Process Control.

Laplace Transform :-

Consider the function $f(t)$. The Laplace transform of the function defined as follows.

$$L[f(t)] = \bar{f}(s) = \int_0^\infty f(t) e^{-st} dt.$$

L-T is a transform of a function from the time domain (where time is the independent variable) to the s-domain (with s the independent variable). S is a variable defined in the complex plane.

$$S = a + jb.$$

Condition:

L-T exist if the integral $\int_0^\infty f(t) e^{-st} dt$ takes a finite value. Consider the function $f(t) = e^{at}$ where $a > 0$ then.

$$L[e^{at}] = \int_0^\infty e^{at} e^{-st} dt = \int_0^\infty e^{(a-s)t} dt$$

Now if $a-s > 0$ or $s < a$ the integral becomes unbounded. Consiquently the L-T of e^{at} is only defined for $s > a$.

L-T is a Linear operation. Justify.

$$L[a_1 f_1(t) + a_2 f_2(t)] = a_1 L[f_1(t)] + a_2 L[f_2(t)]$$

where a_1 and a_2 are constant parameters. The proof is straight forward.

$$L[a_1 f_1(t) + a_2 f_2(t)] = \int_0^\infty [a_1 f_1(t) + a_2 f_2(t)] e^{-st} dt$$

$$= a_1 L[f_1(t)] + a_2 L[f_2(t)]$$

Laplace Transformation of Some basic functions:

1. Exponential function:

$$f(t) = e^{-at} \quad \text{for } t \geq 0$$

The $L[e^{-at}] = \dfrac{1}{s+a}$.

$$L[e^{-at}] = \int_0^{\alpha} e^{-at} e^{-st} dt$$

$$= \int_0^{\alpha} e^{-(a+s)t} dt$$

$$= -\frac{1}{(s+a)} \left[e^{-(s+a)t} \right]_0^{\alpha}$$

$$= -0 + \frac{1}{s+a} \cdot 1 = \frac{1}{s+a}.$$

2. Ramp function:

This function is defined as:

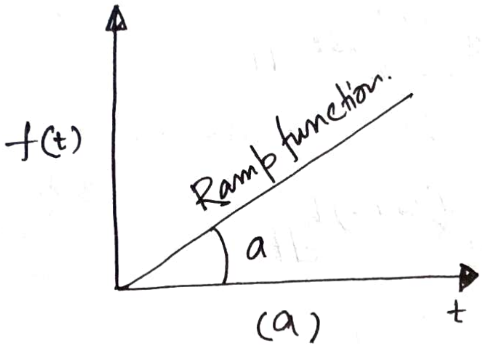
$$f(t) = at \quad \text{for } t \geq 0 \quad \text{with } a = \text{constant}$$

$$L[at] = \int_0^{\alpha} at \, e^{-st} dt$$

$$= \left[at \cdot \frac{(-1)}{-s} e^{-st} \right]_0^{\alpha} - \int_0^{\alpha} a \cdot \frac{(-1)}{s} e^{-st} dt$$

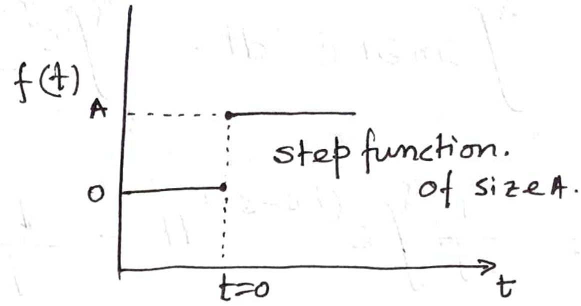
$$= 0 + \frac{a}{s} \int_0^{\alpha} e^{-st} dt = \frac{a}{s} \left[\frac{-1}{s} e^{-st} \right]_0^{\alpha}$$

$$\boxed{L[at] = a/s^2}$$



(a)

Ramp function



(b)

Step function

3. Step function:

$$f(t) = \begin{cases} A & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$$

$$\mathcal{L}\,[\text{step function of size A}] = \frac{A}{s}$$

Proof: We notice that a discontinuity in the value of the function exists at $t=0$ Such that $f(t=0)$ undifined.

The defination of L-T ~~from~~ requires the knowledge of the function at $t=0$

$$\mathcal{L}\,[f(t)] = \int_{\epsilon}^{T} f(t)\, e^{-st} dt$$

$\epsilon \to 0+$
$T \to \infty$

Thus for step function the upper limit $T = \infty$ but the lower limit is $t = 0+$ (very small but finite possitive time) instead of $t=0$.

$$\mathcal{L}\,[A] = \int_{0}^{\infty} A \cdot e^{-st} dt = \frac{A}{s}$$

Trigonometric functions:

Consider Sinousodial function $f(t) = \sin \omega t$.

$$\mathcal{L}\,[\sin \omega t] = \int_{0}^{\infty} \sin \omega t\, e^{-st} dt.$$

$$\int_0^{\infty} \sin\omega t \, e^{-st} dt = \int_0^{\infty} \frac{e^{j\omega t} - e^{-j\omega t}}{2j} e^{-st} dt$$

$$= \frac{1}{2j}\int_0^{\infty} e^{(j\omega - s)t} dt - \frac{1}{2j}\int_0^{\infty} e^{-(j\omega + s)t} dt$$

$$= \frac{1}{2j}\int_0^{\infty} e^{-(s-j\omega)t} dt - \frac{1}{2j}\int_0^{\infty} e^{-(s+j\omega)t} dt$$

$$= \frac{1}{2j}\left[\frac{e^{-(s-j\omega)t}}{-(s-j\omega)} - \frac{e^{-(s+j\omega)t}}{-(s+j\omega)}\right]_0^{\infty}$$

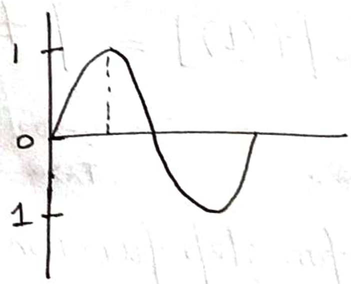
$$= \frac{1}{2j}\left[\frac{1}{(s-j\omega)} - \frac{1}{(s+j\omega)}\right]$$

$$= \frac{1}{2j} \frac{\cancel{s}+j\omega - \cancel{s}+j\omega}{(s-j\omega)(s+j\omega)}$$

$$= \frac{\cancel{2j}\omega}{\cancel{2j}} \times \frac{1}{(s^2+\omega^2)}$$

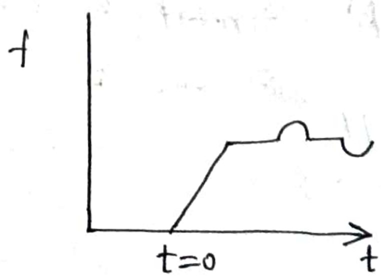
$$\boxed{L(\sin\omega t) = \frac{\omega}{(s^2+\omega^2)}}$$

$$\boxed{L(\cos\omega t) = \frac{s}{s^2+\omega^2}}$$

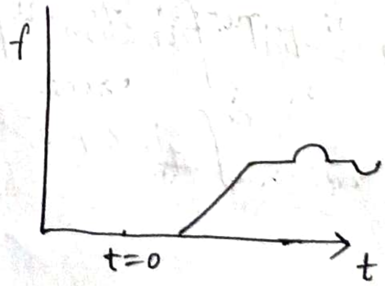


Sinusodial function.

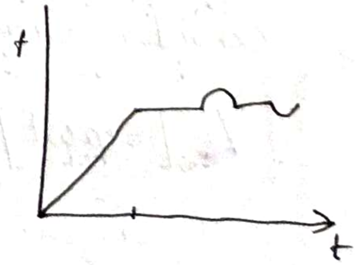
Translated function:



Normal function
$f(t)$



Delayed function.
$f(t-t_0)$



Advanced function
$f(t+t_0)$

Relationship of these three function:

$$f(t+t_0) = f(t) = f(t-t_0)$$

Now $\mathcal{L}[f(t)] = \bar{f}(s)$

For delayed function the laplace transform,

$$\mathcal{L}[f(t-t_0)] = \int_0^{\infty} f(t-t_0)\, e^{-st}\, dt$$

$$= e^{-st_0} \int_0^{\infty} f(t-t_0)\, e^{-s(t-t_0)}\, dt$$

$$\rightarrow\ = e^{-st_0} \int_{-t_0}^{\infty} f(\tau)\, e^{-s\tau}\, d\tau$$

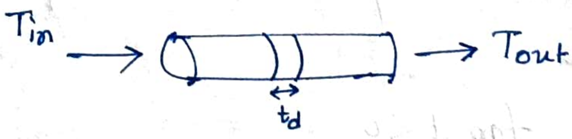
Let $t - t_0 = \tau$

$$\boxed{\mathcal{L}[f(t-t_0)] = e^{-st_0}\, \bar{f}(s)}$$

For advance function similarly.

$$\boxed{\mathcal{L}[f(t+t_0)] = e^{st_0}\, \bar{f}(s)}$$

Ex-1 Let the flow of an incompressible liquid through a pipe is shows



$$T_{out} = T_{in}(t-t_d)$$

$$\Rightarrow \quad t_d = \frac{\text{Volume of pipe}}{\text{Volumetric flowrate}} = \frac{AL}{A \cdot U_{AV}} = \frac{L}{U_{AV}}$$

The temperature of the outlet is equal to the temperature of inlet by delayed by t_d where t_d is the dead time that is the time required for a change in the inlet to reach the outlet.

