In[1]:=
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(* MA39110 / Assignment 4.3 / 16MA20053 / NER ROHIT *)
ClearAll["Global`*"];
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In[2]:=
```
BlockThomas[a_, b_, c_, d_] :=
  Module[{b1 = Array[0 &, {Length[b], 1, 2, 2}], c1 = Array[0 &, {Length[c], 1, 2, 2}],
    d1 = Array[0 &, {Length[d], 1, 2, 1}], x = Array[0 &, {Length[b], 1, 2, 1}]},
   c1[[1, 1]] = Dot[Inverse[b[[1, 1]]], c[[1, 1]]];
   d1[[1, 1]] = Dot[Inverse[b[[1, 1]]], d[[1, 1]]];
   Do[
    b1[[i, 1]] = b[[i, 1]] - Dot[a[[i - 1, 1]], c1[[i - 1, 1]]];
    If[i ≠ Length[d], c1[[i, 1]] = Dot[Inverse[b1[[i, 1]]], c[[i, 1]]];];
    d1[[i, 1]] =
     Dot[Inverse[b1[[i, 1]]], d[[i, 1]] - Dot[a[[i - 1, 1]], d1[[i - 1, 1]]]];
    , {i, 2, Length[d]}];
   x[[Length[b], 1]] = d1[[Length[b], 1]];
   Do[
    x[[i, 1]] = d1[[i, 1]] - Dot[c1[[i, 1]], x[[i + 1, 1]]];
    , {i, Length[b] - 1, 1, -1}];
   x];
Model[n0_] := Module[{n = n0},
   x0 = 0; xf = 10; h = (xf - x0) / n;
   a = Array[{{0, 0}, {0, 0}} &, {n - 2, 1}];
   b = Array[{{0, 0}, {0, 0}} &, {n - 1, 1}];
   c = Array[{{0, 0}, {0, 0}} &, {n - 2, 1}];
   d = Array[{0, 0} &, {n - 1, 1}];
   eps = 0.01;
   X = Table[x0 + x * h, {x, 1, n - 1}];
   XT = Table[x0 + x * h, {x, 0, n}];
   (* Initial Approximation: f(x) = (19x^3-317x^2+1288x)/648 *)
   fT = N[(19 XT^3 - 317 XT^2 + 1288 XT) / 648];
   FT = Flatten[{{0}, N[(57 X^2 - 634 X + 1288) / 648], {1}}];
   While[{
     f = fT; F = FT;
     For[i = 1, i < n, i++,
      {
       im = i + 1;
       b[[i, 1]] =
        {{1, -h/2}, {(F[[im + 1]] - F[[im - 1]]) / (2 h), -2/h^2 - 2 F[[im]]}};
       d[[i, 1]] = {-f[[im]] + f[[im - 1]] + (h/2) (F[[im]] + F[[im - 1]]),
         -(1/h^2) (F[[im - 1]] - 2 F[[im]] + F[[im + 1]]) -
          (1/(2 h)) f[[im]] (F[[im + 1]] - F[[im - 1]]) + F[[im]]^2 - 1};
       If[i ≠ n - 1, {a[[i, 1]] = {{-1, -h/2}, {0, 1/h^2 - f[[im + 1]] / (2 h)}},
         c[[i, 1]] = {{0, 0}, {0, 1/h^2 + f[[im]] / (2 h)}}}];
```

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        }];
       FT = F + Flatten[{{0}, Flatten[BlockThomas[a, b, c, d]][[2 ;; ;; 2]], {0}}];
       fT = f + Flatten[{{0}, Flatten[BlockThomas[a, b, c, d]][[1 ;; ;; 2]], {0}}];};
     N[Max[Abs[fT - f]]] > eps];
    XT = XT[[1 ;; -2 ;; 1]];
    fT = fT[[1 ;; -2 ;; 1]];
    fT];
```

In[4]:=
```
in = {20, 40, 80};
sol = Model[in[[1]]];
solt = NDSolve[
   {y'''[x] + y[x] y''[x] + 1 - y'[x]^2 == 0, y[0] == y'[0] == 0, y'[10] == 1}, y[x], x];
err1 = Abs[NDSolveValue[{y'''[x] + y[x] y''[x] + 1 - y'[x]^2 == 0,
      y[0] == y'[0] == 0, y'[10] == 1}, y[XT], x] - sol];
perr1 = ListPlot[Transpose[{XT, err1}], PlotStyle → Red];
Print[N[Max[err1]]];
p1 = Show[{Plot[Evaluate[y[x] /. solt], {x, x0, xf},
     PlotLabel → Style[StringForm["h = ``", N[h]], FontSize → 18]]},
   {ListPlot[Transpose[{XT, sol}], PlotStyle → Red]},
   {Plot[(19 x^3 - 317 x^2 + 1288 x) / 648, {x, 0, 10}, PlotStyle → {Dashed, Black}]}];
```

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0.0534011
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In[11]:=
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sol = Model[in[[2]]];
solt = NDSolve[
   {y'''[x] + y[x] y''[x] + 1 - y'[x]^2 == 0, y[0] == y'[0] == 0, y'[10] == 1}, y[x], x];
err2 = Abs[NDSolveValue[{y'''[x] + y[x] y''[x] + 1 - y'[x]^2 == 0,
      y[0] == y'[0] == 0, y'[10] == 1}, y[XT], x] - sol];
perr2 = ListPlot[Transpose[{XT, err2}], PlotStyle → Green];
Print[N[Max[err2]]];
p2 = Show[{Plot[Evaluate[y[x] /. solt], {x, x0, xf},
     PlotLabel → Style[StringForm["h = ``", N[h]], FontSize → 18]]},
   {ListPlot[Transpose[{XT, sol}], PlotStyle → Red]},
   {Plot[(19 x^3 - 317 x^2 + 1288 x) / 648, {x, 0, 10}, PlotStyle → {Dashed, Black}]}];
```

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0.0163164
```

In[17]:=
```
sol = Model[in[[3]]];
solt = NDSolve[
    {y'''[x] + y[x] y''[x] + 1 - y'[x]^2 == 0, y[0] == y'[0] == 0, y'[10] == 1}, y[x], x];
err3 = Abs[NDSolveValue[{y'''[x] + y[x] y''[x] + 1 - y'[x]^2 == 0,
       y[0] == y'[0] == 0, y'[10] == 1}, y[XT], x] - sol];
perr3 = ListPlot[Transpose[{XT, err3}], PlotStyle → Blue];
Print[N[Max[err3]]];
p3 = Show[{Plot[Evaluate[y[x] /. solt], {x, x0, xf},
      PlotLabel → Style[StringForm["h = ``", N[h]], FontSize → 18]]},
    {ListPlot[Transpose[{XT, sol}], PlotStyle → Red]},
    {Plot[(19 x^3 - 317 x^2 + 1288 x) / 648, {x, 0, 10}, PlotStyle → {Dashed, Black}]}];
```

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0.00470185
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In[23]:=
```
p4 = Show[{Plot[y[x] = 10^-2, {x, 0, 10}, PlotStyle → {Dashed, Black}]}, {perr1},
    {perr2}, {perr3}, PlotLabel → Style["Truncation Error", FontSize → 18],
    PlotRange → {{0, 10}, {0, 0.05}}];
GraphicsGrid[{{p1, p2}, {p3, p4}}]
```

Out[24]=