Converse, Contrapositive, and Inverse :

Consider the statements:

- If the physical environment changes, then the biological environment changes. : $p \rightarrow q$: Conditional statement
- If the biological environment does not change, then the physical environment does not change. : $\neg q \rightarrow \neg p$: Contrapositive of $p \rightarrow q$
- If the biological environment changes, then the physical environment changes. : $q \rightarrow p$: Converse of $p \rightarrow q$
- If the physical environment does not change, then the biological environment does not change. : $\neg p \rightarrow \neg q$: Inverse of $p \rightarrow q$

Q. Prove that the conditional statement and its contrapositive are equivalent.

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

EX: Check whether the following statement is true or false by proving its contrapositive.

If $x, y \in \mathbb{Z}$ such that if xy is odd, then both x and y are odd.

SOL. Let p: xy is odd

q: x is odd

r: y is odd

then the statement is symbolic form:

$p \rightarrow (q \wedge r) \equiv \neg(q \wedge r) \rightarrow \neg p$

LHS

$\equiv (\neg q \vee \neg r) \rightarrow \neg p$

RHS

$[\neg q \vee \neg r] \equiv \neg(x \text{ is odd}) \vee \neg(y \text{ is odd})$

$\equiv x$ is even or y is even

$\equiv x = 2m$ or $y = 2n$ or both

then $xy = 2my$ or $2nx$ or $4mn$

$\Rightarrow xy$ is even

$\Rightarrow \neg(xy \text{ is odd})$

Thus, $\Rightarrow \neg p$

$\therefore (\neg q \vee \neg r) \rightarrow \neg p$

By the method of contraposition the given statement is true.

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Examine the validity of the argument:

$$\begin{array}{c} p \to r \\ \neg p \to q \\ q \to s \\ \hline \therefore \neg r \to s \end{array}$$

Method-1: Construct the truth table for three premises and the conclusion. If in the rows in which all the premises have truth value T, the conclusion also has the truth value T, then the argument is valid, otherwise it is invalid.

Method-2 (method of derivation): Use Rule P, Rule T and laws of logical equivalence and the laws of logical implication.

$\{1\}$	(1) $p \to r$	Rule P
$\{1\}$	(2) $\neg r \to \neg p$	Rule T, (1), $p \to q \equiv \neg q \to \neg p$
$\{3\}$	(3) $\neg p \to q$	Rule P
$\{1,3\}$	(4) $\neg r \to q$	Rule T, (2), (3), $p \to q, q \to r \Rightarrow p \to r$
$\{5\}$	(5) $q \to s$	Rule P
$\{1,3,5\}$	(6) $\neg r \to s$	Rule T, (4), (5), $p \to q, q \to r \Rightarrow p \to r$

Method-3 (Rule CP):

$\{1\}$	(1) $\neg r$	Rule P (additional/assume)
$\{2\}$	(2) $p \to r$	Rule P
$\{2\}$	(3) $\neg r \to \neg p$	Rule T, (1), (2), $p \to q \equiv \neg q \to \neg p$
$\{1,2\}$	(4) $\neg p$	Rule T, (1), (3). $p, p \to q \Rightarrow q$
$\{5\}$	(5) $\neg p \to q$	Rule P
$\{1,2,5\}$	(6) q	Rule T, (4), (5), $p, p \to q \Rightarrow q$
$\{7\}$	(7) $q \to s$	Rule P
$\{1,2,5,7\}$	(8) s	Rule T, (6), (7), $p, p \to q \Rightarrow q$
$\{1,2,5,7\}$	(9) $\neg r \to s$	Rule CP, (1), (8)

EX: Examine the validity of the argument:

$$\begin{array}{c} (p \vee q) \to r \\ r \to s \\ \neg s \\ \hline \therefore \neg p \end{array}$$

Method-1: Use the method of truth table.

Method 2 (Method of Derivation):

$\{1\}$	(1) $(p \lor q) \to r$	Rule P	
$\{2\}$	(2) $r \to s$	Rule P	
$\{1,2\}$	(3) $(p \lor q) \to s$	Rule T, (1), (2),	$p \to q,\ q \to r \Rightarrow p \to r$
$\{1,2\}$	(4) $\neg s \to \neg(p \lor q)$	Rule T, (3),	$p \to q \equiv \neg q \to \neg p$
$\{5\}$	(5) $\neg s$	Rule P	
$\{1,2,5\}$	(6) $\neg(p \lor q)$	Rule T, (4), (5),	$p,\ p \to q \Rightarrow q$
$\{1,2,5\}$	(7) $\neg p \land \neg q$	Rule T, (6),	$\neg(p \lor q) \equiv \neg p \land \neg q$
$\{1,2,5\}$	(8) $\neg p$	Rule T, (7),	$p \land q \Rightarrow p$

Method 3 (the method of contradictions):

If possible let the conclusion of the argument i.e. $\neg p$ be false.

$\therefore$ p is true

Take p as an additional premise.

$\{1\}$	(1) p	Rule P (additional premise)	
$\{1\}$	(2) $p \lor q$	Rule T, (1),	$p \Rightarrow p \lor q$
$\{3\}$	(3) $(p \lor q) \to r$	Rule P	
$\{1,3\}$	(4) r	Rule T, (3),	$p,\ p \to q \Rightarrow q$
$\{5\}$	(5) $r \to s$	Rule P	
$\{1,3,5\}$	(6) s	Rule T, (4), (5),	$p,\ p \to q \Rightarrow q$
$\{7\}$	(7) $\neg s$	Rule P	
$\{1,3,5,7\}$	(8) $s \land \neg s \equiv F$	Rule T, (6), (7),	$p, q \Rightarrow p \land q$

Thus a contradiction is derived, therefore our assumption is wrong / impossible.

$\therefore$ The conclusion $\neg p$ is true

$\therefore$ The Given argument is valid.

EX: Test the validity of the argument

$$\begin{array}{c} p \to q \\ r \to q \\ \hline \therefore r \to p \end{array}$$

method-1: Use truth-table method.

method-2: The argument is INVALID.

$$\begin{array}{lcl} p \to q & : & T \equiv F \to T \\ r \to q & : & T \equiv T \to T \\ \hline \therefore r \to p & : & F \equiv T \to F \end{array}$$

$v(r) = T$

$v(p) = F$

$v(q) = T$

Thus, $p \equiv F$, $q \equiv T$, $r = T$ (a truth-value assignment for which the conclusion is false.)