SOLUTIONS

1. (b) $a + b = 10, ab = 6$
$a^3 + b^3 = (a + b)\{(a + b)^2 - 3ab\}$
$\Rightarrow 10\{(10)^2 - 3 \times 6\} = 10 \times 82$
$= 820$

2. (a) ATQ,
$\frac{x}{y} + \frac{y}{x} = 1, \; x + y = 2$
$\Rightarrow x^2 + y^2 = xy, x^2 + y^2 - xy = 0$
$x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$
$= 2(0) = 0$

3. (a) $\frac{A}{L} + \frac{M}{B} = 1$
Let,
$\frac{A}{L} = x, \; \frac{M}{B} = y$ and $\frac{C}{N} = z$
ATQ,
$\frac{A}{L} + \frac{M}{B} = 1$
$x + y = 1$
$x = 1 - y$
$\frac{1}{x} = \frac{1}{1-y}$ .....(i)
Again,
$\frac{B}{M} + \frac{N}{C} = 1$
$\frac{1}{y} + \frac{1}{z} = 1$
$\frac{1}{z} = 1 - \frac{1}{y} = \frac{y-1}{y}$
$z = \frac{y}{y-1}$ .....(ii)
Put the value from equation (i) and (ii)
$\frac{L}{A} + \frac{C}{N}$
$= \frac{1}{x} + z = \frac{1}{1-y} + \frac{y}{y-1}$
$= \frac{1-y}{1-y} = 1$

4. (d) $x\left(5 - \frac{2}{x}\right) = \frac{5}{x}$
$\Rightarrow 5x - 2 = \frac{5}{x}$
$\Rightarrow 5x - \frac{5}{x} = 2$
ie, $x - \frac{1}{x} = \frac{2}{5}$
$\Rightarrow x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$
$= \frac{4}{25} + 2 = \frac{54}{25}$

5. (d) If $a + \frac{1}{a} = x$
then $a^2 + \frac{1}{a^2} = x^2 - 2$
$a + \frac{1}{a} = 3$
Square both side
$a^2 + \frac{1}{a^2} = 7$
Again,
Square both side
$a^4 + \frac{1}{a^4} = 47$

6. (a) $a + b + c = 5$
$ab + bc + ca = 7$
$\because a^3 + b^3 + c^3 - 3abc = (a + b + c)\{(a + b + c)^2 - 3(ab + bc + ca)\}$
$= 5 \times (25 - 3 \times 7) = 5 \times 4 = 20$

7. (c) $a - \frac{1}{a} = 4$
$a + \frac{1}{a} = \sqrt{\left(a - \frac{1}{a}\right)^2 + 4}$
$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$

8. (a) Given,

$$a-\frac{1}{a-5}=10$$

$$(a-5)-\frac{1}{(a-5)}=5$$

We know that, $a-\frac{1}{a}=k$ then

$$a^3-\frac{1}{a^3}=k^3+3k.$$

Hence,

$$(a-5)^3-\frac{1}{(a-5)^3}=5^3+3\times5$$

= 125 + 15 = 140

9. (d) Given,

$$x^4+\frac{1}{x^4}=1154$$

$$\Rightarrow x^2+\frac{1}{x^2}=\sqrt{1154+2}$$

$$\Rightarrow x^2+\frac{1}{x^2}=\sqrt{1156}$$

$$\Rightarrow x^2+\frac{1}{x^2}=34$$

Similarly,

$$x+\frac{1}{x}=\sqrt{34+2}$$

$$\Rightarrow x+\frac{1}{x}=6$$

Hence, $x^3+\frac{1}{x^3}=(6)^3-3\times6=198$

10. (a) Given,

$x^4 + x^{-4} = 7$

$x^2+x^{-2}=\sqrt{7+2}$

$x^2 + x^{-2} = 3$

The value of $x^2+\frac{1}{x^2}-2=3-2=1$

11. (b) Given,

$a + b + c = 10$

$a^2 + b^2 + c^2 = 48$

We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow (10)^2 = 48 + 2(ab + bc + ca)$

$\Rightarrow 100 - 48 = 2(ab + bc + ca)$

$\Rightarrow 52 = 2(ab + bc + ca)$

$\Rightarrow ab + bc + ca = 26$

SMART APPROACH:-

Let, $c = 0$

then, $a + b = 10$, $a^2 + b^2 = 48$

$(a + b)^2 = a^2 + b^2 + 2ab$

$2ab = (10)^2 - 48$

$ab = 26$

12. (c) We know that,

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

$$=\frac{(x-y)^3+(y-z)^3+(z-x)^3}{(x^2-y^2)+(y^2-z^2)^3+(z^2-x^2)^3}$$

$$=\frac{3(x-y)(y-z)(z-x)}{(x^2-y^2)(y^2-z^2)(z^2-x^2)}$$

$$=\frac{3(x-y)(y-z)(z-x)}{3(x-y)(y-z)(z-x)(x+y)(y+z)(z+x)}$$

$$=\frac{1}{(x+y)(y+z)(z+x)}$$

13. (c) Given,

$a + b + c = 10$

$a^2 + b^2 + c^2 = 38$

We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow (10)^2 = 38 + 2(ab + bc + ca)$

