STATEMENT (OR PROPOSITION):

A sentence which is either true or false but not both at the same time is called a mathematically acceptable statement.

Ex. Check whether the following sentences are statements.

(i) In 2018, the president of India was a woman.
(ii) Woman are more intelligent than men.
(iii) Chennai is the capital of Tamil Nadu.
(iv) There is no rain without clouds.
(v) New Delhi is in Pakistan.
(vi) The sun is a star.
(vii) Give me a glass of water.
(viii) How are you?
(ix) How beautiful!
(x) May you live long!
(xi) Tomorrow is Wednesday.
(xii) mathematics is fun.
(xiii) How far is Agra from here?
(xiv) Australia is a continent.
(xv) She is a mathematics graduate.
(xvi) Kashmir is far from here.
(xvii) There are 30 days in a month.
(xviii) there are 40 days in a month.
(xix) She is more intelligent than you.
(xx) Pfizer's Covid-19 vaccine is 90% efficacious.

gation of a statement : The denial of statement.

The negation of the statement : "Australia is a continent" is

It is not the case that Australia is a continent or

It is false that Australia is a continent or

Australia is not a continent.

Special Words | Phrases :

(i) The word "And" , (ii) The word "Or" → Inclusive "Or" / → Exclusive "Or"

(iii) Phrase "If ----, then ---" (iv) Phrase "---- if and only if ---"

Consider the statements.

(i) All living things have two legs and two eyes.

(ii) A mixture of alcohol and water can be separated by chemical methods.

(iii) To enter a country, you need a passport or a voter registration card.

(iv) The university is closed if it is a holiday or a Sunday.

(v) Two lines intersect at a point or are parallel.

(vi) Students can take French or Sanskrit as their third language.

(vii) If you are born in some country then you are citizen of that country.

(viii) A tumbler is half empty if and only if it is half full.

Compound statement / Proposition / Statement formula / well-formed formula

'Formula' is defined recursively as follows:

1. Each propositional variable is a formula
2. If φ and ψ are formulae, so are $(\neg\varphi)$, $(\varphi \wedge \psi)$, $(\varphi \rightarrow \psi)$, and $(\varphi \vee \psi)$
3. A string of symbols is a formula only as determined by (finitely many applications of) the first two clauses/steps

			Inclusive	Exclusive			NOR	NAND
P	Q	$P \wedge Q$	$P \vee Q$	$P \veebar Q$ ($P \oplus Q$ or)	$P \rightarrow Q$	$P \leftrightarrow Q$	$P \downarrow Q$	$P \uparrow Q$
T	T	T	T	F	T	T	F	F
T	F	F	T	T	F	F	F	T
F	T	F	T	T	T	F	F	T
F	F	F	F	F	T	T	T	T

P	$\neg P$
T	F
F	T

P, Q : Individual 'propositional/statement variables' which are incapable of further analysis : Primitive statements

Tautology: A statement formula $\varphi(P, Q, R, \cdots)$ which is true under/for all possible assignments of truth values to its statement variables $P, Q, R, \cdots$, is called a tautology, or is said to be valid or universally true.

Contradiction: A formula is contradictory (or contradiction) if it is false under all assignments.

Contingency: A formula is satisfiable/contingency if it is true under at least one assignment

Thus, $\varphi(P, Q, R, \cdots)$ is contradictory if and only if $\neg\varphi(P, Q, R, \cdots)$ is a tautology, and $\varphi(P, Q, R, \cdots)$ is satisfiable if and only if it is not contradictory.

Logical Equivalence: Two statement formulae $\varphi(P, Q, R, \cdots)$ and $\psi(P, Q, R, \cdots)$ are said to be logically equivalent if they have the same/identical truth-tables - i.e. if the set of assignments for which φ is is true is the same as that for ψ.

Logical Implication: If Γ is a set of formulae and φ is a single formula, then φ is a logical consequence/implication of Γ, written $\Gamma \vDash \varphi$, if, for any assigment making all members of Γ true, φ is also true.

A formula φ logically implies ψ, written as $\varphi \Rightarrow \psi$, if for any assigment making φ true, ψ is also true.

Thus, two formulas are logically equivalent if and only if each is is a logical consequence of the other.

EX: Construct the truth-table for the formula: $[(p\to r)\wedge(\neg q\to p)\wedge\neg r]\to q$, examine it for a tautology or contradiction or contingency.

EX: Construct the truth-tables for

(i) $p\wedge(q\vee r)$ and $(p\wedge q)\vee(p\wedge r)$; $[p\wedge(q\vee r)]\leftrightarrow[(p\wedge q)\vee(p\wedge r)]$

(ii) $p\vee(q\wedge r)$ and $(p\vee q)\wedge(p\vee r)$; $[p\vee(q\wedge r)]\leftrightarrow[(p\vee q)\wedge(p\vee r)]$

EX: Construct the truth-table for $[p\to(q\to r)]\to[(p\to q)\to(p\to r)]$ and examine it for a tautology.

EX: Construct the truth-table for each of the following and examine for a tautology, contradiction, satisfiable.

(i) $[(p\to q)\wedge(q\to r)]\to(p\to r)$

(ii)

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1. Draw up truth tables for the following formulae:
   (a) $p\wedge(\neg p\to(p\vee\neg q))$
   (b) $(p\to q)\to((p\to(q\to r))\to(p\to r))$
   (c) $(p\to(q\to(r\to s)))\to(((p\to q)\to r)\to s)$
   (d) $\neg(p\vee q\vee\neg r)\wedge((r\to p)\vee(r\to q))$

   Which are tautologies, which are satisfiable, and which are contradictory?
2. Examine which of the following formulae are logically equivalent:
   (a) $(q\to(r\vee s))\wedge((q\wedge r)\to s)$; (b) $((p\vee r)\vee(s\to p))\wedge(p\to(s\to r))$
   (c) $q\to s$; (d) $(s\to(q\vee r))\wedge((q\wedge s)\to r)$
   (e) $((p\vee s)\vee(q\to p))\wedge(p\to(q\to s))$
3. Determine which of $q\to s$, $\neg p\vee(q\wedge\neg r)$ is a logical consequence of $\Gamma=\{p\to q, q\to\neg r, r\to(p\vee s)\}$

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Construct truth table for $[p \wedge (q \vee r)] \leftrightarrow [(p \wedge q) \vee (p \wedge r)]$

1	2	3	4	5	6	7	8	9
p	q	r	$p \wedge q$	$p \wedge r$	$q \vee r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$	$[p \wedge (q \vee r)] \leftrightarrow [(p \wedge q) \vee (p \wedge r)]$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	F	F	T	F	F	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	T	F	F	T
F	F	F	F	F	F	F	F	T

From column 7 and column 8 of the truth table.
$p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ have identical truth-table

$\therefore$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ [Distributive Law]

From column 9. $[p \wedge (q \vee r)] \leftrightarrow [(p \wedge q) \vee (p \wedge r)]$ have truth value T for all possible truth value assignments to the primitive statements / statement variables p, q, r.

$\therefore$ $[p \wedge (q \vee r)] \leftrightarrow [(p \wedge q) \vee (p \wedge r)]$ is a tautology

Also, $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

Ex: For any primitive statements p, q, r, construct the truth table for statement formula: $[p \wedge ((p \wedge r) \to s)] \to (r \to s)$

p
$(p \wedge r) \to s$
$\therefore r \to s$

H_1				H_2	C		$H_1 \wedge H_2 \to C$
p	r	s	$p \wedge r$	$(p \wedge r) \to s$	$r \to s$	$p \wedge ((p \wedge r) \to s)$	$[p \wedge ((p \wedge r) \to s)] \to (r \to s)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	F	T	T	F	T
F	T	F	F	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

Given formula $[p \wedge ((p \wedge r) \to s] \to (r \to s)$ is a tautology.

$\therefore$ $p \wedge ((p \wedge r) \to s) \Rightarrow (r \to s)$ i.e. $p, (p \wedge r) \to s \Rightarrow (r \to s)$

EX: Construct truth table for $[[p \vee (q \vee r)] \wedge \neg q] \rightarrow (p \vee r)$

1 p	2 q	3 r	4 $q \vee r$	5 $p \vee (q \vee r)$	6 $\neg q$	7 $[p \vee (q \vee r)] \wedge \neg q$	8 $p \vee r$	
T	T	T	T	T	F	F	T	T
T	T	F	T	T	F	F	T	T
T	F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	F	F	T	T
F	T	F	T	T	F	F	F	T
F	F	T	T	T	T	T	T	T
F	F	F	F	F	T	F	F	T

Column 9 has all truth values T.

$\therefore [[p \vee (q \vee r)] \wedge \neg q] \rightarrow (p \vee r)$ is a tautology

$$\therefore \quad [[p \vee (q \vee r)] \wedge \neg q] \Rightarrow (p \vee r)$$

The rows: 3, 4 & 7 in which $p \vee (q \vee r)$ and $\neg q$ have truth value T, $(p \vee r)$ also has truth value T.

$$\therefore \quad p \vee (q \vee r) \wedge \neg q \Rightarrow p \vee r$$

It can also written that $p \vee (q \vee r), \neg q \Rightarrow p \vee r$

Also as

premise ← $p \vee (q \vee r)$
premise ← $\neg q$
conclusion ← $\therefore p \vee r$

This, called an argument, is valid.

ARGUMENT (General Form):

Consider the implication:

$(H_1 \wedge H_2 \wedge H_3 \wedge \cdots \wedge H_n) \rightarrow C$

Here n is a positive integer

The statements $H_1, H_2, \ldots, H_n$ are called the Premises of the argument, and the statement C is the Conclusion for the argument.

$$\begin{array}{c} H_1 \\ H_2 \\ \vdots \\ H_n \\ \hline \therefore C \end{array}$$