CS & IT ENGINEERING

GRAPH THEORY

Lecture No. 1

By- SATISH YADAV SIR

TOPICS TO BE COVERED

01 Definition of Graph

02 Handshaking Lemma

03 Types of Graphs

04 No of Graphs

05 Simple Graphs theorem

DM:

(Rosen)

- Graph Theory (4 - 6)
- logic (2 - 4)
- Set theory (2 - 4)
- Combinatorics (2 - 4)

Graph Theory

Basics of graphs

Degree sequence

Types of graphs

Connectivity

coloring

Independence no

matching

covering

Planarity

DM → $\boxed{150}$

Basics of Graph

point / joint → o → vertex / vertices

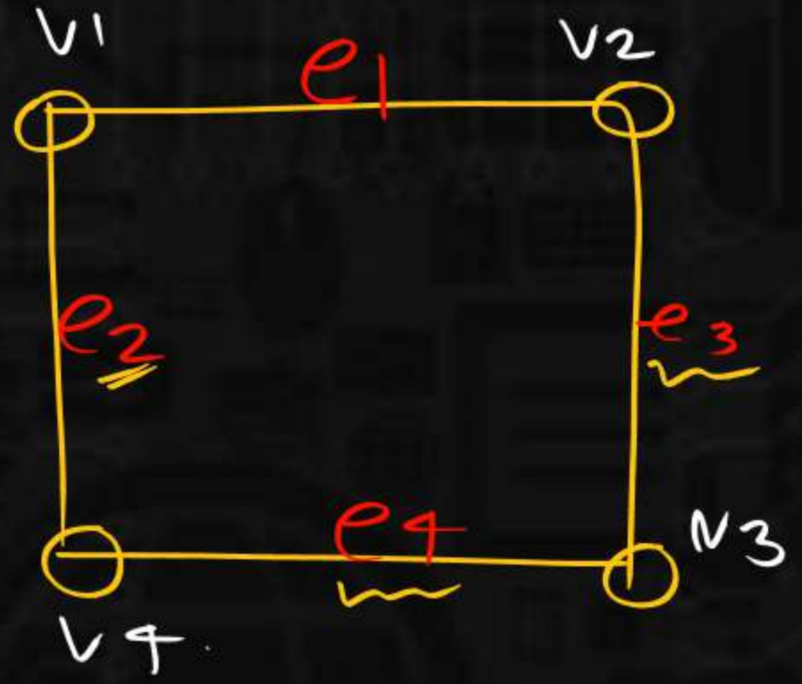
line / branch → — → edge / edges

Graph $G = (V, E)$

E → set of edges

V → set of vertices

Basics of Graph

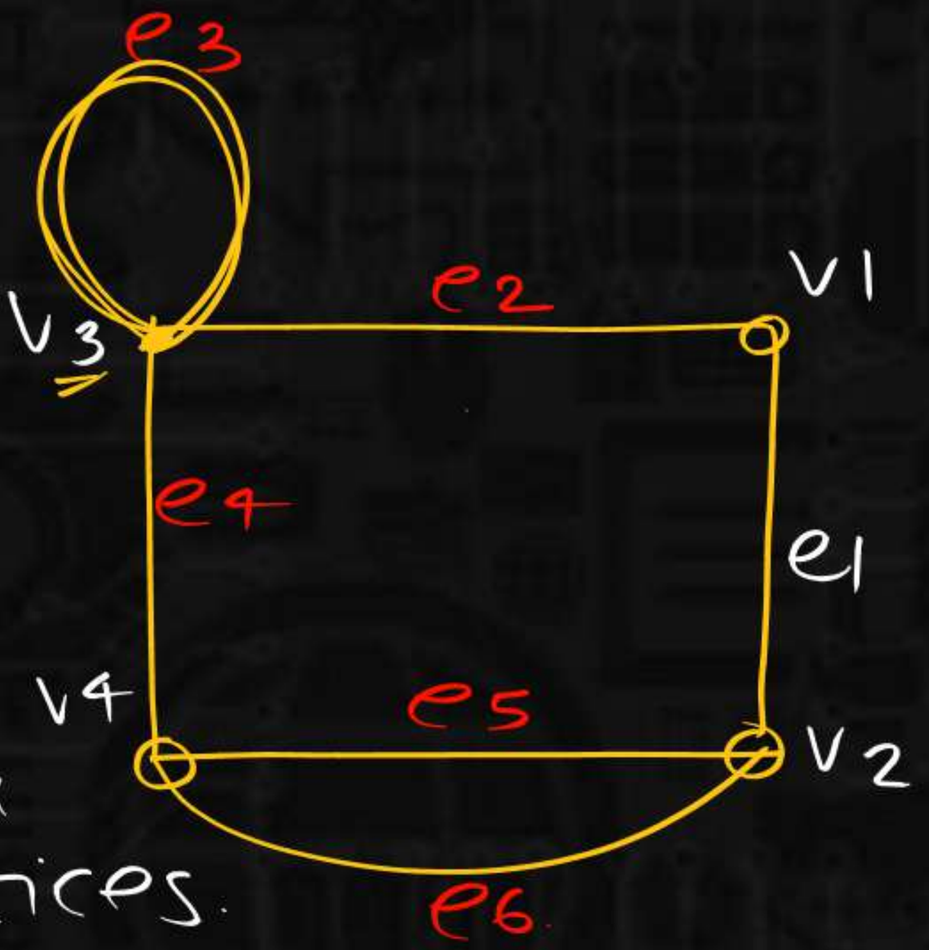


$G = (V, E)$

1. $V = \{v1, v2, v3, v4\}$
2. $E = \{e1, e2, e3, e4\}$
3. each edge is associated with unordered pair of vertices.

$e1 \rightarrow (v1, v2) \mid (v2\ v1)$

Basics of Graph



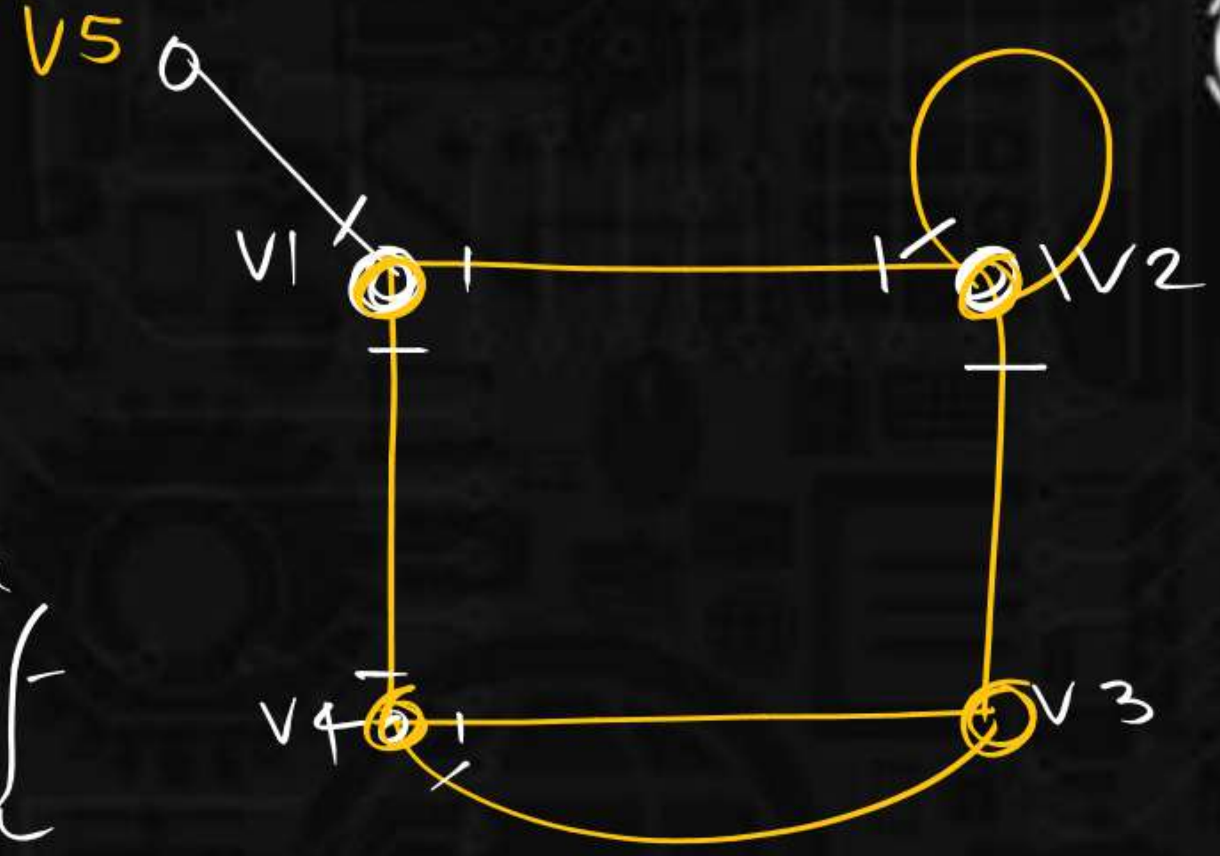
end vertices: e1 → (v1, v2)

each edge is associated with unordered pair of vertices called as end vertices.

