

JII<sup>th</sup> CS, H MCA

MCA  
FIRST SEMESTER II MID-SEMESTER EXAMINATION, 2011-12  
DISCRETE STRUCTURES

HMA-102

M.M.: 15

Time: 1 hour

NOTE: Attempt All questions.

1. (a) Design a circuit for a light fixture controlled by three switches, where flipping one of the switches turns the light on when it is off and turns it off when it is on.

- (b) Use generating functions to find the number of integer solutions to the equation:

$$x + y + z + w = 20 \\ \text{where } -3 \leq x \leq 3, -3 \leq y \leq 3, -5 \leq z \leq 5 \text{ and } 0 \leq w$$

2. (a) Develop a program in C to examine whether a given binary relation on a finite set is a partial order relation.

- (b) Illustrate with example

- (i) One Graph-coloring model,      (ii) One recurrence relation model

3. (a) Illustrate the application of graphs in modeling Local Area Networks.

- (b) Find a minimal-sum-of-products representation for the function:

$$f(w, x, y, z) = wx\bar{y}\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + \bar{w}xyz + \bar{w}x\bar{y}z + \bar{w}\bar{x}yz + \bar{w}\bar{x}\bar{y}z$$

using Quine-McClusky method.

4. (a) Determine the number of relations on a finite set X which are

- (i) symmetric but not reflexive  
(ii) reflexive and antisymmetric.

- (b) Determine which amounts of postage can be formed using just 4-cents and 11-cents stamps only. Prove your answer using strong induction.

5. (a) Negate the following,

- (i)  $(\exists x)(|x| = x)$ , where x is an arbitrary integer  
(ii) Every supercomputer is manufactured in Japan  
(iii) No person has green eyes.  
(iv) Some girls are blondes.

- (b) Use the law of the contrapositive to prove:

If the square of an integer is odd, then the integer is odd.

## MCA Odd Semester Examination, 2011-12

## DISCRETE STRUCTURES

Time: 3 Hours

Max. Marks: 100

Note: 1 Attempt any five questions. Question No. 1 is compulsory.

2. All questions carry equal marks.

1. (a) Determine whether the assignment statement,  $sum \leftarrow sum + i + j$  will be executed in the following sequence of statements:

$$\begin{array}{l} i \leftarrow 3 \\ j \leftarrow 5 \\ sum \leftarrow 0 \\ \text{if } (i < 4) \wedge (j \leq 5) \text{ then} \\ \quad sum \leftarrow sum + i + j \end{array}$$

- (b) Express the following in predicate logic.  
"Everyone has exactly two biological parents."

- (c) Give inductive/recursive definition of the set of even integers.  
(d) State two efficiency measures for the analysis of algorithms.  
(e) State two applications of Lagrange's theorem.  
(f) State two applications of the semigroups in Computer Science.  
(g) State two important application of the trees in Computer Science.  
(h) State two real-life problems that Boolean algebra handles well.  
(i) Identify the general form of the adjacency matrix for the graph  $K_{m,n}$ .  
(j) If  $G(z)$  is the generating function for the sequence  $\{a_n\}$ , find the generating function for the sequence: 0,  $a_0, \frac{a_1}{2}, \frac{a_2}{3}, \frac{a_3}{4}, \dots$

2. (a) Establish the validity of the following argument or give a counter example to show that it is invalid:

$$\begin{array}{l} P \leftrightarrow Q \\ Q \rightarrow R \\ R \vee \neg S \\ \neg S \rightarrow Q \end{array}$$

$v$	$m$	$m \wedge n$
$m$	$n$	$0$
$n$	$0$	$0$

$\neg S - \text{True}$   
 $\neg S - \text{False}$

- (b) Test the following statements for consistency:  
If Aditi takes the job offer, then she will get a signing bonus.

$$(A \leftarrow B) \wedge (B \leq S)$$

$$\neg A \wedge F$$

$$\begin{array}{l} \neg A \wedge F \\ \neg A \wedge (B \leq S) \\ \neg A \wedge F \wedge (B \leq S) \end{array}$$

To show:

- If she takes the job offer, then she will receive a higher salary.  
 If she gets a signing bonus, then she will not receive a higher salary.  
 She takes the job offer.

(c) Examine the validity of the following arguments:

(i) "All horses are animals"

Therefore, the head of a horse is the head of an animal."

(ii)

$$\forall x [P(x) \rightarrow [Q(x) \wedge R(x)]]$$

$$\forall x [P(x) \wedge S(x)]$$

Ans

$$\therefore \forall x [R(x) \wedge S(x)]$$

3. (a) Define the number of times the assignment statement  $x \leftarrow x + 1$  is executed by the following nested FOR loop, recursively:

for  $i = 1$  to  $n$  do

for  $j = 1$  to  $i$  do

for  $k = 1$  to  $j$  do

$x \leftarrow x + 1$

$$n(n+1)/2$$

$$(n+1)n/2$$

$$\begin{matrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{matrix}$$

- (b) Determine which amounts of postage can be formed using just 4-cent and 1-cent stamps only. Prove your answer using strong Induction.

- (c) Let  $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$ , and a relation  $R$  on  $A$  is defined by :

$$(x_1, y_1) R (x_2, y_2) \text{ if } x_1 + y_1 = x_2 + y_2$$

- (i) Examine  $R$  for an equivalence relation on  $A$ .

- (ii) If yes, determine the equivalence classes  $[(1, 3)], [(1, 1)]$

Prove that a countable union of countable sets is countable.

- (d) 4. (a) (i) Let  $k, m$  be fixed integers. Find all values for  $k, m$  for which  $(\mathbb{Z}, \oplus, \odot)$  is a ring under the binary operations:

$$x \oplus y = x + y - k, \quad x \odot y = x + y - mxy, \quad \forall x, y \in \mathbb{Z}$$

For given groups  $G = (\mathbb{Z}_{12}, +_{12})$  and  $H = ([0], [4], [8], +_{12})$

- (I) Examine  $H$  for cyclic group.

- (II) Find the factor group.

- (III) Verify Lagrange's theorem.

$$x \oplus y = x$$

- (b) An encoding function  $E : \mathbb{Z}_2^3 \rightarrow \mathbb{Z}_2^6$  is given by the generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$x+y-k = x$$

- (i) Determine all code words. Comment on the error-detection and error-correction capabilities of this code.

- (ii) Find the associated parity-check matrix,  $H$ .

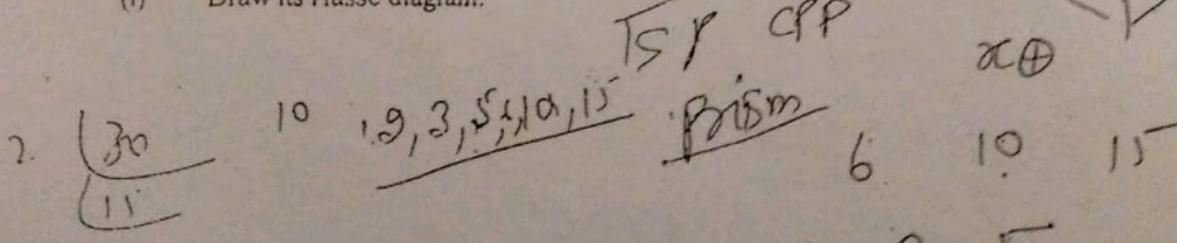
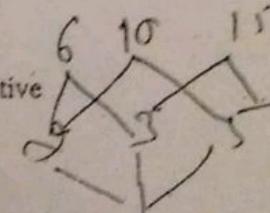
- (iii) Use  $H$  to decode each of the following received words.

110110, 000111

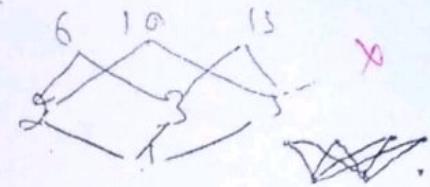
$$y = \underline{\underline{k}}$$

5. (a) Given a Boolean algebra  $B = (S, \vee, \wedge, \neg, 1, 0)$ , where  $S$  is the set of positive divisors of 30,  $x \vee y = \text{lcm}(x, y)$ ,  $(x \wedge y) = \text{gcd}(x, y)$  and  $\bar{x} = \frac{30}{x}$ .

- (i) Draw its Hasse diagram.



- (ii) Examine it for a lattice, and its types.  
 (iii) Investigate the relation " $\leq$ " on  $S$  defined by  
 $x \leq y$  if  $x \wedge y = x$

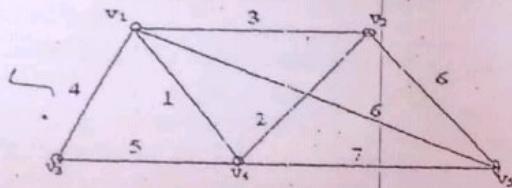


for a partial order relation.

- (b) (i) Design a circuit for a light fixture controlled by three switches when flipping one of switches turns the light on when it is off and turns it off when it is on.  
 (ii) Find a minimal-sum-of-products representation for the function:

$$f(a, b, c, d) = \sum m(0, 1, 2, 8, 10, 11, 14, 15)$$

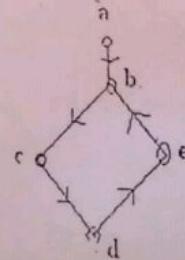
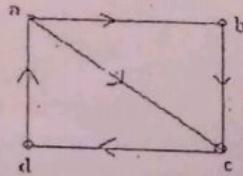
6. (a) (i) Identify a complete graph with 28 edges and a complete bipartite graph with minimum number of vertices and 21 edges.  
 (ii) State important applications of the minimal spanning trees. Find an MST for the weighted graph using matrix version of Prim's algorithm:



K

- (b) A computer system considers a string of decimal digits a valid code word if it contains an even number of 0 digits.  
 (i) Develop a recurrence relation model for the number of valid  $n$ -digit codewords.  
 (ii) Use generating functions to find an explicit formula for it.

7. (a) Examine the following digraphs for the types of connectedness and for Euler digraph/unicursal digraph / Hamiltonian digraph:



$$x \oplus k = x + k/2$$

$$x + p_1 - k = k$$

$$x + y = 2k$$

$$k = \frac{x+y}{2}$$

- (b) Prove that the kernel of a group homomorphism is a normal subgroup of the group.  
 (c) Illustrate two applications of Polya's enumeration theorem.  
 (d) Use the law of the contrapositive to prove the statement:

If the square of an integer is odd, then the integer is odd.

(3) 3

$$ac + a'c'$$

$$b'd' + b'd$$

b

$$a'c + a'c' + b$$

$$x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23}$$

MCA Odd Semester Examination, 2013-14

DISCRETE STRUCTURES

Time: 3 Hours

Note: 1. Attempt all parts.  
2. All questions carry marks as shown against them.

Max. Marks: 100

1.(a) Attempt all parts.

(i) Name two persons who contributed to the development of logic. [5]

(ii) Express in predicate logic :

"Some people tell the truth some of the time, others never lie".

(iii) Write a parse tree of:  $\rightarrow \wedge P \rightarrow QR \rightarrow \wedge P \wedge Q \vee \neg Q \wedge R$ .

(iv) Use Venn diagram to examine the validity of:  $A \rightarrow B, B \rightarrow C \Rightarrow A \rightarrow C$ .

(v) Use the following in predicate logic and by Venn diagram :

"No beggar is a thief".

(b) Write a program in C/C++ to examine whether the following system specifications are consistent:

"Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save files, then the system software is not being upgraded". [5]

(c) Use rules of inference to show that if  $\forall x(P(x) \vee Q(x)), \forall x(\neg Q(x) \vee S(x)), \forall x(R(x) \rightarrow \neg S(x))$ , and  $\exists x \neg P(x)$  are true, then  $\exists \neg R(x)$  is true. [5]

(d) Use a proof by contraposition and a proof by contradiction to examine the statement : "If n is an integer and  $3n + 2$  is even, then n is even". [5]

2.(a) Attempt all parts.

(i) Use quantifiers to define :  $A \subset B$ . [4]

(ii) Use quantifiers to define : f is one-to-one.

(iii) Express mathematical induction as a rule of inference

(iv) Who introduced big -  $\Omega$  and big -  $\Theta$  notations to describe the growth of functions.

(b) Examine the following for a countable set :

(i) the set of all positive integers. [4]

(ii) the set of all C/C++ programs.

(c) Develop a program in C/C++ to compute an immediate predecessor relation matrix from the partial order relation matrix : [4]

$$M_s = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) State the principle of inclusion-exclusion and its applications.  
How many ways are there to assign five different jobs to four different employees if [4]  
a every employee is assigned atleast one job?

(2)

(e) State some elegant applications of the pigeon-hole principle.  
How many numbers must be selected from the set {1, 3, 5, 7, 9, 11, 13, 15} to guarantee [4]  
that at least one pair of these numbers add up to 16?

3.(a) An encoding function is defined by the parity-check matrix: [10]

(1)

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

- (i) Examine the encoding function for a group code.
- (ii) If yes, examine the group code for a Hamming code.
- (iii) Encode the message : 1001.
- (iv) Decode the received word : 0011100.
- (v) Construct a decoding table consisting of the syndromes and coset leaders for this code.
- (vi) Use the result in part (v) to decode the received word 1100001.

(b) (i) Let  $(\mathbb{Z}; +)$  be the group of integers under ordinary addition and  $(\mathbb{Z} \times \mathbb{Z}, \oplus)$  be the abelian group where, [5]

$$(a, b) \oplus (c, d) = (a+c, b+d)$$

Define  $f: (\mathbb{Z} \times \mathbb{Z}, \oplus) \rightarrow (\mathbb{Z}, +)$  by  $f(x, y) = x - y$

- (i) Examine  $f$  for a homomorphism onto  $\mathbb{Z}$ .
  - (ii) Determine all  $(a, b) \in \mathbb{Z} \times \mathbb{Z}$  with  $f(a, b) \geq 0$
- (ii) Find all values for fixed integers  $k, m$  for which  $(\mathbb{Z}, \oplus, \odot)$  is a ring under the binary operations : [5]

$$x \oplus y = x + y - k, \quad x \odot y = x + y - mxy, \quad \forall x, y \in \mathbb{Z}.$$

4.(a) Attempt all parts. [6]

(i) State some real-life problems where the notion of partial order/total order arises ?

(ii) Name some applications of trees as a partially ordered set.

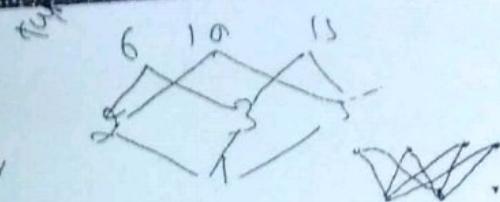
(iii) Name some uses of the lattices.

(iv) How are Boolean algebras useful for the Computer Scientists and Electronic Engineers ?

(b) Develop a partially ordered structure on the set of all positive even factors of 308 and examine it for a totally ordered set, well ordered set, lattice with its types and Boolean algebra. [7]

(3)

- (ii) Examine it for a lattice, and its types.  
 (iii) Investigate the relation " $\leq$ " on S defined by  
 $x \leq y$  if  $x \wedge y = x$



for a partial order relation.

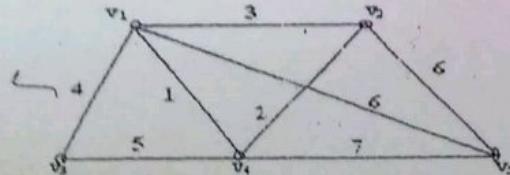
- (b) (i) Design a circuit for a light fixture controlled by three switches when flipping one of switches turns the light on when it is off and turns it off when it is on.

(iii) Find a minimal-sum-of-products representation for the function:

$$f(a, b, c, d) = \sum m(0, 1, 2, 8, 10, 11, 14, 15)$$

$$ab'c'd' + ab'c'd + ab'cd + abc'd'$$

6. (a) (i) Identify a complete graph with 28 edges and a complete bipartite graph with minimum number of vertices and 21 edges.  
 $ab'c'd' + ab'cd + abc'd'$   
 (ii) State important applications of the minimal spanning trees. Find an MST for the weighted graph using matrix version of Prim's algorithm:



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- (b) A computer system considers a string of decimal digits a valid code word if it contains an even number of 0 digits.

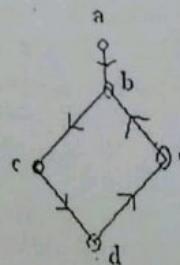
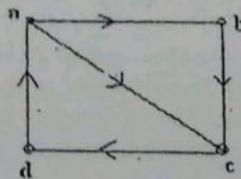
- (i) Develop a recurrence relation model for the number of valid n-digit codewords.  
 (ii) Use generating functions to find an explicit formula for it.

$$x \oplus x = k$$

$$x + y - 1c = 1c$$

$$k = \frac{x+y}{2}$$

- (a) Examine the following digraphs for the types of connectedness and for Euler digraph/unicursal digraph / Hamiltonian digraph:



- (b) Prove that the kernel of a group homomorphism is a normal subgroup of the group.

- (c) Illustrate two applications of Polya's enumeration theorem.

- (d) Use the law of the contrapositive to prove the statement:

If the square of an integer is odd, then the integer is odd.

(3)^3

$$ac + a'c'$$

$$b'd' + b'd$$

$$b'c + a'c' + a$$

$$x_{12} + x_{13} + x_{23} + x_{12} + x_{13}$$

MCA  
FIRST SEMESTER II MID-SEMESTER EXAMINATION, 2013-14  
DISCRETE STRUCTURES

IMA-102

M.M.: 15

Time: 1 hour

NOTE: Attempt All questions.

- 1.(a) Use Karnaugh map or Quine-McClusky method to obtain a minimal product-of-sums representation for

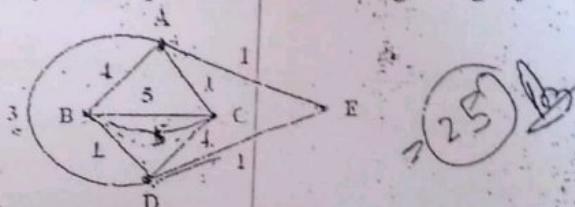
$$f(w, x, y, z) = \prod M(0, 1, 2, 4, 5, 10, 12, 13, 14)$$

- (b) Develop and verify a poset, lattice, and Boolean algebra on the subsets of  $\{a, b, c, d, e\}$ .

- 2.(a) Let  $X = \{o, x, z\}$  and  $A = X \times X$ . Define the relation  $R$  on  $A$  by  
 $(a, b)R(c, d)$  if (i)  $a < c$ ; or (ii)  $a = c$  and  $b \leq d$ .

Examine  $R$  for a partial order for  $A$ .

- (b) Solve the Chinese postman problem for the weighted graph of



- 3.(a) Find an incidence matrix and chromatic number of wheel  $W_n$ .

- (b) Illustrate the application of Hamilton circuits to coding.

- (c) Explain how Euler's formula for planar graphs can be used to show that a simple graph is nonplanar.

- 4.(a) State the applications of mathematical induction.

Which amounts of money can be formed using just two-dollar bills and five-dollar bills? Prove your answer using strong induction.

- (b) State some elegant applications of the pigeon-hole principle.

How many numbers must be selected from the set  $\{1, 3, 5, 7, 9, 11, 13, 15\}$  to guarantee that at least one pair of these numbers add up to 16?

- 5.(a) Negate the following statements :

(i) No person has green eyes.

(ii) Every person dies on one day only.

- (b) Illustrate the local area network graph models.

- (c) Prove that if  $n$  is an integer and  $3n+2$  is even, then  $n$  is even using

- (i) a proof by contraposition, (ii) a proof by contradiction.

FIRST SEMESTER I MID-SEMESTER EXAMINATION, 2013-2014DISCRETE STRUCTURES

Time: 1 Hour

M.M.: 15

Note: Attempt all questions.

- 1.(a) Establish the validity of the following argument:  
 Only citizens are voters.  
 Not all residents are citizens.  
 Therefore, some residents are not voters.
- (b) Establish the validity of the following argument  
 "There is a man whom all men despise."  
 "Therefore, at least one man despises himself."
- 2.(a) Examine the validity of the following argument or give a counter example to show that it is invalid:  
 "All whales are heavy."  
 "All elephants are heavy."  
 "Therefore, all whales are elephants."
- (b) Develop a program in C/C++ to examine the logical equivalence of the formulae :  
 $P \rightarrow (Q \vee R)$  and  $P \wedge (\neg Q) \rightarrow R$

- 3.(a) Find the parse tree for the formula written in Polish notation as  
 $\rightarrow \wedge P \rightarrow QR \rightarrow \wedge P \neg Q \vee \neg Q \neg R$
- (b) Examine the following sets for an adequate set of connectives  
 (i)  $\{\wedge, \vee, \rightarrow\}$ , (ii) {nor}
4. Solve the recurrence relations :
- (i)  $a_n = \frac{1}{2^n} a_{n-1}$  for  $n \geq 1$  with  $a_0 = 3$ , using O.G.F.  
 (ii)  $a_n = 3^n - 2^n a_0 - 2^{n-1} a_1 - \dots - 2^2 a_{n-2} - 2 a_{n-1}$  for  $n \geq 1$  with  $a_0 = 1$   
 using O.G.F.  
 (iii)  $a_n = n a_{n-1} + 3^n$  with  $a_0 = 0$ , using E.G.F.
- 5.(a) Use generating functions to determine the number of sequences of length r that are made up of the letters  $\{x, y, z, \alpha, \beta\}$  with the first portion of each sequence made up of English letters and the second portion made up of Greek letters.
- (b) A valid codeword is an r-digit number in decimal notation containing an odd number of 0s. Develop a recurrence relation model for the number of valid codewords of length r.

$$\begin{aligned} & \langle 0, \neg \vee \rangle \\ & \neg \langle 0, \wedge \rangle \\ & \neg \langle \neg \rangle \\ & \langle \neg \rangle \end{aligned}$$

MCA Odd Semester Examination, 2014-15

DISCRETE STRUCTURES

Max. Marks: 100

Time: 3 Hours

Note: 1. Attempt all parts.

2. All questions carry marks as shown against them.

- 1.(a) Examine the validity of the following argument: [5]

$$\begin{aligned}(A \wedge Q) &\rightarrow M \\ (P \rightarrow Q) \wedge (\neg P \rightarrow A)\end{aligned}$$

$$\therefore \neg M \rightarrow (\neg P \vee P)$$

- (b) Establish the validity of the following argument: [5]

"A mother is a loving person."

"Therefore, the heart of a mother is the heart of a loving person."

- (c) Establish the validity of the following argument or give a counter example to show that it is invalid: [5]

$$\begin{aligned}R &\rightarrow (CVL) \\ \neg C &\rightarrow \neg N \\ \neg T &\rightarrow \neg L\end{aligned}$$

$$\therefore R \rightarrow (NVT)$$

- (d) Develop a program in C/C++ to find the principal conjunctive normal form of the statement formula:  $(\neg P \rightarrow R) \wedge (Q \leq P)$  [5]

- 2.(a) Examine the set  $N \times N \times N$  for a countable set. [5]

- (b) Use a proof by contraposition to prove the pigeonhole principle. Use it to prove or disprove the statement: "If 10 points are selected inside an equilateral triangle of unit side, then atleast two of them are no more than  $1/3$  of a unit apart". [5]

- (c) Determine which amounts of postage can be formed using just 4-cents and 11-cents stamps only. Prove your answer using strong Induction. [5]

- (d) Examine the relation  $\sim$  on  $N \times N$  given by  $(a, b) \sim (c, d)$  if  $a + d = b + c$  for an equivalence relation. If yes, identify the equivalence classes. [5]

- 3.(a) An encoding function is defined by the generator matrix: [10]

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{matrix} (00011) \\ = 000111 \end{matrix}$$

- (i) Examine the encoding function for a group code.

- (ii) If yes, examine the group code for a Hamming code and find the rate of this code.

- (iii) Encode the message: 1111.

- (iv) Decode the received word: 1110111.

- (v) Construct a decoding table consisting of the syndromes and coset leaders for this code.

- (vi) Use the result in part (v) to decode the received word 0010001.

- (b) (I) The operations  $\oplus$  and  $\odot$  on  $R = \{a, b, c, d\}$  given by the table below: [5]

$\oplus$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$\odot$	a	b	c	d
a	a	a	a	a
b	b	a	b	b
c	c	b	c	d
d	d	a	d	a

- (i) Examine  $(R, \oplus, \odot)$  for a commutative ring. (ii) Does R has a unity?  
 (ii) Find a pair of zero divisors. (iv) Is R an integral domain/field? [5]

(II) Let G be the symmetric group  $(S_3, o)$ .

- (i) Examine  $H = \{(1)(2)(3), (132), (123)\}$  for an abelian subgroup of G.  
 (ii) Find all cosets of H in G and examine H for normal subgroup of G.  
 (iii) Verify Lagrange's theorem and find index of H in G.

4.(a) Illustrate how partially ordered structures are useful in modeling. [10]  
 Develop and verify a structure on the set of positive divisors of 646 which is a poset, totally ordered set, well-ordered set, a lattice and its types, and Boolean algebra.

(b) (i) Use Quine-McClusky method to find a minimal-product-of-sums representation for [5]

$$f(w, x, y, z) = \prod M(1, 5, 7, 9, 10, 13, 14, 15)$$

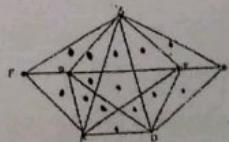
(ii) Design a circuit for a light fixture controlled by three switches, where flipping one of the switches turns the light on when it is off and turns it off when it is on. [5]

5.(a) There are two kinds of particles inside a nuclear reactor. In every second, an  $\alpha$  particle will split into two  $\alpha$  particles and one  $\beta$  particle, and a  $\beta$  particle will split into one  $\alpha$  particle and one  $\beta$  particle. If there is a single  $\alpha$  particle in a reactor initially, develop a recurrence relation model for the number of  $\alpha$  and  $\beta$  particles in the reactor at time n and solve the model. [4]

(b) A company hires 25 new employees. Use generating functions to find the number of ways to assign these people to the four subdivisions so that each subdivision receives atleast 3, but no more than 10 new people. [4]

(c) Illustrate with examples how Polya's enumeration theorem can be used to solve combinatorial problems/graph enumeration problems. [4]

(d) Use Kuratowski's theorem to examine the planarity of the graph: [4]



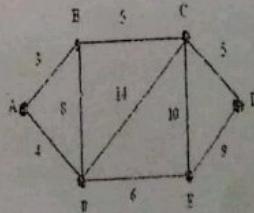
$$14 + 8 -$$

$$6 + 3 + 5 + 5 + 3 + 20 + 30$$

[4]

(e) Solve the Chinese postman problem for the weighted graph:

C15



Time: 1-Hour

Note: Attempt all questions.

1. Solve the system of simultaneous recurrence relations :

$$a_n = \frac{(n+1) | 1 + (-1)^n | - \sum_{i=1}^{n-1} a_i a_{n-i} - \sum_{i=0}^{n-1} b_i b_{n-i}}{2}, n \geq 1$$

$$b_n = \frac{| 1 + (-1)^n |}{2} - \sum_{i=1}^n a_i b_{n-i}, n \geq 1$$

$$\text{with } a_0 = 1, a_1 = -1, b_0 = 1$$

2. There are two kinds of particles inside a nuclear reactor. In every second, an  $\alpha$  particle will split into three  $\alpha$  particles and one  $\beta$  particle, and a  $\beta$  particle will split into two  $\alpha$  particles and one  $\beta$  particle. There is a singly  $\beta$  particle in the reactor at  $t = 0$ .

- (i) Develop a recurrence relation model for the number of  $\alpha$  and  $\beta$  particles in the reactor at time  $t$ .  
(ii) Use generating functions to solve the recurrence relation model.

- 3.(a) Use Prim's algorithm (matrix version) to find MST for the weighted graph :



- (b) Find chromatic polynomial for the graph  $K_{2,5}$ .

- (c) Examine  $K_{m,n}$  for an Eulerian graph, a Hamiltonian graph and a planar graph.

- 4.(a) Establish the validity of the following argument or give a counter example to show that it is invalid :

$$R \rightarrow \{\text{CVL}\}$$

$$\neg C \rightarrow \neg N$$

$$\neg T \rightarrow \neg L$$

$$\therefore R \rightarrow (\neg V T)$$

- (b) Develop a program in C/C++ to examine the statement formula:

$$(p \rightarrow (Q \rightarrow (R \rightarrow S))) \rightarrow (((P \rightarrow Q) \rightarrow R) \rightarrow S) \text{ for a tautology.}$$

- 5.(a) Establish the validity of the following argument:

"A mother is a loving person."

"Therefore, the heart of a mother is the heart of a loving person."

- (b) Test the validity of the following argument:

"If a man is a bachelor, he is unhappy. If a man is unhappy, he dies young. Therefore, bachelors die young."

MCA ODD SEMESTER EXAMINATION, 2016-17

DISCRETE STRUCTURES

Time: 3 Hrs.

Max. Marks: 100

- Note: 1. Attempt all questions.  
2. All questions carry marks as shown against them.

1. (a) Develop a program in C/C++ to find the principal conjunctive normal form of the statement formula: 5

- (b) Establish the validity of the argument: 5

$$\begin{array}{c} P \rightarrow R \\ \neg P \rightarrow \neg Q \\ Q \rightarrow S \end{array}$$

$$\therefore \neg R \rightarrow S$$

- (c) Establish the validity of the argument or give a counter example to show that it is invalid: 5

$$\begin{array}{c} (A \wedge Q) \rightarrow M \\ (F \rightarrow Q) \wedge (\neg P \rightarrow A) \end{array}$$

$$\therefore \neg M \rightarrow (\neg F \vee P)$$

- (d) Establish the validity of the argument: 5

All mothers are loving persons.

Therefore, the heart of a mother is the heart of a loving person.

2. (a) Determine which amounts of postage can be formed using just 4-cent and 5-cent stamps only. Prove your answer using strong induction. 5

- (b) Examine the set  $Z^+ \times Z^+ \times Z^+$  for a countable set. 5

- (c) Examine the relation R on  $\mathbb{R} \times \mathbb{R}$  defined by: 5

$$(a,b)R(c,d) \text{ if } a^2 + b^2 = c^2 + d^2$$

for an equivalence relation. Find  $[(3,4)]$  and give its interpretation. 5

- (d) Prove the pigeon-hole principle and state its applications. 5

There are 38 different time periods during which B.Tech. classes can be scheduled. If there are 677 different classes, how many different rooms will be needed? 5

3. (a) An encoding function is defined by the generator matrix: 10

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ \hline 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- (i) Examine whether the matrix is useful in correcting single errors in transmission.
- (ii) Examine the encoding function for a group code.
- (iii) If yes, examine the group code for a Hamming code.
- (iv) Comment on the error-detection and error-correction capabilities of the code and find its rate.
- (v) Construct a decoding table consisting of the syndromes and coset leaders for this code.
- (vi) Decode the received word : 1110111.
- (b) (i) Let  $G = (\mathbb{Z}_{12}, +_{12})$  and  $H = ([0], [4], [8]), +_{12}$
- (i) Examine  $G$  for a cyclic group.
- (ii) Examine  $H$  for a subgroup of  $G$ .
- (iii) If yes, define the factor group  $(G/H, \oplus_{12})$
- (iv) Examine  $H$  for a normal subgroup of  $G$ .

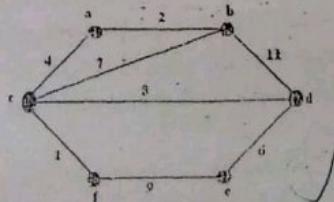
(ii) Examine  $(\mathbb{R}^+, \oplus, \odot)$  for a commutative ring, integral domain and field, where operations  $\oplus$  and  $\odot$  on  $\mathbb{R}^+$  are defined by:

$$a \oplus b = ab, \quad a \odot b = a^{\log_2 b}$$

4. (a) Develop a partially ordered structure on the set of all even positive integer divisors of 308 and examine it for a partially ordered set, well ordered set, lattice with its types and Boolean algebra. 10
- (b) State the purpose of minimization of the Boolean functions and two methods for the purpose. Find minimal-sum-of-products representation for the function: 10

$$f(v, w, x, y, z) = \sum m(1, 2, 3, 4, 10, 17, 18, 19, 22, 23, 27, 28, 30, 31).$$

5. (a) A coding system encodes messages using strings of octal (base 8) digits. A codeword is considered valid if and only if it contains an even number of 7s. Develop a recurrence relation model for the number of valid code words of length  $n$ . 5
- (b) A ship carries 48 flags, 12 each of the colors red, white, blue and black. 12 of these flags are placed on a vertical pole in order to communicate a signal to other ship. Determine the number of signals which use at least one flag of each color. 5
- (c) (i) Use Kuratowski's theorem to examine the planarity of Petersen graph. 3
- (ii) Illustrate how the graph coloring can be used in animal habitats' requirement in a zoo. 2
- (d) Use Prim's algorithm (matrix version) to find MST for the weighted graph: 5



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(3)

IMA-102

MCA FIRST SEMESTER I MID-SEMESTER EXAMINATION, 2016-2017  
DISCRETE STRUCTURES

M.M.: 15

Time: 1 Hour

Note: Attempt all questions.

- 1.(a) Use generating function to prove :  $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$ .  
(b) Illustrate how the graph coloring can be used in animal habitats construction in a zoo.
- 2.(a) Write the adjacency and incidence matrices for  $K_{m,n}$ .  
(b) Illustrate three different algorithms to examine the planarity of a graph.
3. There are two kinds of particles inside a nuclear reactor. In every second , an  $\alpha$  particle will split into three  $\alpha$  particles and two  $\beta$  particle, and a  $\beta$  particle will split into two  $\alpha$  particles and one  $\beta$  particle. There is a single  $\beta$  particle in the reactor initially.
  - (i) Develop a recurrence relation model for the number of  $\alpha$  and  $\beta$  particles in the reactor at time r.
  - (ii) Use generating functions to solve the recurrence relation model.
- 4.(a) Examine the validity of the following argument. Give a counter example if it is invalid.  
"All whales are heavy. All elephants are heavy. Therefore, all whales are elephants."  
(b) Establish the validity of the following argument.  
"There is a man whom all men adore. Therefore; atleast one man adores himself."
- 5.(a) Write a program in C/C<sup>++</sup> to Establish the validity of the argument :

$$\begin{array}{c} P \rightarrow Q \\ R \rightarrow \neg Q \\ \hline \therefore P \rightarrow \neg R \end{array}$$

- (b) Use a proof by contraposition and a proof by contradiction to examine the statement:  
"If  $m + n \geq 73$ , then  $m \geq 37$  or  $n \geq 37$ , where m and n are positive integers."

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✓

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FIRST SEMESTER II MID- SEMESTER EXAMINATION, 2016-17  
DISCRETE STRUCTURES

Time: 1hour

M.M.: 15

NOTE: Attempt All questions.

- 1.(a) Use the Quine-McClusky method to obtain a minimal product-of-sums representation for:

$$f(w, x, y, z) = \prod M(1, 5, 7, 9, 10, 13, 14, 15)$$

- (b) Examine  $\{ \langle a, b \rangle, \langle c, d \rangle \mid a + c = b + d \}$  on  $Z \times Z$  for an equivalence relation. If yes, determine  $[<3,5>]$ .

- 2.(a) Develop and verify a poset, lattice, and Boolean algebra on the set of even positive integers of 308.

- (b) Given the matrix representing a relation on a finite set, develop a program in C/C++ to find the matrix representing the transitive closure of the relation.

- 3.(a) Illustrate with at least one example how the principle of mathematical induction can be used in computer programming.

- (b) Explain Boolean isomorphism. Examine  $(D_{70}, |)$  and  $(P(2, 3, 7), \subseteq)$  for isomorphic Boolean algebras.

- 4.(a) Examine the set:  $Z^+ \times Z^+ \times Z^+$  for a countable set.

- (b) Use the proof of contraposition to prove the pigeon-hole principle.

How many numbers must be selected from the set  $\{1, 3, 5, 7, 9, 11, 13, 15\}$  to guarantee that at least one pair of these numbers add up to 16?

- 5.(a) Establish the validity of the argument.

A student in this class has not read the book.

Each student in this class passed the first exam.

Therefore, someone who passed the first exam has not read the book.

- (b) Find the discrete numeric function corresponding to EGF:  $E(Z) = e^{Z^3}$ .

- (c) Illustrate one application of the Euler circuit/graph.

MCA ODD SEMESTER EXAMINATION, 2017-18

DISCRETE STRUCTURES

Time: 2:30 Hrs.

Max. Marks: 50

- Note: 1. Attempt *all* questions.  
2. All questions carry marks as shown against them.

- 
1. (a) Use a proof by contraposition to prove the theorem: 2  
“If  $3n + 2$  is odd, then  $n$  is odd”, where  $n$  is an integer.  
(b) Establish the validity of the argument: 2  
“If today is Tuesday, I have a test in Mathematics or Economics. If my Economics Professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics Professor is sick. Therefore, I have a test in Mathematics.”  
(c) Examine  $P \rightarrow Q, Q \rightarrow R, S \rightarrow \neg R$  and  $P \wedge S$  for consistency. 2  
(d) Establish the validity of the argument: 2  
“Damini, a student in MCA class knows how to write programs in Java. Every one who knows how to write programs in Java can get a high-paying job. Therefore, someone in this class can get a high-paying job.”  
(e) Write a program in C/C++ to examine the logical equivalence of the statements: 2  
$$(P \rightarrow Q) \wedge (P \rightarrow R) \text{ and } P \rightarrow (Q \wedge R)$$
2. (a) Illustrate the application of mathematical induction in the analysis of computer algorithms/programs. 2  
(b) Examine the set  $N \times N \times N$  for a countable set. 2  
(c) Examine the relation  $R$  on  $\mathbb{R} \times \mathbb{R}$  defined by : 2  
$$(a,b)R(c,d) \text{ if } a+2b = c+2d$$
  
for an equivalence relation. Find  $[(3,4)]$ .  
(d) Use the pigeon-hole principle to prove or disapprove the statement: 2  
“If five points are selected in the interior of a square with side = 1 unit, there are atleast two whose distance apart is less than  $\frac{1}{\sqrt{2}}$ .”  
(e) State the principle of inclusion-exclusion and illustrate its applications. 2
3. (a) An encoding function is defined by the generator matrix: 5  
$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$
  
(i) Examine whether the matrix is useful in correcting single errors in transmission.  
(ii) Examine the encoding function for a group code and a Hamming

code. Find its rate.

- (iii) Use the syndrome table to decode 101001 and find original message.  
 (iv) Decode the received word : 110001 and find original word.

- (b) The operations  $\oplus$  and  $\odot$  on  $R = \{s, t, x, y\}$  of a ring  $(R, \oplus, \odot)$  are given by the table below:

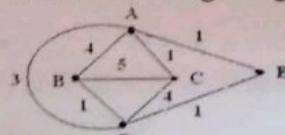
$\oplus$	s	t	x	y
s	y	x	s	t
t	x	y	t	s
x	s	t	x	y
y	t	s	y	x

$\odot$	s	t	x	y
s	y	y	x	x
t	y	y	x	x
x	x	x	x	x
y	x	x	x	x

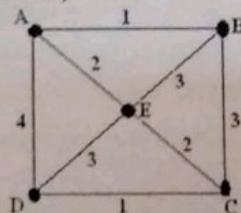
- (i) Examine  $(R, \oplus, \odot)$  for a commutative ring. (ii) Does it has a unity?  
 (iii) Find a pair of zero divisors. (iv) Is it an integral domain/field?

4. (a) Develop a partially ordered structure on the set of all positive integer divisors of 42 and examine it for a partially ordered set, well ordered set, lattice with its types and Boolean algebra.  
 (b) Use Quine-McCluskey's method to find minimal-sum-of-products representation for
- $$f(a, b, c, d) = \sum m(2, 3, 7, 9, 11, 13, ) + \sum \emptyset (1, 10, 15).$$

5. (a) A coding system encodes messages using strings of octal (base 8) digits. A codeword is considered valid if and only if it contains an even number of 7s. Develop a recurrence relation model for the number of valid code words of length n.  
 (b) Determine discrete numeric function corresponding to EGF :  $e^{z^5}$ .  
 (c) Solve the Chinese postman problem for the weighted graph :



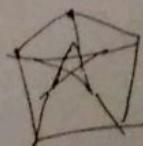
- (d) Use Prim's algorithm (matrix version) to find MST for the weighted graph :



- (e) Find (i) chromatic number of  $W_n$  (ii) crossing number of Petersen graph.

= (3)-1  
= (2)

Just laugh on myself



2

MCA MID-ODD SEMESTER EXAMINATION, 2017-2018DISCRETE STRUCTURESTime:  $1\frac{1}{2}$  Hour

M.M.: 30

Note: Attempt all questions.

- 1.(a) Examine  $P \rightarrow Q, Q \rightarrow R, P \rightarrow \neg R$  and  $P \wedge S$  for consistency.  
 (b) Examine  $\{\uparrow\}$  and  $\{\downarrow\}$  for a functionally complete set of connectives.  
 (c) Does  $P$  follow logically from  $(\neg P \vee \neg Q) \rightarrow (R \wedge S), R \rightarrow T$  and  $\neg T$ ?
  
- 2.(a) Develop a recurrence relation model for the number of valid n-digit codewords, each of which is a valid codeword if it contains an even number of 0 digits. Use generating functions to solve the model.  
 (b) Illustrate with examples one application of: (i) Hamilton circuit, (ii) Graph coloring.  
 (c) Write adjacency matrices for  $K_n$  and  $K_{m,n}$ .
  
- 3.(a) Determine which amounts of postage can be formed using just 4-cent and 11-cent stamps only. Prove your answer using strong induction.  
 (b) Examine a relation defined on  $\mathbb{R} \times \mathbb{R}$  by  $(a,b)R(c,d)$  iff  $a+2b = c+2d$  for an equivalence relation. Determine  $[(3,5)]_R$ .  
 (c) Examine the set of positive rational numbers for a countable set.
  
- 4.(a) Examine  $(D_{70}, \sqsubseteq)$  for a poset, a lattice and its types and a Boolean algebra.  
 (b) Illustrate the use of Quine-McCluskey's method to find minimum SOP for:  

$$f(a,b,c,d) = \sum m(2,3,7,9,11,13) + \sum \Phi(1,10,15).$$
  
 (c) Explain Boolean isomorphism. Examine  $(D_{70}, \sqsubseteq)$  and  $(P(2,3,7), \subseteq)$  for isomorphic Boolean algebras.
  
- 5.(a) Establish the validity of the following argument.  
 "There is a man whom all men adore. Therefore, atleast one man adores himself."  
 (b) Find the discrete numeric function corresponding to EGF:  $E(Z) = e^{Z^3}$ .  
 (c) Develop a program in C/C++ to compute a transitive closure of a relation on the set  $A = \{1,2,3\}$  given by  $R = \{<1,2>, <2,3>, <3,1>\}$ .

THIRD SEMESTER II MID- SEMESTER EXAMINATION, 2017-18  
DISCRETE MATHEMATICAL STRUCTURES

M.M.: 15

Time: 1 Hour

NOTE: Attempt all questions.

- 1.(a) Use Quine - McCluskey's method to minimise:  
 $f(\bar{c}, a, b, c, d, e) = \sum m(0, 1, 3, 8, 9, 13, 14, 15, 16, 17, 19, 24, 25, 27, 31)$
  - (b) Illustrate with familiar examples how Boolean algebra can be used in simplifying circuit design problems.
  
  - 2.(a) Develop a partially ordered structure on the set of all positive divisors of 2310 and verify it for a poset, lattice and its types and Boolean algebra.
  - (b) Use strong mathematical induction to examine the uniqueness of the function  $\varphi(n) = 2(3^n) - 5$  defined by :  
 $\varphi(0) = -3, \varphi(1) = 1, \text{ and } \varphi(n) = 4\varphi(n-1) - 3\varphi(n-2), \forall n \geq 2.$
  
  - 3.(a) Examine the set of positive rational numbers for a countable set.  
(b) Use the method of contraposition to prove the pigeonhole principle.  
How many numbers must be selected from the set  $\{1, 3, 5, 7, 9, 11, 13, 15\}$  to guarantee that atleast one pair of these numbers add upto 16.
  
  - 4.(a) Use a proof by contraposition and a proof by contradiction to prove the statement :  
“ If  $m + n \geq 73$ , then  $m \geq 37$  or  $n \geq 37$ , where  $m$  and  $n$  are positive integers.”
  - (b) Establish the validity of the argument or give a counter example to show that it is invalid :
- $$\frac{\begin{array}{c} (A \wedge Q) \rightarrow M \\ (F \rightarrow Q) \wedge (\neg P \rightarrow A) \end{array}}{\therefore \neg M \rightarrow (\neg F \vee P)}$$

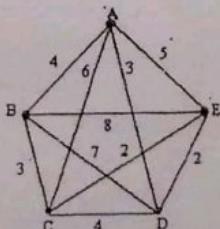
- 5.(a) Solve the system of simultaneous recurrence relations :

$$a_n = \frac{(n+1)[1+(-1)^n] - \sum_{i=1}^{n-1} a_i a_{n-i} - \sum_{i=0}^n b_i b_{n-i}}{2}, n \geq 1$$

$$b_n = \frac{[1+(-1)^n]}{2} - \sum_{i=1}^n a_i b_{n-i}, n \geq 1$$

with  $a_0 = 1, a_1 = -1, b_0 = 1$

- (b) Solve the travelling salesman problem for the weighted graph:



$C/C^f \neq \emptyset$

## MCA ODD SEMESTER EXAMINATION, 2018-19

180231034

## DISCRETE STRUCTURES

Time: 2:30 Hrs.

Max. Marks: 50

- Note:
1. Attempt all questions.
  2. All questions carry marks as shown against them.

1. (a) Use a proof by contraposition to prove the statement:  
"If  $m + n \geq 73$ , then  $m \geq 37$  or  $n \geq 37$ , where m and n are positive integers". 2  
 (b) Establish the validity of the argument:  
"No politicians are corrupted. Some politicians are ministers. Therefore, some ministers are not corrupted". 2  
 (c) Find the PDNF of the statement formula :  $P \wedge \neg(Q \wedge R) \vee (P \rightarrow Q)$ . 2  
 (d) Establish the validity of the argument:  
"All engineers are either M Tech or B Tech. No officer who is an engineer is M Tech. Therefore, if all officers are engineers, then all officers are B Tech". 2  
 (e) Write a program in C/C++ to examine the logical equivalence of the statements:  
 $\neg P \rightarrow (Q \rightarrow R)$  and  $Q \rightarrow (P \vee R)$  2
  
2. (a) Illustrate with example how the mathematical induction can be used in the verification of computer algorithms/programs. 2  
 (b) Examine the set  $Z^+ \times Z^+ \times Z^+$  for a countable set. 2  
 (c) Examine the relation R on  $\mathbb{R} \times \mathbb{R}$  defined by :  
$$(a,b)R(c,d) \text{ if } a^2 + b^2 = c^2 + d^2$$
 for an equivalence relation. Find  $[(3,4)]$ . 2  
 (d) Use the pigeon-hole principle to prove or disapprove the statement:  
"If 10 points are selected in the interior of an equilateral triangle of unit side, there must be atleast two whose distance apart is less than  $\frac{1}{3}$ ". 2  
 (e) Determine the number of antisymmetric relations on a finite set. 2
  
3. (a) An encoding function is defined by the generator matrix: 5  

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$
  - (i) Examine whether the matrix is useful in correcting single errors in transmission.
  - (ii) Examine the encoding function for a group code and a Hamming code.
  - (iii) Decode 101111,111111 and find original messages.
  - (iv) Comment on the error-detection and error-correction capabilities of

1

the code.

- (b) The operations  $\oplus$  and  $\odot$  on  $R = \{s, t, x, y\}$  are defined by the tables:

$\oplus$	s	t	x	y
s	y	x	s	t
t	x	y	t	s
x	s	t	x	y
y	t	s	y	x

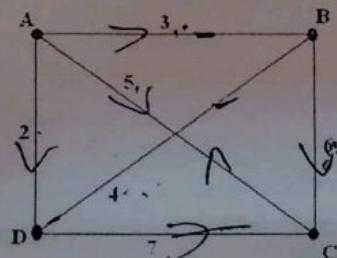
$\odot$	s	t	x	y
s	y	y	x	x
t	y	y	x	x
x	x	x	x	x
y	x	x	x	x

- (i) Examine  $(R, \oplus, \odot)$  for a commutative ring. (ii) Does it have a unity?  
 (iii) Find a pair of zero divisors. (iv) Is it an integral domain/field?

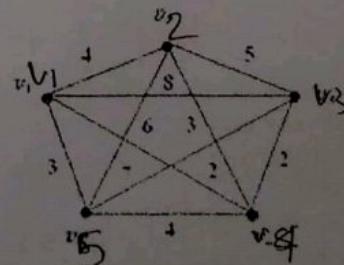
4. (a) Develop a partially ordered structure on the set of all positive integer divisors of 70 and examine it for a partially ordered set, well ordered set, lattice with its types and Boolean algebra. 5  
 (b) Use Quine-McCluskey's method and Karnaugh map to find minimal-sum-of-products representation for

$$f(a, b, c, d) = \sum m(2, 3, 7, 9, 11, 13, ) + \sum \emptyset (1, 10, 15).$$

5. (a) A coding system encodes messages using strings of decimal digits. A codeword is considered valid if and only if it contains an even number of 0s. Develop a recurrence relation model for the number of valid code words of length  $n$  and find the number of code words using generating functions. 2  
 (b) State Pólya's enumeration theorem and illustrate its one application. 2  
 (c) Solve the travelling salesman problem for the weighted graph :



- (d) Use Prim's algorithm (matrix version) to find MST for the weighted graph : 2



- (e) Illustrate (i) one application of graph coloring,  
 (ii) significance of Kuratowski's theorem. 2

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BMA-401

MCA MID-ODD SEMESTER EXAMINATION, 2018-2019

DISCRETE STRUCTURES

*P. N = 8*  
*P. N = 2*

Time:  $1\frac{1}{2}$  Hour

*P → Q ∨ P M ≤ 37*  
*¬P ∨ P M ≤ 37*

M.M.: 30

**Note:** Attempt any *Five* questions. All questions carry marks as shown against them.

- 1.(a) Use a proof by contraposition and a proof by contradiction to examine the statement:  
 “If  $m + n \geq 73$ , then  $m \geq 37$  or  $n \geq 37$ , where  $m$  and  $n$  are positive integers.” [3]
- (b) Establish the validity of the argument or give a counter example to show that it is invalid :

$$\begin{array}{c} (A \wedge Q) \rightarrow M \\ (F \rightarrow Q) \wedge (\neg P \rightarrow A) \\ \hline \therefore \neg M \rightarrow (\neg F \vee P) \end{array}$$

[3] ✓

- 2.(a) Establish the validity of the following argument or give a counter example to show that it is invalid :

$$\begin{array}{c} P \rightarrow Q \\ Q \rightarrow \neg R \\ R \rightarrow (P \vee S) \\ \hline \therefore \neg P \vee (Q \wedge \neg R) \end{array}$$

[3] ✓

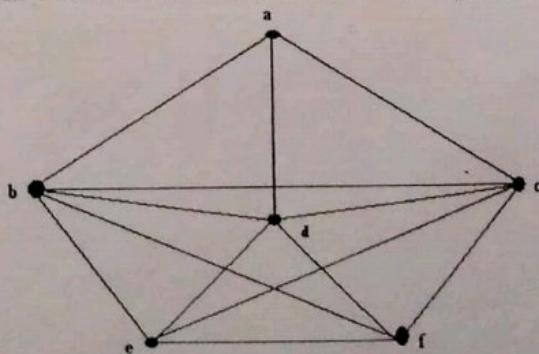
- (b) Examine the validity of the argument :

“All radioactive substances either have a very short life or have medical value. No uranium isotope that is radioactive has very short life. Therefore, if all uranium isotopes are radioactive, then all uranium isotopes have medical value.” [3] ✓

3. There are two kinds of particles inside a nuclear reactor. In every second, an  $\alpha$  particle will split into three  $\alpha$  particles and two  $\beta$  particles, and a  $\beta$  particle will split into two  $\alpha$  particles and one  $\beta$  particle. If there is a single  $\alpha$  particle in the reactor initially.

- (i) Develop a recurrence- relation model for the number of  $\alpha$  and  $\beta$  particles in the reactor at time  $n$ . [3]  
 (ii) Use generating functions to determine the number of  $\alpha$  and  $\beta$  particles at time  $n$ . [3]

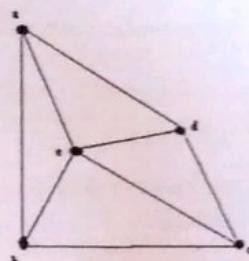
4. (a) State Kuratowski's Theorem and use it to examine the planarity of the graph : [3]



1

mid sem - 2018 - 2019

- (b) Use decomposition theorem to find the chromatic polynomial of the graph : [3]



- 5.(a) Use quantifiers to express the principle of mathematical induction. Examine the correctness of the following program segment by mathematical induction :

```
while n ≠ 0 do  
begin  
    x := x * y  
    n := n - 1  
end  
answer := x
```

[3]

- (b) Determine the numbers of relations which are :

(i) symmetric but not reflexive (ii) reflexive but not symmetric

[3]

6. (a) Prove that if 10 points are selected in the interior of an equilateral triangle of unit side, there must be atleast two whose distance apart is less than  $\frac{1}{3}$ .

[3]

- (b) If  $S, T$  are infinite and countable , examine  $S \times T$  for a countable set.

[3]

FIRST SEMESTER II MID- SEMESTER EXAMINATION  
DISCRETE STRUCTURES

M.M.: 15

Time: 1 hour

NOTE: Attempt all questions.

- 1.(a) Define and verify on the set of positive divisors of 70 a poset, a lattice with its types, and a Boolean algebra.  
(b) Use Karnaugh map or Quine-McClusky method to obtain a minimal sum-of-products representation for  
 $f(x,y,z,w,t) = \sum m(0, 2, 4, 6, 8, 10, 12, 13, 14, 15, 16, 17, 29, 31).$
- 2.(a) Examine  $(Z^+ \times Z^+, \leq)$  for a well-ordered set, where  $\leq$  is a lexicographic order.  
(b) Develop a program in C/C++ to determine the transitive closure of a relation.
- 3.(a) Illustrate with example how mathematical induction can play a major role in computer programs verification.  
(b) Find the smallest relation containing the relation  $\{(1,2), (1,4), (3,3), (4,1)\}$  that is  
(i) symmetric and transitive.  
(ii) reflexive, symmetric and transitive.
- 4.(a) Establish the validity of the following argument:  
"There is a man whom all men despise."  
"Therefore, at least one man despises himself."  
(b) Examine (i)  $K_{2,2,3}$  for a planar graph.  
(ii)  $K_{m,n}$  for Eulerian and Hamiltonian graphs.
- 5.(a) Let  $X = \{0, 1, 2\}$  and  $A = X \times X$ . Define the relation  $R$  on  $A$  by  
 $(a, b)R(c, d)$  if (i)  $a < c$ ; or (ii)  $a = c$  and  $b \leq d$ .  
Examine  $R$  for a partial order on  $A$ .  
(b) Illustrate the use of the time-complexity functions in the analysis (comparison) of algorithms.