Research Statement – Rohit Nagpal

My research interests include algebraic combinatorics, modular representation theory, homological stabilization and its applications to geometry and topology. In particular, my focus has been the study of $\mathcal{C}$*-modules* which are functors from a small category $\mathcal{C}$ to the category k-Mod of modules over a ring k.

1 FI-MODULES.

One of the basic examples of a small category is the category FI.

Definition 1.1 (The category FI [CEF]). The category FI is the category whose objects are finite sets and whose morphisms are injections. We denote the set $\{1, 2, \ldots, n\}$ by $[n]$ and the empty set by $[0]$. The endomorphism group $\mathrm{End}_{\mathrm{FI}}([n])$ is naturally isomorphic to the permutation group on n letters and is denoted by S_n. $\triangleleft$

A functor $V\colon \mathrm{FI} \to k$-Mod is an FI-module. The endomorphism group S_n acts naturally on $V_n := V([n])$. So FI-modules provide a setup to study coherent sequences of S_n representations which were first observed as a ubiquitously occurring phenomenon by Church and Farb in [CF]. Soon after, the theory of FI-modules was introduced and developed by Church, Ellenberg and Farb in [CEF] to systematize the study of such sequences.

Here are some examples of FI-modules over a Noetherian ring k:

Example 1.2 ([CEF]). Let k be the ring of integers $\mathfrak{O}_K$ of a number field K. For any proper ideal $\mathfrak{p} \subset k$, let the congruence subgroup $\Gamma_n(\mathfrak{p})$ is the kernel of the natural reduction map $\mathrm{GL}_n(k) \to \mathrm{GL}_n(k/\mathfrak{p})$. Then the map $[n] \mapsto H_m(\Gamma_n(\mathfrak{p}), \mathbb{Z})$ is a functor from FI to k-Mod and hence defines an FI-module. $\triangleleft$

Example 1.3 ([CEF]). Let M be a compact, connected, orientable manifold of dimension ≥ 2 and let $\mathrm{Conf}_n(M)$ be the configuration space of n distinct ordered points on M. Then the map $[n] \mapsto H^m(\mathrm{Conf}_n(M), k)$ is a functor from FI to k-Mod and hence defines an FI-module. $\triangleleft$

The notion of coherence of a sequence of S_n representation is made precise by the natural notion of finite generation of an FI-module. In general, finite generation is defined as follows.

Definition 1.4 (Finitely generated functor [SS3]). Let $\mathcal{C}$ be a small category. A functor $V\colon \mathcal{C} \to k$-Mod is *finitely generated* if there exists a finite set (called a finite set of generators) $S \subset \bigsqcup_{x \in \mathrm{Obj}(\mathcal{C})} V(x)$ of elements such that no proper sub-functor $U \subset V$ contains S. $\triangleleft$

Example 1.5 ([CEF]). Let W be a finitely generated $k[S_m]$-module. The map

$$[n] \mapsto \operatorname{Ind}_{S_m \times S_{n-m}}^{S_n} W \boxtimes k$$

defines a finitely generated FI-module and is denoted by $M(W)$. Here the second factor k is the trivial representation of S_{n-m}. Any finite set S of generators of W can be thought of as a finite set of generators for $M(W)$ via the identification $W \cong M(W)_m := M(W)([m])$ (see Definition 1.10, Theorem 1.11). $\triangleleft$

Church, Ellenberg and Farb showed in [CEF] that finitely generated FI-modules over k satisfy a Noetherian property and provided applications, including the proof that the FI-module in Example 1.3 is finitely generated if k is a field of characteristic 0. In my paper with Church, Ellenberg and Farb ([CEFN]) characteristic free methods were developed which lead to the proof of Noetherian property over an arbitrary Noetherian ring k and an inductive description of an FI-module.