If $\mathcal{L}[T_{in}(t)] = \bar{T}_{in}(s)$

&

$$\mathcal{L}[T_{out}(t)] = \bar{T}_{out}(s)$$

$$= \mathcal{L}[T_{in}(t - t_d)]$$

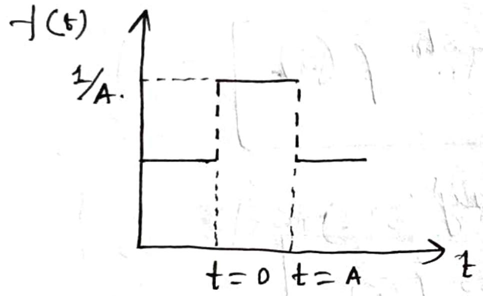
$$= e^{-t_d s}\, \bar{T}_{in}(s)$$

using the properties of delayed function.

Unit pulse function:

Consider a function $f(t) = \frac{1}{A}$ for $t > 0$

$= 0$ for $t < 0$



The height $1/A$ and with A the area under the curve is $\frac{1}{A} \times A = 1$

This function is called unit pulse function of duration A and is defined as,

$$\delta_A(t) = \begin{cases} 0 & \text{for } t < 0 \\ \frac{1}{A} & \text{for } 0 < t < A \\ 0 & \text{for } t > 0 \end{cases}$$

It can be described as the difference of two step function of equal size $1/A$. The first step function occurs at $t = 0$ while the second is delayed by A unit of time.

1st step function : $f_1(t) = \begin{cases} 0 & t<0 \\ 1/A & \text{when } t>0 \end{cases}$

2nd step function : $f_2(t) = \left\{ \begin{array}{ll} 0 & t>A \\ 1/A & t>A \end{array} \right\} = f_1(t-A)$

Then $\delta_A(t)$ = unit pulse of duration A.

$$= f_1(t) - f_2(t)$$

$$= f_1(t) - f(t-A)$$

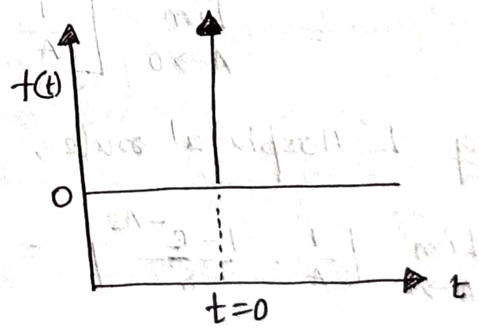
$$\mathcal{L}[\delta_A(t)] = \mathcal{L}[f_1(t)] - \mathcal{L}[f_2(t)]$$

$$= \frac{1}{AS} - e^{-AS}\frac{1}{AS}$$

$$\boxed{\mathcal{L}(\delta_A(t)) = \frac{1}{A} \cdot \frac{(1-e^{-AS})}{S}}$$

Unit impulse function :-

Consider that the duration A of unit pulse function is allowed to shrink approaching zero, while the height $1/A$ approaches infinity and the area under the curve is always 1.



$$\lim_{A \to 0} \left(A \cdot \frac{1}{A}\right) = 1.$$

As $A \to 0$ we take the function shown in. This function is called unit impulse or dirac function. It is usually represented as $\delta(t)$

The function is defined as equal to zero for all times except $t=0$. Since the area under the curve is equal to 1 it is clear that this is true for unit impulse.

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$L[\delta(t)] = 1$$

$$\therefore \boxed{\delta(t) = \lim_{A\to 0} \delta_A(t)}$$ → Relation of unit pulse and unit impulse.

$$L[\delta(t)] = L\left[\lim_{A\to 0} \delta_A(t)\right]$$

$$= \int_0^{\infty} \lim_{A\to 0} \delta_A(t) e^{-st}\,dt$$

$$= \lim_{A\to 0} \int_0^{\infty} \delta_A(t) e^{-st}\,dt$$

$$= \lim_{A\to 0} \int_0^{\infty} \frac{1}{A} \cdot \frac{(1-e^{-As})}{s} \cdot e^{-st}\,dt.$$

$$= \lim_{A\to 0} \left[\frac{1}{A} \cdot \frac{1-e^{-As}}{s}\right]$$

using L'Hosphital rule,

$$\lim_{A\to 0} \left[\frac{1}{A} \cdot \frac{1-e^{-As}}{s}\right] = \lim_{A\to 0} \left[\frac{Se^{-As}}{s}\right] = 1$$

Remark :- $f(s) = L[f(t)] = \dfrac{q_1(s)}{q_2(s)}$

where $q_1(s)$ and $q_2(s)$ are two polynomials in s

$$q_1(s) = K_m s^m + K_{m-1} s^{m-1} + \cdots + K_1 s + K_0$$

$$q_2(s) = l_n s^n + l_{n-1} s^{n-1} + \cdots + l_1 s + l_0$$

Ex-2

If $f(t) = \cos \omega t$ then $\bar{f}(s) = ?$

$$\bar{f}(s) = \frac{s}{s^2+\omega^2} = \frac{q_1(s)}{q_2(s)}$$

$$q_1(s) = 1.s + 0$$

$$q_2(s) = 1.s^2 + \omega^2$$

First order and Second order Transfer function.

Transfer function:

$$G_p = \frac{O}{I}$$

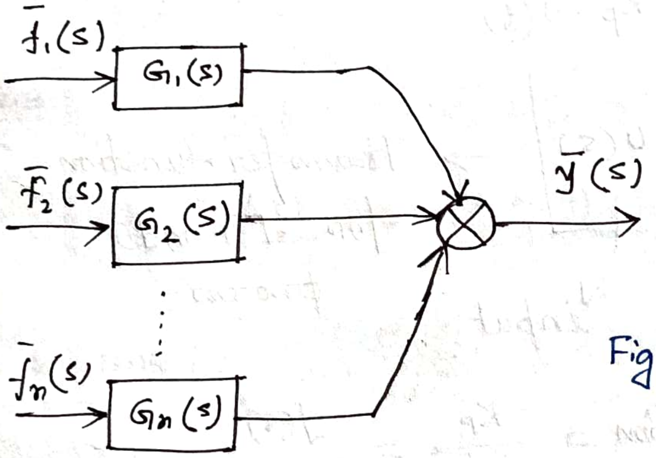


Fig: Several input and single ~~input~~ output.

Transfer function $(G_s) = \dfrac{\text{Laplace transform of output}}{\text{Laplace transform of input}}$

First order process:

Output $y(t)$ is modeled by a first order differential equation:

$$a_1 \frac{dy}{dt} + a_0 y = \cancel{bf(t)}\; bf(t)$$

$$\boxed{\frac{a_1}{a_0}\frac{dy}{dt} + y = \frac{b}{a_0} f(t)}$$

First order process.

$$\tau_p \frac{dy}{dt} + y = K_p u(t) \quad \text{---- (i)}$$

y = output $\quad u(t)$ = input.

at initial ~~$\tau_p = 0$~~ and y is also $y(0) = 0$

Now $L\left(\frac{dy}{dt}\right) = s.Y(s) - y(0)$

From equation (i)

$$\tau_p \left(sY(s) - \cancel{y(0)}^{0}\right) + Y(s) = K_p \, u(s)$$

$$Y(s)(\tau_p s + 1) = K_p . U(s)$$

$$\boxed{Y(s) = \frac{K_p}{(\tau_p s + 1)} . U(s)}$$ → Transfer function for 1st order process.

output

input

Now, transfer function $= \frac{K_p}{\tau_p s + 1} = \frac{Y(s)}{U(s)}$

Second order process :-

For second order process

This is our output

This is our input

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b f(t)$$

if $a \neq 0$ then the equation changes to

$$\tau^2 \frac{d^2y}{dt^2} + 2\xi\tau \frac{dy}{dt} + y = K_p f(t)$$

Where $\tau^2 = \frac{a_2}{a_0}$, $2\xi\tau = \frac{a_1}{a_0}$ $K_p = \frac{b}{a_0}$