$\Rightarrow 100 - 38 = 2(ab + bc + ca)$

$\Rightarrow 62 = 2(ab + bc + ca)$

$\Rightarrow 31 = ab + bc + ca$

$\therefore (a-b)^2+(b-c)^2+(c-a)^2$

$= a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ca$

$= 2(a^2 + b^2 + c^2) - 2(ab + bc + ca)$

$= 2(38) - 2(31) = 76 - 62 = 14$

SMART APPROACH:-

By value putting,

$a = 5, b = 3, c = 2$

$\therefore (a-b)^2 + (b-c)^2 + (c-a)^2$

$= 2^2 + 1^2 + 3^2 = 14$

14. (c) $\dfrac{\{(m^2+n^2)(m-n)-(m-n)^3\}}{(m^2n-mn^2)}$

$$=\frac{(m-n)\{(m^2+n^2)-(m-n)^2\}}{mn(m-n)}$$

$$=\frac{m^2+n^2-(m^2+n^2-2mn)}{mn}$$

$$=\frac{2mn}{mn}=2$$

15. (a) We know, $(a + b)(a - b) = a^2 - b^2$

$$16y^2-k=\left(4y+\frac{3}{2}\right)\left(4y-\frac{3}{2}\right)$$

$$\Rightarrow 16y^2-k=(4y)^2-\left(\frac{3}{2}\right)^2$$

On comparing

$$k=\frac{9}{4}$$

16. (d) Given,

$x + y + z = 8$

$x^2 + y^2 + z^2 = 20$

We know that,

$(x + y + z)^2$

$= x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$\Rightarrow 64 = 20 + 2(xy + yz + zx)$

$\Rightarrow 64 - 20 = 2(xy + yz + zx)$

$\Rightarrow 44 = 2(xy + yz + zx)$

$\Rightarrow 22 = xy + yz + zx$

$\because x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)\{x^2 + y^2 + z^2 - (xy + yz + zx)\}$

$= 8\{20 - 22\} = -16$

SMART APPROACH:-

Here the given equation is of three variables and the number of the equation is two.

In this case we assume the value of 1 term is 0.

Asssume $z = 0$

Now, $x + y = 8$

$x^2 + y^2 = 20$

$(x + y)^2 = x^2 + y^2 + 2xy$

$\Rightarrow 64 = 20 + 2xy$

$\Rightarrow 64 - 20 = 2xy$

$\Rightarrow 44 = 2xy$

$\Rightarrow xy = 22$

Hence,

$x^3 + y^3 + z^3 - 3xyz = x^3 + y^3$

$x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

$= 8(20 - 22) = 8 \times -2 = -16$

17. (a) We know that,

$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

$\therefore (7x + 4y)^2 + (7x - 4y)^2$

$= 2\{(7x)^2 + (4y)^2\}$

$= 98x^2 + 32y^2$

18. (c) $6x + 7y = 5xy$ ......(1)

$10y - 4x = 4xy$ ......(2)

On dividing equation (1) and (2) by xy, we get

$$\frac{6}{y}+\frac{7}{x}=5 \quad ......(3)$$

and $\dfrac{10}{x}-\dfrac{4}{y}=4$ ......(4)

Again multiplying equation (3) and equation (4) by 10 and 7 respectively

$$\frac{60}{y}+\frac{70}{x}=50$$

$$\frac{70}{x}-\frac{28}{y}=28$$

On substracting, we get:

$$\frac{60}{y}+\frac{70}{x}-\left(\frac{70}{x}-\frac{28}{y}\right)=50-28$$

$$\Rightarrow \frac{88}{y}=22$$

$\Rightarrow y = 4$

On putting y = 4 in equation (3), we get $x = 2$

Hence, $x = 2$ and y = 4

SMART APPROACH:-

We can Assume value of x and y respectively 2 and 4
Now
Put the value of x and y 2 and 4 then satisfy the eqⁿ. (i)
$6x + 7y = 5xy$
$12 + 28 = 5 \times 2 \times 4$
40 = 40 Satisfy
eqn. (ii)
$10 \times 4 - 4 \times 2 = 4 \times 24$
32 = 32 Satisfy
Hence,
The value of x and y is 2 and 4

19. (b) Given,

$x + y = 10$ ....(1)

$2xy = 48$

$\Rightarrow xy = 24$ ....(2)

Assume, such a value of x and y which statisfies equation (1) & (2).

