SOLUTIONS

1. (d) $\tan^2\theta + \tan^4\theta$
$= \tan^2\theta(1 + \tan^2\theta)$
$[\because \sec^2\theta - \tan^2\theta = 1]$
$= \tan^2.\sec^2\theta$
$= (\sec^2\theta - 1)\sec^2\theta$
$= \sec^4\theta - \sec^2\theta$

2. (a) $\sin\theta + \operatorname{cosec}\theta = 2$
Let $\theta = 90°, \Rightarrow$ L.H.S = R.H.S
$\therefore \sin^5\theta + \operatorname{cosec}^5\theta = 2$

Alternate Method:

$\sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow \sin\theta + \dfrac{1}{\sin\theta} = 2$

$\Rightarrow \dfrac{\sin^2\theta + 1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$
$\Rightarrow (\sin\theta - 1)^2 = 0$
$\Rightarrow \sin\theta = 1$
$\Rightarrow \theta = 90°$
$\therefore \sin^5\theta + \operatorname{cosec}^5\theta$
$= \sin^5 90° + \operatorname{cosec}^5 90°$
$= 1 + 1 = 2$

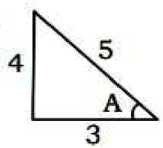
3. (c) If, $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$
Comparing with $a\cos\theta - b\sin\theta = c$
$a = 1, b = 1, c = \sqrt{2}\sin\theta$
$\cos\theta + \sin\theta = \sqrt{a^2 + b^2 - c^2}$
$= \sqrt{1 + 1 - 2\sin^2\theta} = \sqrt{2}\cos\theta$

Alternate Method:

$\cos\theta - \sin\theta = \sqrt{2}\sin\theta$
Squaring both sides,
$\cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta = 2\sin^2\theta$
$\Rightarrow 1 - 2\sin\theta\cos\theta = 2(1 - \cos^2\theta)$
$\Rightarrow 1 + 2\sin\theta\cos\theta = 2\cos^2\theta$
$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$
$\Rightarrow (\sin\theta + \cos\theta)^2 = 2\cos^2\theta$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta$

4. (c)



$\tan A = \dfrac{4}{3}$

$\therefore \sin A = \dfrac{4}{5}$

5. (d) $(1 + \sin^4 A - \cos^4 A)\operatorname{cosec}^2 A$
Let, $A = 45°$
$\Rightarrow (1 + \sin^4 45° - \cos^4 45°)\operatorname{cosec}^2 45°$

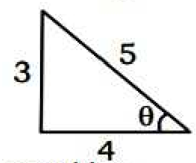
$= \left[1 + \left(\dfrac{1}{\sqrt{2}}\right)^4 - \left(\dfrac{1}{\sqrt{2}}\right)^4\right](\sqrt{2})^2 = 2$

Alternate Method:

Consider,
$(1 + \sin^4 A - \cos^4 A)\operatorname{cosec}^2 A$
$= (1 + (\sin^2 A - \cos^2 A))\operatorname{cosec}^2 A$
$= (1 + \sin^2 A - \cos^2 A)\operatorname{cosec}^2 A$
$= (1 - \cos^2 A + \sin^2 A)\operatorname{cosec}^2 A$
$= 2\sin^2 A\operatorname{cosec}^2 A = 2$

6. (c) Given,

$\cot\theta = \dfrac{4}{3}$

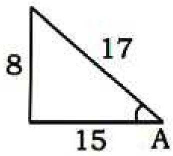


consider,
$5p\cos^2\theta\sin\theta = \cot^2\theta$

$\Rightarrow 5p\left(\dfrac{4}{5}\right)^2 \times \dfrac{3}{5} = \left(\dfrac{4}{3}\right)^2$

$\Rightarrow p = \dfrac{16}{9} \times \dfrac{5}{3} \times \dfrac{25}{16} \times \dfrac{1}{5} = \dfrac{25}{27}$

7. (a)



$\cos A = \frac{15}{17}$

$\cot(90° - A) = \tan A$

$\therefore \tan A = \frac{8}{15}$

8. (a) $\frac{\cos 65°}{\sin 25°} + \frac{5\sin 19°}{\cos 71°} - \frac{3\cos 28°}{\sin 62°}$

$= \frac{\cos(90° - 25°)}{\sin 25°} + \frac{5\sin(90° - 71°)}{\cos 71°} - \frac{3\cos(90° - 62°)}{\sin 62°}$

$= \frac{\sin 25°}{\sin 25°} + \frac{5\cos 71°}{\cos 71°} - \frac{3\sin 62°}{\sin 62°}$

$= 1 + 5 - 3 = 3$

9. (d) $\sin\theta + \cos\theta = \frac{\sqrt{11}}{3}$

Comparing with $a\sin\theta + b\cos\theta = c$

$a = 1, b = 1, c = \frac{\sqrt{11}}{3}$

$\cos\theta - \sin\theta = \sqrt{a^2 + b^2 - c^2}$

$= \sqrt{1 + 1 - \frac{11}{9}} = \sqrt{\frac{7}{9}} = \frac{\sqrt{7}}{3}$

10. (b) $A + B = 90°$

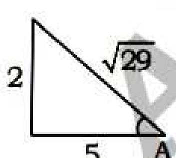
$\frac{\cot A}{\cot B} + \cos^2 A + \cos^2 B$

$= \frac{\cot(90° - B)}{\cot B} + \cos^2(90° - B) + \cos^2 B$

$= \tan^2 B + 1 = \sec^2 B = \sec^2(90° - A)$

$= \operatorname{cosec}^2 A$

11. (b) $\tan A = \frac{2}{5}$



$\therefore \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A = \frac{4}{25}$

12. (a) $k(\tan 45° \sin 60°)$

$= \cos 60° \cot 30°$

$\Rightarrow k\left(1 \times \frac{\sqrt{3}}{2}\right) = \frac{1}{2} \times \sqrt{3}$