Now

$$L\left(\tau^2\frac{d^2y}{dt^2} + 2\xi\tau\frac{dy}{dt} + y\right) = L\left(K_p \cdot f(t)\right)$$

$$\tau^2 s^2 y(s) + 2\xi\tau s\, y(s) + y(s) = K_p \cdot f(s)$$

$$y(s)\left[\tau^2 s^2 + 2\xi\tau s + 1\right] = K_p \cdot f(s)$$

$$y(s) = \frac{K_p}{(\tau^2 s^2 + 2\xi\tau s + 1)} \cdot f(s)$$

Hence the transfer function is given by:

$$\boxed{G(s) = \frac{K_p}{(\tau^2 s^2 + 2\xi\tau s + 1)}}$$

N.B

$$L\left[\frac{d^2y}{dt^2}\right] = s^2 y(s) - s\,y(0) - y'(0)$$

$$L\left[\frac{dy}{dt}\right] = s\,y(s) - y(0)$$

at any time: $\tau_p^2\frac{d^2y(t)}{dt^2} + 2\xi\tau_p\frac{dy(t)}{dt} + y(t) = K_p\, x(t)$ --- (i)

at steadystate: $\tau_p^2\frac{d^2y(s)}{dt^2} + 2\xi\tau_p\frac{dy(s)}{dt} + y(s) = K_p\, x(s)$ --- (ii)

Substituting (ii) from (i)

$$\tau_p^2\left\{\frac{d^2(y(t) - y(s))}{dt^2}\right\} + 2\xi\tau_p\left\{\frac{d}{dt}(y(t) - y(s))\right\} + (y(t) - y(s))$$

$$= K_p\left[x(t) - x(s)\right]$$

$$\tau_p^2\frac{d^2Y(t)}{dt^2} + 2\xi\tau_p\frac{dY(t)}{dt^2} + Y(t) = K_p\, X(t)$$

Taking Laplace,

$$\tau_p^2\left[s^2Y(s) - s\cancel{Y(0)}^{0} - \cancel{Y'(0)}^{0}\right] + 2\xi\tau_p\left[sY(s) - \cancel{Y(0)}^{0}\right] + Y(s) = K_p X(s)$$

$$\left[\text{As. } Y(0) = y'(t=0) - y(s)\right.$$
$$= y(0) - y(0)$$
$$= 0$$

$$Y(s)\left[\tau_p^2 s^2 + 2\xi\tau_p s + 1\right] = K_p X(s)$$

$$\boxed{Y(s) = \frac{K_p}{(\tau_p^2 s^2 + 2\xi\tau_p s + 1)} X(s)}$$

Transfer function $= \dfrac{Y(s)}{X(s)} = \dfrac{K_p}{\tau_p^2 s^2 + 2\xi\tau_p s + 1}$

where τ_p = Time constant for second order.

ξ = damping function. it indicates how our system faster or slower.

Roots of second order transfer function:

$$\frac{Y(s)}{X(s)} = \frac{K_p}{(\tau^2 s^2 + 2\xi\tau_p s + 1)}$$

To determine the roots.

$$\tau^2 s^2 + 2\xi\tau_p s + 1 = 0$$

let two roots be s_1, s_2

then $s_1, s_2 = \dfrac{-2\xi\tau_p \pm \sqrt{(2\xi\tau_p)^2 - 4\tau_p^2}}{2\tau_p^2}$

$$\boxed{s_1, s_2 = -\frac{\xi}{\tau_p} \pm \frac{\sqrt{\xi^2 - 1}}{\tau_p}}$$

Q Let we have a transfer function defined as..,

$\frac{7}{5s^2+3s+1}$. Determine it's time constant, Damping factor.

Decay ratio, overshoot.

$\Rightarrow$ From second order transfer function.

$$G(s) = \frac{K_p}{\tau_p^2 s^2 + 2\xi\tau_p s + 1}$$

comparing this with given equation.

(i) $\tau_p^2 = \sqrt{5}$ = time constant.

(ii) $2\xi\tau_p = 3$

$$\xi = \frac{3}{2\sqrt{5}}$$

(iii) Decay ratio, c/a.

$$= \exp\left(\frac{-2\pi\xi}{\sqrt{1-\xi^2}}\right)$$

$$= \exp\left(-\frac{2\pi\xi}{\sqrt{1-\frac{9}{2\times5}}}\right)$$

$$= \cancel{31.4}\ \cancel{e^{-9.92}}$$

(iv) Overshoot

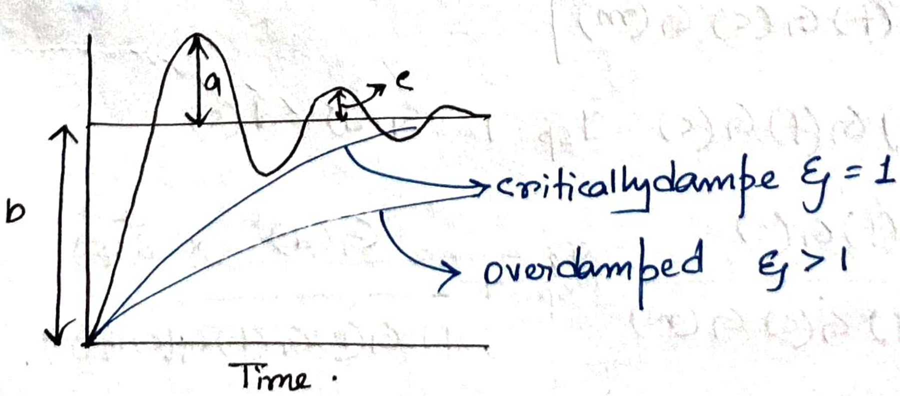
$$b/a = \exp\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right)$$

$$= \cancel{\exp\left(-\frac{\pi\xi}{\sqrt{0.1}}\right)}$$

$$= \cancel{\exp(-19.84)})$$

$\therefore$ Decay Ratio = (Overshoot)^2

Concept of Decay ratio and Overshoot :-



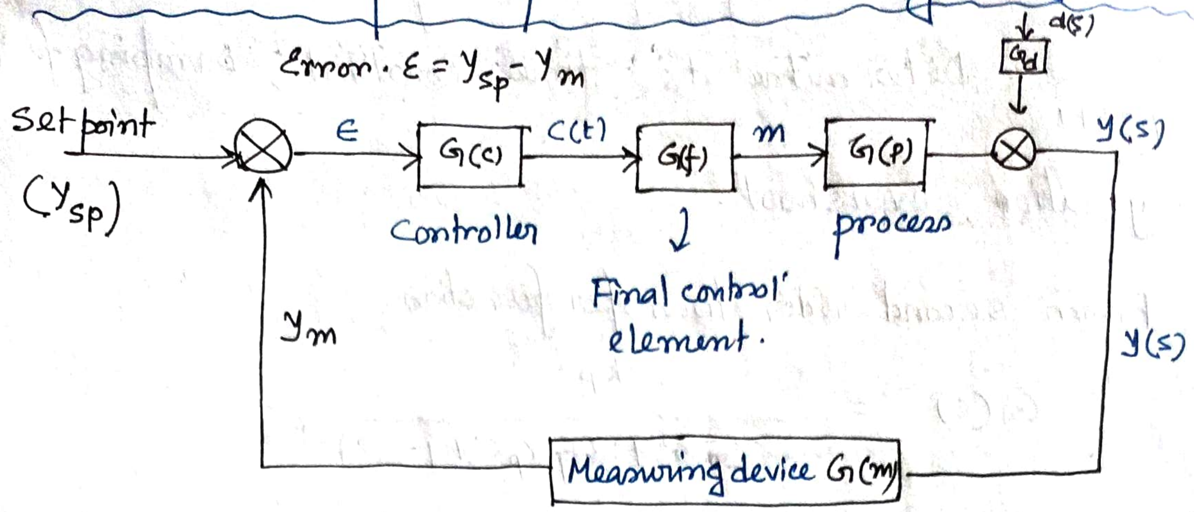
$$\boxed{\text{Decay ratio} = (\text{overshoot})^2}$$

$$\text{Decay ratio} = \frac{C}{A} = \exp\left(-\frac{2\pi\xi}{\sqrt{1-\xi^2}}\right)$$

$$\text{Overshoot} = \frac{A}{B} = \exp\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right)$$

Characteristic Equations for Servo and regulatory control:

Error. $\varepsilon = Y_{sp} - Y_m$



$$y(s) = \frac{G(c)\,G(f)\,G(P)}{1 + G(c)\,G(f)\,G(P)\,G(m)}\,\bar{y}_{sp}(s) + \frac{1}{1 + G(s)\,G(f)\,G(P)\,G(m)}\,\bar{d}(s)$$

Derivation:

$$y(s) = m\,G(P) + G(d).\bar{d}(s)$$

$$y(s) = G(f)\,c(t)\cdot G(P) + G(d)\,\bar{d}(s)$$

$$= G(P)\,G(f)\,\varepsilon\cdot G(c) + G(d)\,\bar{d}(s)$$

$$= G(P)\,G(f)\,G(c)\left[\bar{y}_{sp} - Y_m\right] + G(d)\,\bar{d}(s)$$

$$= G(P)\,G(f)\,G(c)\left[\bar{y}_{sp} - G(m)\,Y_s\right] + G(d)\,\bar{d}(s)$$

$$y(s)\left[1 + G(P)\,G(f)\,G(c)\,G(m)\right]$$

$$= G(P)\,G(f)\,G(c)\cdot\bar{y}_{sp} + G(d)\cdot\bar{d}(s)$$

$$y(s) = \frac{G(P)\,G(f)\,G(c)}{1 + G(P)\,G(f)\,G(c)\,G(m)}\cdot\bar{y}_{sp} + \frac{G(d) \times \bar{d}(s)}{1 + G(P)\,G(f)\,G(c)\,G(m)}$$

This is Servo function (when there is no disturbance)

This is Regulatory function (when $Y_{sp} = 0$)

There are some assumptions.

The final control measuring device's transfer function $G(m) = 1$ because we want our final measuring to be accurate.

Routh stability analysis :-

This basically a stability factor of a system.

Let at system is defined by a polynomial,

$$a_5 s^5 + a_4 s^4 + a_3 s^3 + a_1 s + a_0 = 0$$

This term should be +ve and non zero

if it is -ve the multiply both side by -1.

	Col-1	Col-2	Col-3	
Row-1	a_5	a_3	a_1	----
Row-2	a_4	a_2	a_0	----
Row-3	A_1	A_2	A_3	----
Row-4	B_1	B_2	B_3	----

All the element in the first column must be +ve.

if any one of them negative whole system is unstable.

$$A_1 = \frac{a_4 a_3 - a_2 a_5}{a_4} \qquad A_2 = \frac{a_2 a_1 - a_0 a_3}{a_2}$$

$$B_1 = \frac{A_1 a_2 - A_2 a_4}{A_1} \qquad B_2 = \frac{A_2 a_0 - A_3 a_2}{A_2}$$

Q Analyse wheather the system is stable or not

$$s^3 + 2s^2 + (2+K_c)s + \frac{K_c}{\tau_1} = 0$$

Row-1	1	$(2+K_c)$	0
Row-2	2	$\frac{K_c}{\tau_I}$	0
Row-3	A_1	0	
Row-4	B_1	0	

$$A_1 = \frac{2(2+K_c) - K_c/\tau_I}{2}$$

$$B_1 = \frac{A_1 \times \frac{K_c}{\tau_I}}{A_1}$$

$$B_1 = \frac{K_c}{\tau_I}$$

The system will be stable is $\frac{2(2+K_c) - K_c/\tau_I}{2} > 0$

Here we have two parameters

K_c = gain (we always want high gain so we will increase it gradually)..

τ_I = Time (we want less time, we will start the analysis with less time)

• Case-1

let $K_c = 5$ $\tau = 0.1$

$$\frac{2(2+5) - 5/0.1}{2} = \frac{14-50}{2} = -ve.$$

Case-2

$K_c = 5$ $\tau = 0.4$

$$\frac{2(2+5) - 12.5}{2} = +ve$$

Case-3

$K_c = 7$ $\tau = 0.4$

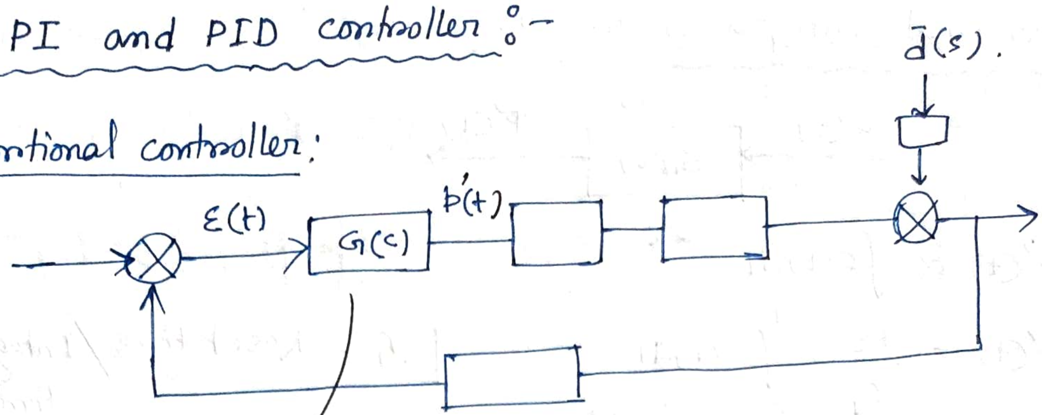
$$\frac{2(2+7) - 17.5}{2}$$

$= +ve.$

The system will be stable if $K_c > 7$ and $\tau \geq 0.4$

P, PI and PID controller :-

Proportional controller:



we want to determine the transfer function of this controller.

For proportionality controller

output $\propto$ error

$p'(t) \propto \varepsilon(t)$

at any time $p'(t) = K_c \, \varepsilon(t)$

at steady state $p'(s) = K_c \, \varepsilon(s)$

$p'(t) - p'(s) = K_c(\varepsilon(t) - \varepsilon(s))$

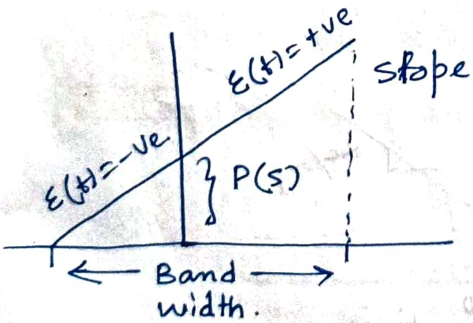
$p'(t) = K_c \, \varepsilon(t)$

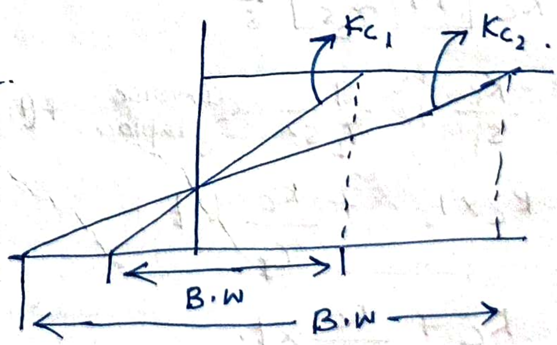
Taking Laplace, $p'(s) = K_c \, \varepsilon(s)$

$$\boxed{\frac{p'(s)}{\varepsilon(s)} = K_c = G(c)}$$

Transfer function.

Graphical analysis: $p(t) = p(s) + K_c \varepsilon(t)$.





For higher K_c Bandwidth decreases.

$$K_c \propto \frac{1}{B.W}.$$

Proportional Integral controller :-

$$\varepsilon(t) \longrightarrow \boxed{G(s)} \longrightarrow P'(t)$$

$$P'(t) \propto \int \varepsilon(t)\,dt$$

$$P'(t) = \frac{K_c}{\tau_I} \int_0^t \varepsilon(t)\,dt$$

[τ_I = Reset time / Integral time]

$$\boxed{P'(t) = K_c\,\varepsilon(t) + \frac{K_c}{\tau_I} \int_0^t \varepsilon(t)\,dt}$$

→ performance error.

Taking Laplace.

$$P(s) = K_c\,\varepsilon(s) + \frac{K_c}{\tau_I}\left[\frac{\varepsilon(s)}{s}\right]$$

$$P(s) = K_c\,\varepsilon(s)\left[1 + \frac{1}{\tau_I s}\right]$$

$$\frac{P(s)}{\varepsilon(s)} = K_c\left[1 + \frac{1}{\tau_I s}\right]$$

$$\boxed{G(s) = K_c\left[1 + \frac{1}{\tau_I s}\right]}$$

→ Transferfunction for P-I controller.

For unit step

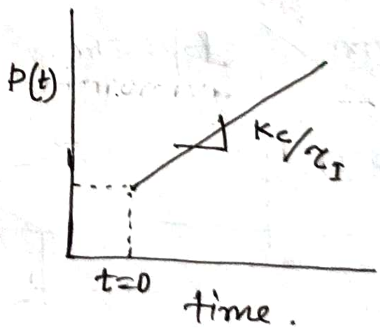
$$P(s) = K_c\left[1 + \frac{1}{\tau_I s}\right]\frac{1}{s}$$

$$P(s) = \frac{K_c}{s} + \frac{1}{T_I s^2}$$

Inverse laplace.

$$P(t) = K_c \times 1 + \frac{K_c}{\tau_I} \times t$$

$$P(t) = K_c + \frac{K_c}{\tau_I} \times t.$$



N.B: Inverse laplace of $\frac{1}{s} = 1$.