Theorem 1.6 (**Noetherian property, Nagpal et al.** [CEFN]). *If* $V\colon \text{FI} \to k\text{-Mod}$ *is a finitely generated* FI*-module and* $U\colon \text{FI} \to k\text{-Mod}$ *is a sub-*FI*-module of* V*, then* U *is finitely generated.*

Theorem 1.7 (**Inductive Description, Nagpal et al.** [CEFN]). *Let* $V\colon \text{FI} \to k\text{-Mod}$ *be a finitely generated* FI*-module over a Noetherian ring* k*. Then there exists an* $N > 0$ *such that*

$$V_n = \operatorname*{colim}_{\substack{S \subset [n] \\ |S| \leq N}} V_S$$

for all $n > N$*, where the colimit is taken over the poset of subsets under inclusion.*

There are analogs of Theorem 1.6 for finitely generated $\mathcal{C}$-modules for several different small categories $\mathcal{C}$. Here are some such examples of $\mathcal{C}$:

- The category $\text{FI}_{\mathcal{BC}}$ of whose objects are sets $\{\pm 1, \pm 2, \ldots, \pm n\}$ and whose morphisms are absolute value preserving injections ([Wi]). This corresponds to studying coherent sequences of representations of Weyl groups of type $\mathcal{BC}$.

- The category FA of finite sets with maps of finite sets ([WG]).

- For a finite ring R, the category $\mathcal{V}(R)$ of finite rank free R-modules whose morphisms are splittable linear maps ([PS]).

- Gröbner and quasi-Gröbner categories ([SS3]). All of the above – FI, FA, $\text{FI}_{\mathcal{BC}}$ and $\mathcal{V}(R)$ (when R is a finite field) are examples of quasi-Gröbner categories. See [SS3] and [SS4] for several other examples. Gröbner categories serve as a bridge between the theory of well-quasi-orders, automatas and the study of functor categories.

Theorem 1.6 shows that finitely generated FI-modules form an abelian category. In particular, one can transition from r-th page to $(r+1)$-th page of a spectral sequence staying within the category of finitely generated FI-modules. This fact was used to show the following result.

Theorem 1.8 (Nagpal et al. [CEFN]). FI*-modules in Example 1.2 and 1.3 are finitely generated.*

Theorem 1.7 provides a structure theorem for a finitely generated and shows that finitely generated FI-modules statisfy Putman's notion of central stability ([Pu]) which he defined to study the inductive structure of the homology groups $H_m(\Gamma_n(\mathfrak{p}), k)$. Other interesting results on homological stability and the growth of Betti numbers of congruence subgroups include [Char] and [Ca].

One of the main tools that lead to the development of characteristic free methods in [CEFN] is the shift functor. I made extensive use of this functor in my thesis (see Theorem 1.11).

Definition 1.9 (Shift functor $\mathcal{S}_{+a}$ [CEF]). Let V be an FI-module. For any $a \geq 0$ we define the FI-module $\mathcal{S}_{+a}V$ by

$$(\mathcal{S}_{+a}V)_n := \operatorname{Res}^{S_{n+a}}_{S_n} V_{n+a}.$$

Equivalently, $\mathcal{S}_{+a}V$ is the composition of the functor $V\colon \text{FI} \to k\text{-Mod}$ and the functor $\sqcup_a\colon \text{FI} \to \text{FI}$ given by $T \mapsto T \sqcup [a]$. Also, there is a natural map $X_a\colon V \to \mathcal{S}_{+a}V$ where for any $T \in \operatorname{Obj}(\text{FI})$, $X_a(T)$ is induced by the natural map $T \hookrightarrow T \sqcup [a]$. $\triangleleft$

In my thesis, I studied the cohomology groups $H^t(S_n, V_n)$ as n varies, where V is a finitely generated FI-module over a field of characteristic $p > 0$. Parallel theories for the twisted cohomology (or homology) of several different sequences of groups exist or are being worked out. Some examples include:

- Cohomology of free groups via polynomial functors ([DPV]).
- Rational cohomology of classical groups via polynomial functors ([To], [Ku]).
- $H_t(\text{GL}_n(R), T(\text{Ad}_n(R))$ where $T\colon \mathbb{Z}\text{-Mod} \to \mathbb{Z}\text{-Mod}$ is a functor of finite degree ([Dw]).

The first step to analyze the cohomology groups $H^t(S_n, V_n)$ is to obtain a resolution (Theorem 1.11) for FI-modules and for this purpose I defined the filtered FI-modules.

Definition 1.10 (Filtered FI-modules [Thesis, §1]). Let V be a finitely generated FI-module over a commutative ring k. Then V is a *filtered* FI-*module* if it admits a filtration

$$0 = V^0 \subset V^1 \subset \ldots \subset V^d = V$$

with graded pieces of the form $M(W)$ (see Example 1.5), that is, the filtration satisfies $\frac{V^r}{V^{r-1}} \cong M(W_r)$ where each W_r is some finitely generated $k[S_{m_r}]$-modules. $\triangleleft$

Theorem 1.11 (**Resolution of a finitely generated** FI**-module, Nagpal** [Thesis, §2])**.** *Let V be a finitely generated* FI*-module over a Noetherian ring k. Then*

1. *For large enough a, $\mathcal{S}_{+a}V$ is filtered.*
2. *There exists a sequence*

$$0 \to V_n \to J_n^0 \xrightarrow{\phi^0} J_n^1 \xrightarrow{\phi^1} \ldots \xrightarrow{\phi^{n-1}} J_n^N \to 0$$

which is exact if $n \geq C$ such that for each $0 \leq i \leq N$, J^i is a filtered FI*-module.*

Remark 1.12. A theory of FI-module homology is being developed by Church and Ellenberg ([CE]). Eric Ramos and I are preparing a paper that provides a bound on the constants N and C in Theorem 1.11 in terms of the FI-module homology groups. $\triangleleft$

Remark 1.13. If k is a field, then it is easy to check that $\dim M(W)_n$ is a polynomial in n. Therefore, Theorem 1.11 provides an effective version of Theorem B in [CEFN] which states that $\dim V_n$ is eventually a polynomial in n. $\triangleleft$

Remark 1.14. Over a field of characteristic 0, Sam and Snowden analyzed the structure of the category of finitely generated FI-modules in great detail ([SS1]) showing in particular that every object has a finite injective resolution and that the filtered FI-modules are injective. Theorem 1.11 provides an approximate characteristic free version of their result. $\triangleleft$

Theorem 1.15 (**Periodicity of the cohomology for filtered** FI**-modules, Nagpal** [Thesis, §3])**.** *Let k be a field of positive characteristic p and V be a filtered* FI*-module over k. Then there exists a map $\mathfrak{R}^{n,a}$ and constants M,* SD *such that*

$$\mathfrak{R}^{n,a}_{t,V} : H^t(S_n, V_n) \to H^t(S_{n-a}, V_{n-a})$$

is an isomorphism whenever $n - a \geq \mathrm{SD}$ and $p^M \mid a$.

The proof of Theorem 1.15 requires working over a field of positive characteristic. I have the following conjecture that should hold over $\mathbb{Z}$.

Conjecture 1.16 (**Nagpal**)**.** *Let V be a filtered* FI*-modules over $\mathbb{Z}$. Then there exists constant M and* SD *such that $H^t(S_n, V_n) \cong H^t(S_{n-a}, V_{n-a})$ whenever $n - a \geq \mathrm{SD}$ and $M \mid a$.*

Theorem 1.15 in conjunction with Theorem 1.11 lead to the proof of periodicity of the cohomology groups for any finitely generated FI-module V. Let $\mathcal{B}_\bullet(S_n) \to \mathbb{Z} \to 0$ with differential ∂ be the bar resolution of S_n. With the notations of Theorem 1.11, consider the spectral sequences $E^{\bullet,\bullet}(n)$ supported on columns $0 \leq x \leq N$, with the following data:

$$\begin{aligned} E^{x,y}(n) &= \mathrm{Hom}_{S_n}(\mathcal{B}_y(S_n), J_n^x), && \\ {}_{\rightarrow}d^{x,y}(n) &: E^{x,y}(n) \to E^{x+1,y}(n), && \text{induced by } \phi^x \text{ and,} \\ {}_{\uparrow}d^{x,y}(n) &: E^{x,y}(n) \to E^{x,y+1}(n), && \text{induced by } \partial_{y+1}. \end{aligned} \tag{E}$$

Analyzing this spectral sequence yields:

Theorem 1.17 (Periodicity of the cohomology for finitely generated FI-modules, Nagpal [Thesis, §3]). *Let V be a finitely generated FI-module over a field of characteristic p. Then for $n \geq C$, $H^t(S_n, V_n)$ admits a filtration of length $N+1$ with $({}_{\uparrow}E_\infty^{x,y}(n))_{x+y=t, 0\leq x \leq N}$ as graded pieces (here N and C are constants as in Theorem 1.11). And there are constants M_∞^t, SD_∞^t and maps*

$$\mathfrak{R}_\infty^{x,y}(n,a) : {}_{\uparrow}E_\infty^{x,y}(n) \to {}_{\uparrow}E_\infty^{x,y}(n-a)$$

such that if $p^{M_\infty^t} \mid a$ and $n - a \geq \mathrm{SD}_\infty^t$ then $\mathfrak{R}_\infty^{x,y}(n,a)$ is an isomorphism. In particular, $\dim H^t(S_n, V_n)$ is eventually periodic in n with period $p^{M_\infty^t}$.

In my thesis, I have provided algorithms to calculate the stable range SD_∞^t and the period $p^{M_\infty^t}$ defined in Theorem 1.17 and showed that the theory of FI-modules is in a sense optimal setup to study the cohomological stabilization for the symmetric groups: A map $f : U \to V$ of finitely generated FI-modules determine maps $f_n : U_n \to V_n$. The kernel and the image of the map $f_{\star,n} : H^t(S_n, U_n) \to H^t(S_n, V_n)$ are periodic in n. Similarly, if

$$0 \to U \to V \to W \to 0$$

is an exact sequence of finitely generated FI-modules then the connecting homomorphisms in the corresponding cohomology long exact sequence are periodic in n.

Theorem 1.17 is a generalization of Nakaoka's stability theorem:

Theorem 1.18 ([Nak]). $H^t(S_n, k) \cong H^t(S_{n-1}, k)$ *for $n > 2t$, where S_n acts trivially on k.*

Unlike Nakaoka's result where the cohomology groups actually stabilize there are finitely generated FI-modules ([Thesis, §3]) such that the period of $H^t(S_n, V_n)$ is an arbitrary large power of p. One of the possible reasons behind this phenomenon is that there is no natural map $H^t(S_n, V_n) \to H^t(S_{n-a}, V_{n-a})$ exhibiting the isomorphism. The isomorphisms in Theorem 1.17 are constructed at the level of complexes.

Church ([Ch2]) and Wahl ([Wah]) have independently shown the following homology version of Theorem 1.17. Note here that the universal coefficient theorem cannot be used to deduce the cohomology version from the homology version and vice-versa because the coefficients are non-trivial representations of S_n.

Theorem 1.19 (Stability of homology of FI-modules [Ch2], [Wah]). *Let V be a finitely generated FI-module over $\mathbb{Z}$ then the groups $H_t(S_n, V_n)$ eventually stabilize in n.*

2 Configuration spaces.

The classical configuration space of unordered n-tuples of distinct points in $\mathcal{M}$ is defined as:

$$\mathrm{conf}_n(\mathcal{M}) := \{(P_1, P_2, \ldots, P_n) \in M^n \mid P_i \neq P_j\}/S_n = \mathrm{Conf}_n(\mathcal{M})/S_n.$$

In the past, there have been several investigations on the cohomology $H^t(\mathrm{conf}_n(\mathcal{M}), k)$ as n varies but all of them required either an assumption on the manifold $\mathcal{M}$ (restricting to an open or a punctured manifold or an odd-dimensional manifold) or an assumption on the characteristic of the field k (restricting to $\mathbb{F}_2$ or a field of characteristic 0), see- [BCT], [Nap], [Ch1], [RW]. Benson Farb was invited to speak at this topic at the ICM-2014. He has notes on his talk ([Fa]) explaining the history of the problem and the connection between FI-modules and the configuration spaces in great detail.

In my thesis, I was able to show periodicity of the mod-p cohomology of unordered configuration spaces of a manifold $\mathcal{M}$ under the assumption:

(*) $\mathcal{M}$ is an orientable, compact and connected manifold of dimension ≥ 2.

The result is new when the manifold is even-dimensional and the field is different from $\mathbb{F}_2$ and is unique in the sense that it shows periodicity and not stability (see Remark 2.2). The methods used to get the result are similar in spirit to Church's method in [Ch1] which works over characteristic 0 fields. Non-exactness of the functor $H^0(S_n, .)$ in positive characteristic forces the use of a generalized version of Theorem 1.17.

Theorem 2.1 (**Periodicity of the cohomology of unordered configuration spaces, Nagpal** [Thesis, §4]**).** *Let k be a field of characteristic $p > 0$ and let $\mathcal{M}$ satisfies* (*). *Then there exist constants $\vec{\mathrm{M}}^t$, $\vec{\mathrm{SD}}^t$ such that*

$$\dim_k H^t(\mathrm{conf}_n(\mathcal{M}), k) = \dim_k H^t(\mathrm{conf}_{n-a}(\mathcal{M}), k)$$

whenever $p^{\vec{\mathrm{M}}^t} \mid a$ and $n - a \geq \vec{\mathrm{SD}}^t$.

Remark 2.2. The first example where the period is not 1 is the 2-sphere, where we have [Bi, Theorem 1.11]:

$$H_1(\mathrm{conf}_n(S^2), \mathbb{Z}) = \mathbb{Z}/(2n-2)\mathbb{Z}$$

and by the universal coefficient theorem

$$H^1(\mathrm{conf}_n(S^2), \mathbb{Z}/p\mathbb{Z}) = \begin{cases} \mathbb{Z}/p\mathbb{Z}, & \text{if } p \mid 2n-2 \\ 0, & \text{otherwise} \end{cases}$$

Thus, when $p \neq 2$, the smallest period is p. ◁

Problem 2.3 (**Nagpal**)**.** *Find a bound on the constants $\vec{\mathrm{M}}^t$, $\vec{\mathrm{SD}}^t$ in Theorem 2.1 in terms of intrinsic invariants like the* dim $\mathcal{M}$ *or the Betti numbers of $\mathcal{M}$.*

3 TWISTED COMMUTATIVE ALGEBRAS.

Sam and Snowden have introduced and developed the theory of twisted commutative algebras ([SS2]). They explain in [SS3, §7.3] that a twisted commutative algebra (tca) can be seen from the point of view of functors from a small category. For example, they showed that a tca finitely generated in degree 1 over a Noetherian ring k is Noetherian by showing that the category of finitely generated functors $\mathrm{FI_d} \to k$-Mod is Noetherian. Here the category $\mathrm{FI_d}$ is a generalization to the category FI.

Definition 3.1 (**The category** $\mathrm{FI_d}$ [SS3]). Let $\mathrm{FI_d}$ be the category whose objects are finite sets and morphisms are injections with d-coloring of the complement. ◁

The following conjecture is an analog of Theorem 1.17 in the category $\mathrm{FI_d}$. If true, this conjecture would have application to the cohomology of unordered configuration spaces (see [SS3, §7.5]).

Conjecture 3.2 (**Nagpal**). *Let k be an arbitrary field and V be a finitely generated $\mathrm{FI_d}$-module. Then $\dim H^t(S_n, V_n)$ is bounded by a polynomial in n of degree at most $d-1$. If k is a field of characteristic 0, $\dim H^0(S_n, V_n)$ is eventually a polynomial.*

It is not known whether a tca finitely generated in degree ≥ 2 over a Noetherian ring is Noetherian. To make precise one of the first steps towards this open problem consider the following category.

Definition 3.3 (**The category** FP [SS3]). Let FP be the category whose objects are finite sets and whose morphisms are injections with perfect matching of the complement. ◁

Conjecture 3.4 (**Sam and Snowden** [SS2]). *Let V be a finitely generated FP-module over a Noetherian ring. Then V is Noetherian.*

A partial result towards proving Conjecture 3.4 can be found in [Abe]. The same result can also be deduced using Pieri maps defined in [Olv].

Finding the Krull dimension of a category is in general a harder problem than showing that a category is Noetherian. It is known that the Krull dimension of the category of finitely generated FA-modules over an arbitrary field is 0 ([WG]). The following problem is open over a field of positive characteristic except in the case $d = 1$ where the Krull dimension is known to be 1.

Problem 3.5 (**Sam and Snowden** [SS3]). *Find the Krull dimension of the category of finitely generated $\mathrm{FI_d}$-modules over an arbitrary field.*

4 SYZYGIES.

Twisted commutative algebras are important tools that can produce bounds on the syzygies. For example, Snowden used tcas with some additional structure called Δ-modules in [Sn1] to obtain information about the syzygies of the Segre embedding. For another example, consider a finite group G of order g acting on a finite dimensional vector space V over a field k. It is known that the ring $T := k[V]^G$ is finitely generated in degree $\leq g$ which is a bound independent of the dimension of the vector space V (see [Fo] for a short proof). Let $f_1, f_2, \ldots, f_r$ be a minimal set of generators of T and let $S := k[y_1, \ldots, y_r] \twoheadrightarrow T$ be the natural surjection. The space $\mathrm{Tor}_t^S(T, k)$ is then a graded vector space, called the space of t-syzygies of T. Let s_t be the highest degree occuring in $\mathrm{Tor}_t^S(T, k)$. The following conjecture is due to Derksen.

Conjecture 4.1 (Derksen [De]). *We have $s_t \leq (t+1)g$ for all V and $t \geq 0$.*

Derksen proved the conjecture for $t = 1$. The conjecture as stated, is false for $t = 2$ because of the following example.

Example 4.2 (Nagpal). Let T be the Veronese ring corresponding to the action of the cyclic group $G = \mathbb{Z}_4$ on a vector space of dimension 3 over the field $k = \frac{\mathbb{Z}}{5\mathbb{Z}}$. Then, a calculation using Macaulay2 yields $s_2 = 16 > 12 = (t+1)g$. $\lhd$

However, in [Sn2], Snowden used twisted commutative algebras to get bounds on s_t independent of $\dim(V)$ assuming that k has characteristic 0. The analogue of his result over an arbitrary field is open.

Problem 4.3 (Snowden [Sn2]). *Assume that k is a field of positive characteristic. Find an upper bound on s_t depending only on g and t.*

I have formulated a version of this conjecture for affine semigroup rings, with partial results. For that let $\mathcal{A} \subset \mathbb{N}^m$ be a finitely generated sub-semigroup satisfying

$$(**) \qquad \begin{aligned} \mathrm{Cone}(\mathcal{A}) &= \mathrm{Cone}(\mathbb{N}^m) \\ \mathbb{N}\mathcal{A} &= \mathbb{Z}\mathcal{A} \cap \mathbb{N}^m \end{aligned}$$

and let $T := k[\mathcal{A}]$ be the corresponding affine semigroup ring. By (**) there is a natural norm on $\mathcal{A}$ given by $|x| := \sum_{1 \leq i \leq m} x_i$ for any $x = (x_i)_{1 \leq i \leq m}$ and there exists a constant L such that every $x \in \mathbb{N}^m$ can be written as $x = y + z$ with $y \in \mathcal{A}$ and $|z| < L$. The L-th Veronese ring in m variables is an example of such an affine semigroup ring.

Let $G := \{u_1, u_2, \ldots, u_r\}$ be a minimal generating set of $\mathcal{A}$ and let $S := k[y_1, \ldots, y_r] \twoheadrightarrow T$ be the natural surjection. The space $\mathrm{Tor}_t^S(T, k)$ is then a $\mathbb{N}^m$ graded vector space, called the space of t-syzygies of T. We have the following conjecture.

Conjecture 4.4 (Nagpal). *With the notation as above and an arbitrary field k, let $\alpha \in \mathbb{N}^m$ and $\mathrm{Tor}_t^S(T, k)_\alpha$ be the α-th graded piece of the space of syzygies. If $|\alpha| > 2tL$, then $\mathrm{Tor}_t^S(T, k)_\alpha = 0$.*

It is known that syzygies of affine semigroup rings can be related to reduced homology of certain simplicial complexes ([BH]). In [Pa], Paul has defined the *pile complex*

$$\Gamma_\alpha(\mathcal{A}) := \left\{ S \subset G : \sum_{u \in S} u \leq \alpha \right\}.$$

and proved the following theorem.

Theorem 4.5 ([Pa])**.** *If $\alpha \in \mathcal{A}$, then* $\dim \operatorname{Tor}_t^S(T,k)_\alpha = \dim \tilde{H}_{t-1}(\Gamma_\alpha, k)$.

By Theorem 4.5, the following conjecture implies Conjecture 4.4 and can be used to obtain partial results for $t < 3$.

Conjecture 4.6 (Nagpal). *With notations as above, we have* $\tilde{H}_{t-1}(\Gamma_\alpha, k) = 0$ *if* $|\alpha| > 2tL$.

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