PSA ASSIGNMENT 4: REPORT
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Tasks To be Performed:

1. Implement a height-weighted Quick Union and ensure all unit tests are successful.

2. To Create a Union Find interface that inputs a number n and generates random pairs of integers ranging from 0 to n-1, while keeping track of the number of connections needed to connect all sites.

3. Determine the relationship between the number of objects (n) and the number of pairs (m) generated to accomplish this (i.e. to reduce the number of components from n to 1). Justify your conclusion in terms of your observations and what you think might be going on.

Derived Conclusion: $m = c \times n\ ln(n)$

Reasons:

1. The amount of connections needed to connect all n objects is equal to (n-1), as every connection reduces the number of separate components by one.

2. The chance of choosing two objects that are not connected in each step of the algorithm is given by p = (n - 1)/n^2, which is calculated as 1/n * (n - 1)/n. This is because there are (n - 1) objects that are not connected, and the possibility of selecting each one is 1/n.

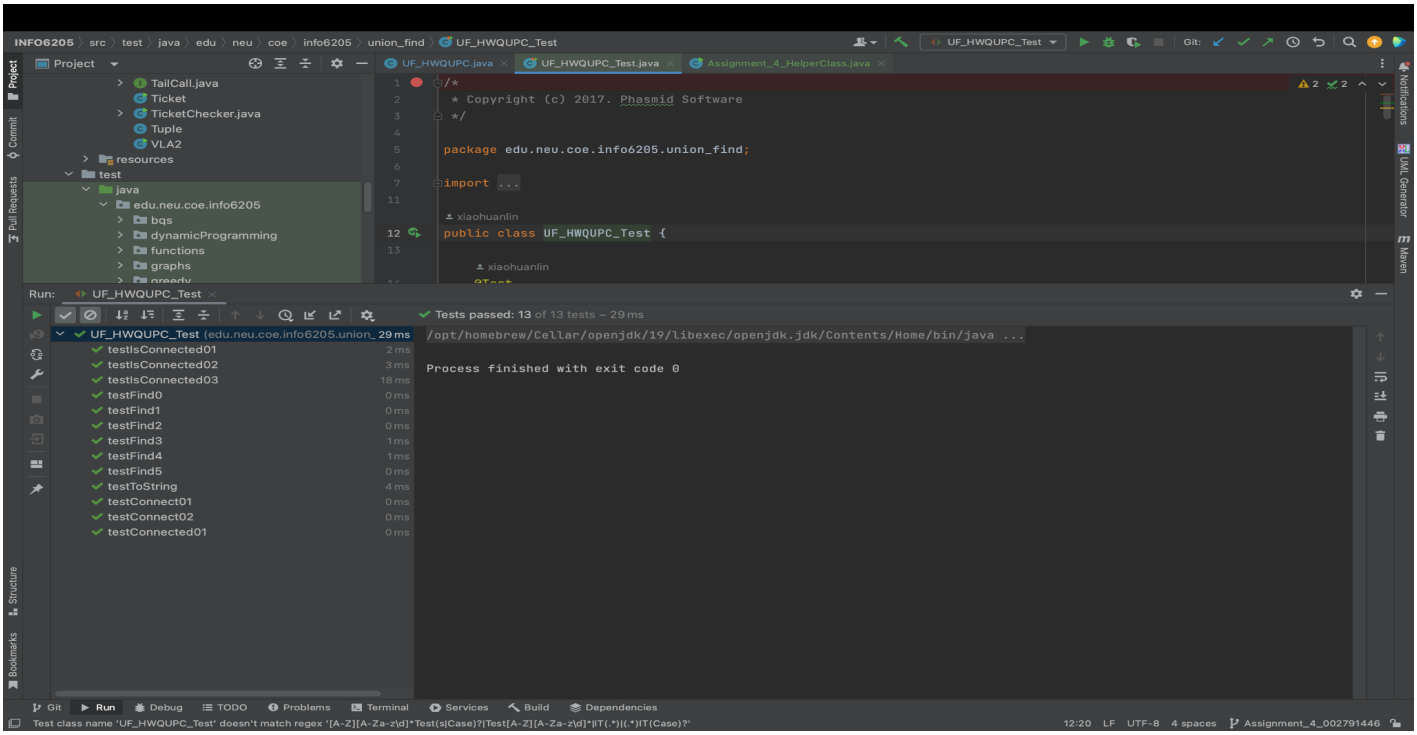
3. In every step of the algorithm, the expected number of connections is represented by m/n = (n – 1)/n^2 = p.