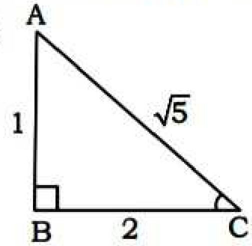
$\Rightarrow k = 1$

13. (d) $\frac{\cos 37°}{\sin 53°} - \cos 47° \operatorname{cosec} 43°$

$= \frac{\cos(90° - 53°)}{\sin 53°} - \cos 47° \operatorname{cosec}(90° - 47°)$

$= 1 - \cos 47° \sec 47° = 1 - 1 = 0$

14. (d)



$\therefore \cos A + \tan C$

$= \frac{1}{\sqrt{5}} + \frac{1}{2} = \frac{2 + \sqrt{5}}{2\sqrt{5}}$

15. (c) $\sin A + \sin^2 A = 1$

$\sin A = 1 - \sin^2 A = \cos^2 A$

and $\cos^6 A = \sin^3 A$

$\cos^4 A = \sin^2 A$

$\therefore \cos^4 A + \cos^6 A = \sin^2 A + \sin^3 A$

$= \sin A(\sin A + \sin^2 A)$

$= \sin A \times 1 = \sin A$

16. (c) $\sec A - \tan A = p$ ......(i)

$\Rightarrow \sec A + \tan A = \frac{1}{p}$ ......(ii)

Adding equation (i) and (ii)

$2\sec A = p + \frac{1}{p}$

$\sec A = \frac{p^2 + 1}{2p}$

17. (d) $2(\sin 1° \times \sec 89°) + 3(\cos 11° \times \operatorname{cosec} 79°) + 5(\tan 21° \times \tan 69°)$

$= 2(\cos 89° \times \sec 89°) + 3(\sin 79° \times \operatorname{cosec} 79°) + 5(\tan 21° \times \cot 21°)$

$= 2 + 3 + 5 = 10$

18. (d) $\sqrt{\frac{1 - \tan A}{1 + \tan A}}$

$= \sqrt{\frac{1 - \frac{\sin A}{\cos A}}{1 + \frac{\sin A}{\cos A}}} = \sqrt{\frac{\cos A - \sin A}{\cos A + \sin A}}$

$= \sqrt{\frac{\cos^2 A + \sin^2 A - 2\sin A\cos A}{\cos^2 A - \sin^2 A}}$

$= \sqrt{\frac{1 - 2\sin A\cos A}{\cos^2 A - \sin^2 A}} = \sqrt{\frac{1 - \sin 2A}{\cos 2A}}$

19. (c) $7\sin^2 A + 3\cos^2 A = 4$

$\Rightarrow 4\sin^2 A + 3(\sin^2 A + \cos^2 A) = 4$

$\Rightarrow 4\sin^2 A = 1$

$\Rightarrow \sin A = \frac{1}{2} \Rightarrow A = 30°$

$\therefore \cot A = \cot 30° = \sqrt{3}$

20. (d) $16\operatorname{cosec}^2\theta + 25\sin^2\theta$

for $a = 16, b = 25$

$\Rightarrow$ Least value $= 2\sqrt{ab}$

$= 2\sqrt{16 \times 25} = 2 \times 4 \times 5 = 40$

21. (d) $\operatorname{cosec} A + \cot A = 3$ ___ (i)

We know,

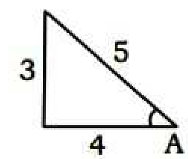
$\operatorname{cosec}^2 A - \cot^2 A = 1$

$\Rightarrow \operatorname{cosec} A - \cot A = \frac{1}{3}$ ___ (ii)

from (i) and (ii)

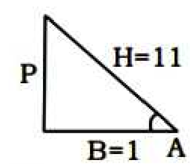
$2\operatorname{cosec} A = \frac{10}{3}$

$\operatorname{cosec} A = \frac{5}{3}$



$\Rightarrow \cos A = \frac{4}{5}$

22. (d) $\cos A = \frac{1}{11}$



$P = \sqrt{11^2 - 1} = \sqrt{120}$

$= 2\sqrt{30}$

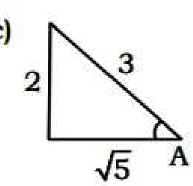
$\therefore \cot A = \frac{B}{P} = \frac{1}{2\sqrt{30}}$

23. (b) $\cos A = \frac{1}{2} \Rightarrow A = 60°$

$\therefore \sin(180° - 60°) = \sin(60°)$

$= \frac{\sqrt{3}}{2}$

24. (c)



$\sin A = \frac{2}{3}$

$\therefore (7 - \tan A)(3 + \cos A)$

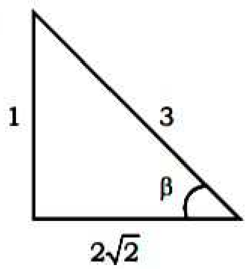
$= \left(7 - \frac{2}{\sqrt{5}}\right)\left(3 + \frac{\sqrt{5}}{3}\right)$

$= \left(\frac{7\sqrt{5} - 2}{\sqrt{5}}\right)\left(\frac{9 + \sqrt{5}}{3}\right)$

$= \frac{63\sqrt{5} + 35 - 18 - 2\sqrt{5}}{3\sqrt{5}}$

$= \frac{61\sqrt{5} + 17}{3\sqrt{5}} = \frac{61}{3} + \frac{17}{3\sqrt{5}}$

25. (b)



$\sin\beta = \frac{1}{3}$,

$(\sec\beta - \tan\beta)^2$

$= \left(\frac{3}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)^2 = \frac{1}{2}$

26. (c) $\frac{\sin^2 39° + \sin^2(90° - 39°)}{\cos^2 35° + \cos^2(90° - 35°)}$

$+ 3\tan 15° \tan 75°$

$= \frac{\sin^2 39° + \cos^2 39°}{\cos^2 35° + \sin^2 35°} + 3\tan 15° \cot 15°$

$= \frac{1}{1} + 3 = 4$

27. (b) $\tan\theta + \cot\theta = 2$

$\Rightarrow \theta = 45°$

$\therefore 2\tan^{25}\theta + 3\cot^{20}\theta + 5\tan^{30}\theta \cot^{15}\theta$

$= (2 \times 1) + (3 \times 1) + (5 \times 1 \times 1)$

$\Rightarrow 2 + 3 + 5 = 10$

28. (b) $\sin(50° + A) - \cos(40° - A)$

$= \cos[90° - (50° + A)] - \cos(40° - A)$

$= \cos(40° - A°) - \cos(40° - A) = 0$

29. (b) $\sin\theta + \cos\theta = \frac{\sqrt{11}}{3}$

Comparing with $a\sin\theta + b\cos\theta = c$

$a = 1, b = 1, c = \frac{\sqrt{11}}{3}$

$\sin\theta - \cos\theta = \sqrt{a^2 + b^2 - c^2}$

$= \sqrt{1 + 1 - \left(\frac{\sqrt{11}}{3}\right)^2} = \sqrt{2 - \frac{11}{9}} = \frac{\sqrt{7}}{3}$

30. (a) $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\cos\theta = \frac{\sin\theta}{\sqrt{2} - 1}$

$\frac{\sin\theta - \cos\theta}{\sin\theta} = \frac{\sin\theta - \frac{\sin\theta}{\sqrt{2} - 1}}{\sin\theta}$

$= \frac{\sqrt{2}\sin\theta - \sin\theta - \sin\theta}{(\sqrt{2} - 1)\sin\theta}$

$= \frac{\sqrt{2}\sin\theta - 2\sin\theta}{(\sqrt{2} - 1)\sin\theta}$

$= \frac{\sqrt{2}\sin\theta(1 - \sqrt{2})}{(\sqrt{2} - 1)\sin\theta} = -\sqrt{2}$

31. (c) $\frac{2\sin A - \cos A}{\sin A + \cos A} = 1$

$\Rightarrow 2\sin A - \cos A = \sin A + \cos A$

$\Rightarrow \sin A = 2\cos A$

$\Rightarrow \tan A = 2$

$\therefore \cot A = \frac{1}{2}$

32. (b) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$\Rightarrow 2 + \frac{3}{4} - \frac{3}{4} = 2$

33. (a) $\cot^2\theta = 1 - e^2$

$[\because \operatorname{cosec}^2\theta - 1 = \cot^2\theta]$

$\operatorname{cosec}^2\theta - 1 = 1 - e^2$

$\operatorname{cosec}^2\theta = 2 - e^2$

$\operatorname{cosec}\theta = \sqrt{2 - e^2}$

$\operatorname{cosec}\theta + \cot^3\theta \sec\theta$

$= \frac{1}{\sin\theta} + \frac{\cos^2\theta}{\sin^3\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin^3\theta} = \operatorname{cosec}^3\theta$

$= \left(\sqrt{2 - e^2}\right)^3 = (2 - e^2)^{\frac{3}{2}}$

34. (b) Given,

$x = \frac{2\sin\theta}{1 + \cos\theta + \sin\theta}$

by the rationalization

$= \frac{2\sin\theta}{1 + \cos\theta + \sin\theta} \times \frac{1 - \cos\theta + \sin\theta}{1 - \cos\theta + \sin\theta}$

$= \frac{2\sin\theta(1 - \cos\theta + \sin\theta)}{(1 + \sin\theta)^2 - (\cos\theta)^2}$

$= \frac{2\sin\theta(1 - \cos\theta + \sin\theta)}{1 + 2\sin\theta + \sin^2\theta - \cos^2\theta}$

$= \frac{2\sin\theta(1 - \cos\theta + \sin\theta)}{\sin^2\theta + \cos^2\theta + 2\sin\theta + \sin^2\theta - \cos^2\theta}$

$= \frac{2\sin\theta(1 - \cos\theta + \sin\theta)}{2\sin\theta + 2\sin^2\theta}$

$= \frac{2\sin\theta(1 - \cos\theta + \sin\theta)}{2\sin\theta(1 + \sin\theta)}$

$= \frac{1 - \cos\theta + \sin\theta}{1 + \sin\theta} = x$

SMART APPROACH:-

We can solve this problem by putting the value of $\theta = 90°$

35. (b) Given

$\left\{\left(\frac{\sec\theta - 1}{\sec\theta + 1}\right)^n\right\} = (\operatorname{cosec}\theta - \cot\theta)$

R.H.S $= \operatorname{cosec}\theta - \cot\theta$

$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$= \frac{1 - \cos\theta}{\sin\theta}$ ....[1]

$= \frac{1 - \cos\theta}{\sin\theta} \times \frac{1 + \cos\theta}{1 + \cos\theta}$

$= \frac{1 - \cos^2\theta}{\sin\theta(1 + \cos\theta)}$

$= \frac{\sin^2\theta}{\sin\theta(1 + \cos\theta)}$

$= \frac{\sin\theta}{1 + \cos\theta}$ ....[2]

Multiplying [1] and [2], we get–

$\frac{1 - \cos\theta}{1 + \cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$

$\frac{\sec\theta - 1}{\sec\theta + 1} = (\operatorname{cosec}\theta - \cot\theta)^2$

$\Rightarrow \left(\frac{\sec\theta - 1}{\sec\theta + 1}\right) = \operatorname{cosec}\theta - \cot\theta$

Hence, $n = \frac{1}{2} = 0.5$

36. (d) We know that, $\sec^2A - \tan^2A = 1$

$16\sec^2A - 16\tan^2A = 16(\sec^2A - \tan^2A)$

$= 16 \times 1 = 16$

37. (a)

$$\left(2\cos^2\theta-1\right)\left[\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}\right]$$

$$=\left(2\cos^2\theta-1\right)\left[\frac{(1+\tan\theta)^2+(1-\tan\theta)^2}{(1-\tan\theta)(1+\tan\theta)}\right]$$

$$=\left(2\cos^2\theta-1\right)\left[\frac{2+2\tan^2\theta}{1-\tan^2\theta}\right]$$

$$=\left(2\cos^2\theta-1\right)\left[\frac{2\left(1+\tan^2\theta\right)}{1-\frac{\sin^2\theta}{\cos^2\theta}}\right]$$

$$=\left(2\cos^2\theta-1\right)\left[\frac{2\sec^2\theta}{\sec^2\theta\left\{\cos^2\theta-\left(1-\cos^2\theta\right)\right\}}\right]$$

$$=\left(2\cos^2\theta-1\right)\left[\frac{2}{\left(2\cos^2\theta-1\right)}\right]=2$$