loop/self loop: e3 → (v3, v3)

if end vertices are same

Basics of Graph



Degree/valency ($d(v_i)$)

no. of edges associated with vertex is degree of vertex

$d(v_1) = 3$

$d(v_2) = 4$

$d(v_4) = 3$

$d(v_3) = 3$

pendant vertex:

$d(v_5) = 1$

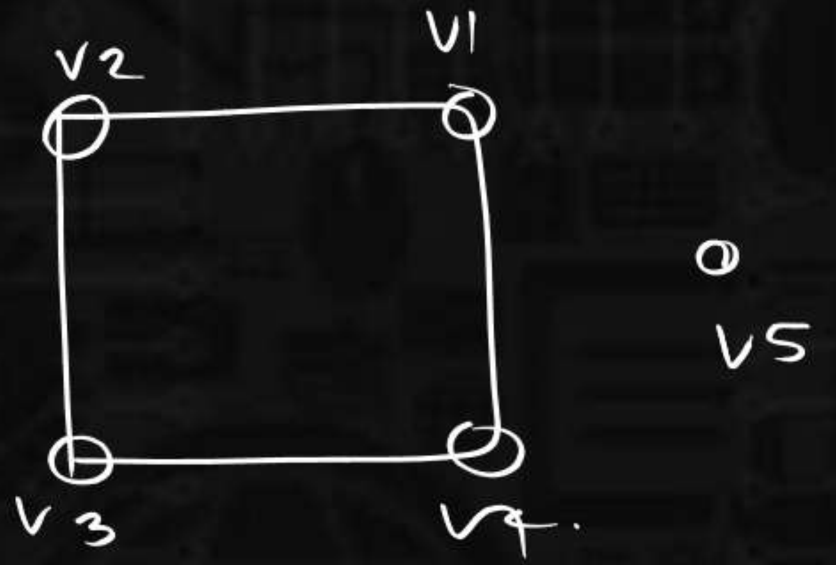
Degree 1 is called pendant vertex

Basics of Graph

isolated vertex:

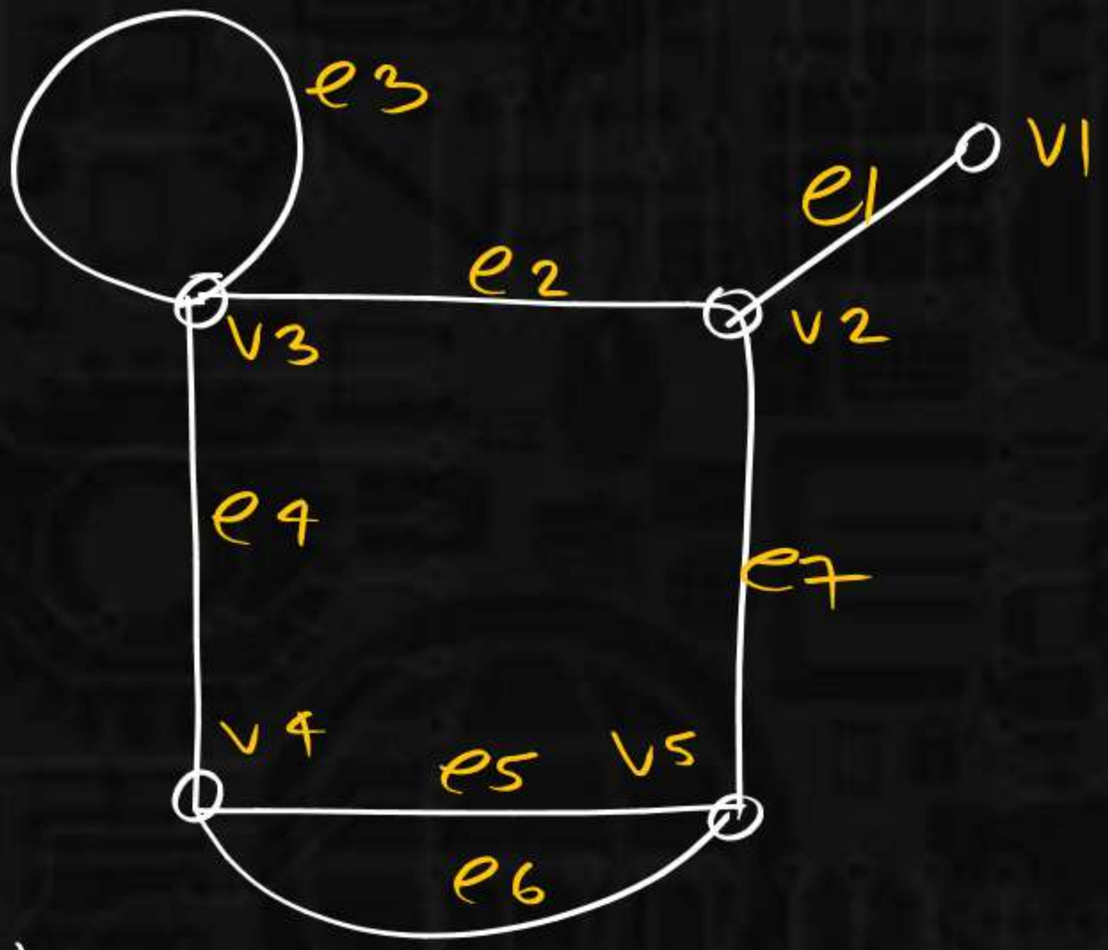
$D(v5) = 0$

if degree 0 is called isolated vertex.



Null Graph: set of isolated vertices.

Basics of Graph



$$
\begin{cases}
d(v_1) = 1 \\
d(v_2) = 3 \\
d(v_3) = 4. \\
d(v_4) = 3 \\
d(v_5) = 3.
\end{cases}
$$

$$d(v_1) + d(v_2) + d(v_3) + d(v_4) + d(v_5)$$

$$= 1 + 3 + 4 + 3 + 3$$

$$= 14 = 2 \times 7 \longrightarrow \text{no. of edges.}$$

$$d(v_1)+d(v_2)+d(v_3)+d(v_4)+d(v_5)=2\times 7 \rightarrow \text{no. of edges.}$$

<u>Thm</u>: Sum of degrees of all vertices is equals to twice the no. of edges.

$$\sum d(v_i)=2e.$$

$$\boxed{L.H.S = R.H.S}$$

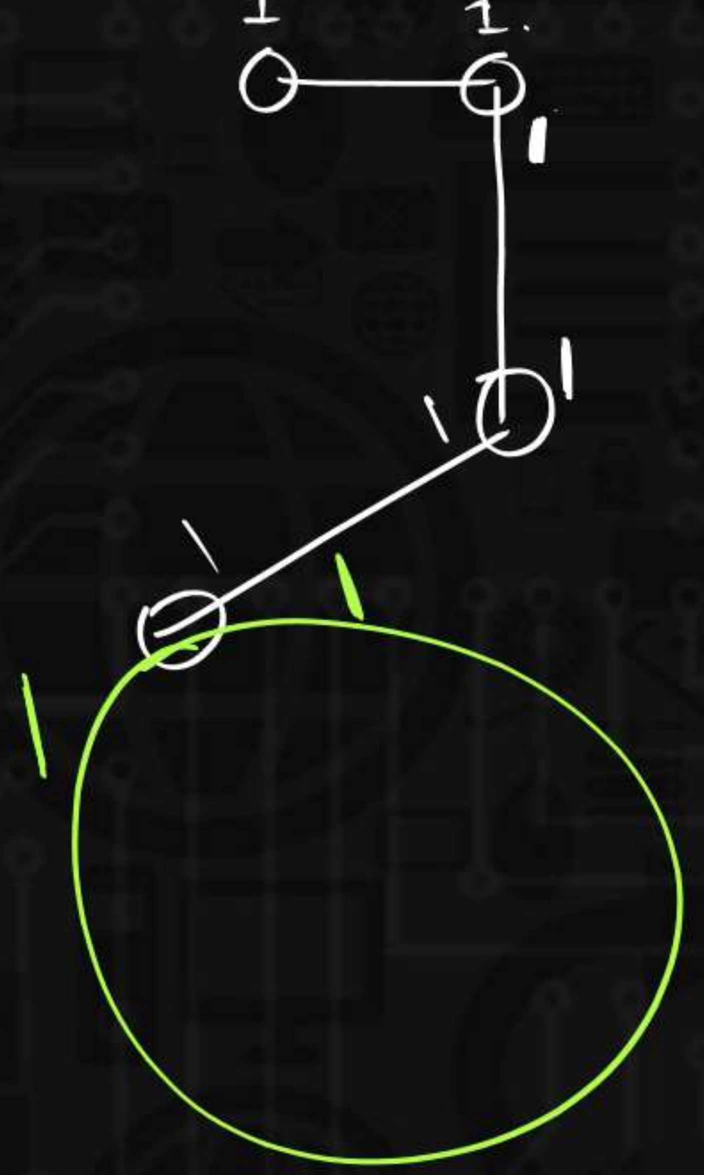
Sum of degrees $= 2 \times$ edges

$2 = 2 \times 1$

$2+2 = 2(1+1)$

$2+2+2 = 2(1+1+1)$

$$\boxed{\sum d(v_i) = 2e}$$



Basics of Graph

Thm1: Sum of degrees of all vertices is equal to twice no. of edges.

