JUNE 1, 2022

A TIME SERIES COMPARISON OF

STOCK AND CRYPTOCURRENCY

USING VOLATILITY MODELLING

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ABSTRACT

"Volatility models have been playing important roles in economics and finance. The comprehension of volatility is a crucial concept in analysing data. The aim of this article is to introduce several volatility models and use these models to predict the conditional variance about the rate of return in mainly two broad markets– Stock and Cryptocurrency. This paper chooses the ARCH, GARCH model, E-GARCH model and TGARCH model to analyse the rate of return. Hence this paper is mainly capturing the forecasting performance with volatility models under different error distributions. Finally, after comparing the Root Mean Square Error (RMSE), choose the best model to predict the conditional variance. Further a real time forecast for next 52 weeks has been done to get some idea on investment decision. This paper selects Berkshire Hathaway Inc. Class A(BRK-A) from the stock market and Bitcoin (BTC-USD) from the cryptocurrency market for the analysis."

KEYWORDS: Volatility, Forecasting. ARCH, GARCH, EGARCH, TGARCH models, Time series, Root Mean Square Error, BRK-A, BTC-USD

INTRODUCTION

A sound investment portfolio should contain a diverse mix of assets. Putting money in different kinds of investments, such as stocks, bonds, real estate, and commodities, spreads risk, there is even room for more speculative investments. In the 20th century, it might have been wildcatters drilling for oil (and not always finding it). In the 1990s, it might have been internet stocks. Today, it's cryptocurrency (also known as crypto). In considering **cryptocurrency vs. stocks**, investors must balance comfort and risk. Investors in digital currencies have had to live with wild swings in value. The roller-coaster ride of stock value can be dizzying, but not quite as wild as crypto's ups and down. Understanding the strengths and weaknesses of each asset and the role they play in a portfolio is necessary to meet the investor's goals. Further investing is not an either-or proposition. It pays to have diverse investments than to have balance safer bets with investments that bear a greater chance of loss. By the same token, investors don't have to decide between cryptocurrency versus stocks – they can pursue both as long as they are comfortable with an element of risk in their portfolio.

In this paper we try to **compare both market in terms of volatility, returns** and then forecast the future weekly closing price for subsequent months so as to understand the nature of each of them. This helps the investor to make a decision on the investment or portfolio he should build. We have taken the **Weekly Close Price data of Berkshire Hathaway Inc. Class A (BRK-A) from the Stock Market and Bitcoin (BTC-USD) from the cryptocurrency market.** The reason for choosing weekly data is daily data for stock markets exclude holidays but cryptocurrency does not have any holiday so data is available for all 7 days. The study is conducted for the time period between 01-01-2015 and 30-12-2022.This paper is mainly talking about the applications on GARCH model and extension GARCH model. It focuses on how to select the appropriate model and use it to predict the future conditional variance.

Full text is organized as follows. First it has a brief introduction. Next we introduce the classic ARCH/GARCH model and the extension GARCH model, error distribution and Root Mean Square Error as the methodology. The data analysis and statistics follows then in the next section. Then we move on to show the estimation results, ARCH-LM test, the out-of sample forecast and the real time forecast for the coming years and finally we point out the conclusion. The reference can be found at the end.

METHODOLOGY

Auto-regressive Conditional Heteroskedastic (ARCH) Model

In econometrics, the autoregressive conditional heteroskedasticity (ARCH) model is a statistical model for time series data that describes the variance of the current error term or innovation as a function of the actual sizes of the previous time periods' error terms; often the variance is related to the squares of the previous innovations. The ARCH model is appropriate when the error variance in a time series follows an autoregressive (AR) model. This model is characterised by its average being equal to zero, with constant variance and is conditional in the past. A p^{th} order Autoregressive Conditional Heteroskedasticity (ARCH) process is given by:

$$r_t = \mu_t + \varepsilon_t$$

$$\sigma_t^2 = c + \sum^{q}_{i=1}\alpha_j \varepsilon^2_{t-j}$$

$$\varepsilon_t = e_t \sigma_t$$

$$e_t \sim N\ (0,1)$$

Where μ_T can be any adapted model of unconditional mean.