$x = 6$ and y = 4

$\therefore 2x - y = 2(6) - 4 = 8$

SMART APPROACH:-

We can Assume value of x and y respectively 6 and 4
Now, putting the value of x and y in eqⁿ.
$x + y = 10$
6 + 4 = Satisfy
$2xy = 48$
$2xy \times 4 = 48$ Satisfy
Now,
$2x - y = 2 \times 6 - 4 = 8$

20. (a) Put b = 0, $a^2 + 0 + 1 = 2a$

$\Rightarrow a^2 - 2a + 1 = 0$

$\Rightarrow (a-1)^2 = 0$

$\Rightarrow a = 1$

Therefore, $a^4 + b^7 = 1^4 + 0^7 = 1$

21. (b) Special Case:

If $x+\frac{1}{x}=2$ then $x = 1$

Put $x = 1$ in $x^4+\frac{1}{x^4}=2$

22. (a) Put a = 2, b = – 1 and c = –1

$$\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=\frac{4}{1}+\frac{1}{-2}+\frac{1}{-2}$$

$= 4 - 1 = 3$

23. (c) Given, $\frac{x^8+1}{x^4}=4$

$$\Rightarrow x^4+\frac{1}{x^4}=14$$

$$\Rightarrow x^2+\frac{1}{x^2}=4$$

$$\therefore x^6+\frac{1}{x^6}=(4)^3-3\times 4$$

$$\Rightarrow \frac{x^{12}+1}{x^6}=64-12=52$$

24. (a) $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2)$

$= x^3 + y^3 + 3xy(x + y) - \{x^3 - y^3 - 3xy(x - y)\} - 6yx^2 + 6y^3$

$= x^3 + y^3 + 3x^2y + 3xy^2 - \{x^3 - y^3 - 3x^2y + 3xy^2\} - 6yx^2 + 6y^3$

$= x^3 + y^3 + 3x^2y + 3xy^2 - x^3 + y^3 + 3x^2y - 3xy^2 - 6yx^2 + 6y^3$

$= 8y^3$

SMART APPROACH:-

Put $x = 0$, y = 1
$(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2)$
1 + 1 + 6 = 8
Putting the value of x and y in option (a) then satisfy the eqⁿ.

25. (d) $x^2 + 6x + 1 = 0$

$x(x + 6) + 1 = 0$

Dividing both side by $(x + 6)$

$$x+\frac{1}{x+6}=0$$

Adding 6 both sides

$$(x+6)+\frac{1}{(x+6)}=6$$

$$(x+6)^3+\frac{1}{(x+6)^3}=6^3-3\times 6$$

$= 216 - 18 = 198$

26. (c) Given,

a + b = 10 and ab = 9

We know,

$(a - b)^2 = (a + b)^2 - 4ab$

$= 100 - 36 = 64$

$\therefore a - b = 8$

SMART APPROACH:-

By value putting,
a = 9, b = 1
$\therefore a - b = 9 - 1 = 8$

27. (d) Given,

$a^2 + b^2 = 82$

$b^2 + c^2 = 65$

Assume, a = 9, b = 1 and c = 8

$\therefore 2a + 7b - 3c = 2(9) + 7(1) - 3(8)$

$= 18 + 7 - 24 = 1$

28. (b) Given, $x(x - 5) = -1$

$$\Rightarrow x-5=\frac{-1}{x}$$

$$\Rightarrow x+\frac{1}{x}=5$$

$$\Rightarrow x^3+\frac{1}{x^3}=5^3-3\times 5$$

$$\Rightarrow x^3+\frac{1}{x^3}=110$$

$\Rightarrow x^6 + 1 = 110x^3$

$\Rightarrow x^6 - 110x^3 = -1$

$\Rightarrow x^3(x^3 - 110) = -1$

29. (d) Given,

r = 55

$\therefore r(r^2 + 3r + 3)$

$= \{r^3 + 3r^2 + 3r + 1\} - 1$

$= \{r^3 + 1 + 3r(r + 1)\} - 1$

$= (r + 1)^3 - 1 = 56^3 - 1$

$= 175616 - 1 = 175615$

SMART APPROACH:-

We can get the direct result by divisibility rule 11 and 5 together
On only option (d) is correct