Inverse laplace of $\frac{1}{s^2} = t$

Proportional Integral Derivative Controller:

$$\varepsilon(t) \longrightarrow \boxed{\text{D.Controller.}} \longrightarrow P(t).$$

$$\boxed{P'(t) \propto \frac{d}{dt}(\varepsilon(t))} \qquad \boxed{P'(t) = K_P \tau_D \frac{d\varepsilon(t)}{dt}.}$$

If $\varepsilon(t)$ error is constant then derivative controller cannot work. It cannot used as single/alone it is often used with PI controller.

Performance equation:

$$P'(t) = K_c \varepsilon(t) + \frac{K_c}{\tau_I}\int_0^t \varepsilon(t)dt + K_c \tau_D \frac{d\varepsilon(t)}{dt}$$

$$P'(t) = \varepsilon(t)\left\{K_c + \frac{K_c}{\tau_I s} + K_c \tau_D \cdot s\right\}$$

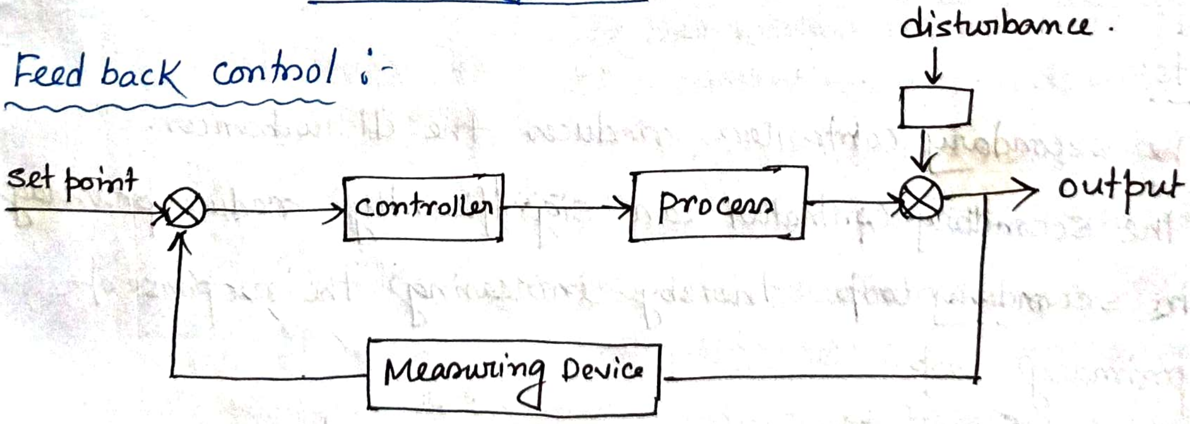
$$\boxed{\frac{P'(s)}{\varepsilon(s)} = K_c\left[1 + \frac{1}{\tau_I s} + \tau_D s\right]}$$

[N.B: for unit step. $\varepsilon(s) = \frac{1}{s}$.]

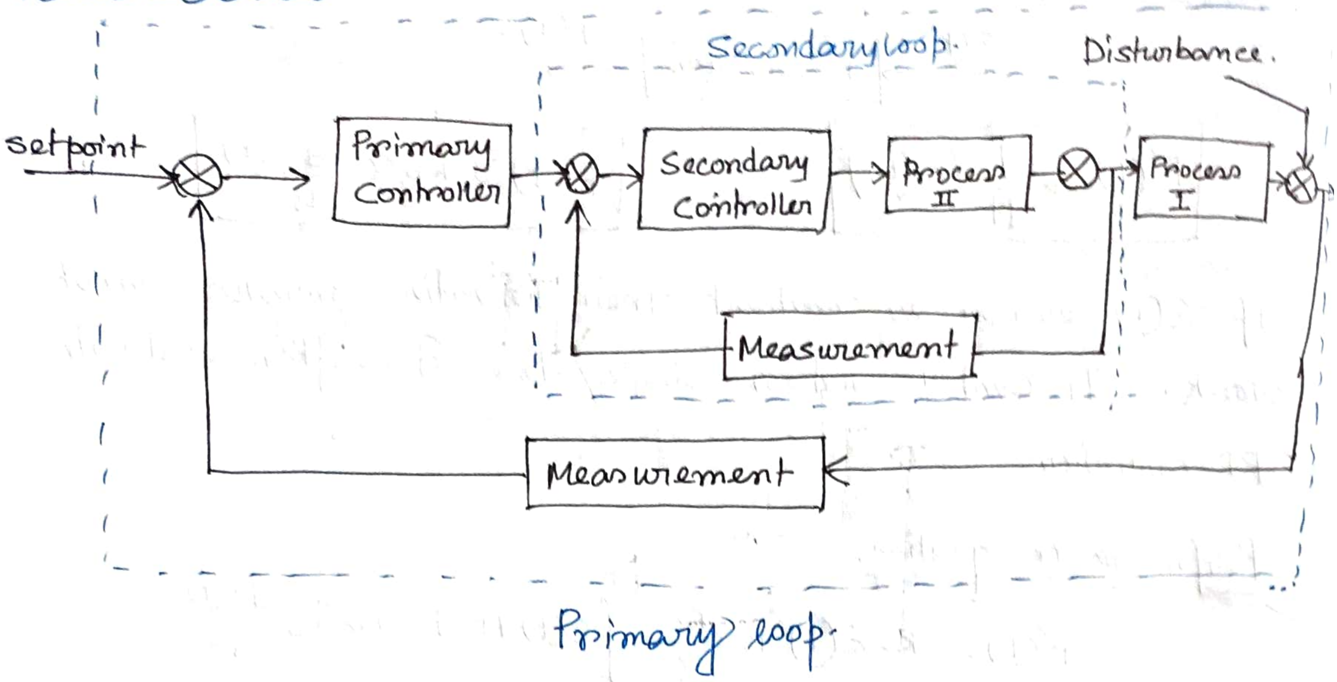
Transfer function for PID.

Block Diagrams

Feed back control :-



Cascade Control :-



Key features:-

i) There is one manupulated variable.

ii) More than one measurements.

iii) There two Controller. Secondary controller acts as the first line of deffance against disturbances. prevent it to affect the primary controller.

iv) The secondary loop should have faster dynamics.

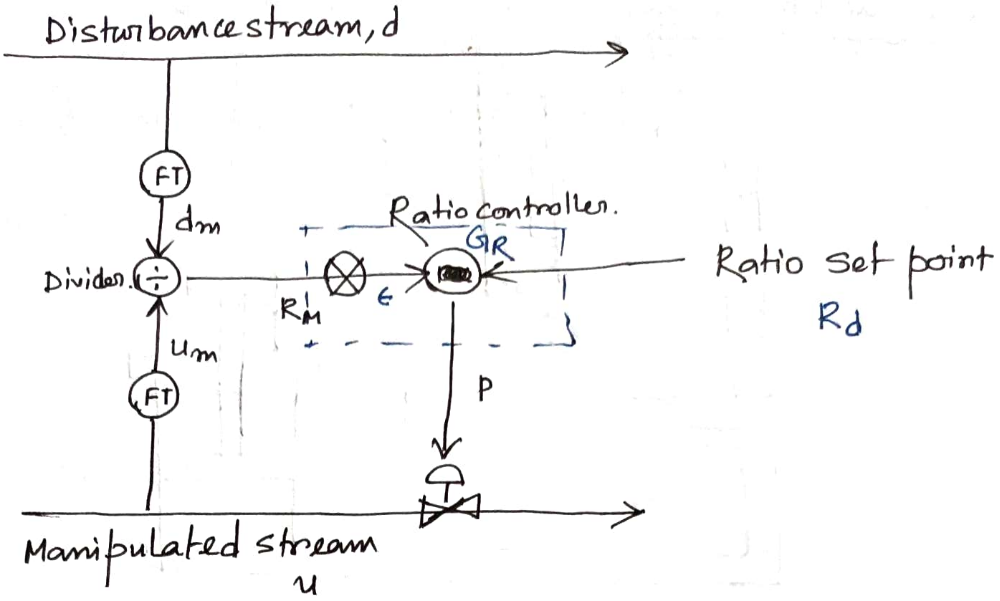
Benifits:

i) The secondary controller reduces the disturbances.

ii) The secondary controller can significantly reduce phase lag in secondary loop thereby increasing the response of primary loop.