An ARCH(P) model is covariance stationary as long as the model for the conditional mean corresponds to a stationary process and $1 - \alpha_1,\ 1- \alpha_2, \ldots\ldots, 1-\alpha_p>0$.

Generalised ARCH (GARCH) Model

GARCH is a statistical model that can be used to analyse a number of different types of financial data, for instance, macroeconomic data. Financial institutions typically use this model to estimate the volatility of returns for stocks, bonds, and market indices. They use the resulting information to determine pricing, judge which assets will potentially provide higher returns, and forecast the returns of current investments to help in their asset allocation, hedging, risk management, and portfolio optimization decisions. Bollerslev (1986) and Taylor (1986) proposed the so-called generalized ARCH (GARCH) model for substituting the ARCH model.

$$\sigma_t^2 = c + \sum^{p}_{j=-1}\beta_j \sigma_{t-j}^2 + \sum^{q}_{i=1}\alpha_j \varepsilon^2_{t-j}$$

Threshold GARCH(TGARCH)model

TGARCH model is an asymmetric model. It extends the standard GARCH (p,q) to include asymmetric terms that capture an important phenomenon in the conditional variance of equities:

the propensity for the volatility to rise more subsequent to large negative shocks than to positive shocks (known as the "leverage effect"). Defining the sequence $\{\varepsilon_t\}$ equals to $z_t\sigma_t$ and $\{\varepsilon_t\}$ is a normal distribution.

$$\varepsilon_t \sim N\ (0,\ \sigma_t^2)$$

The TGARCH model is written by:

$$\sigma_t^2 = c + \sum^{p}_{j=-1} \beta_j\ \sigma_{t-j}^2 + \sum^{q}_{i=1}\alpha_j \varepsilon^2_{t-j} + \sum^{r}_{k=1} \gamma_k \varepsilon^2_{t-k} I_{t-k}(\varepsilon_{t-k} < 0)$$

where the sign of the indicator term captures the asymmetry where

$$I_t = 1,\quad \text{if } \varepsilon_t < 0$$
$$0,\quad \text{otherwise}$$

which implies that when the residual$\{\varepsilon_t\}$ is smaller than zero the indicator term (I_t) equals to one or zero when the residual is not smaller than zero.

Exponential GARCH (EGARCH) model

The exponential general autoregressive conditional heteroskedastic (EGARCH) is another form of the GARCH model. E-GARCH model was proposed by Nelson (1991) to overcome the weakness in GARCH handling of financial time series. In particular, to allow for asymmetric effects between positive and negative asset returns.

Formally, an E-GARCH(p,q):

$$X_t = \mu + a_t$$

$$\ln(\sigma_t^2) = \omega + \alpha(|e_{t-1}| - E[|e_{t-1}|] + \gamma e_{t-1} + \beta \ln(\sigma_{t-1}^2)$$
$$\varepsilon_t = e_t\sigma_t$$

$$e_t \sim N\ (0,1)$$

Root Mean Square

Root Mean Square Error (RMSE) measures the difference between the true values and estimated values, and accumulates all these difference together as a standard for the predictive ability of a model. The criterion is the smaller value of the RMSE, the better the predicting ability of the model. This article uses this method to determine which model has the best forecasting performance.

DATA AND SUMMARY STATISTICS

In this paper two markets are compared – **Cryptocurrency and Stock market** and for the analysis data on **weekly closing price of Berkshire Hathaway Inc. Class A (BRK-A) and Bitcoin (BTC-USD)** has been taken. The study is conducted for the time period between 01-01-2015 and 30-12-2022. Figure 1. And Figure 2 visualizes the plot of weekly closing price of BRK-A and BTC-USD respectively. Clearly from the figure the data on weekly closing price of both BRK-A and BTC-USD is non-stationary which has been further confirmed from the **Augmented Dickey Fuller Test (Unit Root Test)** which gives the value of p-Value as 0.9715 and 0.9120 for BRK-A and BTC-USD respectively accepting the null hypothesis of Having Unit Root.

