

# AS Project

DSBA

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## Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

	Striker	Forward	Attacking Midfielder	Winger	Total
Players Injured	45	56	24	20	145
Players Not Injured	32	38	11	9	90
Total	77	94	35	29	235

Based on the above data, answer the following questions.

### 1.1 What is the probability that a randomly chosen player would suffer an injury?

**Solution :**

► Total number of Injured players (x)

► Total number of players (y)

► Probability that a randomly chosen player would suffer an injury (p)

$$x = 145$$

$$y = 235$$

$$p = x/y$$

$$p = 145/235$$

$$p = 0.6170$$

► The probability that a randomly chosen player would suffer an injury is 0.6170 i.e 61.70%

### 1.2 What is the probability that a player is a forward or a winger?

**Solution :**

- ▶ Total number of Forward players (x)
- ▶ Total number of Winger (y)
- ▶ Total number of players (z)
- ▶ Probability that a player is a forward or a winger (p)

$$x = 94$$

$$y = 29$$

$$z = 235$$

$$p = (x+y)/z$$

$$p = (94+29)/235$$

$$p = 0.5234$$

- ▶ The probability that a player is a forward or a winger is 0.5234 i.e 52.34%

### **1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?**

**Solution :**

- ▶ Total number of Injured Strikers (x)
- ▶ Total number of players (y)
- ▶ Probability that a randomly chosen player plays in a striker position and has a foot injury (p)

$$x = 45$$

$$y = 235$$

$$p = x/y$$

$$p = 45/235$$

$$p = 0.1914$$

- ▶ The probability that a randomly chosen player in a striker position and has a foot injury is 0.3103 i.e 31.03%

### 1.4 What is the probability that a randomly chosen injured player is a striker?

**Solution :**

- ▶ Total number of Injured Strikers (x)
- ▶ Total number of Injured players (y)
- ▶ Probability that a randomly chosen injured player is a striker (p)

$$x = 45$$

$$y = 145$$

$$p = x/y$$

$$p = 45/145$$

- ▶ The probability that a randomly chosen injured player is a striker is 0.3103 i.e 31.03%



## Problem 2

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter.

The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain;

Answer the questions below based on the given information; (Provide an appropriate visual representation of your answers, without which marks will be deducted)

### 2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm?

**Solution :**

►observed value (x)

►mean of the sample ( $\mu$ )

►standard deviation of the sample ( $\sigma$ )

►standard score (Z)

$$x = 3.17$$

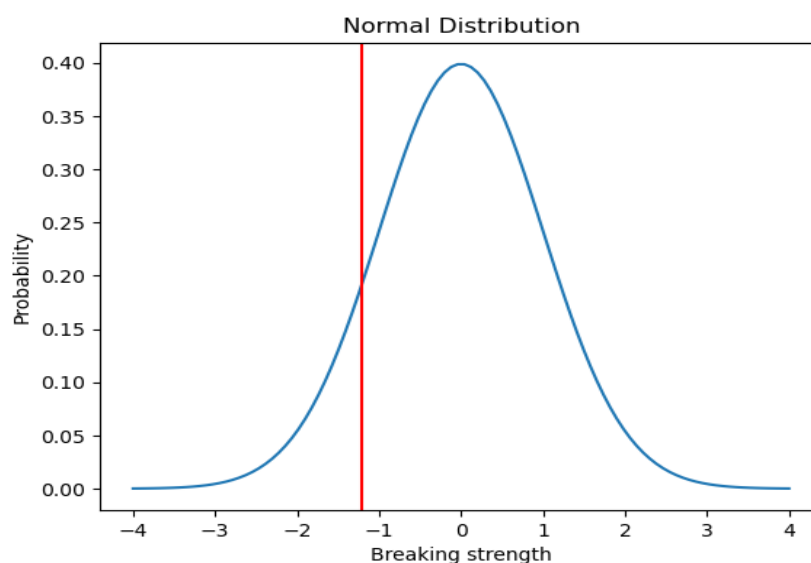
$$\mu = 5$$

$$\sigma = 1.5$$

$$Z = (x - \mu)/\sigma$$

$$Z = (3.17-5)/1.5$$

$$Z = -1.22$$



**Figure 1 : Proportion of gunny bags (strength < 3.17 kg per sq. cm)**

► We find the proportion  $P(Z < -1.22) \approx 0.1112$ . So, approximately 11.12% of the bags have a breaking strength less than 3.17 kg per sq cm.

## 2.2 What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq cm.?

**Solution :**

- observed value ( $x$ )
- mean of the sample ( $\mu$ )
- standard deviation of the sample ( $\sigma$ )
- standard score ( $Z$ )

$$x = 3.6$$

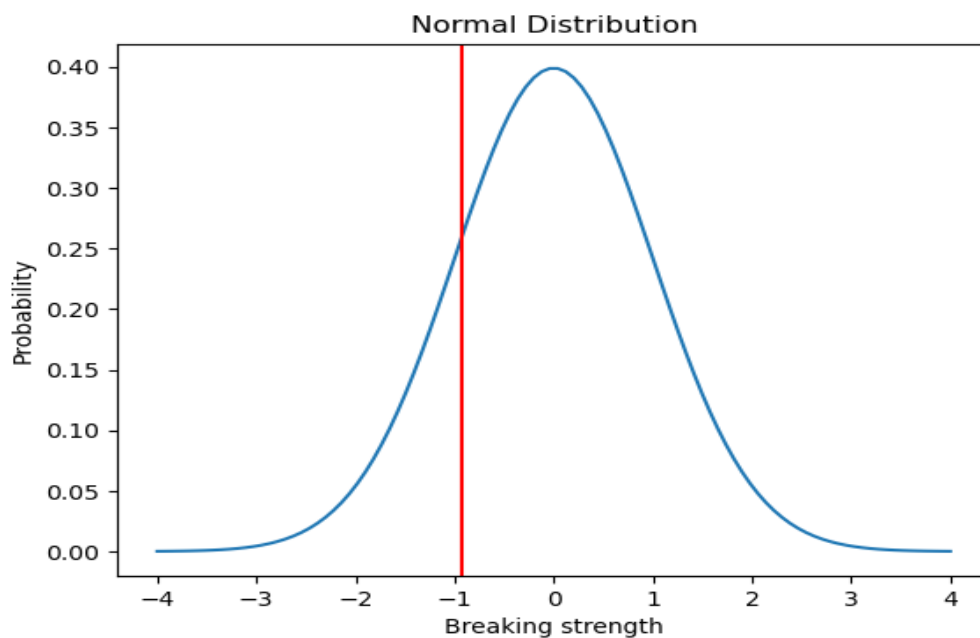
$$\mu = 5$$

$$\sigma = 1.5$$

$$Z = (x - \mu)/\sigma$$

$$Z = (3.6 - 5)/1.5$$

$$Z = -0.93$$



**Figure 2 : Proportion of gunny bags (strength  $\geq 3.6$  kg per sq. cm)**

► We find the proportion  $P(Z > -0.93) \approx 0.8245$ . So, approximately 82.45% of the bags have a breaking strength of at least 3.6 kg per sq cm.

### 2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

**Solution :**

► observed value ( $x_1$ )

► mean of the sample ( $\mu_1$ )

► standard deviation of the sample ( $\sigma_1$ )

► standard score ( $Z_1$ )

$$x_1 = 5$$

$$\mu_1 = 5$$

$$\sigma_1 = 1.5$$

$$Z_1 = (x_1 - \mu_1) / \sigma_1$$

$$Z_1 = (5 - 5) / 1.5$$

$$Z_1 = 0.0$$

Now,

► observed value ( $x_2$ )

► mean of the sample ( $\mu_2$ )

► standard deviation of the sample ( $\sigma_2$ )

► standard score ( $Z_2$ )

$$x_2 = 5.5$$

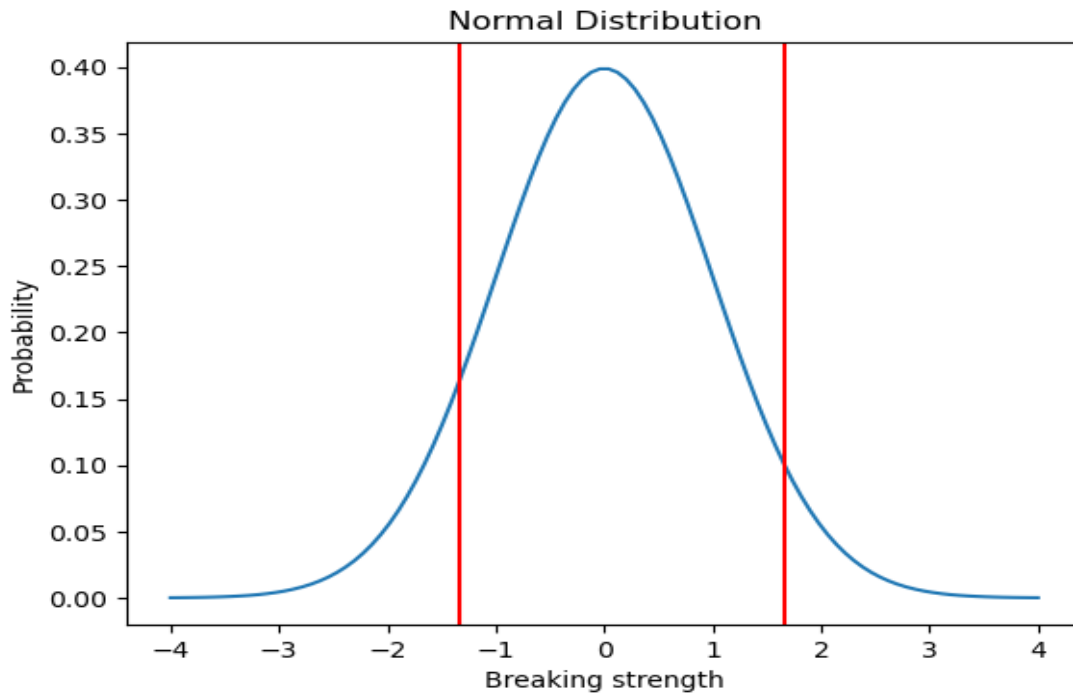
$$\mu_2 = 5$$

$$\sigma_2 = 1.5$$

$$Z_2 = (x - \mu) / \sigma$$

$$Z_2 = (5.5 - 5) / 1.5$$

$$Z_2 = 0.33$$



**Figure 3 : Proportion of gunny bags (strength between 5 and 5.5 kg per sq. cm)**

► We find the proportion  $P(0 < z < 0.3333) = P(Z < 0.3333) - P(Z < 0) \approx 0.1305$ . So, approximately 13.05% of the bags have a breaking strength between 5 and 5.5 kg per sq.cm.

