

CARRY OVER
B.Tech. END SEMESTER EXAMINATION, 2018-19

DISCRETE MATHEMATICAL STRUCTURES

Time: 3 Hrs.

Max. Marks: 100

- Note: 1. Attempt all questions.
 2. All questions carry marks as shown against them.

1. (a) Find the principal disjunctive normal form of the statement formula: $P \wedge \neg(Q \wedge R) \vee (P \rightarrow Q)$ 5
Use the method of contradiction. $P \wedge \neg(Q \wedge R) \vee (P \rightarrow Q)$
- (b) Derive $\neg R \rightarrow S$ from the premises: $P \rightarrow R$, $\neg P \rightarrow Q$ and $Q \rightarrow S$. 5
 $\neg P$ $P \rightarrow R$ $\neg P \rightarrow Q$ $Q \rightarrow S$ $\neg R \rightarrow S$
- (c) Establish the validity of the argument: 5
 No mothers are males. Some males are politicians. Therefore, some politicians are not mothers. *A horse is an animal. Therefore, the head of a horse is the head of an animal.*
- (d) Examine the premises: $P \rightarrow Q$, $Q \rightarrow R$, $S \rightarrow \neg R$ and $P \wedge \neg S$ for consistency. 5
 $a \rightarrow (b \rightarrow c)$, $d \rightarrow (b \wedge \neg c)$ and $a \wedge \neg d$
2. (a) Examine the set $\mathbb{N} \times \mathbb{N}$ for a countable set: 5
- (b) Determine which amounts of postage can be formed using just 4-cent and 5-cent stamps only. Prove your answer using strong induction. 5
- (c) Examine a relation R defined on $\mathbb{R} \times \mathbb{R}$ by $(a, b) R (c, d)$ if $a + b = c + d$ for an equivalence relation. Determine $[(3, 5)]$. 5
 $a + b = c + d$ $c + 2d$
- (d) Explain how the generalized pigeonhole principle can be used to show that among any 91 integers, there are at least ten that end with the same digit. 5
3. (a) An encoding function is defined by the parity-check matrix: 10
 $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ *generator*
 $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$
- (i) Examine whether the matrix is useful in correcting single errors in transmission.
- (ii) Examine the encoding function for a group code and a Hamming code.
- (iii) Comment on the error-detection and error-correction capabilities of the code and find its rate.
- (iv) Decode the received words: 001111, 110001 and find the original words.

- (b) The operations \oplus and \odot on $R = \{s, t, x, y\}$ are defined in the table below:

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\oplus	s	t	x	y
s	s	t	x	y
t	t	s	y	x
x	x	y	s	t
y	y	x	t	s

\odot	s	t	x	y
s	s	t	x	y
t	t	s	y	x
x	x	y	s	t
y	y	x	t	s

- (i) Examine (R, \oplus, \odot) for a commutative ring. (ii) Does it have a unity?
 (iii) Find a pair of zero divisors. (iv) Is it an integral domain/field?

4. (a) (i) Use Quine-McCluskey's method to find a minimal sum-of-products representation for

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$$f(a, b, c, d) = \sum m(2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$$

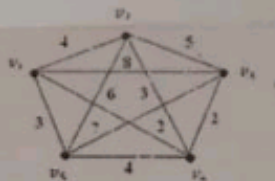
Design a minimal logic circuit for the same.

- (b) Develop a partially ordered structure on the set of all positive even factors of 30 and examine it for a partially ordered set, well ordered set, lattice with its types and Boolean algebra

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5. (a) Use Prim's algorithm (matrix version) to find MST for the weighted graph:

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- (b) There are two kinds of particles inside a nuclear reactor. In every second, an α particle will split into three α particles and one β particle, and a β particle will split into two α particles and one β particle. If there is a single β particle in a reactor initially,

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- (i) Develop a recurrence-relation model for the number of α and β particles in the reactor at time n .

- (ii) Use generating functions to solve the model.

- (c) Illustrate applications of (i) Euler formula for planar graphs (ii) graph coloring.

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- (d) Solve the Chinese postman problem for the weighted graph

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