\bar{X}_n - 1| \geq \varepsilon) \leq \frac{1}{2n\varepsilon^2}$

as $n \to \infty$

$\Rightarrow P(|\bar{X}_n - 1| \geq \varepsilon) \leq 0$ & as $P(\cdot) \geq 0$

$\Rightarrow P(|\bar{X}_n - 1| \geq \varepsilon) = 0$

$\therefore$ Acc. to convergence in prob $\Rightarrow \bar{X}_n \xrightarrow{P} 1$

$\therefore \boxed{C = 1}$

Q.10. As X_n iid $B(a, 1/2)$ & from K-WLLN

$\bar{X}_n \xrightarrow{p} 3$ as $n \to \infty \Rightarrow E(\bar{X}_n) = 3$

Also $E(\bar{X}_n) = E(X) = np = a/2$

$\Rightarrow 3 = a/2 \Rightarrow \boxed{a = 6}$

Q.11. As X_i's iid $B(a, 1/3)$ & from K-WLLN

$\bar{X}_n \xrightarrow{p} 5$ as $n \to \infty \Rightarrow E(\bar{X}_n) = 5$

Also $E(\bar{X}_n) = E(X) = np = a/3$

$\Rightarrow 5 = \frac{a}{3} \Rightarrow \boxed{a = 15}$

Q.12 As X_i's iid $B(10, p)$ & $E(\bar{X}_n) = 2$ (from K-WLLN)

Also $E(\bar{X}_n) = E(X) = np = 10p$

$\therefore 2 = 10p \Rightarrow \boxed{p = \frac{1}{5}}$