$$\sum d(v_i) = 2e$$

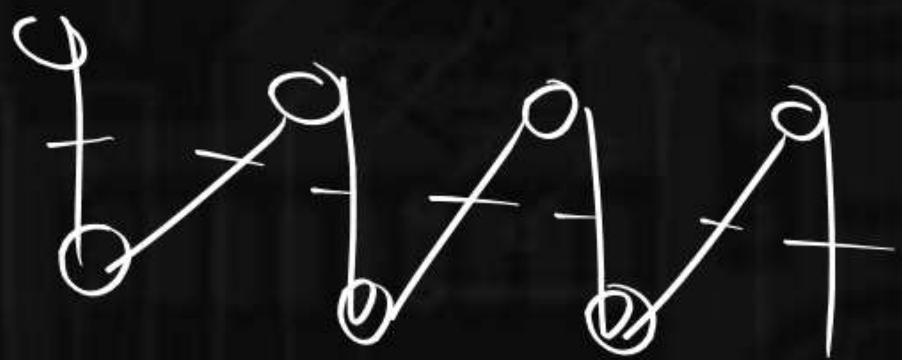
$$\sum d(v_i) = \text{even}$$

eg1:

$\sum d(v_i) = 7$

Graph is not possible

eg2: $e = 7$ $\quad \sum d(v_i) = 2 \times 7 = 14$



Basics of Graph

$1 + \underline{3 + 5}$
$1 + 8$
$= 9$ odd

$d(v_1) + d(v_2) + d(v_3) \ldots\ldots = 2e$

$d(v_1) + d(v_2) + d(v_3) \ldots\ldots = \text{even}$

$\boxed{1, 3, 5 \;,\; 2, 4, 6} = \text{even}$

odd $\qquad$ even

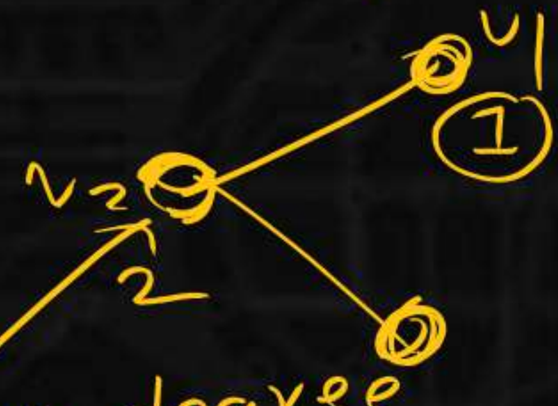
$\boxed{\underline{o_1, o_2, o_3}} + \boxed{e_1, e_2} = \text{even}$

$\downarrow \qquad\qquad \downarrow$

$\underline{\text{odd} + \text{even}}$

$\underline{\underline{\text{odd}}} = \text{even}$

odd degree vertex



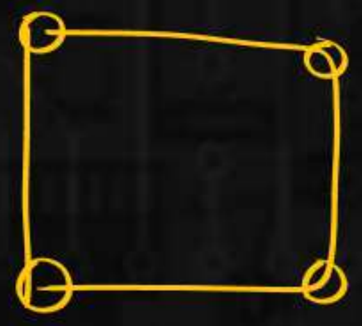
even degree vertex

$d(v_1) = 1$
odd degree vertex

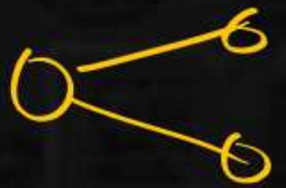
$d(v_2) = 2$
even degree vertex

Basics of Graph

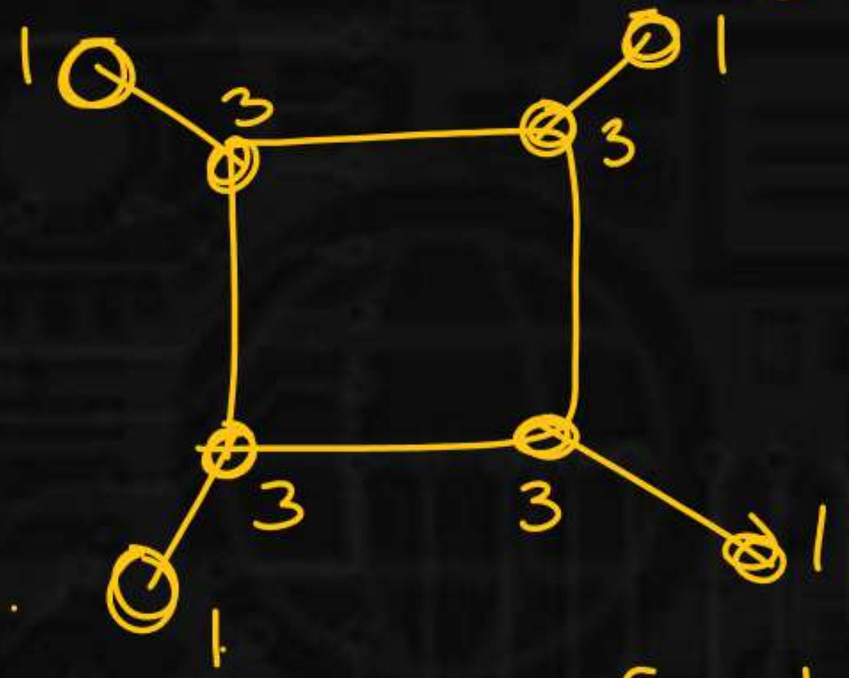
<u>Thm 2</u>: No. of odd degree vertices will be even.



no. of odd vertices = 0



no. of odd vertices = 2.

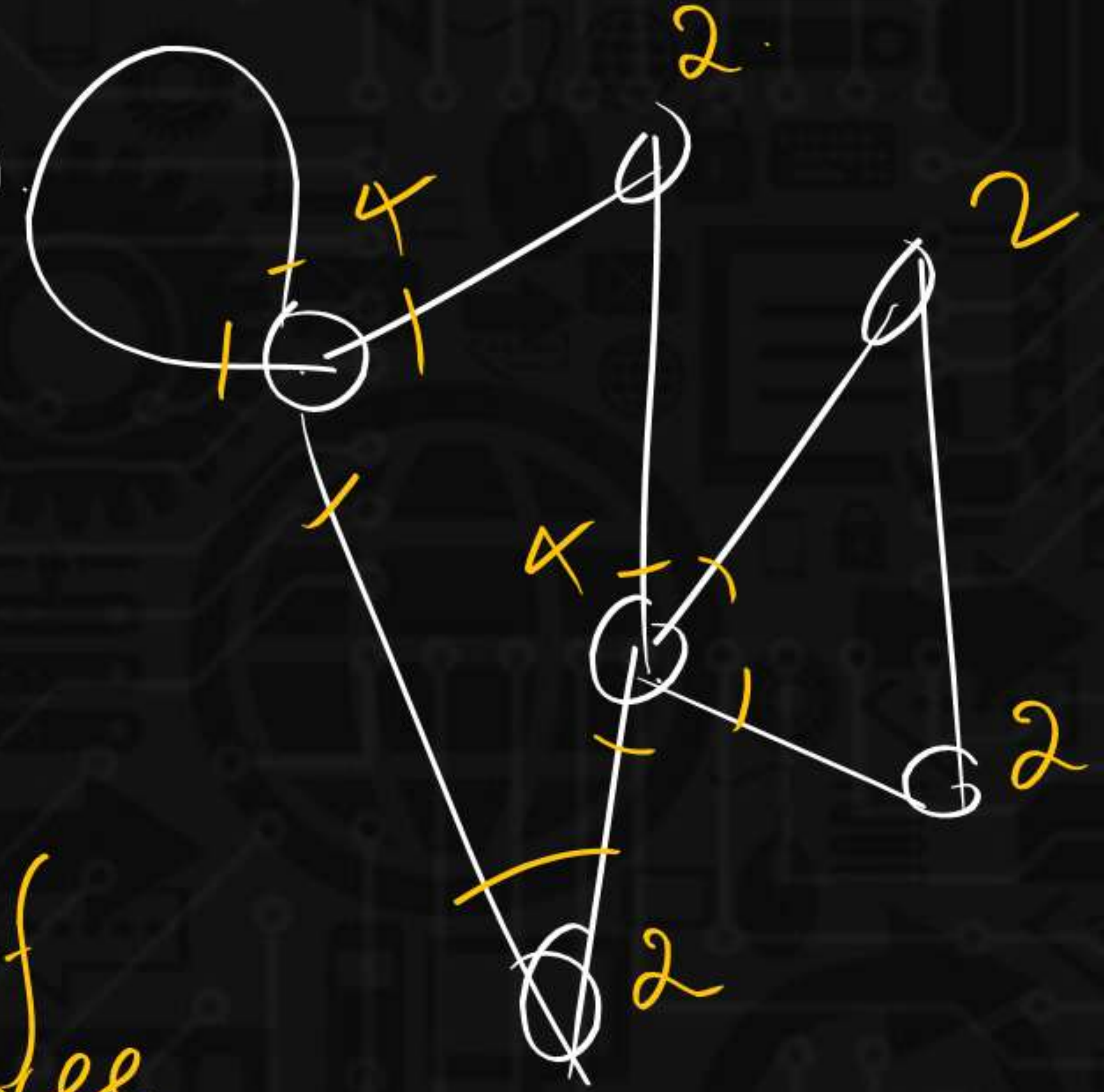


no. of odd vertices = 8

$$O_1 + O_1 + \cancel{O_2} + \underbrace{e_1 + e_2}_{\text{even}} = \text{even}$$

$$\text{odd} + \text{even}$$

$$\text{odd} \neq \text{even}$$



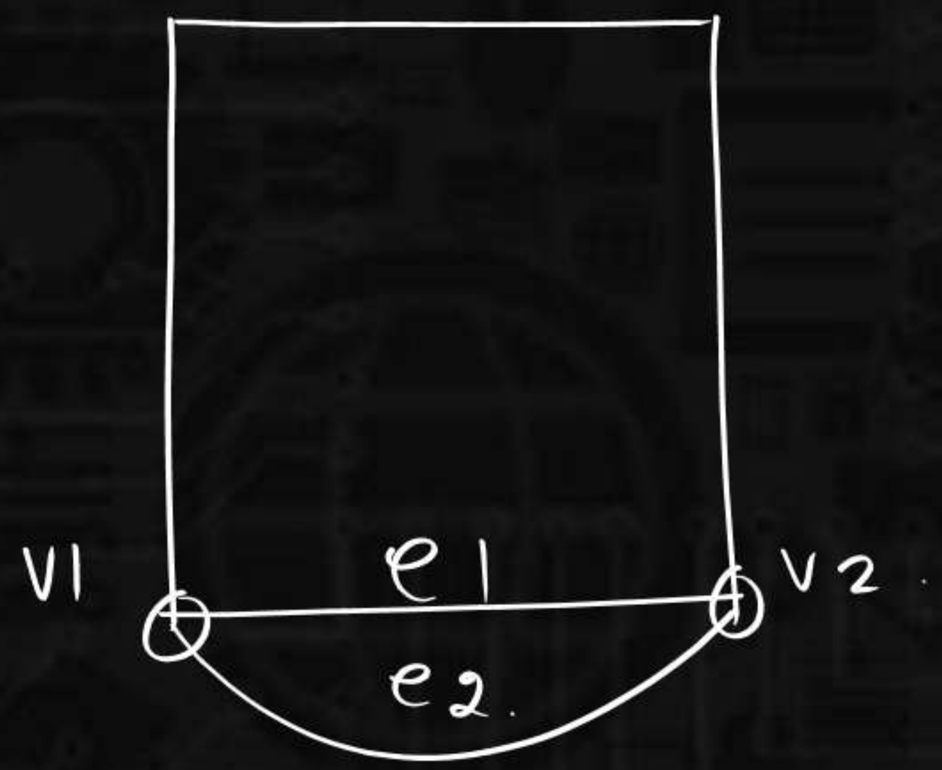
no. of odd degree vertices $= 0 =$ even.

Basics of Graph

<u>|| edges</u>:

$e1 \to \boxed{(v1, v2)}$
$e2 \to \boxed{(v1, v2)}$

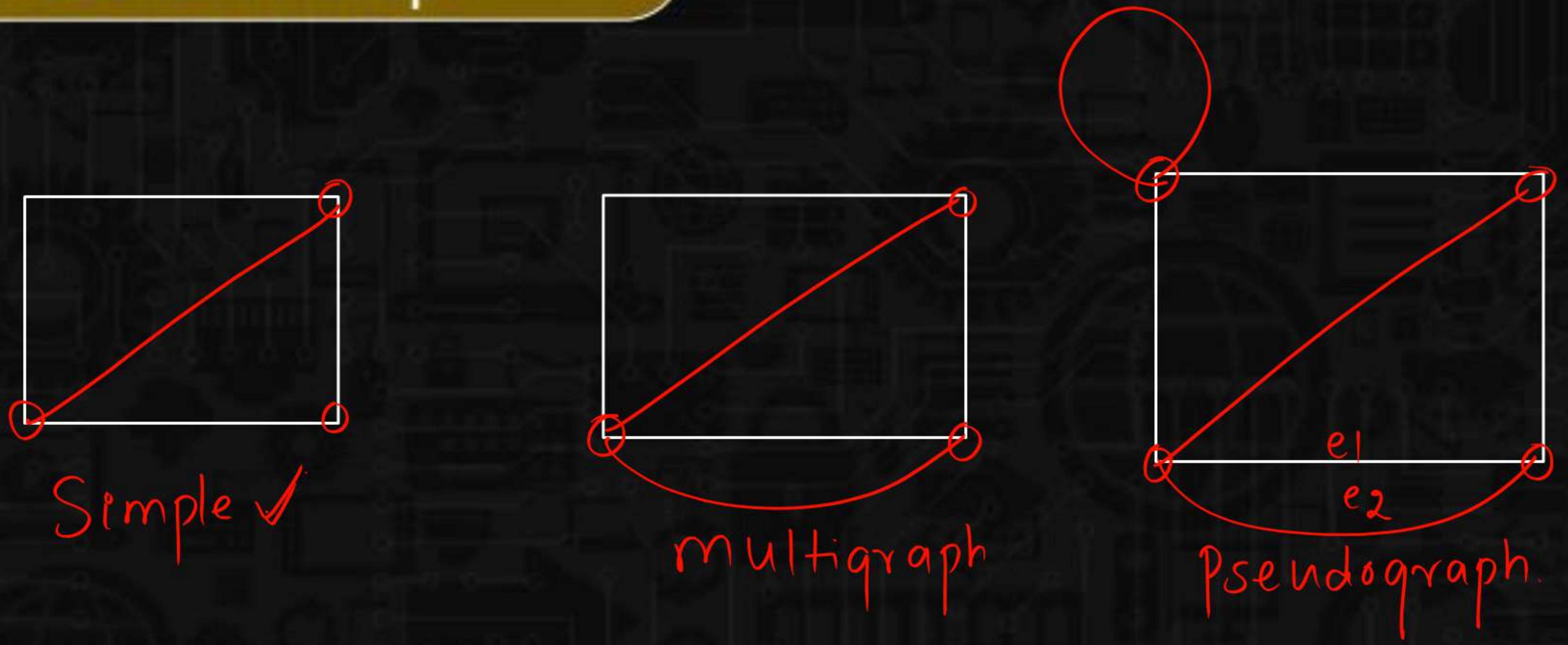
2 or more edges associated with same end vertices.



Basics of Graph

	loop	‖ edges
Simple Graph	✗	✗
multigraph	✗	✓
Pseudograph	✓	✓

Basics of Graph



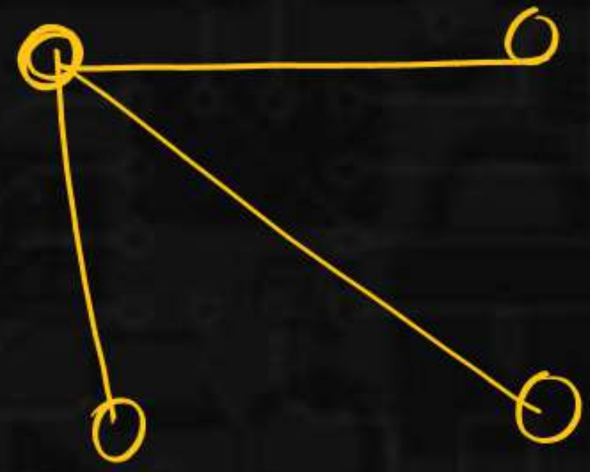
Basics of Graph

Thm 3 :

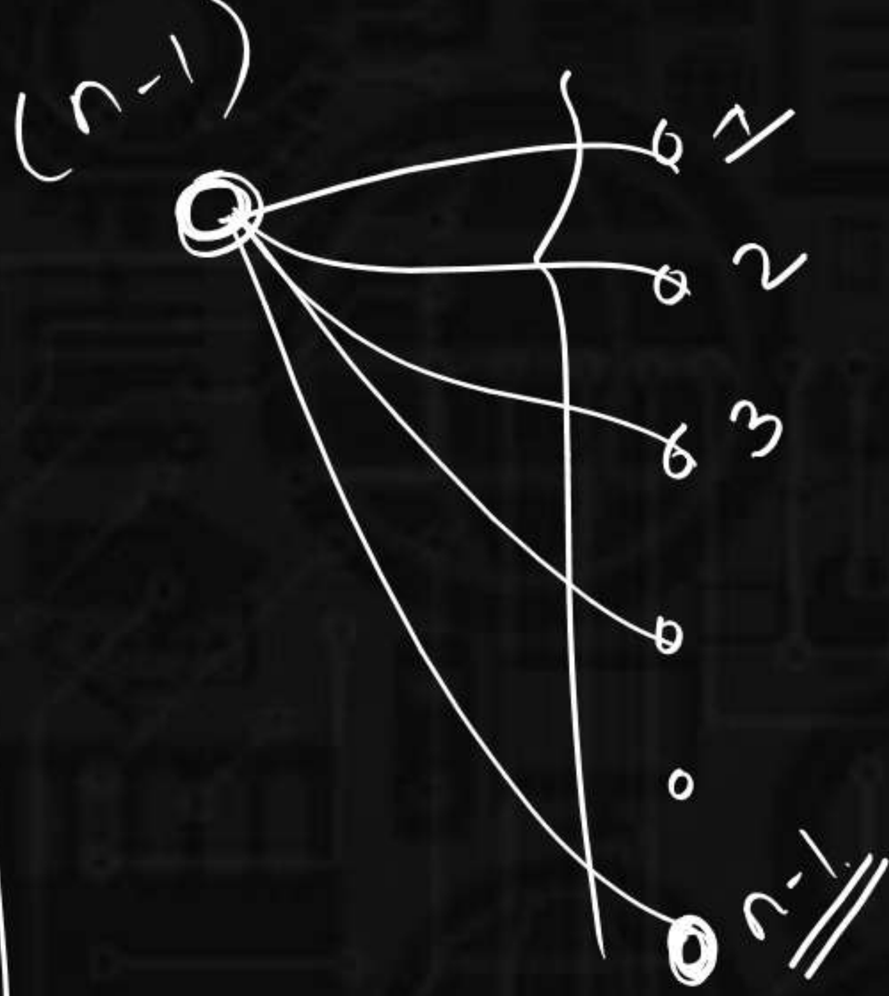
maximum degree in simple graph $\le n-1$.

$n = 4$

x



Total vertices $= n$.

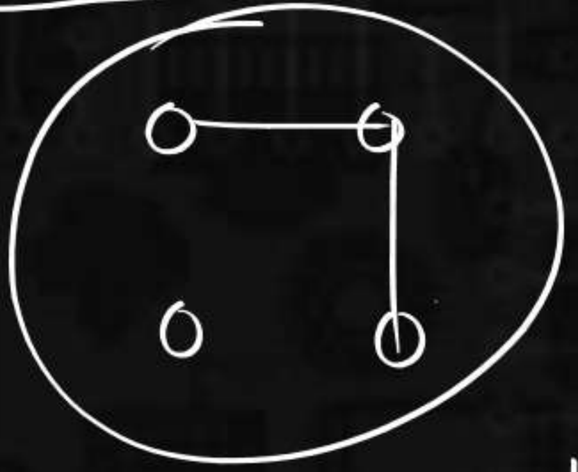


Basics of Graph

maximum degree in simple graph $\leq n-1$.

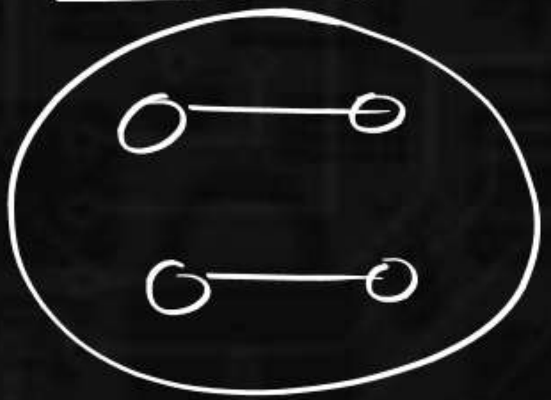
$n = 4$

Case 1:



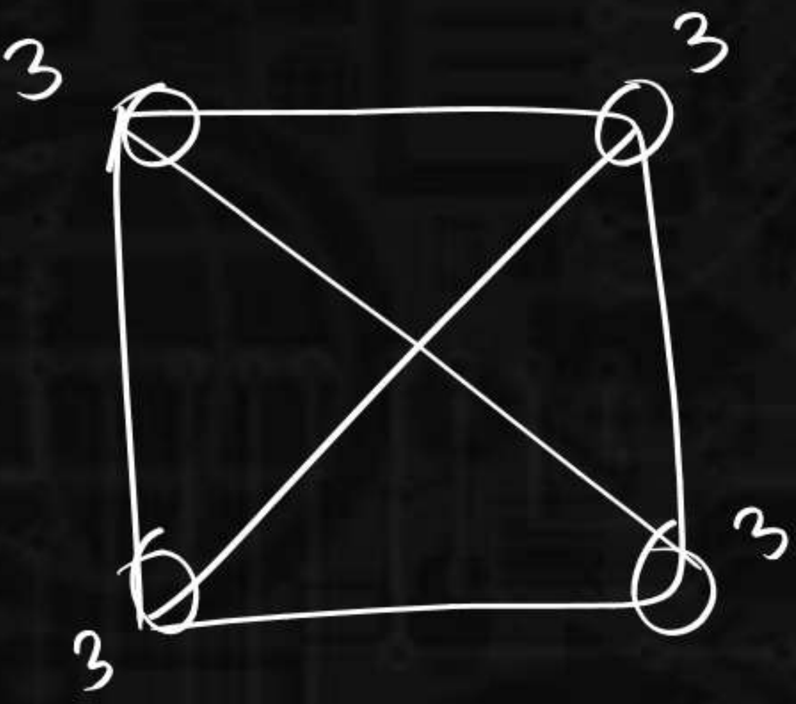
maximum degree = 2

Case 2



maximum degree 1

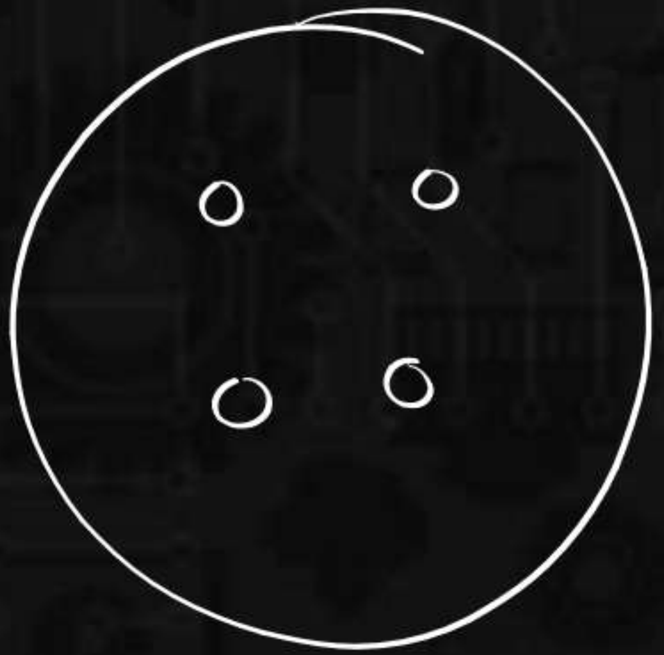
$n = 4$

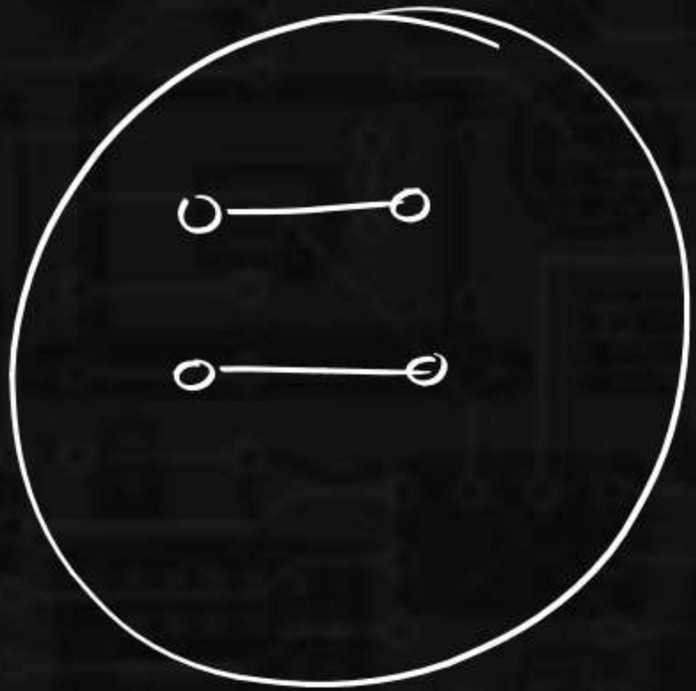


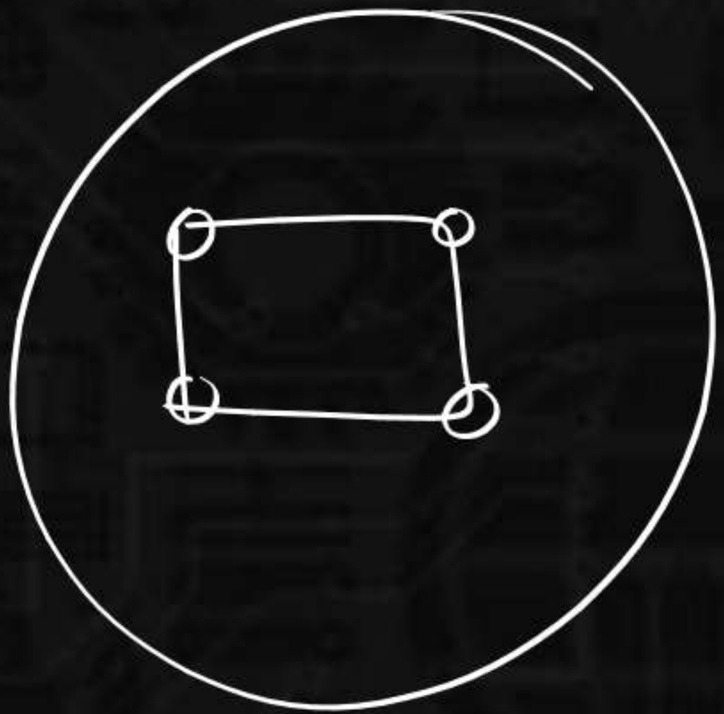
maximum degree 3

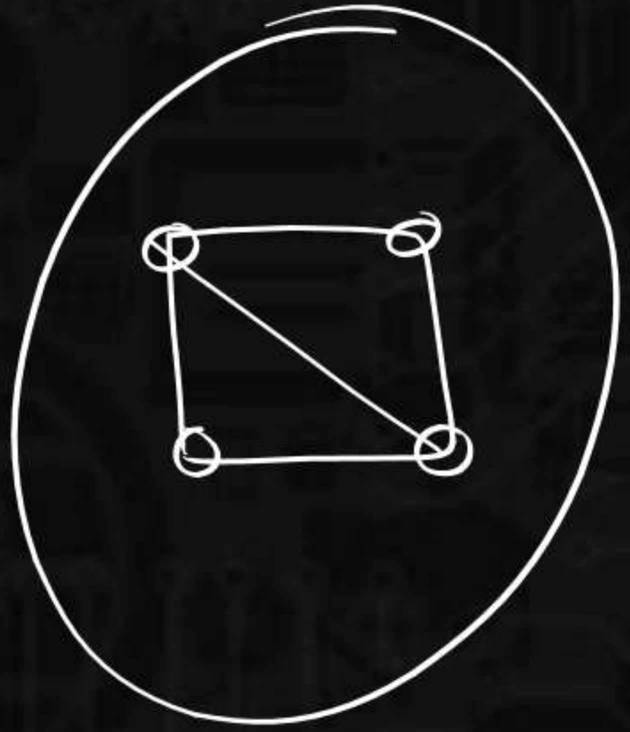
Basics of Graph

$n = 4$



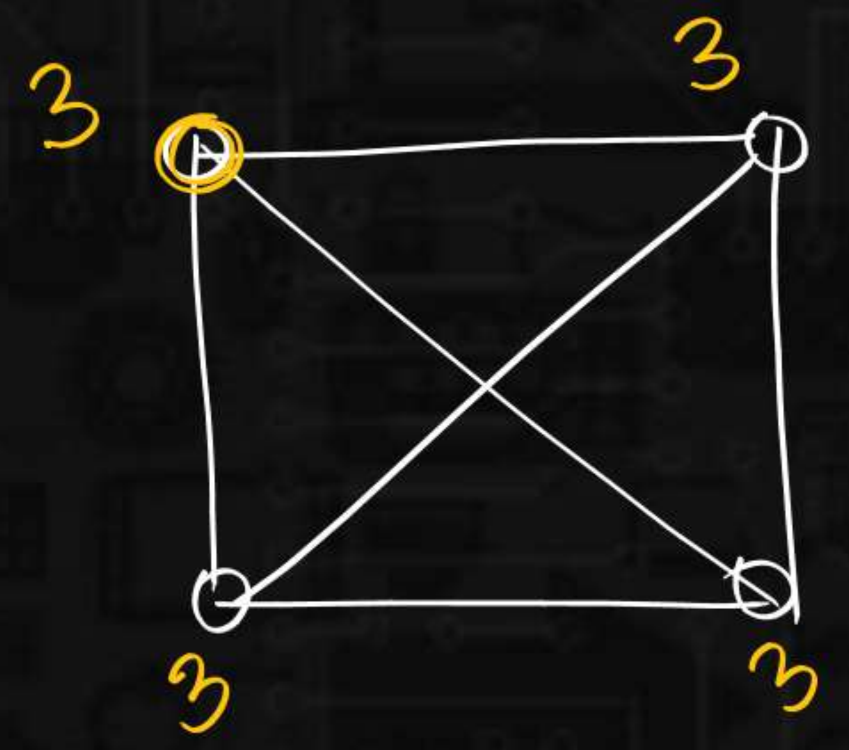






Basics of Graph

$n = 4$



Total vertices $= 4 = n$.

Degree of each vertex $= n-1\ (3)$

$$\sum d(v_i) = 2e$$

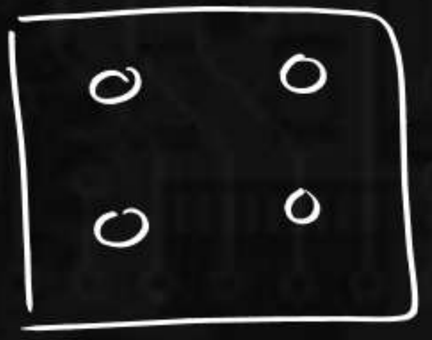
$$n-1+n-1+n-1 \ldots = 2e.$$

$$n(n-1) = 2e.$$

$$\boxed{e = \frac{n(n-1)}{2}}$$

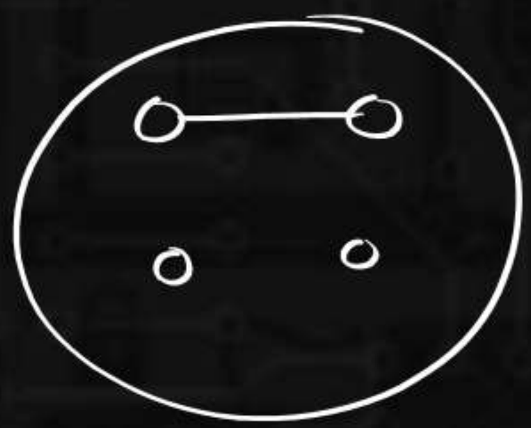
Basics of Graph

$n = 4$



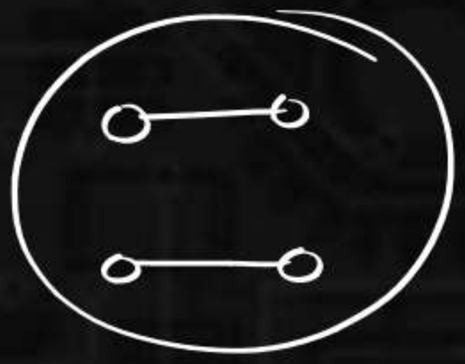
$e = 0$

$n = 4$



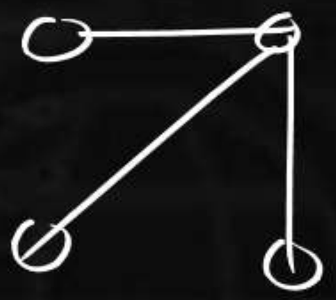
$e = 1$

$n = 4$



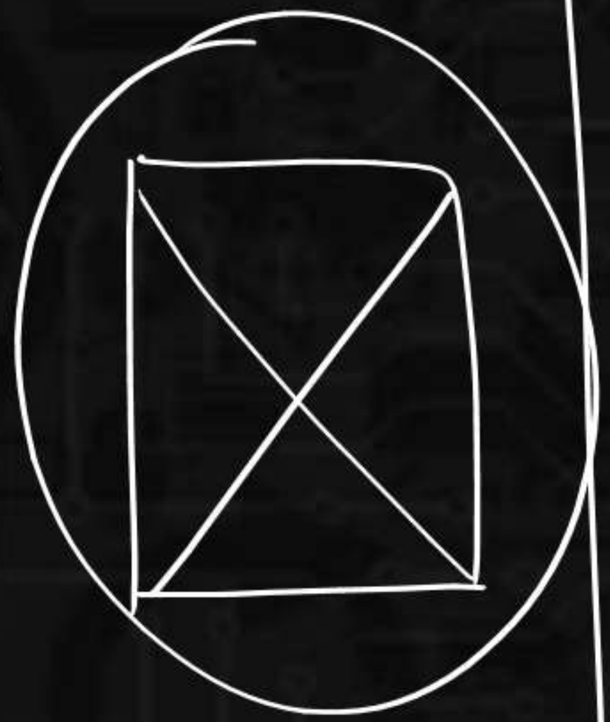
$e = 2$

$n = 4$



$e = 3$

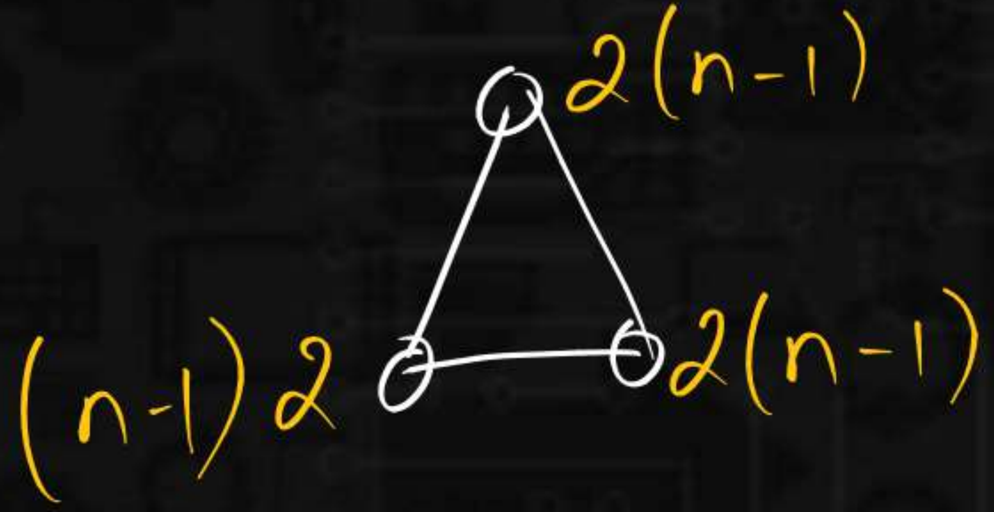
$e = 6$



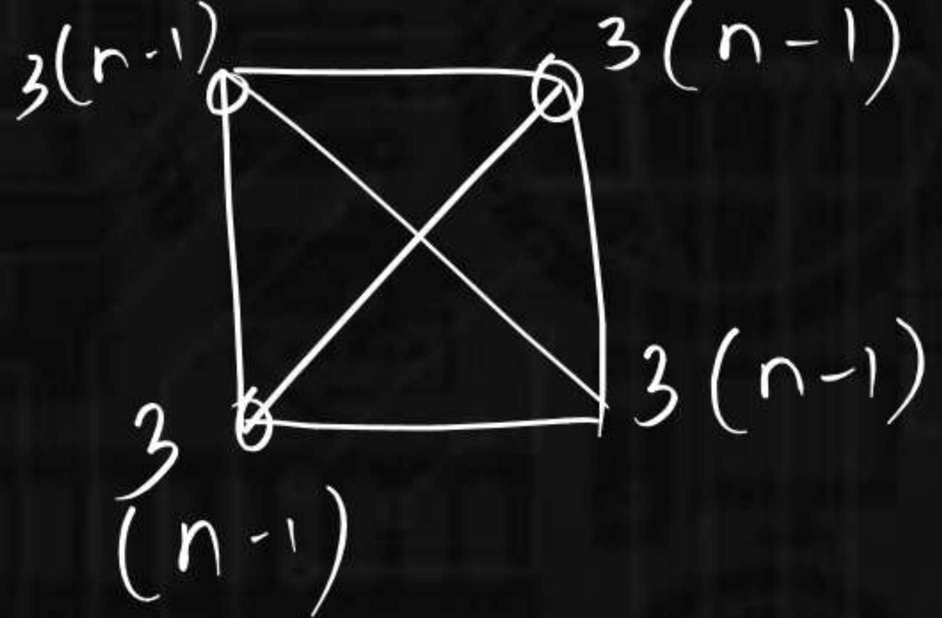
Basics of Graph

Thm 4: In Simple Graph maximum no. of edges $\leq \frac{n(n-1)}{2}$.