To connect all n objects, the average number of connections necessary is estimated as m = (n–1)/n = n × (n–1)/n^2.

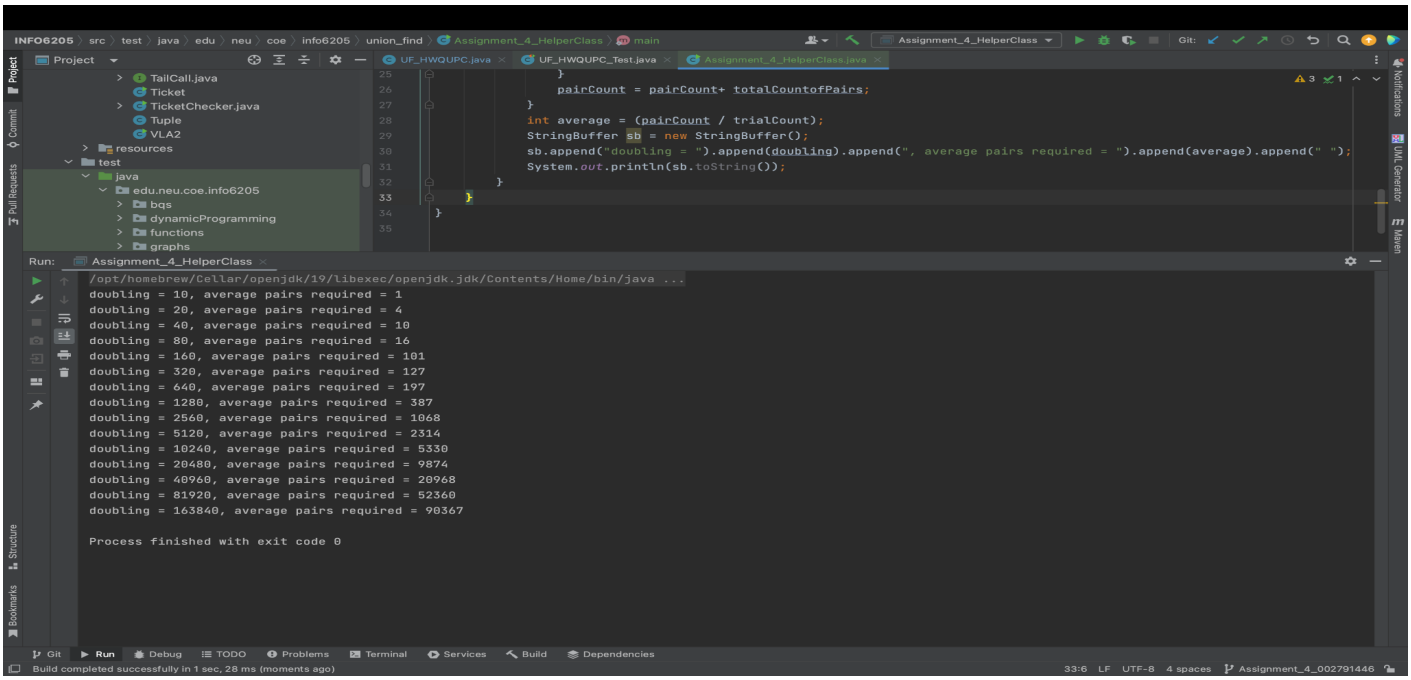
4. By approximating (n - 1)/n as $ln(n)$, we obtain the equation $m = n * ln(n)$.

To account for the constant factors involved in the algorithm, a constant C can be multiplied, which can be experimentally estimated. The final formula then becomes $m = C * n * ln(n)$.

UNIT TEST CASES RESULT:



RANDOM PAIR GENERATION OUTPUT:



GRAPH: