

1. Is it possible that an event is independent of itself? If so, when?

ANS:

The only events that are independent of themselves are those with probability either 0 or 1. That follows from the fact that a number is its own square if and only if it is either 0 or 1. The only way a random variable  $X$  can be independent of itself is if for every measurable set  $A$ , either  $P(X \text{ belongs to } A) = 1$  or  $P(X \text{ belongs to } A) = 0$ .

2. Is it always true that if  $A$  and  $B$  are independent events, then  $A^c$  and  $B^c$  are independent events? Show that it is, or give a counterexample.

ANS:

$A^c$  and  $B^c$  are also independent, which again means that the occurrence of  $B^c$  doesn't affect the probability of  $A$ . Therefore, the occurrence of  $B^c$  also doesn't affect the probability of  $A^c$ . So, by definition,  $B^c$  and  $A^c$  are also independent.