

1. Give an example of 3 events A, B, C which are pairwise independent but not independent. Hint: find an example where whether C occurs is completely determined if we know whether A occurred and whether B occurred, but completely undetermined if we know only one of these things.

ANS:

We throw two dice. Let A be the event “the sum of the points is 7”, B the event “die #1 came up 3”, and C the event “die #2 came up 4”. Now, $P[A] = P[B] = P[C] = 1/6$. Also, $P[A \cap B] = P[A \cap C] = P[B \cap C] = 1/36$, so that all events are pairwise independent. However,

$$P[A \cap B \cap C] = P[B \cap C] = 1/36$$

while

$$P[A]P[B]P[C] = 1/216$$

so they are not independent as a triplet.

First, note that, indeed, $P[A \cap B] = P[B \cap C] = 1/36$, since the fact that A and B occurred is the same as the fact that B and C occurred.

2. A bag contains one marble which is either green or blue, with equal probabilities. A green marble is put in the bag (so there are 2 marbles now), and then a random marble is taken out. The marble taken out is green. What is the probability that the remaining marble is also green?

ANS:

Let G represents green marble and B represents Blue marble. According to the question. After putting a green marble in the bag, we have Let already present marble in the bag is green. Then, after taking green marble. Probability that the remaining marble is also green = 1
Let already present marble in the bag is blue, then after taking green marble. Probability that the remaining marble is also green = 0 So, required probability = $0 + 1 = 1$