30. (c) Given

$$a+\frac{1}{a}=P^2$$

$$\Rightarrow a^2+\frac{1}{a^2}=P^4-2$$

31. (a) Given a = 9.6, b = 4.44 and c = 5.16

We know that

If a + b + c = 0 then, $a^3 - b^3 - c^3 - 3abc = 0$

Here,

a – b – c = 9.6 – 4.44 – 5.16 = 0

Therefore, $a^3 - b^3 - c^3 - 3abc = 0$

SMART APPROACH:-

We know that,
If a – b – c = 0 then $a^3 - b^3 - c^3 - 3abc = 0$
Here,
a = 9.6, b = 4.44 and c = 5.16
a – b – c = 9.6 – 4.44 – 5.16 = 0
Now,
$a^3 + b^3 - c^3 - 3abc = 0$

32. (b) Given

$$S-\frac{1}{S-8}=20$$

$$\Rightarrow (S-8)-\frac{1}{(S-8)}=12$$

$$\therefore (S-8)^3-\frac{1}{(S-8)^3}$$

$= 12^3 + 3 \times 12 = 1728 + 36 = 1764$

33. (a) Given

$x^2+\frac{1}{x^2}=98$

$\Rightarrow x+\frac{1}{x}=\sqrt{98+2} = \sqrt{100} = 10$

34. (a) $k(21x^2+24)+rx+(14x^2-9)=0$

$\Rightarrow 21kx^2+24k+rx+14x^2-9=0$

$\Rightarrow (21k+14)x^2+rx+24k-9=0$ .....(1)

$k(7x^2+8)+px+(2x^2-3)=0$

$\Rightarrow 7kx^2+8k+px+2x^2-3=0$

$\Rightarrow (7k+2)x^2+px+8k-3=0$ ....(2)

On dividing (1) by (2), we get–

$\Rightarrow \frac{21k+14}{7k+2}=\frac{r}{p}=\frac{24k-9}{8k-3}$

$\Rightarrow \frac{r}{p}=\frac{3(8k-3)}{8k-3}$

$\Rightarrow \frac{r}{p}=3 \quad \Rightarrow \frac{p}{r}=\frac{1}{3}$

35. (a) $a^2+b^2+c^2+216=2(6a+6b-12c)$

$(a-6)^2+(b-6)^2+(c+12)^2=0$

then

$a=6,\ b=6,\ c=-12$

Now,

$=\sqrt{ab-bc+ca}$

$=\sqrt{6\times6-(6)\times(-12)+(-12)\times(6)}$

$=\sqrt{36+72-72} = 6$

36. (a) $\left(5\sqrt{5}x^3-3\sqrt{3}y^3\right)\div\left(\sqrt{5}x-\sqrt{3}y\right)$

$=Ax^2+By^2+Cxy$

$\left(3A+B-\sqrt{15}C\right)=?$

$\because a^3-b^3=(a-b)(a^2+b^2+ab)$

$\frac{\left(\sqrt{5}x-\sqrt{3}y\right)\left(5x^2+3y^2+\sqrt{15}xy\right)}{\sqrt{5}x-\sqrt{3}y}$

$=Ax^2+By^2+Cxy$

On comparing

$A=5,\ B=3,\ y=+\sqrt{15}$

$3A+B-\sqrt{15}\,C$

$=3\times5+3-\sqrt{15}\times\sqrt{15}$

$=15+3-15=3$

37. (a) $x^4+\frac{1}{x^4}=194,\quad x+\frac{1}{x}=?$

$x^4+\frac{1}{x^4}+2=196$

$x^2+\frac{1}{x^2}=14$

$x+\frac{1}{x}=4$

38. (a) $x^2+8y^2-12y-4xy+9=0$, $(7x-8y)=?$

$x^2-4xy+4y^2+4y^2-12y+9=0$

$(x-2y)^2+(2y-3)^2=0$

$x-2y=0$

$x=2y$

$2y-3=0$

$y=\frac{3}{2}$

$x=2\times\frac{3}{2} \qquad x=3,\ y=\frac{3}{2}$

$7x-8y=7\times3-8\times\frac{3}{2}$

$=21-12=9$

39. (d) $x^2-5x+1=0$,

$\left(x^4+\frac{1}{x^2}\right)\div(x^2+1)=?$

$x+\frac{1}{x}=5$

$\Rightarrow \frac{\left(x^4+\frac{1}{x^2}\right)\times\frac{1}{x}}{\left(x^2+1\right)\times\frac{1}{x}} \Rightarrow \frac{\left(x^3+\frac{1}{x^3}\right)}{\left(x+\frac{1}{x}\right)}$

$\because x+\frac{1}{x}=5$

$x^3+\frac{1}{x^3}=110$

$\frac{\left(x^3+\frac{1}{x^3}\right)}{\left(x+\frac{1}{x}\right)}=\frac{110}{5}=22$

40. (c) $(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)$

$361=x^2+y^2+z^2+2\times114$

$x^2+y^2+z^2=133$

$x^3+y^3+z^3-3xyz=(x+y+z)$

$[x^2+y^2+z^2-(xy+yz+zx)]$

$x^3+y^3+z^3+xyz$

$=19\times[133-114]+216+648$

$x^3+y^3+z^3+xyz=1225$

Alterante Method:

$a^3+b^3+c^3-3abc=(a+b+c)[(a+b+c)^2-3(ab+bc+ca)]$

$\therefore$ $x^3+y^3+z^3+xyz=(19)[(19)^2-3\times114]+216+648=1225$

SMART APPROACH:-

Value putting,

$a=6,\ b=4,\ c=9$

$\therefore x^3+y^3+z^3+xyz=1225$

41. (b) $x^2-3x+1=0$, $\frac{\left(x^4+\frac{1}{x^2}\right)}{\left(x^2+1\right)}=?$

$x+\frac{1}{x}=3$

$\because x^3+\frac{1}{x^3}=27-9=18$

$\frac{\left(x^4+\frac{1}{x^2}\right)\times\frac{1}{x}}{\left(x^2+1\right)\times\frac{1}{x}}=\frac{x^3+\frac{1}{x^3}}{x+\frac{1}{x}}=\frac{18}{3}=6$

42. (d) $x^2+y^2+z^2=(x+y+z)^2-2(xy+yz+zx)$

$=289-2\times111$

$=289-222=67$

$x^3+y^3+z^3-3xyz=(x+y+z)[x^2+y^2+z^2-(xy+yz+zx)]$

$x^3+y^3+z^3+xyz=(x+y+z)[x^2+y^2+z^2-(xy+yz+zx)]+4xyz$

$=17[67-111]+4\times171$

$=-17\times44+684$

$=-748+684=-748+684$

$x^3+y^3+z^3+xyz=-64$

$\sqrt[3]{x^2+y^3+z^3+xyz}=\sqrt[3]{-64}=-4$

SMART APPROACH:-

$a^3+b^3+c^3-3abc=(a+b+c)[(a+b+c)^2-3(ab+bc+ca)]$

$\therefore x^3+y^3+z^3+xyz=(17)[(17)^2-3\times111]+171+513=-64$

$\sqrt[3]{x^3+y^3+z^3+xyz}=-4$

43. (a) $x^2+8y^2+12y-4xy+9=0$, $(7x+8y)=?$

$x^2-4xy+4y^2+4y^2+12y+9=0$

$(x-2y)^2+(2y+3)^2=0$

$x=2y,\ y=\frac{-3}{2}$

$x=-3,\ y=\frac{-3}{2}$

$7x+8y=7\times-3+8\times\frac{-3}{2}=-33$

44. (d) $(xy+yz+zx)$

$=\frac{(x+y+z)^2-\left(x^2+y^2+z^2\right)}{2}$

$=\frac{169-133}{2}=18$

$x^3+y^3+z^3-3xyz=(x+y+z)[x^2+y^2+z^2-(xy+yz+zx)]$

$xyz=$

$\frac{x^3+y^3+z^3-(x+y+z)\left[x^2+y^2+z^2-(xy+yz+zx)\right]}{3}$

$=\frac{847-13[133-18]}{3}$

$=\frac{847-1495}{3}$

$xyz = -216$

$\sqrt[3]{xyz} = \sqrt[3]{-216} = -6$

SMART APPROACH:-

$a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c)$

$[3(a^2 + b^2 + c^2) - (a + b + c)^2]$

$847 - 3xyz = \frac{1}{2} \times 13\ [3 \times 133 - 169]$

$\sqrt[3]{xyz} = \sqrt[3]{-216} = -6$

45. (d) $a^3 + b^3 = (a + b)\ [(a+b)^2 - 3ab]$

$217 = 7[49 - 3ab]$

$31 = [49 - 3ab]$

$3ab = 18$

$ab = 6$

SMART APPROACH:-

Value putting,
$a = 6,\ b = 1$
$\therefore ab = 6$

46. (d) $a^2 + b^2 + c^2 + 84 = 2(2a - 4b + 8c)$

$(a - 2)^2 + (b + 4)^2 + (c - 8)^2 = 0$

Then,

$a = 2,\ b = -4,\ c = 8$

$= \sqrt{ab - bc + ca}$

$= \sqrt{2 \times -4 + 4 \times 8 + 8 \times 2}$

$= \sqrt{40} = 2\sqrt{10}$

47. (b) $x + y + z = 19,\ x^2 + y^2 + z^2 = 133,\ xz = y^2,\ (x - z) = ?$

$(xy + yz + zx) =$

$$\frac{(x+y+z)^2 - (x^2+y^2+z^2)}{2}$$

$= \frac{361 - 133}{2}$

$xy + yz + zx = 114$

$\because zx = y^2$

$xy + yz + y^2 = 114$

$y(x + y + z) = 114$

$y = \frac{114}{19} = 6$

$x + 6 + z = 19$

$x + z = 13$

$xz = 36$

$(x - z)^2 = (x + z)^2 - 4xz$

$\Rightarrow 169 - 4 \times 36$

$\Rightarrow 169 - 144$

$(x - z) = 5$

SMART APPROACH:-

Value putting, $x = 9, y = 6, z = 4$
$\therefore x - z = 9 - 4 = 5$

48. (c) $\frac{(\sqrt{5}x - \sqrt{3}y)(5x^2 + 3y^2 + \sqrt{15}xy)}{(\sqrt{5}x - \sqrt{3}y)}$

$= (Ax^2 + By^2 + Cxy)$

On comparing

$A = 5,\ B = 3,\ C = \sqrt{15}$

Now,

$3A - B - \sqrt{15}\ C$

$\Rightarrow 3 \times 5 - 3 - \sqrt{15} \times \sqrt{15}$

$\Rightarrow 15 - 3 - 15$

$\Rightarrow -3$

49. (a) $x^4 + \frac{1}{x^4} = 194,\ x + \frac{1}{x} + 2 = ?$

$\because x^2 + \frac{1}{x^2} = 14$

$x + \frac{1}{x} = 4$

$x + \frac{1}{x} + 2 = 6$

50. (d) $\because (a + b)^2 = a^2 + b^2 + 2ab$

$= 82 + 2 \times 9 = 100$

$a + b = 10$

$a^3 + b^3 = (a + b)\ (a^2 + b^2 - ab)$

$= 10(82 - 9) = 730$

SMART APPROACH:-

Value putting, $a = 9, b = 1$
$\therefore a^3 + b^3 = 730$

51. (d) $x^2 + y^2 + z^2 = (x + y + z)^2 - 2(xy + yz + zx)$

$= 361 - 2 \times 114 = 133$

$x^3 + y^3 + z^3 + xyz = (x + y + z)$
$[x^2 + y^2 + z^2 - (xy + yz + zx) + 4xyz$

$= 19[133 - 114] + 4 \times 216$

$= 19 \times 19 + 864$

$x^3 + y^3 + z^3 + xyz = 1225$

$\sqrt{x^3 + y^3 + z^3 + xyz} = \sqrt{1225} = 35$

SMART APPROACH:-

Value putting, $x = 4, y = 6, z = 9$
$\therefore \sqrt{x^3 + y^3 + z^3 + xyz} = \sqrt{1225} = 35$

52. (b) If $a + b + c = 0$ then,

$\because a^3 + b^3 + c^3 = 3abc$

$\Rightarrow \frac{a^3 + b^3 + c^3}{abc} \Rightarrow \frac{3abc}{abc} = 3$

53. (d) $(4x^3y - 6x^2y^2 + 4xy^3 - y^4)$

We know,

$(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + 4y$

with option $= x^4 - (x - y)^4$ (satisfied)

54. (b) $(2x + 3y + 4)\ (2x + 3y - 5)$

$4x^2 + 6xy - 10x + 6xy + 9y^2 - 15y + 8x + 12y - 20$

$4x^2 + 9y^2 + 12xy - 2x - 3y - 20$

Compare with

$(ax^2 + by^2 + 2hxy + 2gx + 2fy + c)$

$a = 4,\ b = 9,\ h = 6,\ g = -1,\ f = \frac{-3}{2},$

$c = -20$

Then,

$= \frac{g + f - c}{abh} = \frac{-1 - \frac{3}{2} + 20}{4 \times 9 \times 6} = \frac{17.5}{216}$

$= \frac{35}{432}$

55. (c) Given, that

$x^4 + y^4 + x^2y^2 = 21,\ x^2 + y^2 + xy = 3$

Then,

$x^2 + y^2 - xy = 7$

$xy = -2$ then $4xy = -8$

SMART APPROACH:-

We know that,
$x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)\ (x^2 - xy + y^2)$
Given,
$x^4 + x^2y^2 + y^4 = 21$
$x^2 + xy + y^2 = 3$ .... (i)
$x^2 - xy + y^2 = 7$ ....(ii)
(i) – (ii)
$xy = 2$
Now,
$4xy = 4 \times (-2) = -8$

56. (b) $x^2 - \sqrt{7}x + 1 = 0$

$= x + \frac{1}{x} = \sqrt{7}x$

$x^3 + \frac{1}{x^3} = (\sqrt{7})^3 - 3 \times \sqrt{7}$

$= 7\sqrt{7} - 3\sqrt{7} = 4\sqrt{7}$

57. (c) We know,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)$
$[(x + y + z)^2 - 3\ (xy + yz + zx)]$

$x^3 + y^3 + z^3 - 3(100) = 10[(10)^2 - 3 \times 25]$

$x^3 + y^3 + z^3 - 300 = 10\ [100 - 75]$

$x^3 + y^3 + z^3 = 550$

58. (a) As we know,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)\ [(x + y + z)^2 - 3\ (xy + yz + zx)]$

$151 - 3xyz = 1[1 - 3 \times -26]$

$-3xyz = 79 - 151$

$3xyz = 79 - 151$

$3xyz = 72$

$xyz = 24$

59. (d) Given that,
$a + b + c = 6, a^2 + b^2 + c^2 = 38$
Let, $c = 0$
Then,
$a + b = 6, a^2 + b^2 = 38, ab = -1$
then, the value of
$ab^2 + ba^2$ or $ab(a + b)$
$-1(6) = -6$

60. (d) $a^3 - b^3 = (a - b)[(a - b)^2 + 3ab]$
$(2x - 5y)^3 - (2x + 5y)^3 = y(Ax^2 + By^2)$
$(2x - 5y - 2x - 5y)[(-10y)^2 + 3 \times (2x - 5y) \times (2x + 5y)]$
$= -10y[100y^2 + 3 \times ((2x)^2 - (5y)^2)$
$= -10y[100y^2 + 3 \times (4x^2 - 25y^2)]$
$= -10y[100y^2 + 12x^2 - 75y^2]$
$= -10y[25y^2 + 12x^2] = y(Ax^2 + By^2)$
On compring
$= A = -120, B = -250$
$\Rightarrow 2A - B = -240 + 250 = 10$

61. (d) $\sqrt{x} + \frac{1}{\sqrt{x}} = 3$

$x + \frac{1}{x} = 9 - 2 = 7$

$x^2 + \frac{1}{x^2} = (7)^2 - 2 = 47$

$\frac{x^4 + 1}{x^2} = 47$

$x^4 + 1 = 47x^2$
$x^4 - 47x^2 = -1$
$= x^2(x^2 - 47) = -1$

62. (b) $4(x - 2)^2 + (y - 3)^2 - 2(x - 3)^2$
For least possible value put
$(y - 3)^2 = 0$
$4(x - 2)^2 - 2(x - 3)^2$
$= 4(x^2 + 4 - 4x) - 2(x^2 + 9 - 6x)$
$= 4x^2 + 16 - 16x - 2x^2 - 18 + 12x$
$= 2x^2 - 2 - 4x$
$= 2(x^2 - 1 - 2x)$
$= 2[(x - 1)^2 - 2]$
We know,
For least possible value put $(x - 1)^2 = 0$
$= 2[(x - 1)^2 - 2] = 2 \times (-2) = -4$

63. (c) $x = 5.51, y = 5.52, z = 5.57$
value of $x^3 + y^3 + z^3 - xyz$
we know that

$x^3 + y^3 + z^3 - 3xyz = \frac{x + y + z}{2}$
$[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{(5.51 + 5.52 + 5.57)}{2}$
$[(5.51 - 5.52)^2 + (5.52 - 5.57)^2 + (5.57 - 5.51)^2]$

$= \frac{16.60}{2}[(0.01)^2 + (0.05)^2 + (0.06)^2]$

$= \frac{16.6}{2}[0.0001 + 0.0025 + 0.0036]$

$= 8.3[0.0062] = 0.05146$

64. (b) Given,
$x^2 + xy + y^2 = \frac{3}{16}$ ----(1)
As we know, $x^4 + y^4 + x^2y^2$
$= (x^2 + y^2 + xy)(x^2 + y^2 - xy)$
$\Rightarrow \left(\frac{21}{256}\right) = \left(\frac{3}{16}\right)(x^2 - xy + y^2)$
$\Rightarrow (x^2 - xy + y^2) = \frac{7}{16}$ ----(2)
On adding (1) and (2), We get-
$\Rightarrow 2(x^2 + y^2) = \frac{5}{8}$

65. (c) $\Rightarrow \frac{8x}{2x^2 + 7x - 2} = 1$
$\Rightarrow 8x = 2x^2 + 7x - 2$
$\Rightarrow 2x^2 - x - 2 = 0$
$\Rightarrow 2x - \frac{2}{x} = 1 \Rightarrow x - \frac{1}{x} = \frac{1}{2}$