iii)

Ratio~~nal~~ Controller:



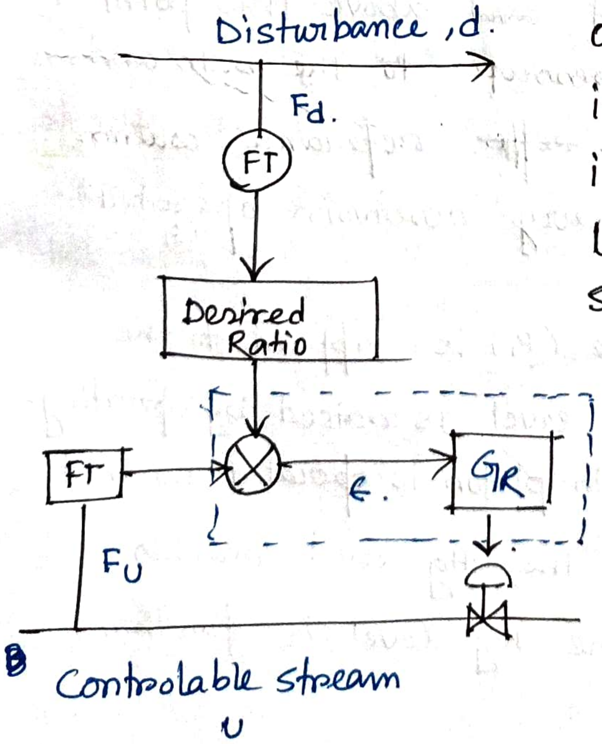
$$R = \frac{U}{d}.$$

Ratio control controls the flow rates of two streams u and d. This ratio is compared to the desired ratio and deviation between desired and measured ratio constitutes.
Here the gain (K_p) is non linear and it is a function of d.

$$K_p = \left(\frac{\partial R}{\partial U}\right) = \frac{1}{d}.$$

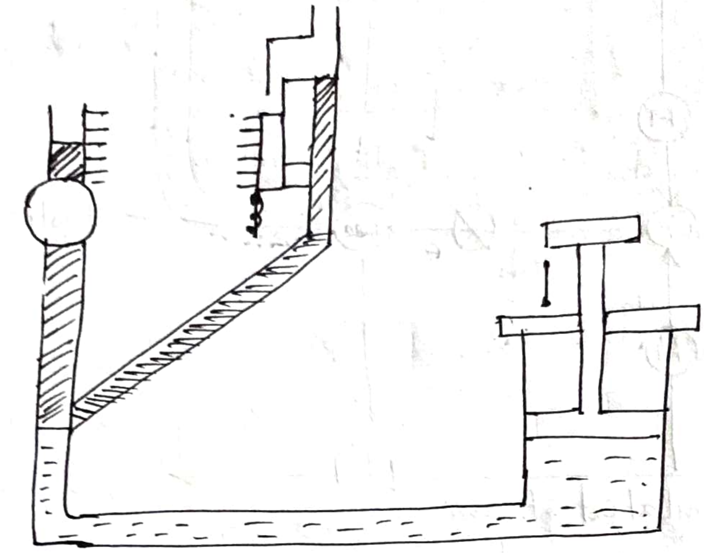
~~Damping factor: (ξ).~~

The actuating signal for ratio control.
In this method we measure flow rate of disturbance stream d and multiply it by the desired ratio. The result is the flow rate that the stream U should have and constitutes the set point value which is compared to the measured flow rate of stream U.



Ratio controller.

Mcleod Vaccum Gauge:



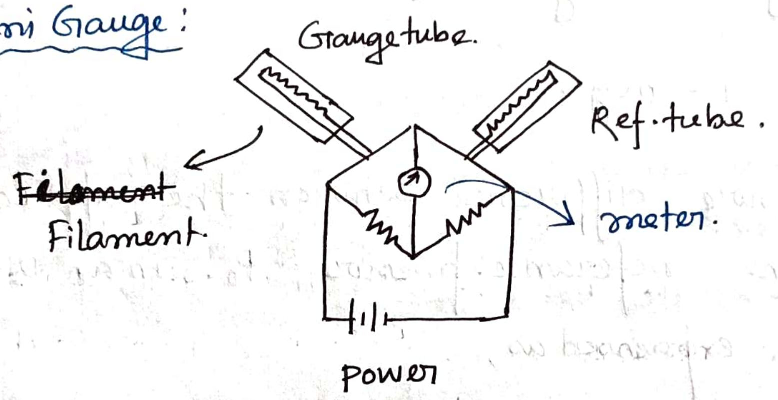
The Mcleod gauge is used to measure vaccum pressure. It also serves as a reference standard to calibrate other low pressure gauges. Mcleud gauge includes a reference column with reference capillary tube. The reference capillary tube has a point called zero reference point. This column is connected to a bulb and measuring capillary and the place of connection of the bulb with reference column is called cutoff point. If the mercury level is raised ~~avd~~ above this point it will cut off entry of applied pressure to the bulb and measuring capillary. Below the ~~reffe~~ referance column and the bulb, there is a mercury reservoir operated by a piston.

Working procedure: The pressure (P_1) is applied to the top of the ref-column. The Hg level is raised by operating piston to fill the volume. Again piston is operated so that mercury level increases. When the Hg level reaches the cut off point known (V_1). The Hg level is further raised by operating the piston so the trapped gas in the bulb and measuring capillary tube is compressed to reaches zero reff point marked on the referance capillary tube. The difference in height (h) in the measuring capillary and ref capillary become measure volume (V_2).

The applied pressure is calculated by Boyle's law

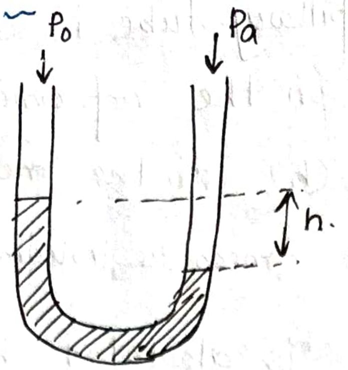
$$P_1 V_1 = P_2 V_2$$

Pirani Gauge:



The pirani gauge consists of a metet wire open to the pressure being measured. The wire is heated by current and cooled by gas surround it. If the gas pressure decreases the cooling will decrease. Hence the equilibrium temperature of the wire will increase. The resistance of the wire is a function of the temp and by measuring the voltage across the wire and the current flow through it, thus resistance can be determined and so the gas pressure is evaluated.

Manometer Gauge:



The pressure (P) excerted by a column of fluid of height(h) and density ρ is given by ~~hrg~~ hydrostatic pressure equation

$$P = h\rho g.$$

Therefore the pressure difference between the applied pressure Pa and the referance pressure P_0 in an U-tube manometer can be expressed as,

$$P_a - P_0 = h\rho g.$$

Since the liq is static, the pressure on either end of the liq must balanced and thus

$$P_a = P_0 + h\rho g.$$

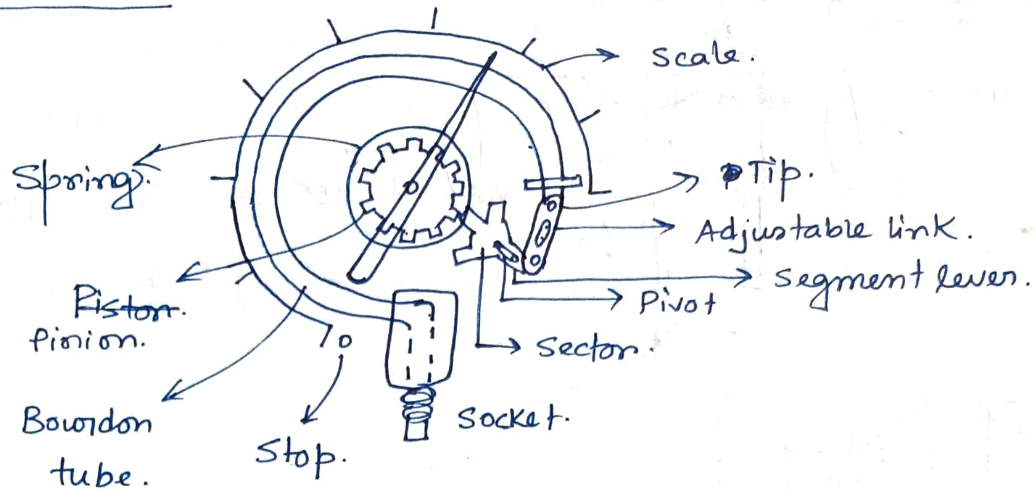
If the fluid being measured is significantly dense, hydrostatic correction may be made for height between the moving surface of the manometer working and the location where pressure measured is designed.

Here the fluid used is Hg ($\rho = 13.6$ g/cc) high density and low vap pressure.

Manometer.

- Simple manometer.
- Micro manometer
- Differential manometer
- Inverted Differential manometer.

Bourdon Tube:



Bourden tube consists of a narrow bore tube of elliptical Crossection, sealed at one end. The tube is formed into a curve. It has a pressurized fluid inside. When the pressure is applied the coil unwinds and changes its shape from eclipse to more circular. The pointer attached at the closed end indicates the pressure value.

Advantages:

i) low cost.
ii) No electrical power supply needed.
iii) It can measure (0.6 – 7000) bar.

Limitations:

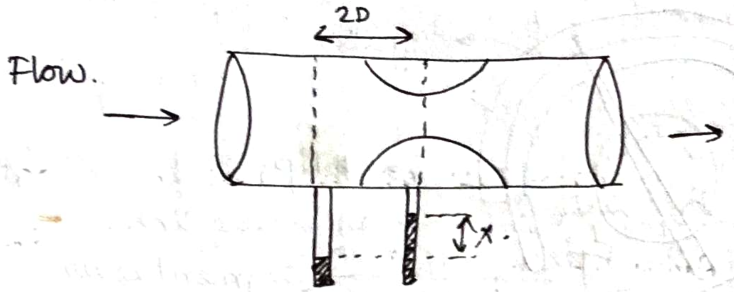
i) slow response.
ii) sensitive to shocks and vibration.

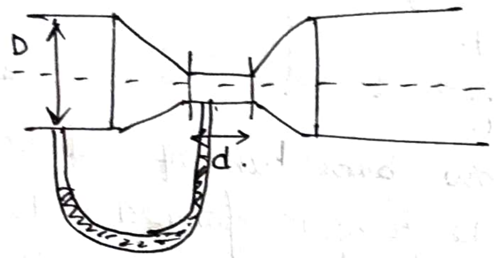
Pressure Gauge:

Many instruments have been invented to measure pressure with different advantages and dis advantages, pressure range sensivity, dynamic response. Pressure gauge is generally a pressure measuring device that measure pressure difference between the unknown pressure and atmospheric pressure. Here the unknown pressure (P) is greater than atmospheric pressure (P_{atm})

$$\boxed{P > P_{atm}}$$

Orificemeter and Venturimeter :-





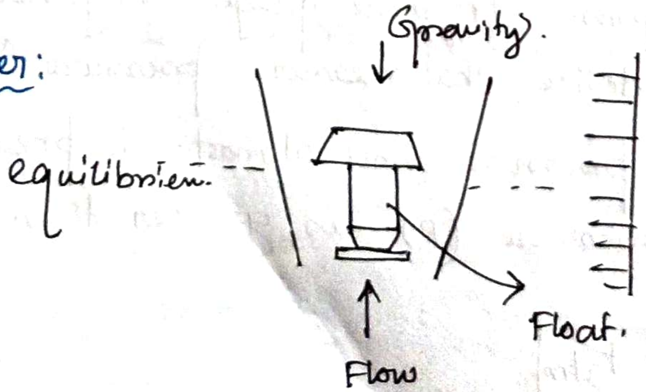
Venturimeter works on the principle of Bernouli's theom. which expressed that the distinction in pressure made over the ~~hyndran~~ hindrance gives the proportion of liq stream rate.

$$\frac{P_1}{\rho g} + \frac{v^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v^2}{2g} + h_2$$

Venturimeter is a type of differential flow meter that generates a flow measurement by measuring the pressure difference at two different location in a pipe.

orifice meter are simplest and cheapest form of the element which inserted in the line and pressure drop is measured.

Rotameter:



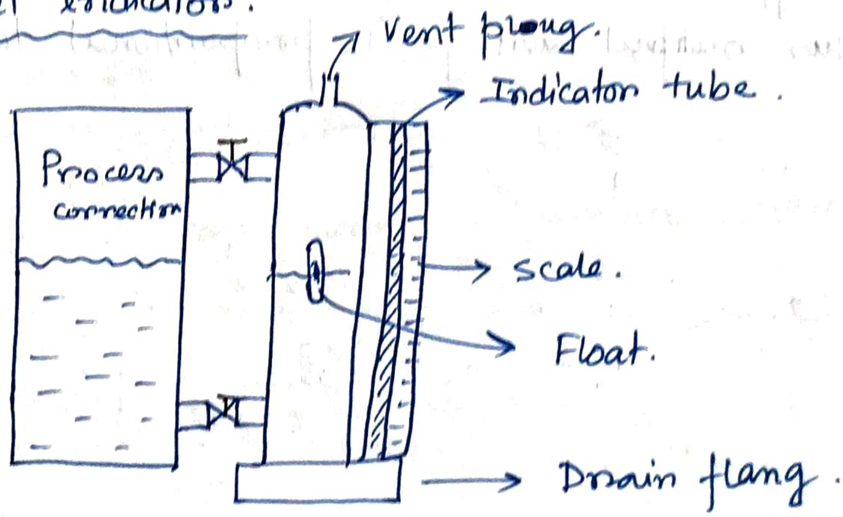
Rotameter is a variable area flowmeter used to measure fluid flow. It works on the principal of upthurst force ~~of gravity~~. excerted by fluid and force of gravity. The buoyant excerted on an immersed object is equal to the wt of liquid displace by the object. Under this principle rotameter works. It consist of tapered tube and a float.

Tapered tube is placed vertically in the flow channel with a conical shape inside. The quantity measured is defined by the hight of float going up.

$$Q = KA\,(gh)^{1/2} \qquad [h = \text{pressure drop}]$$

Flow rate.

Float type level indicator:

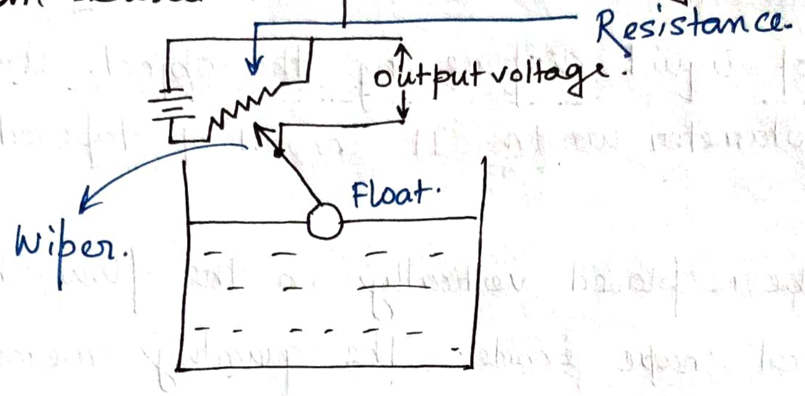


~~The float is connected to the torque tube which twists as the height of the displacement device changes. The twisting force driver the position of a pointer which indicate the liquid level.~~

It consists of a float which rest on the surface of a liquid. The movement of the float is transmitted to a pointer which indicates liquid level. Various floats are used including hollow metal sphere, cylindrical or disc.

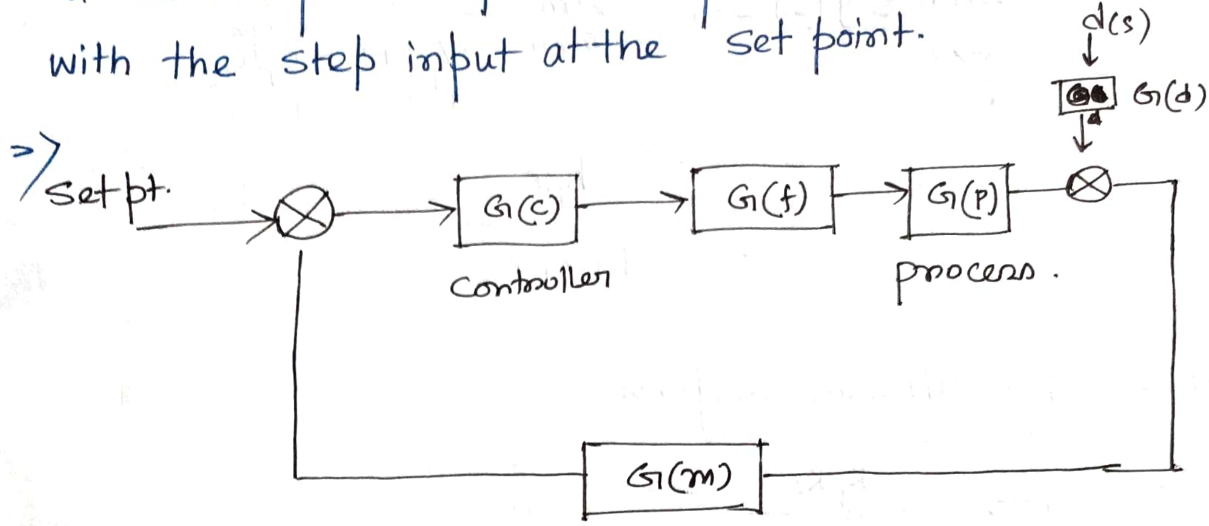
Advantage - low cost, reliable and operate over a large temp range.