SMART APPROACH:-

We can easily solve this problem by putting the value of $\theta = 30°$

38. (b) We know that,

$$\because \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\tan x = \frac{5}{1}$$

Hence, Perpendicular = 5 and Base = 1

By Pythagoras Theorm,

$$H = \sqrt{P^2+B^2} = \sqrt{5^2+1^2} = \sqrt{26}$$

$$\text{Sin}x = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{5}{\sqrt{26}}$$

39. (d) $\dfrac{\sin 8\theta\cos\theta - \sin 6\theta\cos 3\theta}{\cos 2\theta\cos\theta - \sin 3\theta\sin 4\theta}$

Multiplying and dividing by 2

$$=\frac{\sin 8\theta\cos\theta-\sin 6\theta\cos 3\theta}{\cos 2\theta\cos\theta-\sin 3\theta\sin 4\theta}\times\frac{2}{2}$$

$$=\frac{2\sin 8\theta\cos\theta-2\sin 6\theta\cos 3\theta}{2\cos 2\theta\cos\theta-2\sin 3\theta\sin 4\theta}$$

We know,

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) + \cos(A-B)$

$$=\frac{\sin 9\theta\sin 7\theta-(\sin 9\theta\cos 3\theta)}{\cos 3\theta+\cos\theta-(\cos\theta-\cos 7\theta)}$$

$$=\frac{\sin 7\theta-\sin 3\theta}{\cos 3\theta+\cos 7\theta}$$

We know,

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$=\frac{2\cos 5\theta\sin 2\theta}{2\cos 5\theta\cos 2\theta}=\tan 2\theta$$

40. (a) We know,

$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$\cos 60° = \frac{1}{2}$$

$\tan 45° = 1$

$\cot 45° = 1$

$\therefore (\sin^4 45° + \cos^4 60°) + (\tan^4 45° + \cot^4 45°)$

$$=\left\{\left(\frac{1}{\sqrt{2}}\right)^4+\left(\frac{1}{2}\right)^4\right\}+(1+1)$$

$$=\left\{\frac{1}{4}+\frac{1}{16}\right\}+2$$

$$=\left\{\frac{4+1}{16}\right\}+2=\frac{5}{16}+2=\frac{37}{16}$$

41. (a) $(1 + \cot A - \text{cosec}A)(1 + \tan A + \sec A) - 1$

$$=\left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)-1$$

$$=\left\{\frac{(\sin A+\cos A)^2-1^2}{\sin A.\cos A}\right\}-1$$

$$=\left\{\frac{\sin^2A+\cos^2A+2\sin A\cos A-1}{\sin A.\cos A}\right\}-1$$

put $\sin^2A + \cos^2A = 1$

$$=\left\{\frac{1+2\sin A\cos A-1}{\sin A.\cos A}\right\}-1$$

$$=\left\{\frac{2\sin A\cos A}{\sin A.\cos A}\right\}-1$$

$= 2 - 1 = 1$

SMART APPROACH:-

We can easily solve this problem by putting the value of $\theta = 45°$

42. (a) We know that,

$\text{cosec}^2A - \cot^2A = 1$

$23\,\text{cosec}^2A - 23\cot^2A$

$= 23(\text{cosec}^2A - \cot^2A)$

$= 23(1) = 23$

43. (c) Given,

$$\sin A = \frac{1}{3}$$

$$\therefore \cos A = \sqrt{1-\sin^2A}$$

$$=\sqrt{1-\left(\frac{1}{3}\right)^2}=\frac{2\sqrt{2}}{3}$$

$$\sin B = \frac{1}{5}$$

$$\therefore \cos B = \sqrt{1-\sin^2B}$$

$$=\sqrt{1-\left(\frac{1}{5}\right)^2}=\frac{2\sqrt{6}}{5}$$

Since the angles given in the second quadrant.

So,

$$\cos A = \frac{-2\sqrt{2}}{3}$$

$$\cos B = \frac{-2\sqrt{6}}{5}$$

We know that–

$\cos(A - B) = \cos A\cos B + \sin A\sin B$

$$=\frac{-2\sqrt{2}}{3}\times\frac{-2\sqrt{6}}{5}+\frac{1}{3}\times\frac{1}{5}$$

$$=\frac{2\sqrt{2}}{3}\times\frac{2\sqrt{2}\times\sqrt{3}}{5}+\frac{1}{3}\times\frac{1}{5}=\frac{8\sqrt{3}+1}{15}$$

44. (a) We know that–

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$$\therefore \frac{\sin(A+B)}{\sin A\cos B}$$

$$=\frac{\sin A\cos B+\cos A\sin B}{\sin A\cos B}$$

$$=\frac{\sin A\cos B}{\sin A\cos B}+\frac{\cos A\sin B}{\sin A\cos B}$$

$= 1 + \cot A\tan B$

45. (d) $\sqrt{\dfrac{1-\sin 45}{1+\sin 45}}=\sqrt{\dfrac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}}$

$$=\sqrt{\frac{\frac{\sqrt{2}-1}{\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}}}}=\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}}$$

$$=\sqrt{\frac{\left(\sqrt{2}-1\right)^2}{\left(\sqrt{2}\right)^2-1^2}}=\sqrt{2}-1$$

We know, $\sqrt{2} = \sec 45°$ and $1 = \tan 45°$

Hence Answer (d)

46. (a) $\cos^2 45° + \cos^2 135° + \cos^2 225° + \cos^2 315°$
$= \cos^2 45° + \{\cos(180° - 45°)\}^2 + \{\cos(180° + 45°)\}^2 + \{\cos(360° - 45°)\}^2$
$= \cos^2 45° + \{-\cos 45°\}^2 + (-\cos 45°)^2 + \{\cos 45°\}^2$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$
$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$$