$n = 3$



$n = 4$



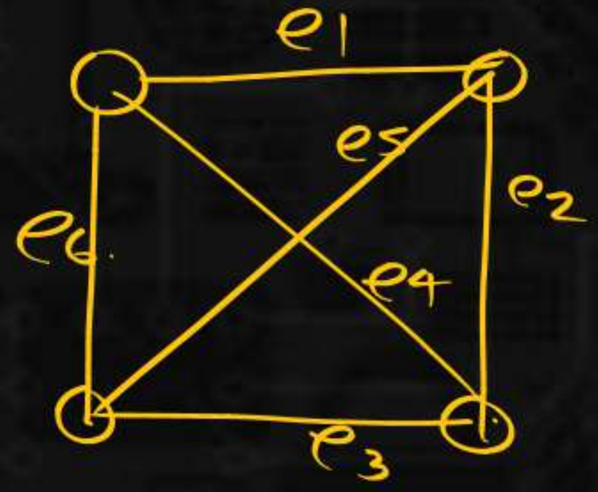
Basics of Graph

$$n = 4$$

$$e = \frac{n(n-1)}{2}$$

$$n = 4$$

$$e = \frac{4 \cdot 3}{2} = 6$$



$$3 + 3 + 3 + 3 = 2e$$

$$4 \times 3 = 2e$$

$$n-1 + n-1 \ldots n-1 = 2e$$

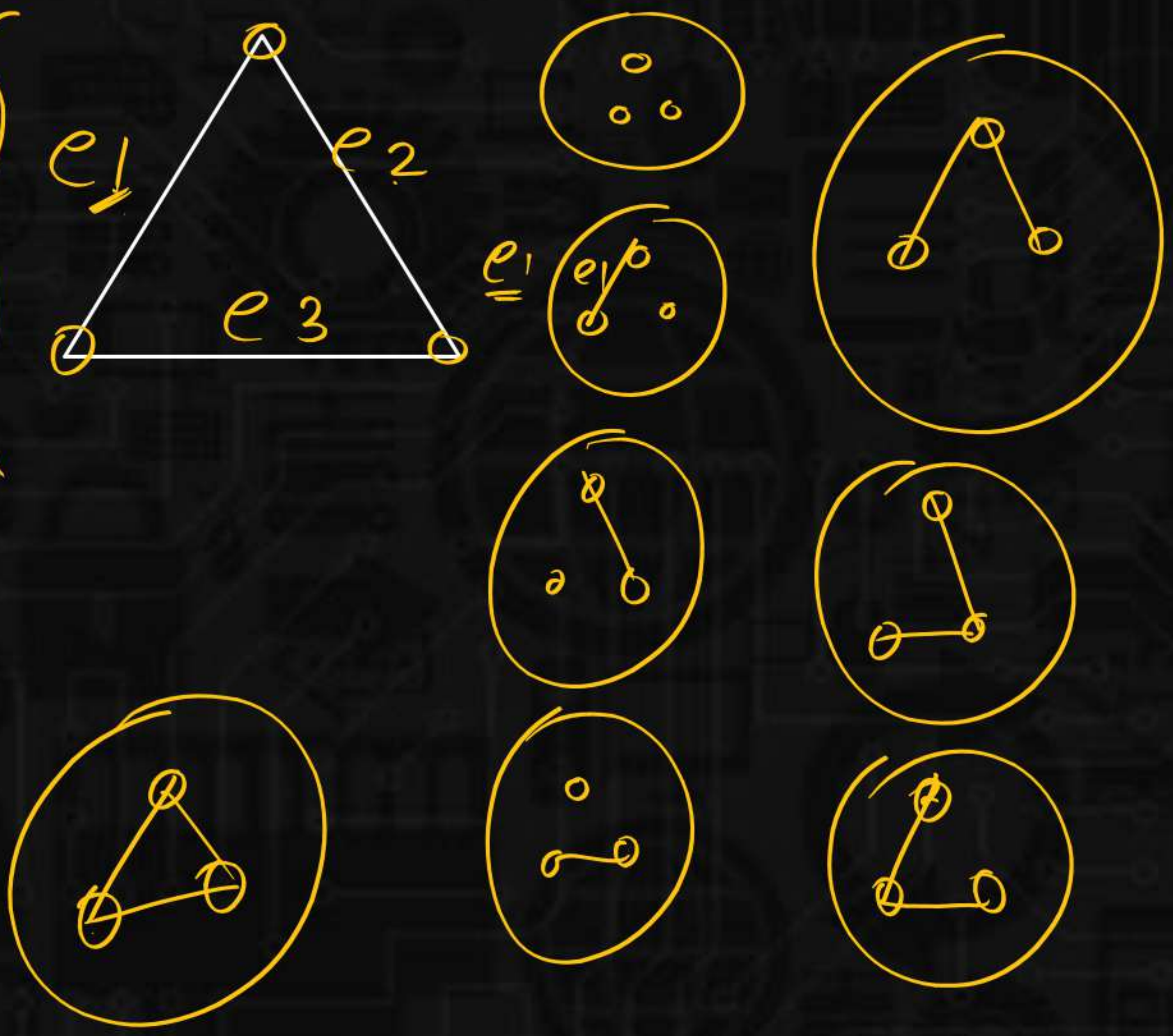
$$n(n-1) = 2e$$

$$e = \frac{n(n-1)}{2}$$

Basics of Graph

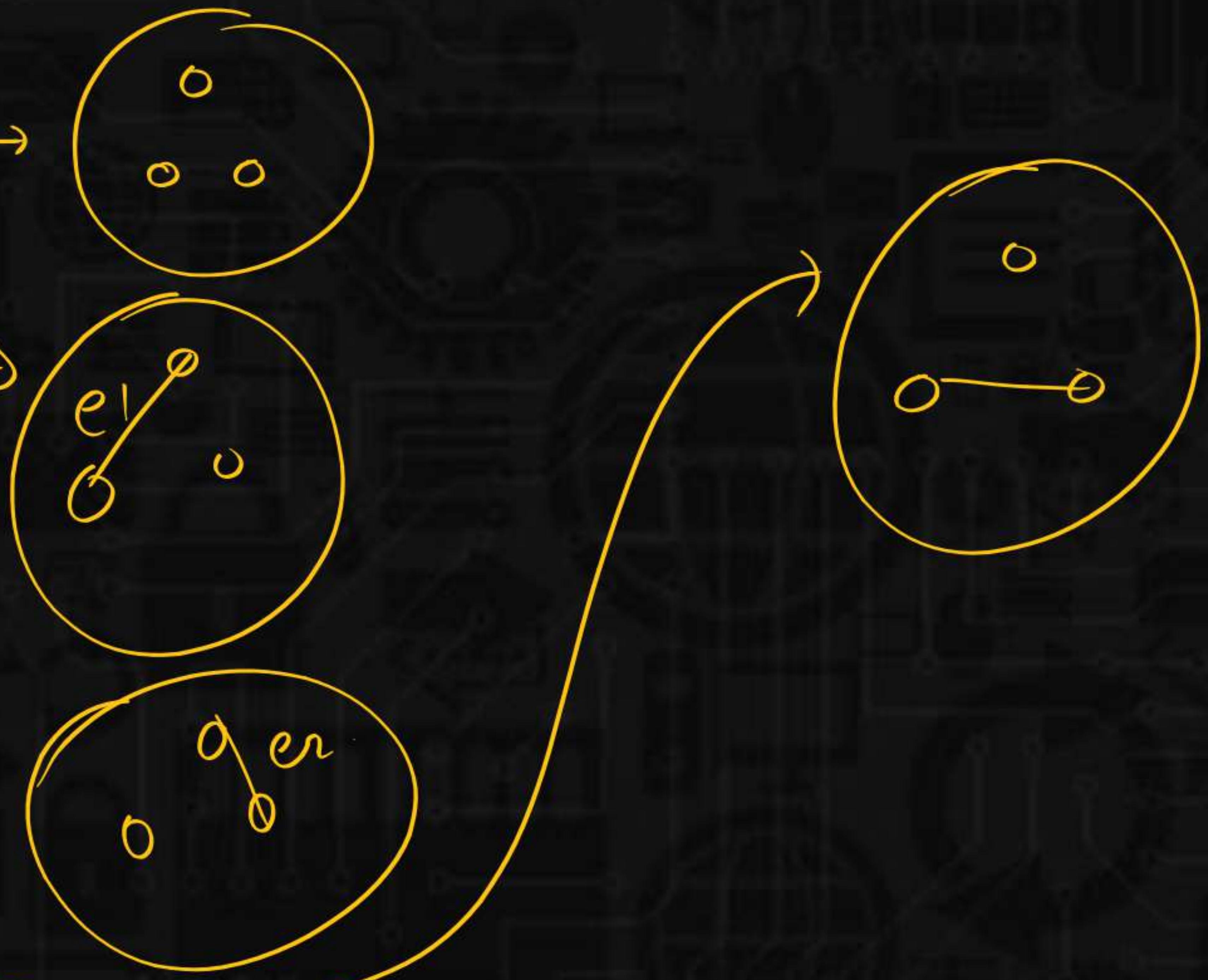
$n = 3$

e_1	e_2	e_3
0	0	0
0	0	1
0	1	0
1	0	0
0	1	1
1	0	1
1	1	0
1	1	1

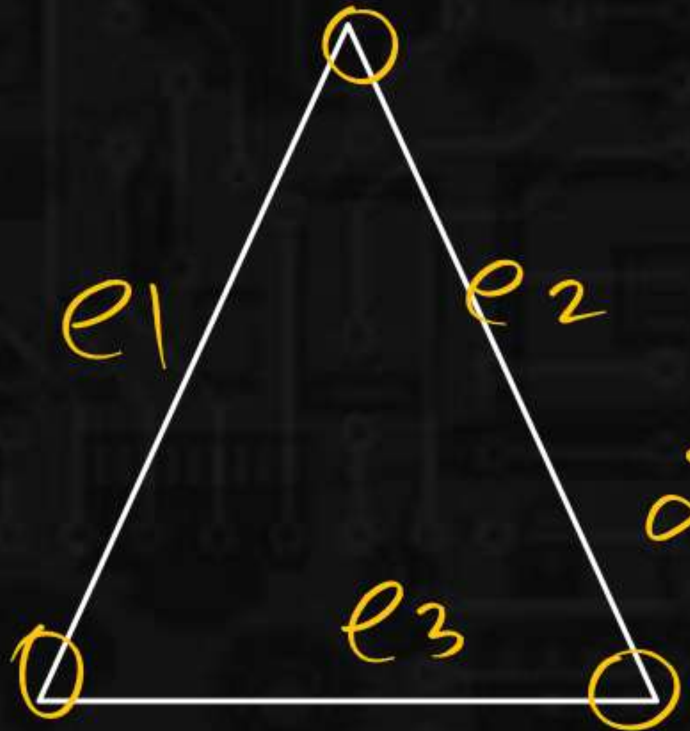


Basics of Graph

e1	e2	e3
0	0	0
1	0	0
0	1	0
0	0	1



Basics of Graph



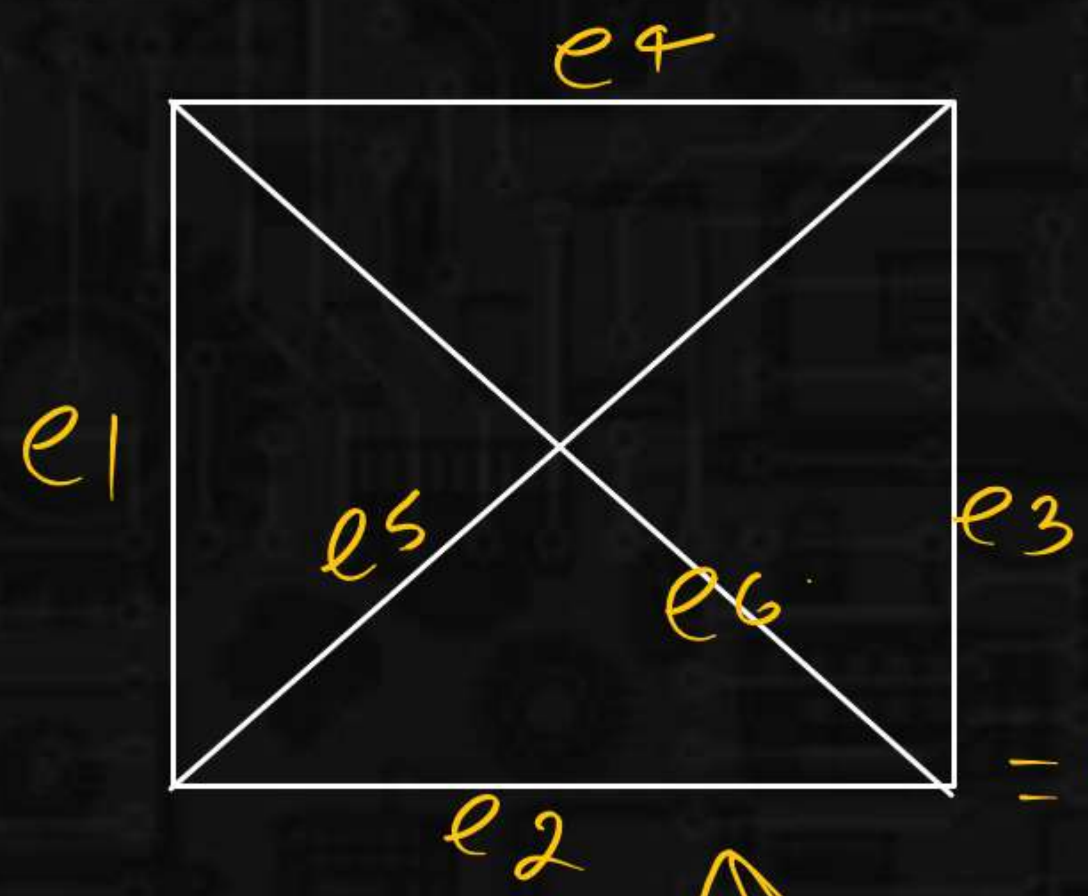
P
a · 2 · 2.

e_1 e_2 e_3.

2^3

$= 8$

Basics of Graph



e1 e2 e3 e4 e5 e6

$2^6 = 64$

$n = 4$ Total graphs $= 2^6$

$= 2^{\frac{4 \cdot 3}{2}}$

vertices is not changing

edges are changing

each edge $\rightarrow$ 2 possibilities

present absent

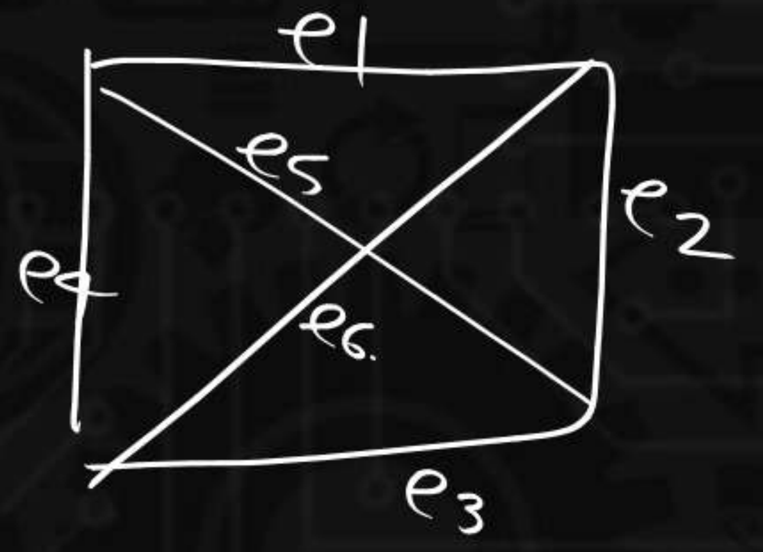
n = Total vertices

Total no. of graphs $2^{\frac{n(n-1)}{2}}$

$n = 4$.

Total no. of graphs $= 2^{6}$.

Total vertices $= 2^{\frac{4 \times 3}{2}}$



Total vertices $= n$.

Total no. of graphs $2^{\frac{n(n-1)}{2}}$

Total no. of graphs $= 2^{\text{Total no. of edges}}$

$= 2^{\frac{n(n-1)}{2}}$

edg <

Vertices.


THANK YOU GW
SOLDIERS !