## **2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?**

**Solution :**

- observed value ( $x_1$ )
- mean of the sample ( $\mu_1$ )
- standard deviation of the sample ( $\sigma_1$ )
- standard score ( $Z_1$ )

$$x_1 = 3$$

$$\mu_1 = 5$$

$$\sigma_1 = 1.5$$

$$Z_1 = (x_1 - \mu_1) / \sigma_1$$

$$Z_1 = (3 - 5) / 1.5$$

$$Z_1 = -1.33$$

Now,

► observed value ( $x_2$ )

► mean of the sample ( $\mu_2$ )

► standard deviation of the sample ( $\sigma_2$ )

► standard score ( $Z_2$ )

$$x_2 = 7.5$$

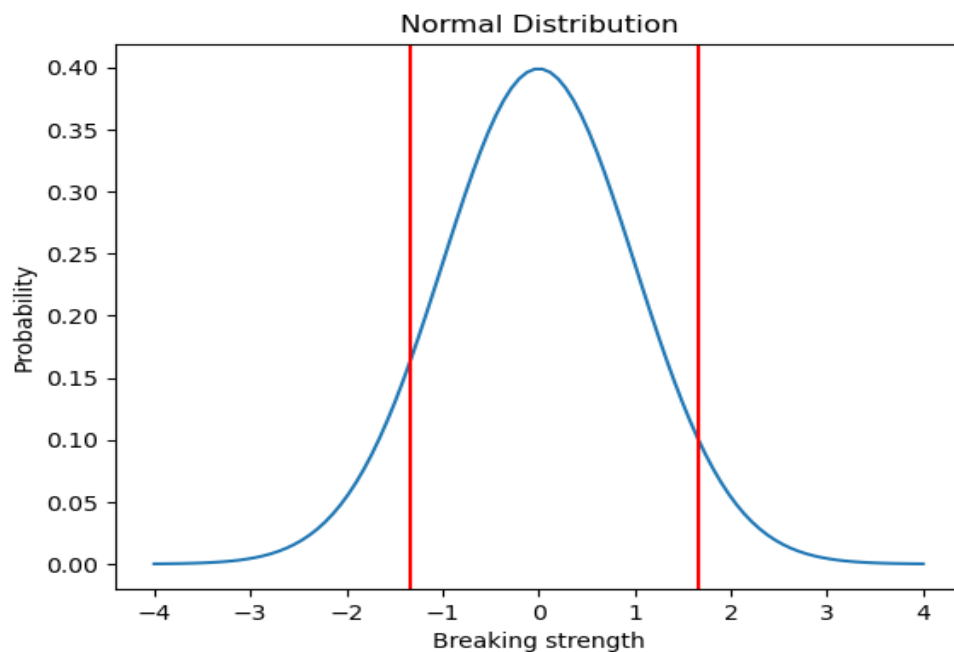
$$\mu_2 = 5$$

$$\sigma_2 = 1.5$$

$$Z_2 = (x - \mu)/\sigma$$

$$Z_2 = (7.5 - 5)/1.5$$

$$Z_2 = 1.66$$



**Figure 4 : Proportion of gunny bags (strength not between 3 and 7.5 kg per sq. cm)**

$$\text{► } Z_1 = (3 - 5)/1.5 = -1.33, Z_2 = (7.5 - 5)/1.5 = 1.66$$

$$\text{cdf}(Z_2) - \text{cdf}(Z_1) = 0.8609$$

► Thus we conclude that the proportion of gunny bags having strength not between 3 and 7.5 kg per sq. cm is  $(1 - 0.8609 \approx 0.1390)$  i.e 13.90%

## Problem 3

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients.

Use the data provided to answer the following (assuming a 5% significance level);

**3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?**

**"- State the null and alternate hypotheses - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results Note: Consider the level of significance as 5%."**

**Solution :**

In this hypothesis test, Zingaro wants to determine whether the unpolished stones have a Brinell's hardness index (mean) that is less than 150. Here are the null and alternative hypotheses:

► Null Hypothesis ( $H_0$ ) :  $\mu_1 \geq 150$  (The mean hardness index of unpolished stones ( $\mu_1$ ) is greater than or equal to 150)

► Alternative Hypothesis ( $H_A$ ) :  $\mu_1 < 150$  (The mean hardness index of unpolished stones ( $\mu_1$ ) is less than 150)

► We have,

Sample size ( $n$ ) = 75

Sample mean ( $\bar{x}$ ) = 134.11

Sample standard deviation ( $s$ ) = 33.04

We will conduct a one-sample t-test to test these hypothesis.

► Now,

$$t = (\bar{x} - \mu_1) \cdot \sqrt{n/s}$$

where:

$\bar{x}$  is the sample mean.

$\mu_1$  is the hypothesized population mean under the null hypothesis.

$s$  is the sample standard deviation.

$$t = (134.11 - 150) \cdot \sqrt{75/33.04}$$

► Now, calculate the t-statistic:

$$t \approx -15.89/3.82$$

$$t \approx -4.164629601426757$$

$$t = -4.16$$

► Since the alternative hypothesis is that the average hardness index is less than 150, we will go for the left-tailed p-value.

► By using the scipy library, we have already calculated the p value associated with t-statistic.

$$\text{P value} = 4.171286997419652e-05$$

The p-value is approximately 0.00004 (rounded to five decimal places).

► With a significance level of 5%, the critical value ( $t_{\text{critical}}$ ) for a one-tailed test is approximately -1.66

► Since the t-statistic (-4.16) is less than the critical value (-1.66) and the p-value (0.00004) is much smaller than the significance level (0.05), we have enough evidence to reject the null hypothesis.

► Based on the hypothesis test results, Zingaro has strong evidence to believe that the unpolished stones are not suitable for printing because the average Brinell's hardness index of the unpolished stones (134.11) is significantly lower than the required level of 150, at a 5% significance level.