FIGURE 1

CLOSING PRICE OF BRK-A

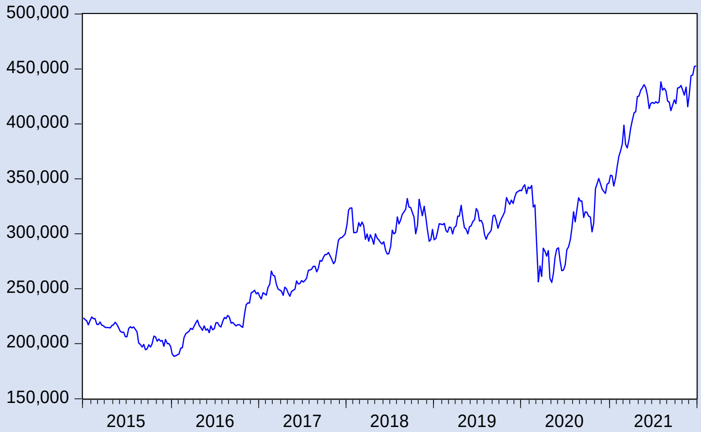


FIGURE 2

CLOSING PRICE OF BTC-USD

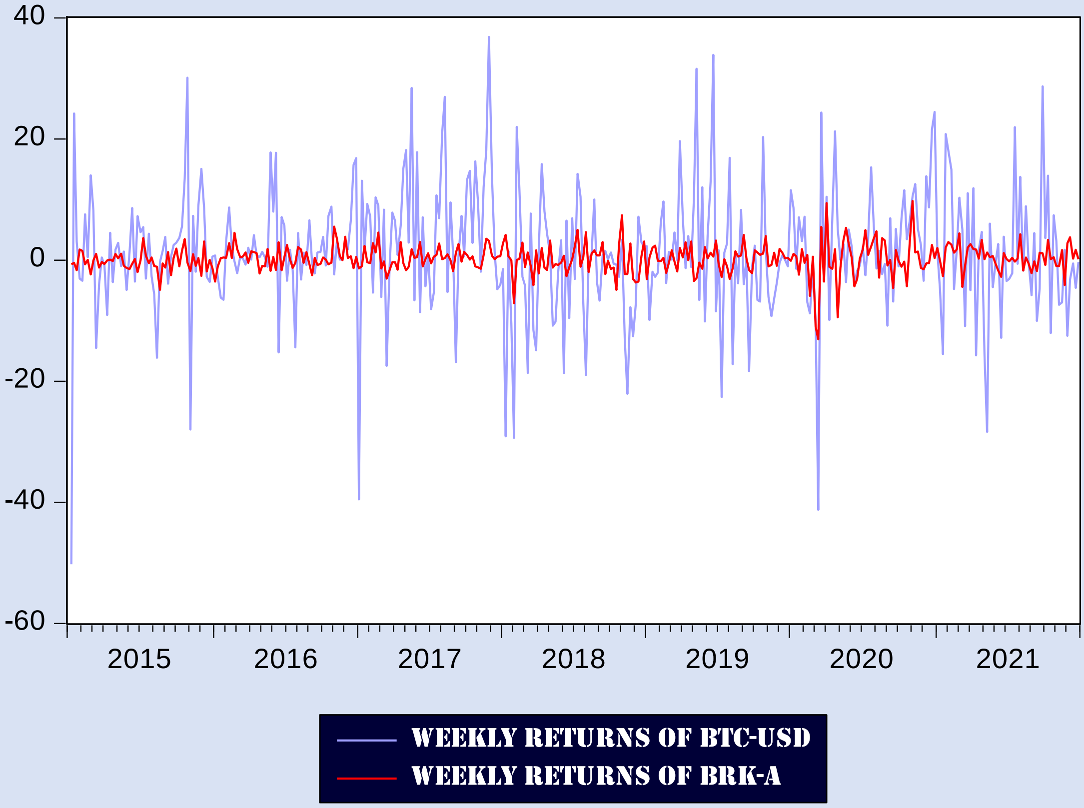
To make the data stationary log differentiation has been taken and this gives us the Return of each asset given by the equation:

$$r_t = \log(close_t/close_{t-1}) * 100$$

The stationary of r_t can be validated by Augmented Dickey-Fuller Test which gives the p-value as 0.0000 for both BRK-A and BTC-USD signifying there is **NO UNIT ROOT**.

Figure 3 shows the graph of Return Data of BRK-A and BTC-USD in a single graph which helps in comparison between the two which can be interpreted as High Volatility with High Return in case of the Crypto-market and Low Returns with relatively low volatility in Stock Market. Figure 3 also visualises **volatility clustering** in both the market representing the fact that a large fluctuation is followed by another large fluctuation or a small fluctuation if followed by another small fluctuation. Thus the variance is autocorrelated and we encounter the problem of heteroskedasticity.

FIGURE 3



DESCRIPTIVE STATISTICS

From the Table 1 it can be interpreted that the mean of returns for both data is positive indicating that closing price of BRK-A and BTC-USD both have increased over the period. Statistically, the risk or volatility is the dispersion of the returns. Daily performance or the average probability for the period is 1.385839 for BTC-USD and 0.193399 which clearly follows that the return from cryptocurrency market is relatively higher on average than the stock market and the risk or standard deviation per week is 10.68087 for BTC-USD and 2.391341 for BRK-A which reflects high volatility of BTC data compared to BRK data. The descriptive statistic show that the returns are negatively skewed, indicating that there is a high probability of earning returns that is greater than the mean in both the market. The value of Kurtosis of the series is also greater than 3 which implies that the return series is fat-tailed and does not follow a normal distribution which is also evident from the Jarque-Bera Test statistic which indicate the returns time series does not follow the normal distribution at a significant level (0.05). This finding is a general feature of every financial time series data.

TABLE 1

	BTC-USD RETURN	BRK-A RETURN
Mean	1.385839	0.193399
Median	1.207925	0.199484
Maximum	36.82813	9.767686
Minimum	-50.23634	-13.08503
Std. Dev.	10.68087	2.391341
Skewness	-0.401342	-0.546696
Kurtosis	6.023046	7.956095
Jarque-Bera	148.7848	391.7420
Probability	0.000000	0.000000
Sum	505.8311	70.59077
Sum Sq. Dev.	41525.46	2081.538
Observations	365	365

Having considered the descriptive statistics the next step to reach our objective is Estimation.

ESTIMATION RESULTS

As our next step GARCH models are used to estimate and forecast the different rates of returns under a normal error distributions, and then compare the results and choose the appropriate model to forecast the conditional variance.

We start with selection of appropriate mean equation for both the market.

BRK-A: Mean Model Selection

To get a rough idea about the mean model for Returns data of BRK-A, a quick snap of correlogram has been taken. From the correlogram it is interpreted that the better model would be an ARMA (4,4) or ARIMA (6,6) because of a similar pattern in both autocorrelation function (ACF) plot and (PACF) plot. However it has been concluded that the **best model is ARMA (3,4)** as it gives the **minimum Akaike info criterion (AIC)** value among all the combination taken. From the following function:

$$r_t^2 = c + \alpha_1 r^2_{t-1} + \alpha_2 r^2_{t-2} + \alpha_2 \varepsilon^2_{t-3} + \beta_1 \varepsilon^2_{t-1} + \beta_2 \varepsilon^2_{t-2} + \beta_3 \varepsilon^2_{t-3} + \beta_4 \varepsilon^2_{t-4}$$

The estimated parameters are c=0.191581 α_1= 0.935455 α_2=-0.162364 α_3=-0.559913 β_1=-0.232853 β_2=0.158343 β_3=0.636916 β_4=-0.169638 and then calculate and estimate GARCH model on estimated ε.

BTC-USD: Mean Model Selection

Again to get a rough idea about the mean model for Returns data of BTC-USD, a quick snap of correlogram has been taken. From the correlogram it is interpreted that the better model would be an ARMA (1,1) because of a similar pattern in both autocorrelation function (ACF) plot and (PACF) plot and the conclusion has been validated that **the best model is ARMA (1,1)** as it gives the **minimum Akaike info criterion (AIC)** value among all the combination taken.

From the following function:

$$r_t^2 = c + \alpha_1 r^2_{t-1} + \beta_1 \varepsilon^2_{t-1}$$

The estimated parameters are c=1.391668 α_1=-0.930308 β_1= 0.883200 and then calculate and estimate GARCH model on estimated ε.

ARCH LM TEST

The next step in our analysis is to check for the ARCH effect in our data on returns of both assets. For this ARCH-LM Test has been performed with –

Null Hypothesis Ho: No ARCH EFFECT

Alternative Hypothesis H1: ARCH EFFECT

Table 2 validates that the data on both returns data has ARCH effect at 5% level of significance.

TABLE 2

	F-statistic	Prob. F	Obs*R-squared	Prob. Chi-Square
BRK-A	43.85431	0.0000	39.33177	0.0000
BTC-USD	19.95950	0.0002	19.95950	0.0002

Since the ARCH effect is detected in the model, GARCH family model is therefore used for modelling the volatility of return series in the BRK-A and BTC-USD data.

A comparison between **ARCH (1) model, EGARCH (1, 1) model and TGARCH (1,1,1) model** under normal error terms distributions for each BRK-A and BTC-USD has been performed and all of the estimated parameters have been shown in the Table 3 and Table 4 respectively:

TABLE 3 : Estimation Results Of BRK-A

	ARCH(1)	TGARCH(1,1,1)	EGARCH(1,1,1)
c	0.249896 (0.0032)	0.174452 (0.0914)	0.173283 (0.0796)
α_1	1.350435 (0.0000)	1.199964 (0.0022)	1.155276 (0.0020)
α_1	-1.269011 (0.0000)	-1.077286 (0.0018)	-1.017591 (0.0017)
α_2	0.522130 (0.0022)	0.365060 (0.0276)	0.314591 (0.0361)
β_1	-1.324424 (0.0000)	-1.168375 (0.0026)	-1.103177 (0.0018)
β_2	1.230436 (0.0000)	1.064614 (0.0022)	0.994737 (0.0018)
β_3	-0.469958 (0.0321)	-0.330206 (0.0374)	-0.259890 (0.0454)
β_4	-0.136931 (0.0566)	-0.112517 (0.0651)	-0.132460 (0.0597)
Variance Equation			
w	2.812453 (0.0000)	0.425717 (0.0052)	-0.021296 (0.0748)
σ_{t-1}^2	0.074824 (0.0495)	0.018555 (0.0556)	0.271063 (0.0000)
δ	0.201771 (0.0000)	0.834507 (0.0000)	0.885045 (0.0000)
η	0.183632 (0.0194)	0.128400 (0.0003)	-0.100858 (0.0024)

(The bracketed term shows the p-value representing the significance of each parameters)

Based on the assumption of 5% significance level, for ARCH (1) model, all of the estimated parameters are significant. For TGARCH (1,1,1) model, coefficient of MA(4) and constant term is not significant. For EGARCH (1,1) model, all of the estimated parameters are significant except for the coefficient of constant term of mean equation. Whatsoever all the estimates of variance equation are significant. We checked again for ARCH effect using the ARCH-LM test for all models and found that the p-value>0.05 resulting in accepting the null hypothesis of No ARCH Effect.

TABLE 3 : Estimation Results Of BTC-USD

	ARCH(1)	TGARCH(1,1,1)	EGARCH(1,1,1)
C	1.150304 (0.0180)	1.34511 (0.0127)	1.282744 (0.0136)
α_1	-0.326849 (0.0124)	-0.315001 (0.0773)	-0.324926 (0.0115)
β_1	0.320862 (0.1441)	0.345814 (0.0637)	0.339039 (0.1110)
Variance Equation			
w	70.22278 (0.0000)	21.14190 (0.0001)	1.804505 (0.0006)
σ_{t-1}^2	0.250900 (0.0000)	0.386731 (0.0000)	0.586392 (0.0000)
δ		0.554323 (0.0000)	0.512009 (0.0000)
η		-0.195036 (0.0248)	0.086889 (0.1250)

(The bracketed term shows the p-value representing the significance of each parameters)

Based on the assumption of 5% significance level, for ARCH (1) model, all of the estimated parameters are significant except for the coefficient of MA component. For TGARCH (1,1,1) model, coefficient of MA (1) is not significant. For EGARCH (1,1,1) model, all of the estimated parameters are significant except for the coefficient of MA component of mean equation. Whatsoever all the estimates of variance equation are significant. We checked again for ARCH effect using the ARCH-LM test for all models and found that the p-value>0.05 resulting in accepting the null hypothesis of No ARCH Effect.

We do not take GARCH (1,1) model in both the cases even though it gives significant estimates of all the parameters because it failed to remove the ARCH effect. So the variance equation is not well specified for returns data of both BRK-A and BTC-USD.

In order to acquire the appropriate model to forecast the conditional variance, the paper uses **out-of-sample forecast to calculate the root mean square error (RMSE).** Estimation is conducted for the period 01-01-2015 to 31-12-2020 and with this estimates a dynamic forecast for the remaining period 01-01-2021 to 30-12-2021 has been done. This applies for both BRK-A and BTC-USD returns data and for each of the model taken into consideration. Each forecast fit gives a RMSE value which has been presented in the Table 4 and we take that model as the best model which has **the least RMSE value.**

TABLE 4

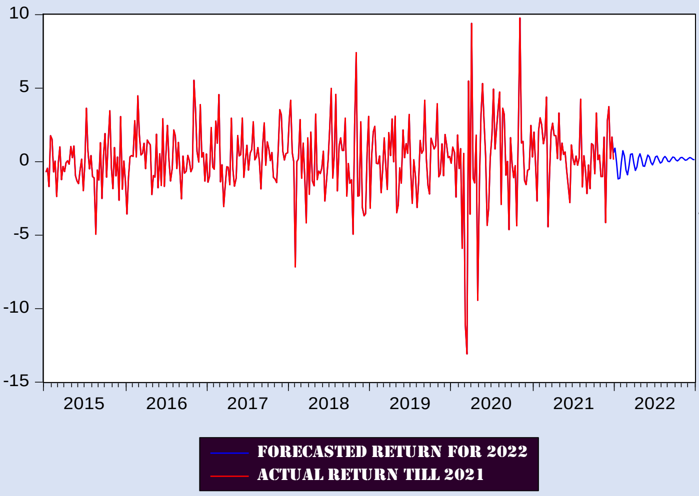
BRK-A	ARCH(1)	0.734228
	TGARCH(1,1,1)	**0.480370**
	EGARCH(1,1,1)	1.923037
BTC-USD	ARCH(1)	11.21733
	TGARCH(1,1,1)	11.14894
	EGARCH(1,1,1)	**11.14774**

Boldfaced word represents the minimal value in each group.

For BRK-A stock return, TGARCH (1,1,1) model gives the minimal RMSE of 0.480370 while for BTC-USD coin return, EGARCH (1,1,1) gives the minimal RMSE of 11.14774. Hence these models will forecast the future conditional variance better than other models for each of them respectively.

Having our models we try to forecast for the future conditional variance for BRK-A stock and BTC-USD coin for the next 1 year i.e. 52 weeks. Figure 4 visualises the actual return till 2021 with forecasted return for 2022 of BRK-A returns which shows that returns tend to converge to conditional mean. Figure 5 shows the forecasted variance of BRK-A returns data where the circle shows the value of variance for the forecasted period. Clearly it seems that the variance tends to become stable.

FIGURE 4 **BRK-A** **FIGURE 5**



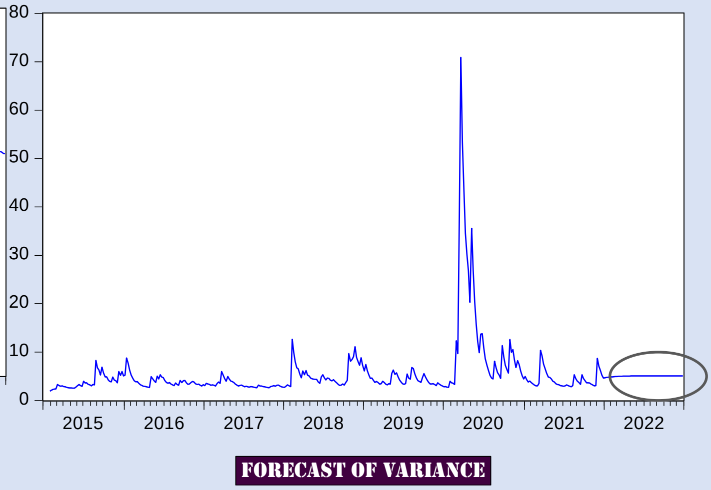
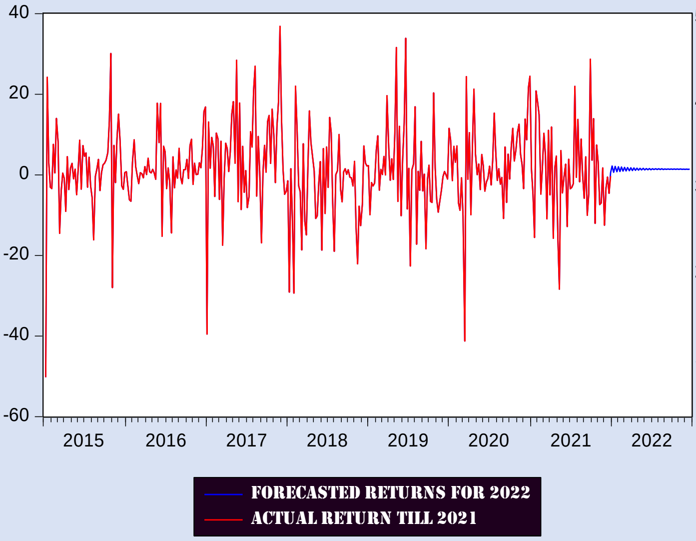
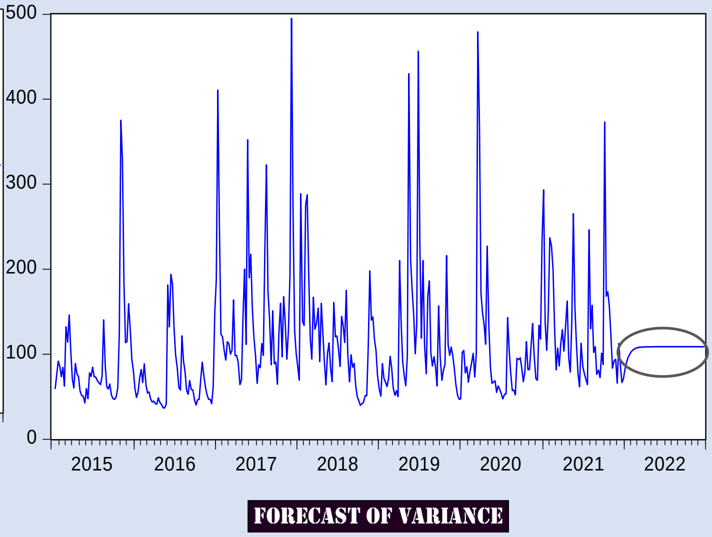


Figure 6 visualises the actual return till 2021 with forecasted return for 2022 of BTC-USD returns which shows that returns almost converge to conditional mean. Figure 7 shows the forecasted variance of BTC-USD returns data where the circle shows the value of variance for the forecasted period. Clearly it seems that the variance tends to become stable.

FIGURE 6 **BTC-USD** **FIGURE 7**

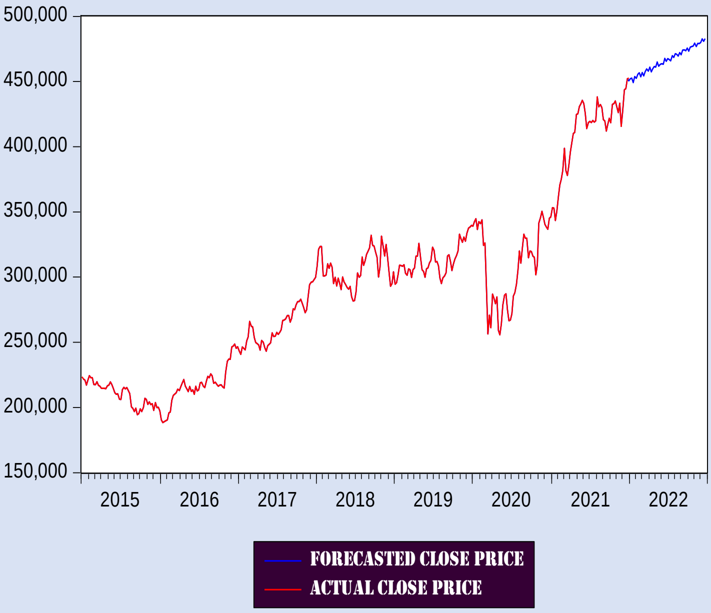




Comparing Figure 5 and 7 we can interpret that stock market is subject to less volatility than cryptocurrency. However both depends on market conditions as we can see that a similarity in both graph is a stark rise in volatility in the year 2020. This might be due to the onset of pandemic and COVID-19 which affected the global market.

Having the estimated mean equation, modelled GARCH model for both type of market and the forecasted variance and return, we try to compute **the forecasted value of Closing Price for both the BRK-A and BTC-USD.** According to the Figure 8 and Figure 9 we see that the Blue line indicates the forecasted value of Weekly Close price for BRK-A and BTC-USD respectively. It is evident from the graph that **Weekly price rises steadily in case of Stock Market** and **exponentially in case of Crypto Market** indicating the uncertain nature of Cryptocurrency market.

FIGURE 8 : BRK-A **FIGURE 9 : BTC-USD**



CONCLUSION

Having high level of volatility in financial markets, it's crucial to understand the risk and return from the asset and then build the portfolio which may be a mix of both stocks and crypto. This paper uses different volatility models to analyse and forecast the conditional variance. The paper shows that both cryptocurrencies and stocks are valid investment options but they serve different purposes in a portfolio. Here both symmetric and asymmetric ARCH family model to analyse the returns data of the both market. The symmetric model used in the analysis are ARCH and GARCH model. The asymmetric model take into account whether changes in the market are negative or positive and respond differently in each case. The asymmetric model used in the statistical analysis is TGARCH and EGARCH.

We used different tests and tools to compute that BRK-A from stock market is best forecasted by a TGARCH(1,1,1) model and BTC-USD from cryptocurrency market has been best modelled by EGARCH(1,1,1) model both are part of the asymmetric family of GARCH. While stocks provide stability, cryptocurrencies are riskier investments with the potential for great rewards.Even the forecasted close price for 2022 shows that the weekly close price rises exponentially for BTC-USD while for the BRK-A , it rises gradually with less volaitilty. Hence to consider how should an investor build his/her portfolio depends on the factor that create volatility - it is influenced by supply and demand – investor and user sentiments – government regulations – media hype. All this combined with some time series analysis may lead to decision-making healthy in terms of investment.

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