CPPL
Date: 19/08/2024
Page No.

classfellow

(CSL003P1M) Assignment - 1 Probability & Statistics
@IIT Jammu

→ All questions solved by myself (with some help from group)

Name: Rohit Tiwari
Dep : CSE
Roll No: 2024PCS0036

Q.1 Let $\Omega = \{1,2,3,4\}$ Check wheather any of the following is a $\sigma$-field of subsets of $\Omega$.

$F_1 = \{ \phi, \{1,2\}, \{3,4\} \}$

Sol: No:- Reasons :→ It ~~voil~~ violates the first necessary property to be a $\sigma$-field which is

$$\Omega \in F_1$$

→ Trivially it also voilates the 2nd property as

$$\forall A \in F_1 , \quad A^c \in F_1$$

which is not true for $\phi \in F_1$ as $\phi^c = \Omega$ and here $\Omega \notin F_1$.

Hence $F_1$ is not a $\sigma$-field.

$F_2 = \{ \phi, \Omega, \{1\}, \{2,3,4\}, \{1,2\}, \{3,4\} \}$

Sol No. Because according to third property $\sigma$-field is closed under union but here $\{1\} \cup \{3,4\} \notin F_2$

That's why $F_2$ is not a $\sigma$-field.

$F_3 = \{\phi, \Omega, \{1\}, \{2,3,4\}, \{1,2\}, \{3,4\}, \{1,3,4\}, \{2\}\}$

$F_3$ is a σ-field because it satisfies all the necessary conditions.

(i) $\Omega \in F_3$.

(ii) $\{1\}^c = \{2,3,4\} \in F_3$    $\{1,2\}^c = \{3,4\} \in F_3$

$\{1,3,4\}^c = \{2\} \in F_3$

iii) Closed under union, & Intersection

Q.2 Prove that if $F_1$ and $F_2$ are σ-fields of subsets of $\Omega$, then $F_1 \cap F_2$ is also a σ-field. Give a counter example to show that similar results for union of σ-field does not hold.

Proof: If $F_1$ & $F_2$ are σ-fields, $F_1 \cap F_2$ also.

- Properties of σ-fields.

i) Contains the sample space $\Omega$:

- $F_1$ and $F_2$ are σ-fi, so $\Omega \in F_1$ & $\Omega \in F_2$
- Therefore $\Omega \in F_1 \cap F_2$

ii) Closed under Complimentation.

- If $A \in F_1 \cap F_2$ then $A \in F_1$ & $A \in F_2$
- Since both $F_1$ and $F_2$ σ-fields $A^c \in F_1$ & $A^c \in F_2$
- Therefore $A^c \in F_1 \cap F_2$

iii) Closed under countable union.

Let $A_1, A_2 \ldots$ be a countable sequence of sets in $F_1 \cap F_2$

Then $A_i \in F_1$ and $A_i \in F_2$ for all i.
Since $F_1$ & $F_2$ are σ-fields $\bigcup_{i=1}^{\infty} A_i \in F_1$ & $\bigcup_{i=1}^{\infty} A_i \in F_2$
Thus $\bigcup_{i=1}^{\infty} A_i \in F_1 \cap F_2$

Since $F_1 \cap F_2$ satisfies all condition of σ-field. intersection of two σ-field is also σ-field.

Counterexample of Union of σ-fields

Let $\Omega = \{1, 2, 3\}$

Consider the following two σ-fields

$F_1 = \{\phi, \{1\}, \{2,3\}, \{1,2,3\}\}$
$F_2 = \{\phi, \{2\}, \{1,3\}, \{1,2,3\}\}$

The union $F_1 \cup F_2 = \{\phi, \{1\}, \{2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$
However this collection is not σ-field because it is not closed under union

$\{1\} \cup \{2\} = \{1, 2\}$ not in $F_1 \cup F_2$

Sol:3 Let F be a $\sigma$-field of subsets of the sample space $\Omega$ and let $A \in F$ be fixed. Show that $F_A = \{C : C = A \cap B, B \in F\}$ is a $\sigma$-field of subsets of A. Q.4

$$\Omega = \{1, 2, 3\}$$

$$F = \{\phi, \{1,2,3\}, \{1,2\}, \{3\}\}$$

$$A = \{1,2\}, \quad \cancel{B \in F = \{3\}}$$

$$F_A = \{\phi, \{1,2\}\}$$

(i) $A \in F$ & $A \cap A = A \Rightarrow A \in F_A$

(ii) Let $C \in F_A$ then $C = A \cap B$ for $B \in F$

$$C^{C_A} \text{ (Compliment w.r.t A)} = A - C$$

$$= A - A \cap B$$

$$= A \cap B^c \in F_A \text{ (as } B^c \in F_A)$$

Sol

(iii) Let $C_1, C_2, \ldots \in F_A$ then

$$C_i = A \cap B_i, \; i = 1, 2, \ldots \text{ for } B_i \in F$$

$$\cup C_i = \cup_i (A \cap B_i) = A \cap \left(\cup_i B_i\right) \in F_A \text{ (as } \cup B_i \in F)$$

$\Rightarrow F_A$ is a $\sigma$-field of subset of A.

Sol ④(a) $\Omega = \{0, 1, 2, \ldots\}$

$$P(A) = \sum_{x \in A} \frac{e^{-\lambda} \lambda^{x}}{x!}, \quad \lambda > 0$$

$\therefore \; P(A) \geq 0 \longrightarrow (i)$

$$P(\Omega) = \sum_{x \in \Omega} \frac{e^{-\lambda} \lambda^{x}}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^{x}}{x!}$$

$$= e^{-\lambda} \left[ 1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots \right] \qquad \left[ e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots \right]$$

$$= e^{-\lambda} \left[ e^{\lambda} \right] = 1$$

$\therefore \; P(\Omega) = 1 \; - (ii)$

$A_i = \{ x \in \Omega : i-1 < x < i+1 \} = \{i\}$

$$P\left( \bigcup_{i=1}^{\infty} A_i \right) = P\{1, 2, \ldots\} = 1 - P(X=0)$$

$$\sum_{1}^{\infty} P(A_i) = P(A_1) + \cdots = 1 - P(X=0)$$

$$P\left( \bigcup_{i=1}^{\infty} A_i \right) = \sum_{1}^{\infty} P(A_i) \quad \text{--- } (iii)$$

classfellow

$\therefore$ According to (i) (ii) (iii) ca.

P(A) is a Prob measure

(b) $\Omega = \{0, 1, 2 ..\}$

$$P(A) = \sum_{x \in A} p(1-p)^x, \quad 0 < p < 1$$

(i) given $0 < p < 1$

$\therefore$ p is +ve

we say $(1-p)$ is also +ve

$\therefore$ $p(1-p) > 0$,

$$p(1-p)^x > 0 \Rightarrow \sum_{x \in A} p(1-p)^x > 0$$

$\therefore$ $P(A) \geq 0$ [1st rule of Axiomatic def of Prob.]

(ii) we need to show $P(\Omega) = 1$

$$P(\Omega) = \sum_{x=0}^{\infty} p(1-p)^x = p \sum_{x=0}^{\infty} (1-p)^x$$ [sum of infinite ~~term~~, this form a G.P. series]

$$= p \cdot \frac{1}{1-(1-p)} \quad \left[S_\infty = \frac{a}{1-r}; \ |r| < 1\right]$$

$$= \frac{p}{p} = 1$$

$\therefore$ $P(\Omega) = 1$ 2nd rule of Axiomatic def of Prob

(iii) let $A_1, A_2, A_3 ..$ be disjoint sets

s.t $A_i \cap A_j = \phi \ \forall i \neq j$.

$\bigcup_{i=1}^{\infty} A_i$ represents the event that atleast one of the events $A_1 A_2 A_3 ...$ occurring, the even are mutually exclusive

classfellow

Date: / /20

Page No.

$$\boxed{P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{x \in \bigcup_{i=1}^{n} A_i}^{n} P(1-P)^x} \quad \text{— (I)}$$

Prob of all union

Sum of Prob of individual event

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = P(A) = \sum_{x \in \Omega} P.(1-P)^x \quad (\text{Given})$$

$$\sum_{i=1}^{\infty} P(A_i) = \sum_{i=1}^{\infty} \sum_{x \in A_i} P(1-P)^x = \sum_{x \in \bigcup_i^{\infty} A_i} P(1-P)^x$$

$$\boxed{\sum_{i=1}^{\infty} P(A_i) = P\left(\bigcup_{i=1}^{\infty} A_i\right)}$$ from eq (I)

↳ 3rd Prop of Axiomatic del sum

(c) $P(A) \geq 0$

$P(\Omega) = 0$ ($\because \Omega$ is infinite)

P is not prob. measure

2nd Part

(a) $P(E) = \sum_{3}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} = 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - \left[e^{-\lambda} + e^{-\lambda}\lambda + \frac{e^{-\lambda}\lambda^2}{2!}\right]$

$= 1 - e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2!}\right]$

$P(F) = \sum_{1}^{2} \frac{e^{-\lambda}\lambda^x}{x!} = P(x=1) + P(x=2)$

$= e^{-\lambda}\lambda = \frac{e^{-\lambda}\lambda^2}{2!} = e^{-\lambda}\lambda\left(1 + \frac{\lambda}{2!}\right)$

$P(G) = \sum_{4}^{5} \frac{e^{-\lambda}\lambda^x}{x!} = P(x=4) + P(x=5)$

$$= \frac{e^{-\lambda}\lambda^4}{4!} + \frac{e^{-\lambda}\lambda^5}{5!}$$

$$= e^{-\lambda}\lambda^4\left[\frac{1}{4!} + \frac{\lambda}{5!}\right]$$

$P(E \cap F) = \phi$

$$P(E \cup F) = \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} = \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} - \sum_{x=0} \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= 1 - \sum_{x=0} \frac{e^{-\lambda}\lambda^x}{x.}$$

$$= 1 - e^{-\lambda}$$

$$P(F \cup G) = \sum_1^5 \frac{e^{-\lambda}\lambda^x}{x!} = P(X=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$$

$$= e^{-\lambda}\lambda + \frac{e^{-\lambda}\lambda^2}{2!} + \frac{e^{-\lambda}\lambda^3}{3!} + \frac{e^{-\lambda}\lambda^4}{4!} + \frac{e^{-\lambda}\lambda^5}{5!}$$

$$= e^{-\lambda}\lambda\left[1 + \frac{\lambda}{2!} + \frac{\lambda^2}{3!} + \frac{\lambda^3}{4!} + \frac{\lambda^4}{5!}\right]$$

$$P(E \cap G) = \sum_4^5 \frac{e^{-\lambda}\lambda^4}{4!} = P(x=4) + P(x=5)$$

$$= \frac{e^{-\lambda}\lambda^4}{4!} + \frac{e^{-\lambda}\lambda^5}{5!} = e^{-\lambda}\lambda^4\left(\frac{1}{4!} + \frac{\lambda}{5!}\right)$$

$P(F \cap G) = \phi,$

classfellow

Date: / /20

Page No.

Q5 (a) elementary Even = {j} Prob $P(\{j\}) = \frac{c2^j}{j!}$

w.k.t sum of all Prob is 1

then, $P(\{0\}) + P(\{1\}) + P(\{2\}) + \dots = 1$

$$\sum_{j=0}^{\infty} \frac{c2^j}{j!} = 1$$

$$\frac{c2^0}{0!} + \frac{c2^1}{1!} + \dots = 1$$

so $c\left(1 + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \dots\right) = 1$

$c[e^2] = 1$ or $\boxed{c = e^{-2}}$

(b) $P(A) = P(2 \le j \le 4) \Rightarrow P(2) + P(3) + P(4)$

$\Rightarrow \boxed{\frac{c2^2}{2!} + \frac{c2^3}{3!} + \frac{c2^4}{4!}} \Rightarrow c = e^{-2}$

$P(B) \Rightarrow P(j \le 3) = P(3) + P(4) + P(5) + \dots$

$\boxed{\frac{c2^3}{3!} + \frac{c2^4}{4!} + \frac{c2^5}{5!} + \dots} \Rightarrow c = e^{-2}$

$P(C) = P(j \text{ is odd no}) = 1$

$= P(1) + P(3) + P(5) + \dots$

$= \frac{c2^1}{1!} + \frac{c2^3}{3!} + \frac{c2^5}{5!} + \dots$

$= c\left(\frac{2^1}{1!} + \frac{2^3}{3!} + \frac{2^5}{5!} + \dots\right)$

p(x=5)