47. (c) $\dfrac{2\cos^3\theta - \cos\theta}{\sin\theta - 2\sin^3\theta}$
$$= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(1 - 2\sin^2\theta)}$$
$$= \frac{\cos\theta \times \cos 2\theta}{\sin\theta \times \cos 2\theta} = \cot\theta$$

48. (d) $\cos 48° = \dfrac{m}{n}$
$$\cos(90° - 42°) = \frac{m}{n}$$
$$\sin 42° = \frac{m}{n}$$
$$\cos 42° = \sqrt{1 - \sin^2 42°}$$
$$= \sqrt{1 - \frac{m^2}{n^2}}$$
$$\Rightarrow \sqrt{\frac{n^2 - m^2}{n^2}} \Rightarrow \frac{\sqrt{n^2 - m^2}}{n}$$
$$\cot 42° = \frac{\cos 42°}{\sin 42°}$$
$$= \frac{\frac{\sqrt{n^2 - m^2}}{n}}{\frac{m}{n}} \Rightarrow \frac{\sqrt{n^2 - m^2}}{m}$$
$$\sec 48° = \frac{n}{m}$$
$$\therefore \sec 48° - \cot 42°$$
$$= \frac{n}{m} - \frac{\sqrt{n^2 - m^2}}{m} \Rightarrow \frac{n - \sqrt{n^2 - m^2}}{m}$$

49. (c) $\therefore \sec^2\theta\left(\sqrt{1 - \sin^2\theta}\right)$
$= \sec^2\theta \times \cos\theta = \sec\theta$

50. (c) Given,
$\cos A + \cos B + \cos C = 3$
$\Rightarrow A = B = C = 0°$
$\sin A + \sin B + \sin C = 0$

51. (a) $\dfrac{\sin A}{1 + \cos A} + \dfrac{1 + \cos A}{\sin A}$
$$= \frac{\sin^2 A + (1 + \cos A)^2}{\sin A(1 + \cos A)}$$
$$= \frac{\sin^2 A + 1 + 2\cos A + \cos^2 A}{\sin A(1 + \cos A)}$$
$$= \frac{2(1 + \cos A)}{\sin A(1 + \cos A)} = 2\operatorname{cosec} A$$

SMART APPROACH:-
We can easily solve this problem by putting the value of $\theta = 30°$

52. (a) We know that–
$\operatorname{cosec}^2 A - \cot^2 A = 1$
$\Rightarrow (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A) = 1$
$\Rightarrow \operatorname{cosec} A - \cot A$
$$= \frac{1}{\operatorname{cosec} A + \cot A}$$
$$\Rightarrow \operatorname{cosec} A - \cot A = \frac{1}{m}$$

53. (a) $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$
squaring both side, we get–
$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$
$\Rightarrow 2\sin\theta\cos\theta = 2\cos^2\theta - \sin^2\theta - \cos^2\theta$
$\Rightarrow 2\sin\theta\cos\theta = \cos^2\theta - \sin^2\theta$
$\Rightarrow 2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\cos\theta - \sin\theta)$
$$\Rightarrow \cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{\cos\theta + \sin\theta}$$
$$\Rightarrow \cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{\sqrt{2}\cos\theta}$$
$$\Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$$

54. (a) $(1 + \cot^2\theta)(1 - \cos\theta)(1 + \cos\theta)$
$= (1 + \cot^2\theta)(1 - \cos^2\theta)$
$= \operatorname{cosec}^2\theta \times \sin^2\theta = 1$

55. (c) Given, $\cos\theta + \sec\theta = 2$
We know,
$\cos 0° = 1$ and $\sec 0° = 1$
put $\theta = 0°$
$\sin^6\theta + \cos^6\theta = (\sin 0°)^6 + (\cos 0°)^6$
$= 0 + 1 = 1$

56. (d) $\cos\left(\dfrac{-7\pi}{2}\right) = \cos\left(4\pi - \dfrac{\pi}{2}\right)$
$$= \cos\frac{\pi}{2} = 0$$

57. (d) $(\operatorname{cosec} A + \cot A)(1 - \cos A)$
$$= \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A}\right)(1 - \cos A)$$
$$= \frac{(1 + \cos A)}{\sin A} \times (1 - \cos A)$$
$$= \frac{1 - \cos^2 A}{\sin A} = \frac{\sin^2 A}{\sin A} = \sin A$$

SMART APPROACH:-
We can easily solve this problem by putting the value of $\theta = 30°$

58. (a) Given,
$a\sin^{12} A + b\sin^{10} A + c\sin^8 A + \sin^6 A = 1$ ....(1)
$\cos A = \sin^2 A$
squaring both sides:
$\Rightarrow \cos^2 A = \sin^4 A$
$\Rightarrow 1 - \sin^2 A = \sin^4 A$
$\Rightarrow 1 = \sin^4 A + \sin^2 A$
cubing both sides:
$\Rightarrow 1 = (\sin^4 A + \sin^2 A)^3$
$\Rightarrow 1 = \sin^{12} A + \sin^6 A + 3\sin^4 A \sin^2 A(\sin^4 A + \sin^2 A)$
$\Rightarrow 1 = \sin^{12} A + \sin^6 A + 3\sin^{10} A + 3\sin^8 A$
$\Rightarrow \sin^{12} A + 3\sin^{10} A + 3\sin^8 A + \sin^6 A = 1$ .....(2)
On comparing (1) and (2), we get
$a = 1$, $b = 3$ and $c = 3$
Hence, $a + b + c = 7$

59. (c) $\cos(36° - A)\cos(36° + A) + \cos(54° - A)\cos(54° + A)$
$= \cos(36° + A)\cos(36° - A) + \cos\{90° - (54° - A)\}\cos\{90° - (54° + A)\}$
$= \cos(36° + A)\cos(36° - A) + \sin(36° + A)\sin(36° - A)$
We know
$[\cos(A - B) = \cos A\cos B + \sin A\sin B]$
$= \cos\{36° + A - (36° - A)\} = \cos 2A$