$x + \frac{1}{x} = \sqrt{\left(\frac{1}{2}\right)^2 + 4} = \frac{\sqrt{17}}{2}$

We know,

If $x + \frac{1}{x} = N$,then $x^3 + \frac{1}{x^3} = N^3 - 3N$

$\Rightarrow x^3 + \frac{1}{x^3} = \frac{17\sqrt{17}}{8} - \frac{3\sqrt{17}}{2} = \frac{5\sqrt{17}}{8}$

66. (c) $a = 500, b = 502, c = 504$
We know,
$a^3 + b^3 + c^3 - 3abc = (a + b + c) \times 3d^2$
$= (500 + 502 + 504) \times 3 \times (2)^2$
$= 1506 \times 12 = 18072$

67. (b) We know,
$(a - b)^2 = a^2 + b^2 - 2ab$
$\Rightarrow 25 = 45 - 2ab$
$\Rightarrow -20 = -2ab$
$\Rightarrow ab = 10$

68. (b) $x^4 + \frac{1}{x^4} = 2599$

$\Rightarrow x^2 + \frac{1}{x^2} = \sqrt{2599 + 2} = \sqrt{2601}$

$x^2 + \frac{1}{x^2} = 51$

$\Rightarrow x - \frac{1}{x} = \sqrt{51 - 2} = \sqrt{49} = 7$

69. (a) We Know, $a^3 + b^3 + c^3 = (a + b + c)$
$\{(a + b + c)^2 - 3(ab + bc + ca)\}$
$= 9\{81 - 3 \times 18\} = 9(27) = 243$

SMART APPROACH:-
By value putting, $a = 3, b = 6$
$\therefore a^3 + b^3 = 243$

70. (b)
$x^2 - 4x + 1 = 0$
$x + \frac{1}{x} = 4$
$x^3 + \frac{1}{x^3} = (4)^3 - 3 \times 4 = 52$
$x^6 + \frac{1}{x^6} = (52)^2 - 2 = 2702$

71. (d) As we know,
If, $(a - b)^2 + (c - d)^2 = 0$
$a = b$ and $c = d$
$\left(x^3 + \frac{1}{x^3} - k\right)^2 + \left(x + \frac{1}{x} - p\right)^2 = 0$
So, $\left(x^3 + \frac{1}{x^3} - k\right)^2 = 0 \Rightarrow x^3 + \frac{1}{x^3} = k$
& $\left(x + \frac{1}{x} - p\right)^2 = 0 \Rightarrow x + \frac{1}{x} = p$
We know,
$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$
$\Rightarrow p^3 = k + 3p$
$\Rightarrow k = p^3 - 3p$
$\Rightarrow \frac{k}{p} = p^2 - 3$

SMART APPROACH:-
Put $x = 1$
$\Rightarrow (2 - k)^2 + (2 - p)^2 = 0$
$\Rightarrow k = 2$ and $p = 2$
$\Rightarrow \frac{k}{p} = \frac{2}{2} = 1$
Option $(P^2 - 3)$ (satisfied)

72. (d) $x^4 + x^2y^2 + y^4 = 133$
$x^2 - xy + y^2 = 7$ ...(1)
We know,
$(x^2 - xy + y^2)(x^2 + xy + y^2) = x^4 + x^2y^2 + y^4$
$\Rightarrow 7(x^2 + xy + y^2) = 133$
$\Rightarrow x^2 + xy + y^2 = 19$ ...(2)
Subtract eqn (1) from eqn (2)
$\Rightarrow 2xy = 12 \Rightarrow xy = 6$

73. (c) We know that, $a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)[(a + b + c)^2 - 3(ab + bc + ca)]$
$= 19(361 - 360) = 19$

74. (c) $x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$
$x^6 - 512y^6 = (x^2 - Ay^2)(x^4 + Bx^2y^2 + Cy^4)$
$= \left[x^2 - \left(\sqrt{8}y\right)^2\right][x^4 + 8x^2y^2 + 64y^4]$
$= (x^2 + Ay^2)(x^4 - Bx^2y^2 + Cy^4)$
On Comparing
$A = -8, B - 8$ and $C = 64$
Now,
$A + B - C = (-8 - 8 - 64) = -80$

75. (d) $(a + b + c)(ab + bc + ca) - abc$
put $a = b = c = 1$
$(1 + 1 + 1)(1 + 1 + 1) - 1$
$= 3 \times 3 - 1 = 8$
Hence, Option 4
$(a + b)(b + c)(c + a)$ (satisfied)