A float can be used to operate a voltage potential divider.



As the liquid level rises the float goes up. It's wiper moves the potential divider whose output terminals are connected to a voltmeter. As float rises greater potential difference needed and the output voltage becomes high. This output voltage is proportional to ~~a~~ liquid level.

Effect of Proportional control on the response of a control process for servo problem and find the offset with the step input at the set point.



$$y(s) = \frac{G(C)\,G(f)\,G(P)}{1 + G(C)\,G(f)\,G(P)\,G(m)} \times \bar{y}_{sp}(s).$$

Servo part.

Now if the controller in Proportional controller,

$G(c) = K_c$.

and $G(f) = 1$, $G(P) = \frac{K_p}{\tau_p s + 1}$ $G(m) = 1$

as we want our measuring devices are accurate.

Thus,

$$y(s) = \frac{K_c}{1+K_c} \cdot \bar{y}_{sp}(s)$$

For unit step $\bar{y}_{sp}(s) = \frac{1}{s}$

$$y(s) = \frac{K_c}{1+K_c} \times \frac{1}{s}$$

$$y(s) = \frac{\frac{K_c K_p}{\tau_p s+1}}{1 + \frac{K_c K_p}{\tau_p s+1}} \cdot \bar{y}_{sp}(s)$$

$$= \frac{K_c K_p}{\tau_p s+1 + K_p K_c} \cdot \frac{1}{s}$$

[For unit step $\bar{y}_{sp}(s) = \frac{1}{s}$]

Ultimate response, [Final value theorem]

$$\lim_{s \to 0} s\, y(s) = \lim_{s \to 0} \frac{K_p K_c}{\tau_p s + 1 + K_p K_c}$$

$$= \frac{K_p K_c}{1 + K_p K_c}$$

Offset = 1 − ultimate response.

$$= 1 - \frac{K_p K_c}{1 + K_p K_c}$$

$$= \frac{1 + \cancel{K_p K_c} - \cancel{K_p K_c}}{1 + K_p K_c} = \frac{1}{1 + K_p K_c}$$

Root Locus analysis method:

For two capacity process.

$$\frac{\bar{y}}{\bar{y}_{sp}} = \frac{\dfrac{K_p}{(\tau_p s + 1)} \times \dfrac{K_p}{(\tau_p s + 1)} K_C}{1 + K_c \dfrac{K_p^2}{(\tau_p s + 1)^2}}$$

$$= \frac{K_p^2 K_c}{(\tau_p s + 1)^2 + K_p^2 K_c}$$

$$= \frac{K_c}{\dfrac{(\tau_p s + 1)^2}{K_c} + K_c}$$

$$= \frac{K_c}{\dfrac{(\tau_p^2 s^2 + 2\xi \tau_p s + 1)}{K_c} + K_c}$$

Root Locus analysis method :-

For two capacity process.

$$\frac{\bar{y}}{\bar{y}_{sp}} = \frac{\frac{K_p}{(\tau_p s+1)} \times \frac{K_p}{(\tau_p s+1)} K_c}{1 + K_c \frac{K_p^2}{(\tau_p s+1)^2}}$$

$$= \frac{K_p^2 K_c}{(\tau_p s+1)^2 + K_p^2 K_c}$$

$$= \frac{K_c}{\frac{(\tau_p s+1)^2}{\tau_p^2} + K_c}$$

$$= \frac{1}{\frac{\tau_p^2 s^2}{K_p^2 K_c} + \frac{2\tau_p s}{K_p^2 K_c} + \frac{1}{K_p^2 K_c} + 1}$$

To find roots.

$$\frac{\tau_p^2 s^2}{K_p^2 K_c} + \frac{2\tau_p s}{K_p^2 K_c} + \frac{1}{K_p^2 K_c} + 1 = 0$$

Roots, $$s_1, s_2 = \frac{-\frac{2\tau_p}{K_p^2 K_c} \pm \sqrt{\left(\frac{2\tau_p}{K_p^2 K_c}\right)^2 - 4\left(\frac{\tau_p^2}{K_p^2 K_c}\right)\left(1 + \frac{1}{K_p^2 K_c}\right)}}{2\frac{\tau_p^2}{K_p^2 K_c}}$$

$$= \frac{-\frac{\tau_p}{K_p^2 K_c} \pm \sqrt{\cancel{\frac{4\tau_p^2}{K_p^4 K_c^2}} - \cancel{\frac{4\tau_p^2}{K_p^4 K_c^2}} - \frac{4\tau_p^2}{K_p^2 K_c}}}{2\frac{\tau_p^2}{K_p^2 K_c}}$$

$$= \frac{-\frac{\tau_p}{K_p^2 K_c} \pm \sqrt{-\frac{4\tau_p^2}{K_p^2 K_c}}}{2\frac{\tau_p^2}{K_p^2 K_c}}$$

$$s_1, s_2 = \frac{-\tau_p \pm \sqrt{-4\tau_p^2 K_p^2 K_c}}{2\tau_p^2}$$

$$s_1 = \frac{-\tau_p + \sqrt{-4\tau_p^2 K_p^2 K_c}}{2\tau_p^2} \qquad s_2 = \frac{-\tau_p - \sqrt{-4\tau_p^2 K_p^2 K_c}}{2\tau_p^2}$$

Damping Factor: (ξ).

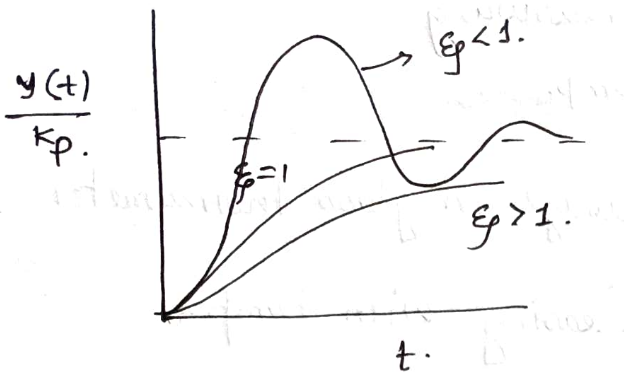
From the second order transfer function.

$$\frac{Y(s)}{X(s)} = \frac{K_p}{\tau_p^2 s^2 + 2\xi\tau_p s + 1}$$

Here ξ = Damping factor.

A damping ~~factor~~ ratio is a dimentionless measure used to describe how with in a system can decompose once a disturbance occurs. (decay)

It is the ratio of the load impedence to the amplifier internal output inpedence.



$\xi > 1$ ~~$\xi < 1$~~ overdamped.

$\xi = 1$ Critically damped

$\xi < 1$ under damped.

underdamped response is initially faster than critically damped and overdamped process ~~but~~ and reaches ultimate value quickly but it doesnot stay there. ~~but~~ It starts oscillating with progressively decreasing amplitude.

Transfer characteristics:

1> **Transferfunction:** It is defined as the relationship between output quntity and input quantity. It describes the input and output behavior of the system.

$$Q_i \longrightarrow \boxed{G} \longrightarrow Q_0 \qquad \frac{Q_0}{Q_i} = G = \text{transfer function.}$$

2> **Sensitivity:** The sensitivity of a transducer is the ration of change in output for a given change in input.

$$S = \frac{dQ_0}{dQ_i}$$

Sensitivity depends on the input quantitys

If seonsivity is constant over the entire range of transducer it shall be defined as $S = \frac{dQ_0}{dQ_i} = \frac{Q_0}{Q_i}$

3> **Error:** Many times the input-output relationship given by $Q_0 = Q_i G$ is not followed by transducer

In such cases error is obtained

$$\varepsilon = Q_0 - Q_i$$

a) Thermocouple is suitable for measuring

– iv) Both high and low temp.

b) Working principal of mercury in glass thermometer is based on the ______ increasing with temp.

— iii) volumetric expansion.

d) A typical example of a physical system with under damped characteristics –

— b) spring loaded diaphragm valve.

e) Mcleod gauge is used to measure – pressure

Module – II

a) A proportional controller with gain of K_c is used to control first order process. The offset will increase if –

i) K_c is reduced.

b) Routh stability method uses ______ loop function

– b) closed loop.

c) A non linear system will have ______ steady state values.

– b) More than one.

d) Which controller have maximux offset – P controller.

e) Transfer function of P-D controller – $K_c(1+\tau_D s)$