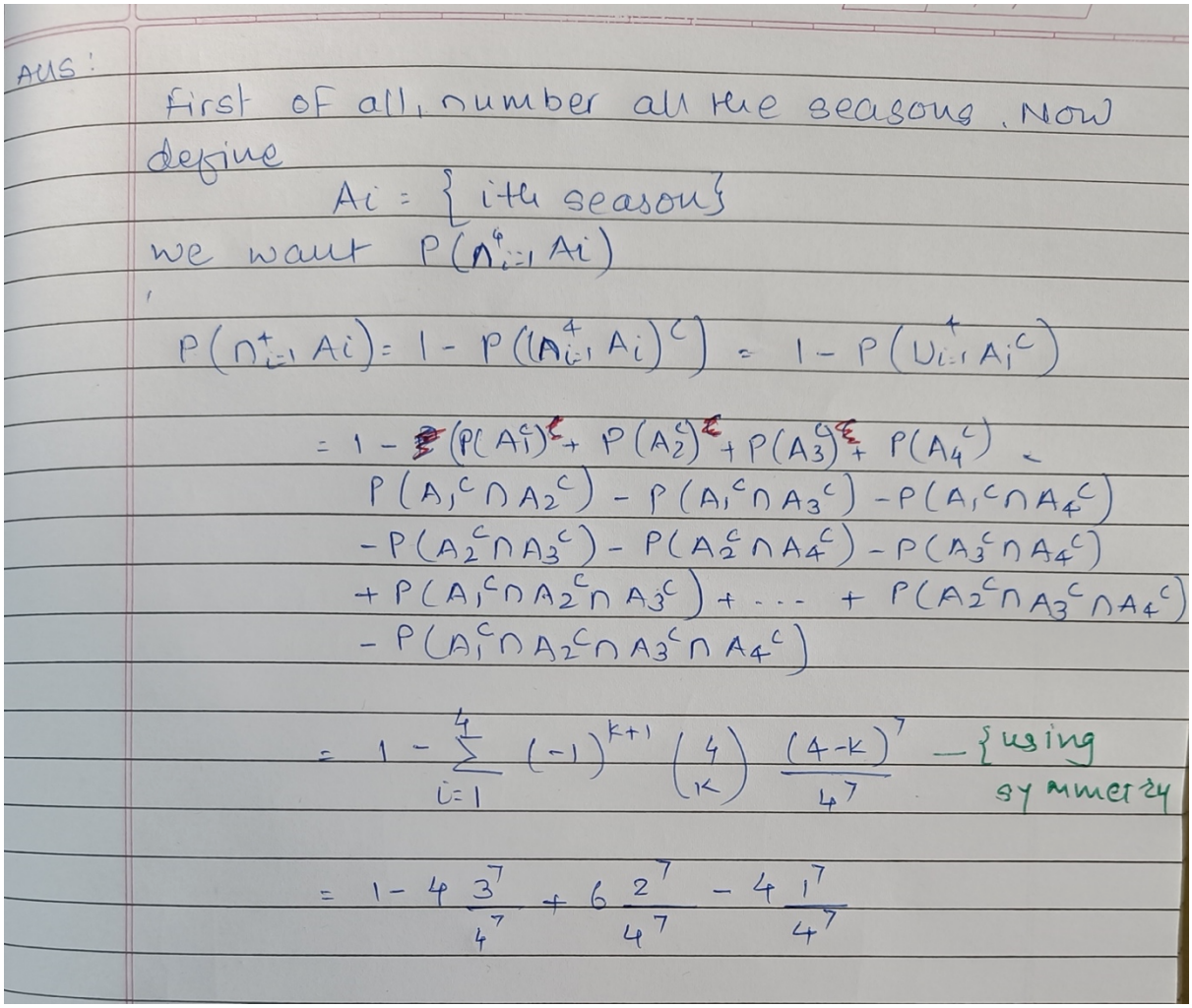
1. For a group of 7 people, find the probability that all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays, assuming that all seasons are equally likely.

ANS:



Ans:

First of all, number all the seasons. Now define

$A_i = \{ i\text{th season} \}$

we want $P(\cap_{i=1}^{4} A_i)$

$$P(\cap_{i=1}^{4} A_i) = 1 - P((\cap_{i=1}^{4} A_i)^c) = 1 - P(\cup_{i=1}^{4} A_i^c)$$

$$= 1 - (P(A_1^c) + P(A_2^c) + P(A_3^c) + P(A_4^c) - P(A_1^c \cap A_2^c) - P(A_1^c \cap A_3^c) - P(A_1^c \cap A_4^c) - P(A_2^c \cap A_3^c) - P(A_2^c \cap A_4^c) - P(A_3^c \cap A_4^c) + P(A_1^c \cap A_2^c \cap A_3^c) + \cdots + P(A_2^c \cap A_3^c \cap A_4^c) - P(A_1^c \cap A_2^c \cap A_3^c \cap A_4^c))$$

$$= 1 - \sum_{i=1}^{4} (-1)^{k+1} \binom{4}{k} \frac{(4-k)^7}{4^7} \quad \text{— using symmetry}$$

$$= 1 - 4\frac{3^7}{4^7} + 6\frac{2^7}{4^7} - 4\frac{1^7}{4^7}$$

2. Alice attends a small college in which each class meets only once a week. She is deciding between 30 non-overlapping classes. There are 6 classes to choose from for each day of the week, Monday through Friday. Trusting in the benevolence of randomness, Alice decides to register for 7 randomly selected classes out of the 30, with all choices equally likely. What is the probability that she will have classes every day, Monday through Friday?

ANS: