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In[1]:= (* MA39110 / Assignment 4.3 / 16MA20053 / NER ROHIT *)
ClearAll["Global`*"];

In[2]:= BlockThomas[a_, b_, c_, d_] :=
Module[{b1 = Array[0 &, {Length[b], 1, 2, 2}], c1 = Array[0 &, {Length[c], 1, 2, 2}],
  d1 = Array[0 &, {Length[d], 1, 2, 1}], x = Array[0 &, {Length[b], 1, 2, 1}]},
  c1[[1, 1]] = Dot[Inverse[b[[1, 1]]], c[[1, 1]]];
  d1[[1, 1]] = Dot[Inverse[b[[1, 1]]], d[[1, 1]]];
  Do[
    b1[[i, 1]] = b[[i, 1]] - Dot[a[[i - 1, 1]], c1[[i - 1, 1]]];
    If[i ≠ Length[d], c1[[i, 1]] = Dot[Inverse[b1[[i, 1]]], c[[i, 1]]];];
  d1[[i, 1]] =
    Dot[Inverse[b1[[i, 1]]], d[[i, 1]] - Dot[a[[i - 1, 1]], d1[[i - 1, 1]]];
  , {i, 2, Length[d]}};
  x[[Length[b], 1]] = d1[[Length[b], 1]];
  Do[
    x[[i, 1]] = d1[[i, 1]] - Dot[c1[[i, 1]], x[[i + 1, 1]]];
    , {i, Length[b] - 1, 1, -1}];
  x];

Model[n0_] := Module[{n = n0},
  x0 = 0; xf = 10; h = (xf - x0) / n;
  a = Array[{{0, 0}, {0, 0}} &, {n - 2, 1}];
  b = Array[{{0, 0}, {0, 0}} &, {n - 1, 1}];
  c = Array[{{0, 0}, {0, 0}} &, {n - 2, 1}];
  d = Array[{0, 0} &, {n - 1, 1}];
  eps = 0.01;
  X = Table[x0 + x * h, {x, 1, n - 1}];
  XT = Table[x0 + x * h, {x, 0, n}];
  (* Initial Approximation:  $f(x) = (19x^3 - 317x^2 + 1288x) / 648$  *)
  fT = N[(19 XT^3 - 317 XT^2 + 1288 XT) / 648];
  FT = Flatten[{ {0}, N[(57 X^2 - 634 X + 1288) / 648], {1} ]};
  While[{
    f = fT; F = FT;
    For[i = 1, i < n, i++,
      {
        im = i + 1;
        b[[i, 1]] =
          { {1, -h/2}, { (F[[im + 1]] - F[[im - 1]]) / (2 h), -2/h^2 - 2 F[[im]] } };
        d[[i, 1]] = { -f[[im]] + f[[im - 1]] + (h/2) (F[[im]] + F[[im - 1]]),
          - (1/h^2) (F[[im - 1]] - 2 F[[im]] + F[[im + 1]]) -
          (1/(2 h)) f[[im]] (F[[im + 1]] - F[[im - 1]]) + F[[im]]^2 - 1 };
        If[i ≠ n - 1, { a[[i, 1]] = { {-1, -h/2}, {0, 1/h^2 - f[[im + 1]] / (2 h)} },
          c[[i, 1]] = { {0, 0}, {0, 1/h^2 + f[[im]] / (2 h)} } }];
      }
    ]
  ]

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    }];
    FT = F + Flatten[{{0}, Flatten[BlockThomas[a, b, c, d][[2 ;; 2]], {0}]}];
    fT = f + Flatten[{{0}, Flatten[BlockThomas[a, b, c, d][[1 ;; 2]], {0}]}];
    N[Max[Abs[fT - f]]] > eps];
    XT = XT[[1 ;; -2 ;; 1]];
    fT = fT[[1 ;; -2 ;; 1]];
    fT];
In[4]:= in = {20, 40, 80};
sol = Model[in[[1]]];
solt = NDSolve[
  {y'''[x] + y[x] y''[x] + 1 - y'[x]^2 == 0, y[0] == y'[0] == 0, y'[10] == 1}, y[x], x];
err1 = Abs[NDSolveValue[{y'''[x] + y[x] y''[x] + 1 - y'[x]^2 == 0,
  y[0] == y'[0] == 0, y'[10] == 1}, y[XT], x] - sol];
perr1 = ListPlot[Transpose[{XT, err1}], PlotStyle -> Red];
Print[N[Max[err1]]];
p1 = Show[Plot[Evaluate[y[x] /. solt], {x, x0, xf},
  PlotLabel -> Style[StringForm["h = ``", N[h]], FontSize -> 18]]],
  {ListPlot[Transpose[{XT, sol}], PlotStyle -> Red]},
  {Plot[(19 x^3 - 317 x^2 + 1288 x) / 648, {x, 0, 10}, PlotStyle -> {Dashed, Black}]}];
0.0534011
In[11]:= sol = Model[in[[2]]];
solt = NDSolve[
  {y'''[x] + y[x] y''[x] + 1 - y'[x]^2 == 0, y[0] == y'[0] == 0, y'[10] == 1}, y[x], x];
err2 = Abs[NDSolveValue[{y'''[x] + y[x] y''[x] + 1 - y'[x]^2 == 0,
  y[0] == y'[0] == 0, y'[10] == 1}, y[XT], x] - sol];
perr2 = ListPlot[Transpose[{XT, err2}], PlotStyle -> Green];
Print[N[Max[err2]]];
p2 = Show[Plot[Evaluate[y[x] /. solt], {x, x0, xf},
  PlotLabel -> Style[StringForm["h = ``", N[h]], FontSize -> 18]]],
  {ListPlot[Transpose[{XT, sol}], PlotStyle -> Red]},
  {Plot[(19 x^3 - 317 x^2 + 1288 x) / 648, {x, 0, 10}, PlotStyle -> {Dashed, Black}]}];
0.0163164

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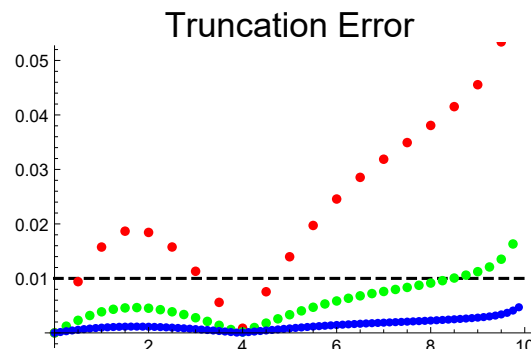
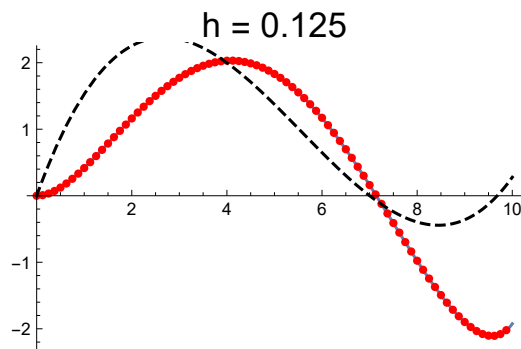
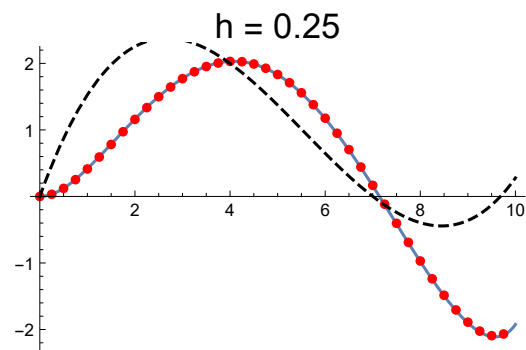
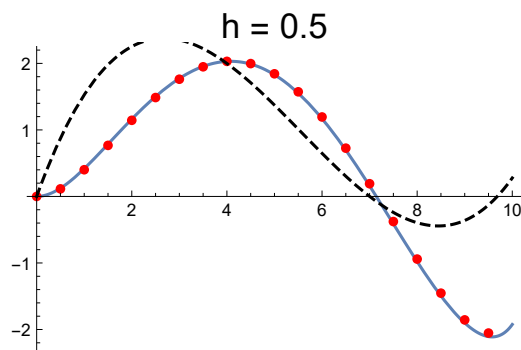
In[17]:= sol = Model[in[[3]]];
solt = NDSolve[
  {y'''[x] + y[x] y''[x] + 1 - y'[x]^2 == 0, y[0] == y'[0] == 0, y'[10] == 1}, y[x], x];
err3 = Abs[NDSolveValue[{y'''[x] + y[x] y''[x] + 1 - y'[x]^2 == 0,
  y[0] == y'[0] == 0, y'[10] == 1}, y[XT], x] - sol];
perr3 = ListPlot[Transpose[{XT, err3}], PlotStyle -> Blue];
Print[N[Max[err3]]];
p3 = Show[Plot[Evaluate[y[x] /. solt], {x, x0, xf},
  PlotLabel -> Style[StringForm["h = ``", N[h]], FontSize -> 18]],
  {ListPlot[Transpose[{XT, solt}], PlotStyle -> Red]},
  {Plot[(19 x^3 - 317 x^2 + 1288 x) / 648, {x, 0, 10}, PlotStyle -> {Dashed, Black}]}];
0.00470185

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In[23]:= p4 = Show[{Plot[y[x] == 10^-2, {x, 0, 10}, PlotStyle -> {Dashed, Black}], {perr1},
  {perr2}, {perr3}, PlotLabel -> Style["Truncation Error", FontSize -> 18],
  PlotRange -> {{0, 10}, {0, 0.05}}];
GraphicsGrid[{{p1, p2}, {p3, p4}}]

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Out[24]=