In[1]:=
```
(* MA39110 / Assignment 4.2 / 16MA20053 / NER ROHIT *)
ClearAll["Global`*"];
```

In[2]:=
```
Thomas[a_, b_, c_, d_] :=
  Module[{c1 = Range[Length[c]], d1 = Range[Length[d]], x = Range[Length[b]]},
   c1[[1]] = c[[1]] / b[[1]]; d1[[1]] = d[[1]] / b[[1]];
   Do[
    If[i ≠ Length[d], c1[[i]] = c[[i]] / (b[[i]] - a[[i - 1]] * c1[[i - 1]])];
    d1[[i]] = (d[[i]] - a[[i - 1]] * d1[[i - 1]]) / (b[[i]] - a[[i - 1]] * c1[[i - 1]]);
    , {i, 2, Length[d]}];
   x[[Length[b]]] = d1[[Length[b]]];
   Do[
    x[[i]] = d1[[i]] - c1[[i]] * x[[i + 1]];
    , {i, Length[b] - 1, 1, -1}];
   x];
Model[n0_] := Module[{n = n0},
   x0 = 0; xf = 1; h = (xf - x0) / n;
   A = Table[0, {x, 1, n - 1}, {y, 1, n - 1}];
   X = Table[x0 + x * h, {x, 1, n - 1}];
   XT = Table[x0 + x * h, {x, 0, n}];
   B = Table[0, {x, 1, n - 1}];
   eps = 0.1;
   f[x_] = x (1 - x);
   Y = f[XT];
   YT = f[XT];
   While[{
     Y = YT;
     For[i = 1, i < n, i++,
      {
       im = i + 1;
       A[[i, i]] = -2 / h^2 - 2 Y[[im]];
       If[i ≠ 1, A[[i, i - 1]] = 1 / h^2];
       If[i ≠ n - 1, A[[i, i + 1]] = 1 / h^2];
       B[[i]] = 2 - (1 / h^2) (Y[[im - 1]] - 2 Y[[im]] + Y[[im + 1]]) + Y[[im]]^2;
      }];
     YT = Y +
       Flatten[{{0}, Thomas[Diagonal[A, -1], Diagonal[A], Diagonal[A, 1], B], {0}}];
    }; N[Max[Abs[YT - Y]]] > eps];
   YT
  ];
```

In[4]:=
```
in = {3, 6, 9};
sol = Model[in[[1]]];
Print[N[sol]];
solt = NDSolve[{y''[x] == 2 + y[x]^2, y[0] == y[1] == 0}, y[x], x];
err1 = Abs[NDSolveValue[{y''[x] - y[x]^2 - 2 == 0, y[0] == y[1] == 0}, y[XT], x] - sol];
perr1 = ListPlot[Transpose[{XT, err1}], PlotStyle → Red];
Print[N[Max[err1]]];
p1 = Show[{Plot[Evaluate[y[x] /. solt], {x, x0, xf},
      PlotLabel → Style[StringForm["h = ``", h], FontSize → 18]]},
   {ListPlot[Transpose[{XT, sol}], PlotStyle → Red]}];
```

```
{0., -0.227944, -0.227944, 0.}
```

```
0.00060093
```

In[12]:=
```
sol = Model[in[[2]]];
Print[N[sol]];
solt = NDSolve[{y''[x] == 2 + y[x]^2, y[0] == y[1] == 0}, y[x], x];
err2 = Abs[NDSolveValue[{y''[x] - y[x]^2 - 2 == 0, y[0] == y[1] == 0}, y[XT], x] - sol];
perr2 = ListPlot[Transpose[{XT, err2}], PlotStyle → Green];
Print[N[Max[err2]]];
p2 = Show[{Plot[Evaluate[y[x] /. solt], {x, x0, xf},
      PlotLabel → Style[StringForm["h = ``", h], FontSize → 18]]},
   {ListPlot[Transpose[{XT, sol}], PlotStyle → Red]}];
```

```
{0., -0.141774, -0.227437, -0.256119, -0.227437, -0.141774, 0.}
```

```
0.000120299
```

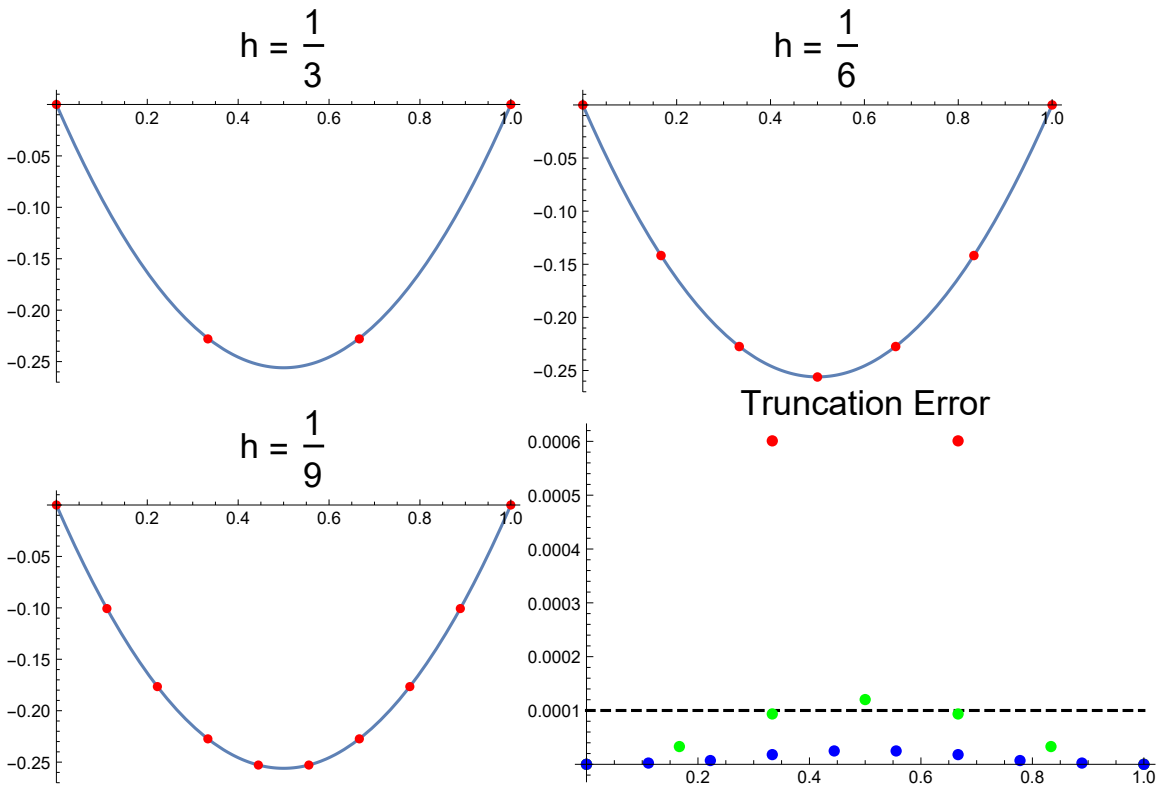
In[19]:=
```
sol = Model[in[[3]]];
Print[N[sol]];
solt = NDSolve[{y''[x] == 2 + y[x]^2, y[0] == y[1] == 0}, y[x], x];
err3 = Abs[NDSolveValue[{y''[x] - y[x]^2 - 2 == 0, y[0] == y[1] == 0}, y[XT], x] - sol];
perr3 = ListPlot[Transpose[{XT, err3}], PlotStyle → Blue];
Print[N[Max[err3]]];
p3 = Show[{Plot[Evaluate[y[x] /. solt], {x, x0, xf},
      PlotLabel → Style[StringForm["h = ``", h], FontSize → 18]]},
   {ListPlot[Transpose[{XT, sol}], PlotStyle → Red]}];
```

```
{0., -0.100689, -0.176562, -0.227362,
 -0.252836, -0.252836, -0.227362, -0.176562, -0.100689, 0.}
```

```
0.0000248485
```

In[26]:=
```
p4 = Show[{Plot[y[x] = 10^-4, {x, 0, 1}, PlotStyle → {Dashed, Black}]}, {perr1},
   {perr2}, {perr3}, PlotLabel → Style["Truncation Error", FontSize → 18],
   PlotRange → {{0, 1}, {0, 0.0006}}];
GraphicsGrid[{{p1, p2}, {p3, p4}}]
```

Out[27]=


h = 1/3
h = 1/6
h = 1/9
Truncation Error
0.2
0.4
0.6
0.8
1.0
−0.05
−0.10
−0.15
−0.20
−0.25
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001