5

$P(A\cap B) = A = \{2 \le j < 4\}$ $B = \{j \ge 3\}$

$A = \{2, 3, 4\}$ $B = \{3, 4, 5 \ldots\}$

$A \cap B = \{3, 4\}$

$P(A\cap B) \Rightarrow \boxed{\frac{c2^3}{3!} + \frac{c2^4}{4!}}$ — $\boxed{c = e^{-2}}$

$P(A\cap C) = A = \{2 \le j \le 4\}$ $C = \{1, 3, 5, 7, \ldots\}$

$A = \{2, 3, 4\}$

$A\cap C \Rightarrow 3$

$P(AC) = P(3) = \boxed{\frac{c2^3}{3!}} \Rightarrow c = e^{-2}$

$P(B\cap C) = B = \{3, 4, 5 \ldots\}$ $C = \{j$ is odd$\}$

$B = \{3, 4, 5 \ldots\}$ $C = \{1, 3, 5, 7 \ldots\}$

$B\cap C = \{$odd no$\} \Rightarrow P(C)$

$P(C) = e^{-2}$ Ans

$P(A\cap B\cap C) \Rightarrow A\ \{2, 3, 4\}$ $B = \{3, 4, 5, \ldots\}$ $C = \{1, 3, 5, 7 \ldots\}$

$P(A\cap B\cap C) = P(\{3\}) \Rightarrow \boxed{\frac{e^{-2} 2^3}{3!}}$

Verify ~~P(A∩B)~~ $P(A\cup B\cup C) \Rightarrow \sum_{i=2}^{\infty} \frac{e 2^i}{i!}$

$= e\left(\frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \ldots\right)$

classfellow

CPPL Date: / /20

Page No.

$= e^{-2}[e^{-2}-1]$

LHS $\to$ $\boxed{1-e^{-2}}$ — (1)

RHS $\to$ $\left(\frac{c2^2}{2!}+\frac{c2^3}{3!}+\frac{c2^4}{4!}\right)+c\left(\frac{2^3}{3!}+\frac{2^4}{4!}+\frac{2^5}{5!}+\cdots\right)+$

$c\left(\frac{2^3}{3!}+\frac{2^5}{5!}+\frac{2^7}{7!}+\cdots\right) \cdots \frac{c2^3}{3!}+\frac{e^{-2}2^3}{3!}$

$= c\left[\frac{2^2}{2!}+\frac{2^3}{3!}+\frac{2^4}{4!}+\cdots\right]$

$\boxed{LHS = RHS}$ $\Rightarrow$ $c\left[\frac{2^2}{2!}+\frac{2^3}{3!}+\frac{2^4}{4!}+\cdots\right]$

$= e^{-2}[e^2-1] = \boxed{1-e^2.}$

$A_1 \quad A_2 \quad A_3$ - - - - - -

$P(\bigcap_{i=1}^{3} A_i^c)$.

Sol) (6) Favorite models of dinosaurs numbered 1,2,3 (say) define events.

$A_i$ = model # I not found in 6 Pockets

$i = 1,2,3.$

Reqd. Prob = $P(A_1^c \cap A_2^c \cap A_3^c) = 1 - P(A_1^c \cap A_2^c \cap A_3^c)^c$

$= 1 - P(A_1 \cup A_2 \cup A_3)$

$= 1 - [P(A_1) + P(A_2) + P(A_3) - P(A_1A_2) - P(A_1A_3) - P(A_2A_3) + P(A_1A_2A_3)]$

$P(A_i) = \left(\frac{4}{5}\right)^6 \ \forall i$

Note that $P(A_iA_j) = \left(\frac{3}{5}\right)^6 \forall i \neq j$

$P(A_1A_2A_3) = \left(\frac{2}{5}\right)^6.$

$= 1 - \left[\left(\frac{4}{5}\right)^6 + \left(\frac{4}{5}\right)^6 + \left(\frac{4}{5}\right)^6 - \left(\frac{3}{5}\right)^6 - \left(\frac{3}{5}\right)^6 - \left(\frac{3}{5}\right)^6 + \left(\frac{2}{5}\right)^6\right]$

$= 1 - [0.26 + 0.26 + 0.26 - 0.046 - 0.046 - 0.046 + 0.004096]$

$= 1 - [0.78 - 0.138 + 0.004]$

$= 1 - 0.646 = 0.35$

$\frac{4^6}{5^6} = \frac{4096}{15625} = 0.26$

$\frac{3^6}{5^6} = 0.046$

Sol 7. Say $A_i$ : Match at ith position

P(at least one match)

reqd Prob = $P(A_1 \cup A_2 \cdots A_n) =$ ~~P~~

$$P(A_1 \cup A_2 \cup A_3 \cdots A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} (P(A_i A_j)) + \cdots$$

$$+ (-1)^{r+1} \sum_{i_1 < i_2 < \cdots i_r} P(A_{i_1} A_{i_2} \cdots A_{i_r})$$

$$+ (-1)^{n+1} P(E_1 E_2 \cdots E_n)$$

$$\sum_{i=1}^{n} P(A_i) = P(A_1) + P(A_2) + P(A_3) \cdots P(A_n)$$

$=$

for the given condition of question

$$P(A_{i_1} \wedge A_{i_2} \wedge \cdots \wedge A_{i_r}) = \frac{(n-r)!}{n!}, \quad 1 \le i < i_2 \cdots < i_r \le n$$

$\&$ $1 \le r \le n$

$\&$

$$\sum P(A_{i_1} \wedge A_{i_2} \wedge \cdots \wedge A_{i_2}) = \frac{1}{r!}$$

$\because$ it is equal to $^nC_r \frac{(n-r)!}{n!}$

$$= \frac{n!}{(n-r)!\,r!} \times \frac{(n-r)!}{n!} = \frac{1}{r!}$$

$\Rightarrow$ $P(\text{atleast one match}) = 1 - \frac{1}{2!} + \frac{1}{3!} + \cdots - (-1)^n \frac{1}{n!}$

Sol 8 (a) Let $A_i$: event that $i^{th}$ bill goes $i^{th}$ envelope

Req. Probability $= P(A_1 \cap A_2 \cap A_3 \cdots A_i) = P(\bigcap_{i=1}^{n} A_i)$

$$P(\bigcap_{i=1}^{n} A_i) = 1 - P(\bigcup_{i=1}^{n} A_i)$$

$$= 1 - \Big[ -(-1)^1 \sum_{i=1}^{n} P(A_i) - (-1)^2 \sum_{1 \le i_1 < i_2 \le n} P(A_{i_1} \wedge A_{i_2}) + \cdots$$

$$\cdots - (-1)^n \sum_{1 \le i < i_2 \cdots < i_n \le n}^{n} P(A_{i_1} \wedge A_{i_2} \wedge \cdots \wedge A_{i_n}) \Big]$$

$$= 1 - \Big[ {}^nC_1 \frac{(n-1)!}{n!} - {}^nC_2 \frac{(n-2)!}{n!} = {}^nC_3 \frac{(n-3)!}{n!} \cdots$$

$$\cdots - (-1)^n \, {}^nC_n \frac{(n-n)!}{n!} \Big]$$

classfellow

Date: / /20

Page No.

$$= 1 - \left[1 - \frac{1}{2!} + \frac{1}{3!} \cdots + (-1)^n \frac{1}{n!}\right] = \sum_{i=0}^{n} (-1)^i \frac{1}{(i)!}$$

(b) $A_i$ = Let be the envelop get $i^{th}$ bill & $i^{th}$ cheque

$\forall\ i = 1, 2, \ldots n$

Required Prob $= P(\cap A_i^c) = 1 - \left[\Sigma P(A_i) - \sum_{i<j} P(A_i A_j) + \sum_{i<j<k} P(A_i A_j A_k) \cdots (-1)^{n-1} \Sigma P(A_1 \ldots A_n)\right]$

$\Rightarrow P(\cap A_i^c) = 1 - \Sigma P(A_i) + \sum_{i<j} P(A_i A_j) + \cdots (-1)^n \Sigma P(A_1 A_2 \ldots A_n)$

$$\sum_{i=1}^{n} P(A_i) = {}^nC_1 \frac{(n-1)!}{n!} \frac{(n-1)!}{n!}$$

→ dearrange (n-1) cheques in (n-1) envelop

Selection of a envelop and filling bill and cheque in correct manner

Rearranging (n-1) bills in (n-1) envelops

$$\sum_{i<j} P(A_i A_j) = {}^nC_2 \frac{(n-2)!}{n!} \frac{(n-2)!}{n!}$$

$$\sum_{\substack{i=0 \text{ to } n \\ i_a < i_b \\ a<b}} \Sigma P(A_1 A_2 A_3 \ldots A_n) = {}^nC_n \frac{(n-n)!}{n!} \frac{(n-n)!}{n!}$$

$$P(\cap A_i^c) = 1 - {}^nC_1 \left(\frac{(n-1)!}{n!}\right)^2 + \cdots (-1)^n\, {}^nC_n \left(\frac{(n-n)!}{n!}\right)^2$$

$$P(\cap A_i^c) = \sum_{i=0}^{n} (-1)^i\, {}^nC_i \left(\frac{(n-i)!}{n!}\right)^2$$

$$= \sum_{i=0}^{n} (-1)^i \frac{\cancel{n!}}{i!\,\cancel{(n-i)!}} \frac{((n-i)!)^2}{(n!)^{\cancel{2}}}$$

$$P(\cap A_i^c) = \sum_{i=0}^{n} (-1)^i \frac{(n-i)!}{i!\, n!}$$ Ans

Q.9 Consider an Urn in which 4 balls have been placed by the following scheme. A fair coin is tossed, if the coin comes up head, a white ball is placed in the urn otherwise a red ball placed in the urn.

(a) what is the probability that the urn will contain exactly 3 white balls.

$$\binom{4}{3}\left(\frac{1}{2}\right)^3 \cdot \frac{1}{2} = \overset{2}{\cancel{4}} \times \frac{1}{\underset{4}{\cancel{8}} \times 2} = 0.25$$

b) What is the prob that urn will contain exactly 3 ~~balls~~ white balls given first ball placed is white

(b)

A: ~~first~~ ball is placed white $P(A) = 1/2$
B: urn contains exactly 3 ~~both~~ white balls

$$P(B|A) = P(AB)/P(A) = \frac{\frac{1}{2}\binom{3}{2}\left(\frac{1}{2}\right)^3}{\frac{1}{2}}$$

$$= \frac{1}{\cancel{2}}\,\frac{3}{8} \times \frac{\cancel{2}}{1} = \frac{3}{8}$$

Q.10 A person has 3 coins in his pocket, two fair coins (head and tails are equally likely) but the third one is biased with probability of heads $\frac{2}{3}$. One coin selected at random drops on the floor, landing head up. How likely it is that it is one of fair coin.

sol. Applying Bayes theorem.

$$\text{Reqd. Probability} = \frac{\frac{1}{2} \times \frac{2}{3}}{\frac{1}{2} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{2}{9}}$$

$$= \frac{1}{\cancel{3}} \times \frac{\cancel{9}\,3}{11} = 3/11.$$

Q.11 A slip of paper is given to A who marks it with either a + or a − sign with probability of 1/3 of writing a + sign. A passes the slip to B, who may either leave it unchanged or change the sign before passing to C. C passes to D after perhaps changing the sign. D passes to referee after " " " ". It is further known that B, C, D each change sign with prob (2/3). Find probability that A ~~is~~ wrote a + given that referee sees + sign on slip

Sol

$A^+, B^+, C^+, D^+$ - events that A, B, C, D pass the paper with (+) sign

bays theorem :

Reqd. prob $= P(A^+ | D^+) = \dfrac{P(A_+) \cdot P(D_+ | A_+)}{P(D^+)}$

$\Rightarrow P(D^+ | A^+) = \left(\frac{1}{3}\right)^3 + \binom{3}{2}\left(\frac{2}{3}\right)^2 \times \frac{1}{3} = \frac{13}{27}$

also $P(D^+) = P(D^+ | A^+) P(A^+) + P(D^+ | A_+^c) P(A_+^c)$

$P(D^+ | A_+^c) =$ P ( D pass with + | A pass - )

$= \binom{3}{1} \frac{2}{3} \left(\frac{1}{3}\right)^2 + \binom{3}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^0 = \frac{14}{27}$

$P(D^+) = \frac{13}{27} \cdot \frac{1}{3} + \frac{14}{27} \cdot \frac{2}{3} = \frac{41}{81}$

$$P(A^+ | D^+) = \frac{\frac{13}{27} \cdot \frac{1}{3}}{\frac{41}{81}} = \boxed{\frac{13}{41}}$$

Q.12 Give a counter example (different from class notes) to show that pairwise independence of a set of events $A_1, A_2, \ldots A_n$ does not imply mutual independence

$\Omega = \{1,2,3,4\}$ 7: Proper set

$P(\{i\}) = \frac{1}{4}$ $i = 1,2,3,4.$

$A = \{1,4\}$, $B = \{2,4\}$, $C = \{3,4\}$.

$P(A) = P(B) = P(C) = 1/2$

$P(AB) = P(AC) = P(BC) = \frac{1}{4}$ . $P(ABC) = \frac{1}{4}$

$P(AB) = P(A)P(B)$, $P(AC) = P(A)P(C)$ &

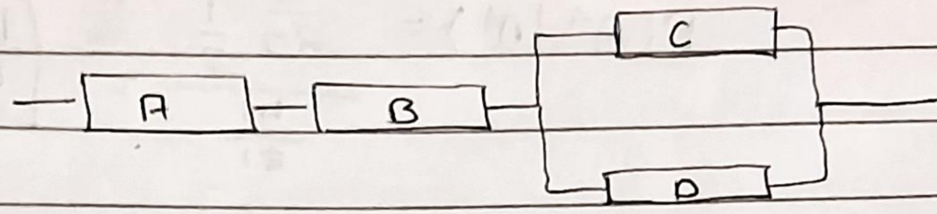
$P(BC) = P(B)P(C)$

i.e. A, B, C are pair wise independent

but $P(ABC) = \frac{1}{4} \neq P(A) \cdot P(B) \cdot P(C) = \frac{1}{8}$

$\Rightarrow$ A, B, C not mutually indep.

Q.13




$P(A) = P(B) = 0.9$ $P(C) = P(D) = 0.8$

Components are independent ∴ so.

$P(\text{circuit will work}) = P(A \cap B \cap (C \cup D))$

$\Rightarrow P(A) \cdot P(B) \cdot P(C \cup D)$

$\Rightarrow P(A) \cdot P(B) \cdot (P(C) + P(D) - P(C \cap D))$

$\Rightarrow 0.9 \times 0.9 \; (0.8 + 0.8 - P(C)P(D))$

$\Rightarrow 0.81 \; (1.6 - 0.64)$

$\Rightarrow 0.81 \times 0.96 = 0.77$ Ans

CPPL
Date: / /20
Page No.
classfellow

3(b) $P\left(\frac{C \text{ not work}}{\text{System works}}\right) = P\left(\frac{\text{only D work}}{\text{Given system work}}\right)$ {As C is not working then system is working only because of D}

$$= \frac{P(A \cap B \cap [(D) - (C \cap D)])}{P(\text{system working})}$$

$$= \frac{P(A) \cdot P(B) \cdot P(D)(1 - P(C))}{P(\text{system working})}$$

$$= \frac{0.9 \times 0.9 \times 0.8 (1 - 0.8)}{0.77} = \frac{0.1296}{0.77} = 0.168$$

classfellow

CPPL Date: / /20

Page No.

(14). During the course of an experiment with a particular brand of a disinfectant on flies, it is found that 80% are killed in the first application. Those which survive develop a resistance so that the percentage of survivors killed in any later application is half. Find Prob that (a) a fly will survive 4 applications (b) it will survive 4 app given it survived 1st

Let's say $A_i$: event that a fly survives ith application
$i = 1, 2, 3, 4$

Note that $A_4 \subset A_3 \subset A_2 \subset A_1$

$\Rightarrow A_4 = A_1 \cap A_2 \cap A_3 \cap A_4$

(a). Required Probability

$P(A_1 A_2 A_3 A_4) = P(A_4)$

$= P(A_1) P(A_2|A_1) P(A_3|A_1 A_2) P(A_4|A_1 A_2 A_3)$

$= (1-0.8) \times (1-0.4) \times (1-0.2) \times (1-0.1)$

$= 0.2 \times 0.6 \times 0.8 \times 0.9$

$= 0.12 \times 0.72 = 0.0864 = 0.0864$ / Ans

(b) $P(A_4|A_1) = \dfrac{P(A_4 \cap A_1)}{P(A_1)} = \dfrac{P(A_4)}{P(A_1)} = \dfrac{0.0864}{0.2}$

$= 0.432$ Ans

CPPL Date: / /20

Page No.

classfellow

~~Ans~~ (15) (a) $y = |x| \ \forall x \in R \ y^{-1}(-\infty, x] = \{\omega : y(\omega) \le x\} = \{\omega : |X(\omega)| \le x\}$

$= \{\omega : -x \le X(\omega) \le x\}$

$= X^{-1}[-x, \infty) \cap X^{-1}(-\infty, x]$

as $X$ is R.V.

$X^{-1}[-x, \infty) \cap X^{-1}(-\infty, x] \in f$

$X$ is r.v $\Rightarrow X^{-1}(B) \in f$

$\Rightarrow X^{-1}[-x, \infty) \in f$ & $X^{-1}(B) \in f$.

$\Rightarrow y^{-1}(-\infty, x] \in f \ \forall x \in R$

$\Rightarrow y = |X|$ is a r.v

(b) $y = X^2, \ \forall x \in R \ y^{-1}(-\infty, x] = \{\omega : y(\omega) \le x\} = \{\omega : X^2(\omega) \le x\}$

$= \{\omega : -\sqrt{x} \le X(\omega) \le \sqrt{x}\}$

$= X^{-1}[-\sqrt{x}, \infty) \cap X^{-1}(-\infty, \sqrt{x}] \in f$

$\Rightarrow y = X^2$ is a r.v

Sol (16) $\Omega = [0,1]$ f: Borel σ-field of subsets of Ω

$X(\omega) = \begin{cases} \omega, & 0 \le \omega \le 1/2 \\ \omega - 1/2, & 1/2 < \omega \le 1 \end{cases}$

$X^{-1}(-\infty, x] = \begin{cases} \phi \in f, & x < 0 \\ [0, x] \cup (\frac{1}{2}, \frac{1}{2} + x] & 0 \le x < \frac{1}{2} \\ \Omega \in f, & x \ge 1/2 \end{cases}$

$\Rightarrow X^{-1}(-\infty, x] \in f \ \forall x \in R \Rightarrow X$ is a r.v.

Sol (17) $\Omega = \{1, 2, 3, 4\}$ and $f = \{\phi, \Omega, \{1\}, \{1, 2, 3, 4\}\}$ is $\sigma$-field subset of $\Omega$.

$X(\omega) = \omega + 1 \rightarrow 2, 3, 4, 5$

$$X^{-1}(-\infty, x] = \begin{cases} \phi \in F, & x < 2 \\ \{1\} \in F, & 2 \le x < 3 \\ \{1, 2\} \notin F, & 3 \le x < 4 \end{cases}$$

$\Rightarrow$ X is not a r.v Ans

Sol (18) From deck of 52 cards
outcome $\omega$ defined on $\Omega$

$$X(\omega) = \begin{cases} 4 & \omega \text{ is an ace} \\ 3 & \omega \text{ is a King} \\ 2 & \omega \text{ is a queen} \\ 1 & \omega \text{ is a jack} \\ 0 & \text{otherwise} \end{cases}$$

$\therefore$ show $X \rightarrow$ R.V. $X^{-1}(-\infty, x] \in$ borel set of $\sigma$-field.

Case 1: $x < 0$

There is not $\omega$ in $\Omega$ s.t $X(\omega) < 0$ as lowest value is zero

$\therefore X^{-1}[(-\infty, x]) = \phi \in f_2$

Case 2 $0 \le x < 1$

for $0 \le x < 1$, $X(\omega) \le x$ where $X(\omega) = 0$ is happening i.e $\omega$ is not ace, not king, not queen, not jack

$X^{-1}[(-\infty, x]]$ is set of all cards that are not ace, king, queen, a jack

Case 3 $1 \leq x < 2$

for $1 \leq x < 2$ $X(\omega) \leq x$ when $X(\omega) = 1$ (all jack)
$X(\omega) = 0$ (no jack)

i.e pre image $X^{-1}[(-\infty, x])$ includes all cards which not King, queen are Ace

Case 4 $2 \leq x < 3$

for $2 \leq x < 3$ $X(\omega) \leq 2$ when $X(\omega) = 0$ (no jack)
$X(\omega) = 1$ (jack)
$X(\omega) = 2$ (queen)

preimage for $X^{-1}((-\infty, x])$ is set of cards with which are not King or Ace.

Similarly Every this case follows that Preimage of $X^{-1}$ $((-\infty, x])$ lies in this borel set of the σ-field. X is a Random Variable