### 3.2 Is the mean hardness of the polished and unpolished stones the same?

**- State the null and alternate hypotheses. - Conduct the hypothesis test. - Write down conclusions from the test results. Note: Consider the level of significance as 5%.**

#### **Solution :**

► Null Hypothesis ( $H_0$ ) :  $\mu(\text{polished}) = \mu(\text{unpolished})$

► Alternative Hypothesis ( $H_A$ ) :  $\mu(\text{polished}) \neq \mu(\text{unpolished})$

► Here, we will perform 2 sample 2 tailed test

► By using scipy library we have calculated the t-stat and p value,

t-stat  $\approx$  3.2422320501414053

t-stat = 3.24

p value  $\approx$  0.001465515019462831

p value = 0.0014

► At 5% significance level p value (0.001465515019462831) < 0.05

So, we reject the null hypothesis.

► We do have statistical evidence to say that the mean hardness of the polished stones is significantly different from the mean hardness of unpolished stones



## Problem 4

Dental implant data: The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists who may favor one method above another and may work better in his/her favorite method. The response is the variable of interest.

### 4.1 How does the hardness of implants vary depending on dentists?

**"- State the null and alternate hypotheses - Check the assumptions of the hypothesis test. - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results - In case the implant hardness differs, identify for which pairs it differs Note: 1. Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys. 2. Even if the assumptions of the test fail, kindly proceed with the test."**

**Soluion :**

#### **Alloy 1]**

- ▶ We are considering the case when dentists are using Alloy=1
- ▶ The null hypothesis states that the mean implant hardness is equal among all the Dentists.
- ▶ the mean implant hardness for each group(dentists) is equal
- ▶  $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$  (the mean implant hardness for each group(dentists) is equal)
- ▶  $H_A$  : Not all the means are equal (At least one group mean is different from the rest)
- ▶  $\alpha = 0.05$
- ▶ We will perform ANOVA to test the hypothesis

We get-

**Table 1 : "ANOVA Table for Alloy 1 Based on Dentist Factor"**

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	1.977112	0.116567
Residual	40.0	539593.555556	13489.838889	NaN	NaN

- ▶ As p-value (0.116) >
- ▶  $\alpha(0.05)$  We failed to reject the null hypothesis
- ▶ This means we don't have sufficient evidence to say that there is a statistically significant

difference between the mean implant hardness of the Five groups(Dentists).

### Alloy 2]

- ▶ We are considering the case when dentists are using Alloy=2
- ▶ The null hypothesis states that the mean implant hardness is equal among all the Dentists.
- ▶  $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$  (the mean implant hardness for each group(dentists) is equal)
- ▶  $H_A$  : Not all the means are equal (At least one group mean is different from the rest)
- ▶ We will perform ANOVA to test the hypothesis

We get-

**Table 2 : "ANOVA Table for Alloy 2 Based on Dentist Factor"**

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	5.679791e+04	14199.477778	0.524835	0.718031
Residual	40.0	1.082205e+06	27055.122222	NaN	NaN

- ▶ As p-value (0.718) > alpha(0.05)
- ▶ We failed to reject the null hypothesis
- ▶ This means we don't have sufficient evidence to say that there is a statistically significant difference between the mean implant hardness of the Five groups(Dentists).

- ▶  $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$  (the mean implant hardness for each group(dentists) is equal)
- ▶  $H_A$  : Not all the means are equal (At least one group mean is different from the rest)
- ▶ Alpha= 0.05

- ▶ Alloy 1 - As p-value (0.116) > alpha(0.05)
- ▶ Alloy 2 - As p-value (0.718) > alpha(0.05)
- ▶ We failed to reject the null hypothesis for both the alloys

- ▶ This means we don't have sufficient evidence to say that there is a statistically significant difference between the mean implant hardness of the Five groups(Dentists) for both alloys (1 and 2).

► Assumptions-

1. The sample drawn from different populations are independent and random.
2. The response variable of all the populations are continuous and ideally normally distributed.
3. The variance of all the populations are equal atleast approximately.

Assumptions are not full filled

### Alloy 1]

► Anderson-Darling Normality Test gives following results

**Table 3 : Anderson darling normality test for alloy 1**

Statistic: 2.561066309273201  
Critical Values: [0.535 0.609 0.731 0.853 1.014]  
Significance Levels: [15. 10. 5. 2.5 1. ]

The test statistic is 2.56 We can compare this value to each critical value that corresponds to each significance level to see if the test results are significant.

The critical value for  $\alpha = 0.025$  is 0.853. Because the test statistic (2.56) is greater than this critical value, the results are significant at a significance level of 0.025.

Same is the case with all the other values-

We can see that the test results are significant at every significance level, which means we would reject the null hypothesis of the test no matter which significance level we choose to use. Thus, we have sufficient evidence to say that the sample data is not normally distributed.

### Alloy 2]

Anderson-Darling Normality Test gives following results

**Table 4 : Anderson darling normality test for alloy 2**

Statistic: 2.561066309273201  
Critical Values: [0.535 0.609 0.731 0.853 1.014]  
Significance Levels: [15. 10. 5. 2.5 1. ]

The test statistic is 1.89 We can compare this value to each critical value that corresponds to each significance level to see if the test results are significant.

The critical value for  $\alpha = 0.025$  is 0.853. Because the test statistic (1.89) is greater than this critical value, the results are significant at a significance level of 0.025.

Same is the case with all the other values-

We can see that the test results are significant at every significance level, which means we would reject the null hypothesis of the test no matter which significance level we choose to use. Thus, we have sufficient evidence to say that the sample data is not normally distributed.

► Both the distributions are not normally distributed , which violates the assumptions.

► Considerable presence of outliers could also be seen for both alloys, and ANOVA is sensitive to Outliers.

## 4.2 How does the hardness of implants vary depending on methods?

**"- State the null and alternate hypotheses - Check the assumptions of the hypothesis test. - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results - In case the implant hardness differs, identify for which pairs it differs Note: 1. Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys. 2. Even if the assumptions of the test fail, kindly proceed with the test."**

**Solution :**

### **Alloy 1]**

- ▶ We are considering the case when dentists are using Alloy=1
- ▶  $H_0 : \mu_1 = \mu_2 = \mu_3$  (the mean implant hardness for each group(Method) is equal)
- ▶  $H_A$  : Not all the means are equal (At least one group mean is different from the rest)
- ▶ Alpha= 0.05

We will perform ANOVA to test the hypothesis-

We get-

**Table 5 : "ANOVA Table for Alloy 1 Based on Method Factor"**

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	148472.177778	74236.088889	6.263327	0.004163
Residual	42.0	497805.066667	11852.501587	NaN	NaN

- ▶ As p-value (0.004163) < alpha(0.05)

We reject the null hypothesis.

This means we have sufficient evidence to say that there is a statistically significant difference between the mean implant hardness of the three groups(Methods)

### **Alloy=2]**

- ▶ We are considering the case when dentists are using Alloy=2
- ▶  $H_0 : \mu_1 = \mu_2 = \mu_3$  (the mean implant hardness for each group(Method) is equal)
- ▶  $H_A$  : Not all the means are equal (At least one group mean is different from the rest)

► Alpha= 0.05

We will perform ANOVA to test the hypothesis-

We get-

**Table 6 : "ANOVA Table for Alloy 2 Based on Method Factor"**

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	499640.4	249820.200000	16.4108	0.000005
Residual	42.0	639362.4	15222.914286	NaN	NaN

► As p-value (0.000005) < alpha(0.05)

We reject the null hypothesis.

This means we have sufficient evidence to say that there is a statistically significant difference between the mean implant hardness of the three groups(Methods)

Since the p value is less than the significance level(0.05), we can reject the null hypothesis and conclude that there is a difference in the mean implant hardness. mean implant hardness is different for at-least one category of methods for both the alloys.

As we have rejected the null hypothesis for both the alloys , lets check for which methods the mean implant hardness is significantly different.

► We can use Tukey's multiple comparison test for this-

**Alloy 1]**

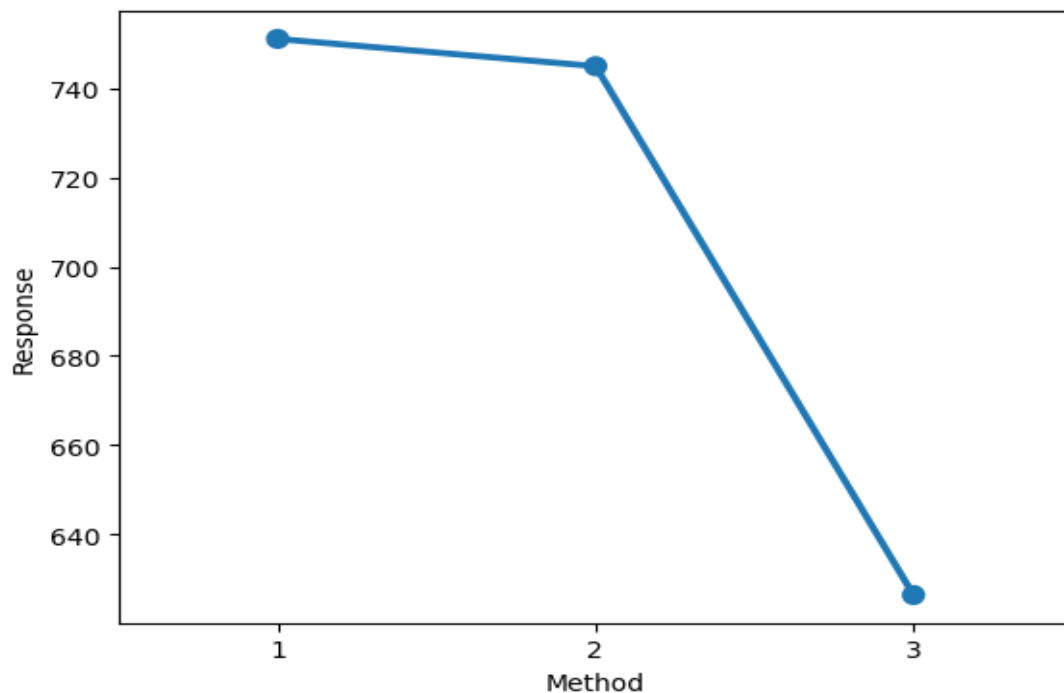
**Table 7 : Tukey' multiple comparison test for alloy 1**

Multiple Comparison of Means - Tukey HSD, FWER=0.05						
group1	group2	meandiff	p-adj	lower	upper	reject
1	2	-6.1333	0.987	-102.714	90.4473	False
1	3	-124.8	0.0085	-221.3807	-28.2193	True
2	3	-118.6667	0.0128	-215.2473	-22.086	True

- Mean difference for method 3 is quite high when compared with both methods 1 & 2.

Tukey's HSD Test for multiple comparisons found that the mean implant hardness was significantly different between Method 1 and Method 3 it is different for Method 2 and Method 3 as well.

- It can also be seen from below plot-



**Figure 5 : Tukey' multiple comparison test for alloy 1**

## Alloy 2]

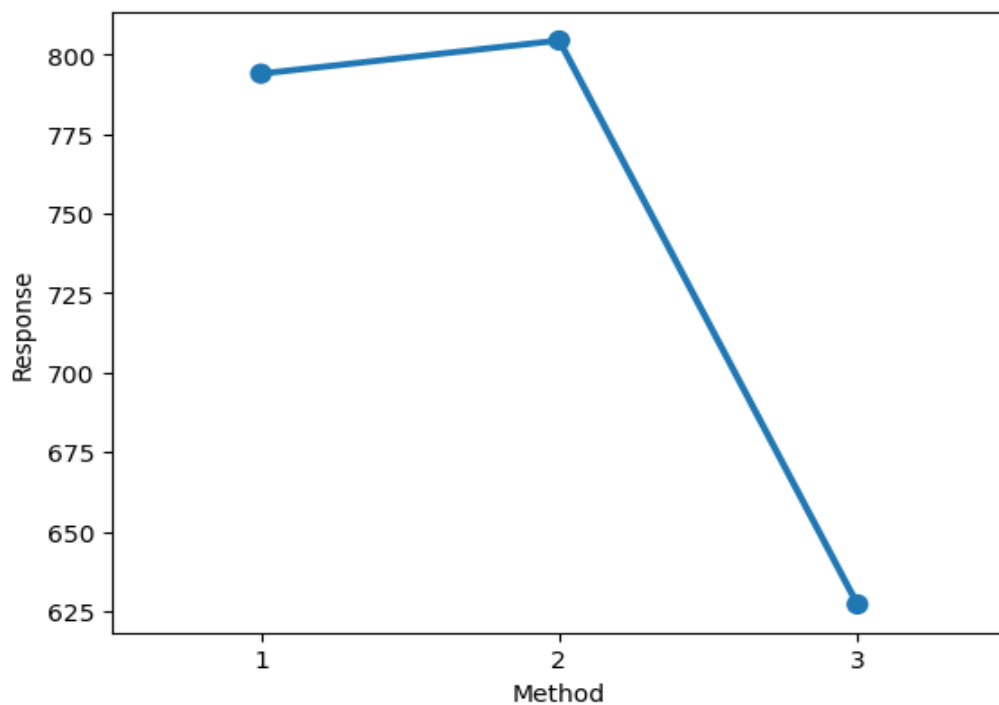
**Table 8 : Tukey' multiple comparison test for alloy 2**

Multiple Comparison of Means - Tukey HSD, FWER=0.05						
group1	group2	meandiff	p-adj	lower	upper	reject
1	2	27.0	0.8212	-82.4546	136.4546	False
1	3	-208.8	0.0001	-318.2546	-99.3454	True
2	3	-235.8	0.0	-345.2546	-126.3454	True

- Mean difference for method 3 is quite high when compared with both methods 1 & 2.

Tukey's HSD Test for multiple comparisons found that the mean implant hardness was significantly different between Method 1 and Method 3 it is different for Method 2 and Method 3 as well.

- It can also be seen from below plot-



***Figure 6 : Tukey' multiple comparison test for alloy 2***



### 4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?

**"- Create Interaction Plot - Inferences from the plot Note: Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys."**

**Solution :**

**Alloy 1]**

Lets, state the hypothesis

- ▶ H<sub>0</sub>- Interection between Dentist and Method does not exist.
- ▶ H<sub>A</sub>- Interection between Dentist and Method exists.
- ▶ Alpha=0.05

We get-

**Table 9 : Anova test for interaction between dentist and Method (for alloy 1)**

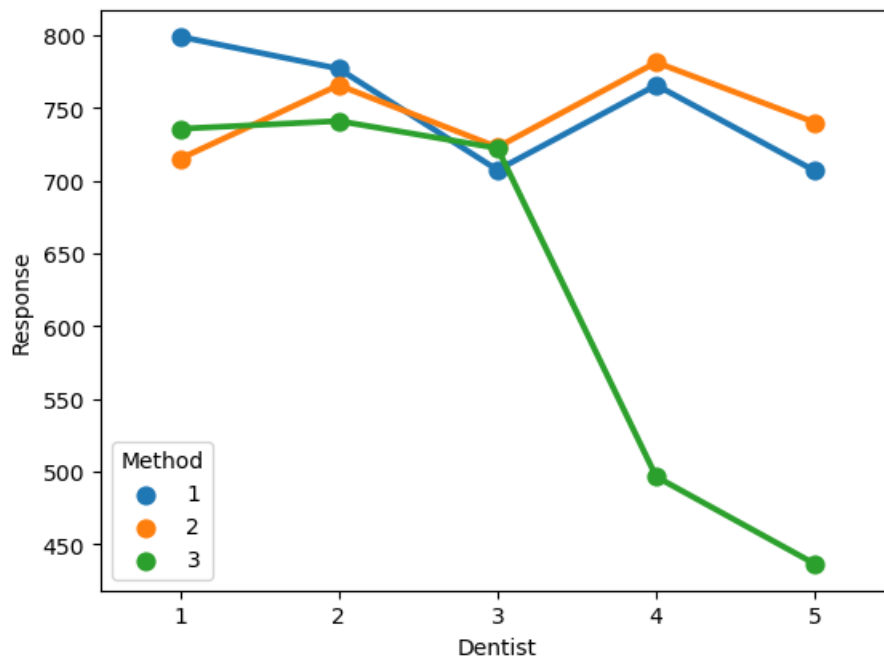
	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	148472.177778	74236.088889	10.854287	0.000284
C(Dentist)	4.0	106683.688889	26670.922222	3.899638	0.011484
C(Method):C(Dentist)	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

- ▶ As p-value (0.0067) < alpha(0.05)

We reject the null hypothesis.

There is statistically significant evedence to say that there is interection between Dentist and Method feature.

We can also see that once we are taking interection of method and dentist features intoconsideration, this also changes the way denstist column can impact mean implant hardness.



**Figure 7 : Anova test for interaction between dentist and Method(for alloy 1)**

► It could be noticed that there is a good interection between dentist and method feature.

## Alloy 2]

Lets, state the hypothesis

► H0- Interction between Dentist and Method does not exist.

► HA- Interction between Dentist and Method exists.

► Alpha=0.05

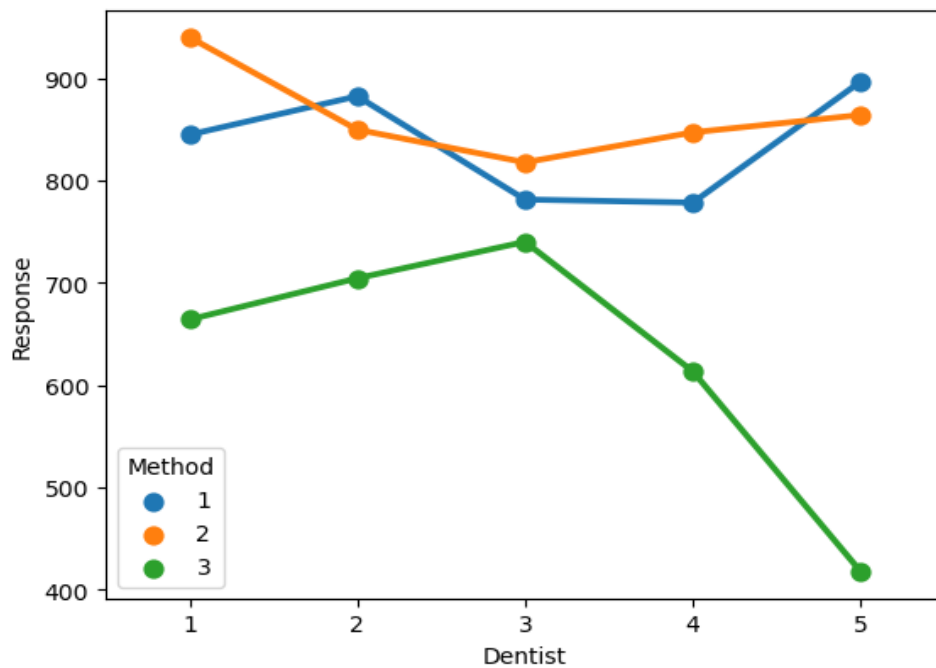
We get-

**Table 10 : Anova test for interaction between dentist and Method (for alloy 2)**

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	499640.400000	249820.200000	19.461218	0.000004
C(Dentist)	4.0	56797.911111	14199.477778	1.106152	0.371833
C(Method):C(Dentist)	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

► As p-value (0.09) > alpha(0.05)

We failed to reject the null hypothesis. There is no significant interaction between Dentist and Method considering only alloy 2.



**Figure 8 : Anova test for interaction between dentist and Method (for alloy 2)**

► There is no as such significant interaction between method and dentist feature for alloy 2.

#### 4.4 How does the hardness of implants vary depending on dentists and methods together?

**"- State the null and alternate hypotheses - Check the assumptions of the hypothesis test. - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results - Identify which dentists and methods combinations are different, and which interaction levels are different. Note: 1. Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys. 2. Even if the assumptions of the test fail, kindly proceed with the test."**

**Solution :**

- ▶ We are considering the case when dentists are using Alloy=1
- ▶ We have to consider 2 hypothesis one for method and one for dentist.
- ▶  $H_0 : \mu_1 = \mu_2 = \mu_3$  (the mean implant hardness for each group(Method) is equal)
- ▶  $H_A$  : Not all the means are equal (At least one group mean is different from the rest)
- ▶  $\alpha = 0.05$
  
- ▶  $H_0 : \mu_1 = \mu_2 = \mu_3$  (the mean implant hardness for each group(Dentist) is equal)
- ▶  $H_A$  : Not all the means are equal (At least one group mean is different from the rest)
- ▶  $\alpha = 0.05$

**Table 11 : Anova test for effect of Dentist and Method separately on alloy 1**

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	148472.177778	74236.088889	7.212522	0.002211
C(Dentist)	4.0	106683.688889	26670.922222	2.591255	0.051875
Residual	38.0	391121.377778	10292.667836	NaN	NaN

- ▶ As p-value (0.002) <  $\alpha(0.05)$  for method feature
- ▶ We reject the null hypothesis

This means we have sufficient evidence to say that there is a statistically significant difference between the mean implant hardness of the three groups(Methods) for alloy 1.

- ▶ As p-value (0.051) >  $\alpha(0.05)$  for dentist feature

- ▶ We fail to reject the null hypothesis
- ▶ This means we don't have sufficient evidence to say that there is a statistically significant difference between the mean implant hardness of the five groups(Dentist) for alloy 1.

### Alloy 2]

- ▶ We are considering the case when dentists are using Alloy=2
  - ▶ We have to consider 2 hypothesis one for method and one for dentist.
  - ▶  $H_0 : \mu_1 = \mu_2 = \mu_3$  (the mean implant hardness for each group(Method) is equal)
  - ▶  $H_A$  : Not all the means are equal (At least one group mean is different from the rest)
  - ▶ Alpha= 0.05
- 
- ▶  $H_0 : \mu_1 = \mu_2 = \mu_3$  (the mean implant hardness for each group(Dentist) is equal)
  - ▶  $H_A$  : Not all the means are equal (At least one group mean is different from the rest)
  - ▶ Alpha= 0.05

**Table 12 : Anova test for effect of Dentist and Method separately on alloy 2**

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	499640.400000	249820.200000	16.295479	0.000008
C(Dentist)	4.0	56797.911111	14199.477778	0.926215	0.458933
Residual	38.0	582564.488889	15330.644444	NaN	NaN

- ▶ As p-value (0.000008) < alpha(0.05)

▶

We reject the null hypothesis

This means we have sufficient evidence to say that there is a statistically significant difference between the mean implant hardness of the three groups(Methods) for alloy 2.

- ▶ As p-value (0.45) > alpha(0.05)
- ▶ We fail to reject the null hypothesis

This means we don't have sufficient evidence to say that there is a statistically significant difference between the mean implant hardness of the five groups(Dentist) for alloy 2.

Its not possible to identify which dentists are different as mean implant hardness is equal among all the Dentists for both the alloys.

Its possible to identify which methods are different.

► We can use Tukey's multiple comparison test for this-

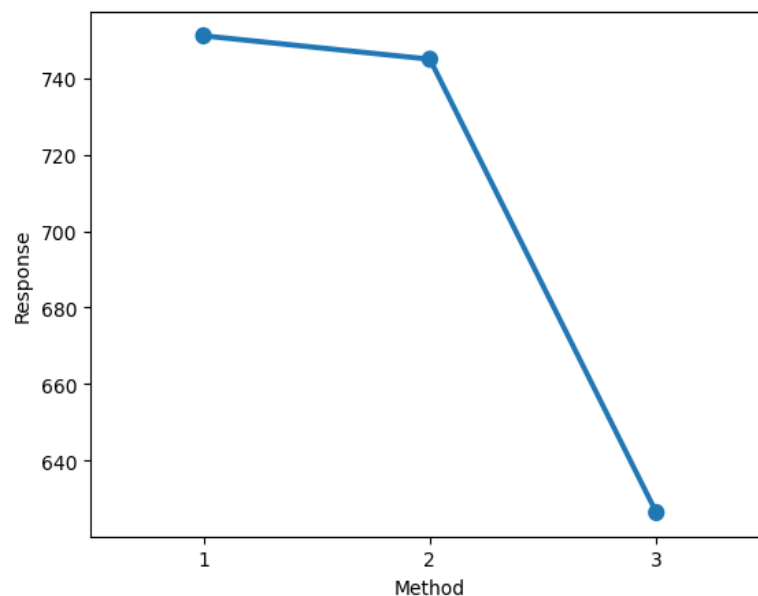
Alloy 1]

**Table 13 : Tukey's multiple comparison test to identify which methods are different(Alloy1)**

Multiple Comparison of Means - Tukey HSD, FWER=0.05						
group1	group2	meandiff	p-adj	lower	upper	reject
1	2	-6.1333	0.987	-102.714	90.4473	False
1	3	-124.8	0.0085	-221.3807	-28.2193	True
2	3	-118.6667	0.0128	-215.2473	-22.086	True

► Mean difference for method 3 is quite high when compared with both methods 1 & 2.

► It can also be seen from below plot-



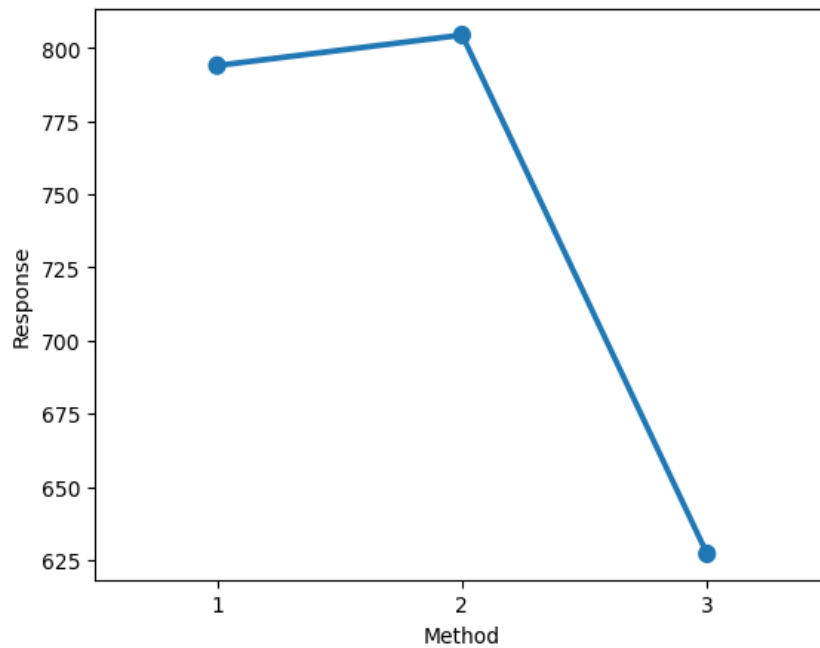
**Figure 9 : Tukey's multiple comparison test to identify which methods are different(Alloy1)**

Alloy 2]

**Table 14 : Tukey's multiple comparison test to identify which methods are different(Alloy2)**

Multiple Comparison of Means - Tukey HSD, FWER=0.05						
group1	group2	meandiff	p-adj	lower	upper	reject
1	2	27.0	0.8212	-82.4546	136.4546	False
1	3	-208.8	0.0001	-318.2546	-99.3454	True
2	3	-235.8	0.0	-345.2546	-126.3454	True

- Mean difference for method 3 is quite high when compared with both methods 1 & 2.
- It can also be seen from below plot-



**Figure 10 : Tukey's multiple comparison test to identify which methods are different(Alloy2)**