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(* MA39110 / Assignment 4.1 / 16MA20053 / NER ROHIT *)
ClearAll["Global`*"];
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In[2]:=
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Thomas[a_, b_, c_, d_] :=
  Module[{c1 = Range[Length[c]], d1 = Range[Length[d]], x = Range[Length[b]]},
   c1[[1]] = c[[1]] / b[[1]]; d1[[1]] = d[[1]] / b[[1]];
   Do[
    If[i ≠ Length[d], c1[[i]] = c[[i]] / (b[[i]] - a[[i - 1]] * c1[[i - 1]])];
    d1[[i]] = (d[[i]] - a[[i - 1]] * d1[[i - 1]]) / (b[[i]] - a[[i - 1]] * c1[[i - 1]]);
    , {i, 2, Length[d]}];
   x[[Length[b]]] = d1[[Length[b]]];
   Do[
    x[[i]] = d1[[i]] - c1[[i]] * x[[i + 1]];
    , {i, Length[b] - 1, 1, -1}];
   x];
Model[n0_] := Module[{n = n0},
   x0 = 0; xf = Pi; h = (xf - x0) / n;
   A = Table[0, {x, 1, n - 1}, {y, 1, n - 1}];
   X = Table[x0 + x * h, {x, 1, n - 1}];
   XT = Table[x0 + x * h, {x, 0, n}];
   B = Table[0, {x, 1, n - 1}];
   eps = 0.0000000000001;
   (*Initial Approximation: Parabola passing through the points (0,1/2),
   (1/2,0) and (Pi,-1/2).*)
   f[x_] = ((Pi - 1) / (Pi^2 - Pi / 2)) x^2 - ((Pi^2 - 1 / 2) / (Pi^2 - Pi / 2)) x + 1 / 2;
   YT = N[f[XT]];
   While[{
     Y = YT;
     For[i = 1, i < n, i++,
      {
       im = i + 1;
       A[[i, i]] = -2 / h^2 - 2 Y[[im]] + 1;
       If[i ≠ 1, A[[i, i - 1]] = 1 / h^2 + (2 / (4 h^2)) (Y[[im + 1]] - Y[[im - 1]])];
       If[i ≠ n - 1, A[[i, i + 1]] = 1 / h^2 - (2 / (4 h^2)) (Y[[im + 1]] - Y[[im - 1]])];
       B[[i]] = -(1 / h^2) (Y[[im - 1]] - 2 Y[[im]] + Y[[im + 1]]) +
          (1 / (4 h^2)) (Y[[im + 1]] - Y[[im - 1]])^2 + Y[[im]]^2 - Y[[im]] - 1;
      }];
     YT = Y +
       Flatten[{{0}, Thomas[Diagonal[A, -1], Diagonal[A], Diagonal[A, 1], B], {0}}];
    }; N[Max[Abs[YT - Y]]] > eps];
   YT
  ];
```

In[4]:=
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in = {50, 100 , 200};
sol = Model[in[[1]]];
(*Print[N[sol]];*)
solt = NDSolve[
   {y''[x] - y'[x]^2 - y[x]^2 + y[x] + 1 == 0, y[0] == 1/2, y[Pi] == -1/2}, y[x], x];
err1 = Abs[NDSolveValue[{y''[x] - y'[x]^2 - y[x]^2 + y[x] + 1 == 0,
      y[0] == 1/2, y[Pi] == -1/2}, y[XT], x] - sol];
perr1 = ListPlot[Transpose[{XT, err1}], PlotStyle → Red];
Print[N[Max[err1]]];
p1 = Show[{Plot[Evaluate[y[x] /. solt], {x, x0, xf},
     PlotLabel → Style[StringForm["h = ``", N[h]], FontSize → 18]]},
   {ListPlot[Transpose[{XT, sol}], PlotStyle → Red]},
   {Plot[((Pi - 1) / (Pi^2 - Pi/2)) x^2 - ((Pi^2 - 1/2) / (Pi^2 - Pi/2)) x + 1/2,
     {x, 0, Pi}, PlotStyle → {Dashed, Black}]}];
```

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0.000474216
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In[11]:=
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sol = Model[in[[2]]];
(*Print[N[sol]];*)
solt = NDSolve[
   {y''[x] - y'[x]^2 - y[x]^2 + y[x] + 1 == 0, y[0] == 1/2, y[Pi] == -1/2}, y[x], x];
err2 = Abs[NDSolveValue[{y''[x] - y'[x]^2 - y[x]^2 + y[x] + 1 == 0,
      y[0] == 1/2, y[Pi] == -1/2}, y[XT], x] - sol];
perr2 = ListPlot[Transpose[{XT, err2}], PlotStyle → Green];
Print[N[Max[err2]]];
p2 = Show[{Plot[Evaluate[y[x] /. solt], {x, x0, xf},
     PlotLabel → Style[StringForm["h = ``", N[h]], FontSize → 18]]},
   {ListPlot[Transpose[{XT, sol}], PlotStyle → Red]},
   {Plot[((Pi - 1) / (Pi^2 - Pi/2)) x^2 - ((Pi^2 - 1/2) / (Pi^2 - Pi/2)) x + 1/2,
     {x, 0, Pi}, PlotStyle → {Dashed, Black}]}];
```

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0.000118608
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In[17]:=
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sol = Model[in[[3]]];
(*Print[N[sol]];*)
solt = NDSolve[
    {y''[x] - y'[x]^2 - y[x]^2 + y[x] + 1 == 0, y[0] == 1/2, y[Pi] == -1/2}, y[x], x];
err3 = Abs[NDSolveValue[{y''[x] - y'[x]^2 - y[x]^2 + y[x] + 1 == 0,
       y[0] == 1/2, y[Pi] == -1/2}, y[XT], x] - sol];
perr3 = ListPlot[Transpose[{XT, err3}], PlotStyle → Blue];
Print[N[Max[err3]]];
p3 = Show[{Plot[Evaluate[y[x] /. solt], {x, x0, xf},
      PlotLabel → Style[StringForm["h = ``", N[h]], FontSize → 18]]},
    {ListPlot[Transpose[{XT, sol}], PlotStyle → Red]},
    {Plot[((Pi - 1)/(Pi^2 - Pi/2)) x^2 - ((Pi^2 - 1/2)/(Pi^2 - Pi/2)) x + 1/2,
      {x, 0, Pi}, PlotStyle → {Dashed, Black}]}];
```

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0.000029815
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In[23]:=
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p4 = Show[{Plot[y[x] = 10^-4, {x, 0, Pi}, PlotStyle → {Dashed, Black}]}, {perr1},
    {perr2}, {perr3}, PlotLabel → Style["Truncation Error", FontSize → 18],
    PlotRange → {{0, Pi}, {0, 0.00045}}];
GraphicsGrid[{{p1, p2}, {p3, p4}}]
```

Out[25]=