60. (c) $\dfrac{\sin 54°}{\cos 36°} + \dfrac{\sec 46°}{\operatorname{cosec} 44°}$
$$= \frac{\cos(90° - 54°)}{\cos 36°} + \frac{\operatorname{cosec}(90° - 46°)}{\operatorname{cosec} 44°}$$
$$= \frac{\cos 36°}{\cos 36°} + \frac{\operatorname{cosec} 44°}{\operatorname{cosec} 44°} = 1 + 1 = 2$$

61. (c) $4\cos\theta + 3\sin\theta = x$
squaring both sides–
$16\cos^2\theta + 9\sin^2\theta + 24\sin\theta\cos\theta = x^2$ ......(1)
Again,
$4\sin\theta - 3\cos\theta = y$
squaring both sides–
$16\sin^2\theta + 9\cos^2\theta - 24\sin\theta\cos\theta = y^2$ ......(2)
Adding equation (1) and (2), we get–
$x^2 + y^2 = 25\sin^2\theta + 25\cos^2\theta$
$= 25(\sin^2\theta + 25\cos^2\theta)$
$= 25(1) = 25$

SMART APPROACH:-
Special Case:
If $A\cos\theta + B\sin\theta = X$ and $A\sin\theta - B\cos\theta = Y$
then –
$X^2 + Y^2 = A^2 + B^2$
Given,
$4\cos\theta + 3\sin\theta = X$
$4\sin\theta - 3\cos\theta = Y$
$\therefore X^2 + Y^2 = 4^2 + 3^2$
$= 16 + 9 = 25$

62. (c) $\dfrac{\cos^4\theta-\sin^4\theta}{\sin^2\theta}$

$= \dfrac{(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)}{\sin^2\theta}$

$= \dfrac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}$

$= \dfrac{\cos^2\theta}{\sin^2\theta}-\dfrac{\sin^2\theta}{\sin^2\theta} = \cot^2\theta-1$

63. (b)

$\dfrac{\sin A}{\cot A+\operatorname{cosec} A}-\dfrac{\sin A}{\cot A-\operatorname{cosec} A}-1$

$=\sin A\left(\dfrac{1}{\cot A+\operatorname{cosec} A}-\dfrac{1}{\cot A-\operatorname{cosec} A}\right)-1$

$=\sin A\left(\dfrac{\cot A-\operatorname{cosec} A-\cot A-\operatorname{cosec} A}{(\cot A+\operatorname{cosec} A)(\cot A-\operatorname{cosec} A)}\right)-1$

$=\sin A\left(\dfrac{-2\operatorname{cosec} A}{\cot^2 A-\operatorname{cosec}^2 A}\right)-1$

$=\dfrac{-2\sin A\operatorname{cosec} A}{-(\operatorname{cosec}^2 A-\cot^2 A)}-1 = 2 - 1 = 1$

SMART APPROACH:-
We can easily solve this problem by putting the value of $\theta = 45°$

64. (a) Given
$\sin^2\theta = \cos^3\theta$
Squaring both sides
$\sin^4\theta = \cos^6\theta$ ....[1]

$\cot^2\theta - \cot^6\theta = \cot^2\theta - \dfrac{\cos^6\theta}{\sin^6\theta}$

From equation [1],

$=\cot^2\theta-\dfrac{\sin^4\theta}{\sin^6\theta}$

$=\cot^2\theta-\dfrac{1}{\sin^2\theta}$

$=\cot^2\theta-\operatorname{cosec}^2\theta$

$=-(\operatorname{cosec}^2\theta-\cot^2\theta)$

$= -1$

65. (c) Given,

$\sin A=\dfrac{1}{2}$

$\Rightarrow A = 30°$
Now,
$\tan A + \cos A = \tan 30° + \cos 30°$

$=\dfrac{1}{\sqrt3}+\dfrac{\sqrt3}{2} = \dfrac{2+3}{2\sqrt3} = \dfrac{5}{2\sqrt3}$

66. (b) $1 + \sin^2\theta - 3\sin\theta\cos\theta = 0$
Dividing the expression by $\sin^2\theta$
$\Rightarrow \operatorname{cosec}^2\theta + 1 - 3\cot\theta = 0$
$\Rightarrow (\operatorname{cosec}^2\theta - 1) + 1 - 3\cot\theta + 1 = 0$
$\Rightarrow \cot^2\theta - 3\cot\theta + 2 = 0$
$\Rightarrow \cot^2\theta - 2\cot\theta - \cot\theta + 2 = 0$
$\Rightarrow \cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$
$\Rightarrow (\cot\theta - 2)(\cot\theta - 1)$
Hence, $\cot\theta = 2$ or 1
Answer (B)

67. (b) Given

$\cos^4\alpha-\sin^4\alpha=\dfrac{5}{6}$

$\Rightarrow(\cos^2\alpha-\sin^2\alpha)(\cos^2\alpha+\sin^2\alpha)=\dfrac{5}{6}$

$\Rightarrow(\cos^2\alpha-\sin^2\alpha)(1)=\dfrac{5}{6}$

$\Rightarrow\cos^2\alpha-(1-\cos^2\alpha)=\dfrac{5}{6}$

$\Rightarrow 2\cos^2\alpha-1=\dfrac{5}{6}$

68. (c) $\tan 45° + \sec 60° = 1 + 2 = 3$
$\Rightarrow x = 3$

69. (d) $4(\operatorname{cosec}^2 57° + \tan^2 33°) - \cos 90° - y\tan^2 66° \times \tan^2 24° = \dfrac{y}{2}$

$4(\operatorname{cosec}^2 57° - \cot^2 57°) - \cos 90° - y\tan^2 66° \cot^2 66° = \dfrac{y}{2}$

$4 - 0 - y = \dfrac{y}{2}$

$4 = \dfrac{y}{2} + y$

$\dfrac{3y}{2} = 4$

$y = \dfrac{8}{3}$

70. (c) $4 - 2\sin^2\theta - 5\cos\theta = 0$, $0° < \theta < 90°$, $\cos\theta + \tan\theta = ?$
Put $\theta = 60°$

$\cos\theta + \tan\theta = \dfrac{1}{2}+\sqrt3 = \left(\dfrac{1+2\sqrt3}{2}\right)$

71. (d) $\sec 3x = \operatorname{cosec}(3x - 45°)$, $3x$ is angle, $x = ?$
$\because A + B = 90$
$\sec A = \operatorname{cosec} B$
$3x + 3x - 45° = 90°$
$6x = 135°$

$x = \dfrac{135°}{6}$

$x = 22.5°$

72. (c)

$7\dfrac{\sin^2 30°+\cos^2 60°-\sec 35°.\sin 55°}{\sec 60°+\operatorname{cosec} 30°}=?$

$\Rightarrow\dfrac{\frac14+\frac14-\operatorname{cosec} 55°\sin 55°}{2+2}$

$\Rightarrow\dfrac{\frac12-1}{4}=\dfrac{-1}{8}$

73. (c) $\sin 3x = \cos(3x - 45°)$, $x = ?$
$\because A + B = 90°$
$\sin A = \cos B$
$3x + 3x - 45° = 90°$
$6x = 135°$
$x = 22.5$

74. (b)

$\dfrac{\sin^2 30°+\cos^2 60°+\sec 45°.\sin 45°}{\sec 60°+\operatorname{cosec} 30°}=?$

$\Rightarrow\dfrac{\frac14+\frac14+\sqrt2\times\frac{1}{\sqrt2}}{2+2}$

$=\dfrac{\frac12+1}{4}=\dfrac{3}{8}$

75. (d) $\dfrac{\sin^2 52°+2+\sin^2 38°}{4\cos^2 43°-5+4\cos^2 47°}=?$

$\Rightarrow\dfrac{\sin^2 52°+\cos^2 52°+2}{4(\cos^2 43°+\sin^2 43°)-5}$

$\Rightarrow\dfrac{3}{-1}=-3$

76. (a) If $A + B = 90°$
$\tan A = \cot B$
$4\theta + \theta - 5 = 90°$
$5\theta = 95°$
$\theta = 19°$

77. (d) $\cos^2\theta - \sin^2\theta = \dfrac{1}{2}$
$\cos 2\theta = \cos 60°$
$\theta = 30°$

78. (c) $\cot\theta = \dfrac{1}{\sqrt3}$
$\theta = 60°$

$\Rightarrow\dfrac{2-\sin^2\theta}{1-\cos^2\theta}+(\operatorname{cosec}^2\theta-\sec\theta)$

$\Rightarrow\dfrac{2-\frac34}{\frac34}+\left(\dfrac43-2\right)$

$\Rightarrow\dfrac{\frac54}{\frac34}+\dfrac{-2}{3} \quad \Rightarrow\dfrac53-\dfrac23=1$

79. (a) $\sec^{107}\theta + \cos^{107}\theta = 2$
put $\theta = 0°$
$= 1^{107} + 1^{107} = 2$
$= 1 + 1 = 2$ (satisfied)
Now,
$\sec\theta + \cos\theta$
$\Rightarrow \sec 0° + \cos 0°$
$\Rightarrow 1 + 1$
$= 2$

80. (c) $2\theta + 50° + 4\theta + 16° = 90$
$= 6\theta + 66 = 90$
$= 6\theta = 24$
$= \theta = 4°$

81. (b) $\frac{1}{\operatorname{cosec}\theta+1}+\frac{1}{\operatorname{cosec}\theta-1} = 2\sec\theta$
Put $\theta = 45°$
$\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{2}-1} = 2\sqrt{2}$
$2\sqrt{2} = 2\sqrt{2}$
(both conditions are satisfied)
Then,
$\frac{\tan\theta+2\sec\theta}{\operatorname{cosec}\theta}$
$\frac{1+2\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}+4}{2} = \frac{4+\sqrt{2}}{2}$

82. (d) $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$
put $\theta = 30°$
$\frac{\cos 30°+\sin 30°}{\cos 30°-\sin 30°} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$
$= \frac{\frac{\sqrt{3}}{2}+\frac{1}{2}}{\frac{\sqrt{3}}{2}-\frac{1}{2}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$
$= \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ (satisfied)
$\sec 30° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

83. (c) $3 + \cos^2\theta = 3(\cot^2\theta + \sin^2\theta)$
put $\theta = 60°$
$= 3 + \frac{1}{4} = 3\left(\frac{1}{3}+\frac{3}{4}\right)$
$= \frac{13}{4} = 3 \times \frac{13}{12}$
$= \frac{13}{4} = \frac{13}{4}$ (satisfied)

So, $(\cos\theta + 2\sin\theta)$
$= (\cos 60° + 2\sin 60°)$
$= \frac{1}{2} + 2 \times \frac{\sqrt{3}}{2}$
$= \frac{1}{2} + \frac{\sqrt{3}}{1} = \frac{1+2\sqrt{3}}{2}$

84. (d) $\tan(11\theta) = \cot(7\theta)$
$\tan(11\theta) = \tan(90° - 7\theta)$
$= 11\theta = 90° - 7\theta = 18\theta = 90°$
$\theta = 5°$
Now,
$\sin^2(6\theta) + \sec^2(9\theta) + \operatorname{cosec}^2(12\theta)$
$= \sin^2(6 \times 5) + \sec^2(9 \times 5) + \operatorname{cosec}^2(12 \times 5)$
$= \sin^2 30° + \sec^2 45° + \operatorname{cosec}^2 60°$
$= \frac{1}{4} + \frac{2}{1} + \frac{4}{3} = \frac{3+24+16}{12} = \frac{43}{12}$

85. (c) $7\sin^2\theta + 3\cos^2\theta = 4$
We have to find value of $(\tan^2 2\theta + \operatorname{cosec}^2 2\theta)$
Let, put $\theta = 30°$
$\Rightarrow 7\sin^2 30° + \cos^2 30° = 4$
$\Rightarrow 7 \times \frac{1}{4} + 3 \times \frac{3}{4} = 4$
$\Rightarrow \frac{16}{4} = 4$
$\Rightarrow 4 = 4$ (Satisfied)
So,
$(\tan^2 2\theta + \operatorname{cosec}^2 2\theta)$
$= (\tan^2 2 \times 30 + \operatorname{cosec}^2 2 \times 30)$
$= (\tan^2 60° + \operatorname{cosec}^2 60°)$
$= 3 + \frac{4}{3} = \frac{13}{3}$

86. (c) $21\tan\theta = 20$
find value of $(1 + \sin\theta + \cos\theta) : (1 - \sin\theta + \cos\theta)$
$= \tan\theta = \frac{20}{21} = \frac{P}{B}$, $h = 29$
$= (1 + \sin\theta + \cos\theta) = (1 - \sin\theta + \cos\theta)$
$= \left(1+\frac{20}{29}+\frac{21}{29}\right) : \left(1-\frac{20}{29}+\frac{21}{29}\right)$
$= \frac{29+20+21}{29} : \frac{29-20+21}{29}$
$= 70 : 30 = 7 : 3$

87. (c) Given, $2\sin\theta + 15\cos^2\theta = 7$
Put $P = 4, B = 3, h = 5$
$\Rightarrow 2 \times \frac{4}{5} + 15 \times \frac{9}{25} = 7$
$\Rightarrow \frac{8}{5} + \frac{27}{5} = 7$
$\Rightarrow \frac{35}{5} = 7$
$\Rightarrow 7 = 7$ (satisfied)
Thus,
$\frac{3-\tan\theta}{2+\tan\theta} = \frac{3-\frac{4}{3}}{2+\frac{4}{3}} = \frac{\frac{5}{3}}{\frac{10}{3}} = \frac{5}{10} = \frac{1}{2}$

88. (c) Given,
$\operatorname{cosec}\theta = 1.25 = \frac{125}{100} = \frac{5}{4} = \frac{H}{P}$
Here, $H = 5$, $P = 4$ & $B = 3$
$\therefore \frac{4\tan\theta - 5\cos\theta + 1}{\sec\theta + 4\cot\theta - 1}$
$= \frac{4\times\frac{4}{3} - 5\times\frac{3}{5}+1}{\frac{5}{3}+4\times\frac{3}{4}-1} = \frac{\frac{16}{3}-3+1}{\frac{5}{3}+3-1} = \frac{10}{11}$

89. (c) Given, $\sin\theta - \cos\theta = 0$
$\Rightarrow \sin\theta = \cos\theta$
Putting $\theta = 45°$ satisfied.
Hence, $\sin^4\theta + \cos^4\theta$
$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

90. (c) $\sqrt{2}\sin(60° - \alpha) = 1$
$\sin(60° - \alpha) = \frac{1}{\sqrt{2}}$
$\sin(60 - \alpha) = \sin 45°$
$\Rightarrow 60° - \alpha = 45°$
$\Rightarrow a = 15°$

91. (b) $3(\cot^2\theta - \cos^2\theta) = 1 - \sin^2\theta$

$\Rightarrow 3(\cot^2\theta - \cos^2\theta) = \cos^2\theta$

Put $\theta = 60°$

$$\Rightarrow 3\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{4}$$

$$\Rightarrow 3 \times \frac{1}{12} = \frac{1}{4}$$

$$\Rightarrow \frac{1}{4} = \frac{1}{4} \text{ (Satisfied)}$$

92. (a) $\therefore \tan^2 48° - \operatorname{cosec}^2 42° + \operatorname{cosec}(67° + \theta) - \sec(23° - \theta)$

$= \tan^2 48° - \operatorname{cosec}^2(90° - 48°) + \operatorname{cosec}(67° + \theta) - \sec[90° - (67° + \theta)]$

$= \tan^2 48° - \sec^2 48° + \operatorname{cosec}(67° + \theta) - \operatorname{cosec}(67° + \theta)$

$= \tan^2 48° - \sec^2 48°$

$= -(\sec^2 48° - \tan^2 48°)$

We know, $\sec^2\theta - \tan^2\theta = -1$

93. (a) $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

put $\theta = 60°$

$$\Rightarrow \frac{\frac{1}{2}}{1-\frac{\sqrt{3}}{2}} + \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} = 4$$

$$\Rightarrow \frac{2+\sqrt{3}-2\sqrt{3}}{1} = 4$$

$\Rightarrow 4 = 4$ (Satisfied)

$\sec\theta + \operatorname{cosec}\theta + \cot\theta$

$$= \frac{2}{1} + \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} = 2 + \sqrt{3}$$

94. (d) $\cos^2\theta - \sin^2\theta = \tan^2\phi$

$$\Rightarrow \frac{(\cos^2\theta - \sin^2\theta)}{1} = \sin^2\phi/\cos^2\phi$$

$$\Rightarrow \frac{(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta + \sin^2\theta)} = \frac{\sin^2\phi}{\cos^2\phi}$$

By Componendo and Dividendo,

$$\Rightarrow \frac{\cos^2\theta}{(-\sin^2\theta)} = \frac{(\sin^2\phi + \cos^2\phi)}{(\sin^2\phi - \cos^2\phi)}$$

$$\Rightarrow \frac{(-\sin^2\theta)}{\cos^2\theta} = \frac{(\sin^2\phi - \cos^2\phi)}{(\sin^2\phi + \cos^2\phi)}$$

$$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = \frac{(\sin^2\phi - \cos^2\phi)}{1}$$

$\Rightarrow \tan^2\theta = (\cos^2\phi - \sin^2\phi)$

SMART APPROACH:-

If $\theta \propto \phi$, then, $\phi \propto \theta$

$\cos^2\theta - \sin^2\theta = \tan^2\phi$

$\therefore \cos^2\phi - \sin^2\phi = \tan^2\theta$

★★★★★