In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats
from scipy.optimize import minimize
```

Problem 1

In [2]:

```
S0 = 50
r = 0.02
T = 0.5
sigma = 0.25
K = 55
M = 100

U1 = np.random.uniform(0, 1, 10)*0.4
U2 = np.random.uniform(0, 1, 30)*0.2 + 0.4
U3 = np.random.uniform(0, 1, 30)*0.2 + 0.6
U4 = np.random.uniform(0, 1, 30)*0.2 + 0.8

S = np.zeros(M)
payoff = np.zeros(M)

for i in range((int)((3*M)/10)):
    S[(int)((i/3)*(1-i%3))] = S0*np.exp((r-sigma**2/2)*T + sigma*np.sqrt(T)*stats.norm.ppf(U1[(int)
((i/3)*(1-i%3))]))
    payoff[(int)((i/3)*(1-i%3))] = np.exp(-r*T)*max(S[(int)((i/3)*(1-i%3))]-K, 0)

    S[i+10] = S0*np.exp((r-sigma**2/2)*T + sigma*np.sqrt(T)*stats.norm.ppf(U2[i]))
    payoff[i+10] = np.exp(-r*T)*max(S[i+10]-K, 0)

    S[i+40] = S0*np.exp((r-sigma**2/2)*T + sigma*np.sqrt(T)*stats.norm.ppf(U3[i]))
    payoff[i+40] = np.exp(-r*T)*max(S[i+40]-K, 0)

    S[i+70] = S0*np.exp((r-sigma**2/2)*T + sigma*np.sqrt(T)*stats.norm.ppf(U4[i]))
    payoff[i+70] = np.exp(-r*T)*max(S[i+70]-K, 0)

mu1 = np.average(payoff[0:9])
mu2 = np.average(payoff[10:39])
mu3 = np.average(payoff[40:69])
mu4 = np.average(payoff[70:99])

sig1 = np.std(payoff[0:9])
sig2 = np.std(payoff[10:39])
sig3 = np.std(payoff[40:69])
sig4 = np.std(payoff[70:99])

price = 0.4*mu1+0.2*(mu2+mu3+mu4)
std_err = np.sqrt((0.4*sig1**2+0.2*(sig2**2+sig3**2+sig4**2))/M)

print("The estimated value of the price of the option is: " + str(price))
print("The estimated standard error of the price of the option is: " + str(std_err))
```

```
The estimated value of the price of the option is: 1.827972901022458
The estimated standard error of the price of the option is: 0.30889960486223494
```

Problem 2

In [3]:

```
def sim_claims(M):
    cost = np.zeros(M)
    loss = np.zeros(M)
    zero_count = 0
    count1000 = 0
```

```
    for i in range(M):
        N = np.random.binomial(12, 0.4)
        prob = np.random.random(N)

        claims = [np.random.gamma(5, 100)*(prob[i]<0.25)+np.random.gamma(4, 50)*(prob[i]>=0.25) for
i in range(N)]
        loss[i] = np.sum(claims)

        if(loss[i]<1000):
            cost[i] = 0
        elif(1000<=loss[i]<2500):
            cost[i] = 0.5*(loss[i]-1000)
        else:
            cost[i] = loss[i]-1750

        if(cost[i] == 0):
            zero_count += 1
        elif(cost[i] >= 1000):
            count1000 += 1

    prob0 = zero_count/M
    prob1000 = count1000/M
    expectation = np.sum(cost)/M
    return [prob0, expectation, prob1000]

M = 10000
results = sim_claims(M)
print("For " + str(M) + " simulations, the probability that the insurance company pays nothing is:
" + str(results[0])
        + ", the expected cost to the insurance company is: " + str(results[1]) + ", and the
probability that the "
     "insurer will have to pay more than $1000 is: " + str(results[2]))
```

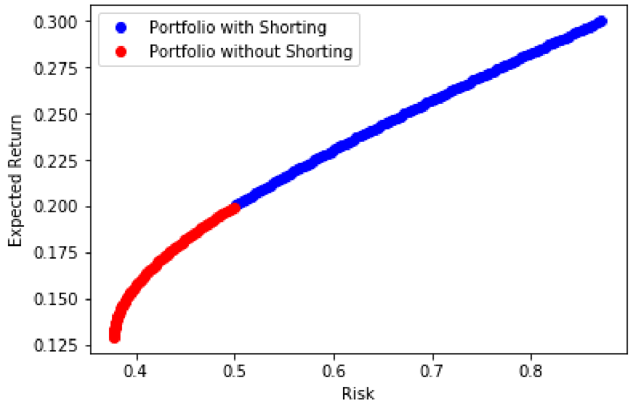
```
For 10000 simulations, the probability that the insurance company pays nothing is: 0.3305, the exp
ected cost to the insurance company is: 222.6892978105333, and the probability that the insurer wi
ll have to pay more than $1000 is: 0.0226
```

Problem 3

In [52]:

```
# Part 1
mu = [0.1, 0.2]
C = np.array([[0.16, 0.1], [0.1, 0.25]])

w1 = np.linspace(-1, 2, 300)
w2 = 1-w1
w = np.transpose(np.array([w1, w2]))
muV = []
volatilities = []
short_indices = []
long_indices = []

for i in range(np.size(w, axis=0)):
    muV.append(np.linalg.multi_dot([w[i], np.transpose(mu)]))
    volatilities.append(np.sqrt(np.linalg.multi_dot([w[i], C, np.transpose(w[i])])))
    if(w[i, 0]<0 or w[i, 1]<0):
        short_indices.append(i)
    else:
        long_indices.append(i)

muV_short = np.array([muV[i] for i in short_indices])
vol_short = np.array([volatilities[i] for i in short_indices])
muV_long = np.array([muV[i] for i in long_indices])
vol_long = np.array([volatilities[i] for i in long_indices])

w_min = np.linalg.multi_dot([np.transpose(np.ones(np.size(C, axis=0))), np.linalg.inv(C)])
w_min = w_min/np.linalg.multi_dot([np.transpose(np.ones(np.size(C, axis=0))), np.linalg.inv(C), np.
ones(np.size(C, axis=0))])

mu_min_port = np.linalg.multi_dot([w_min, np.transpose(mu)])
```

```
plt.plot(vol_short[muV_short>mu_min_port], muV_short[muV_short>mu_min_port], 'bo')
plt.plot(vol_long[muV_long>mu_min_port], muV_long[muV_long>mu_min_port], 'ro')
plt.xlabel("Risk")
plt.ylabel("Expected Return")
plt.legend(["Portfolio with Shorting", "Portfolio without Shorting"])
plt.show()
```



In [59]:

```
# Part 2
portfolio_to_maximize = np.array(muV)-0.3*np.array(volatilities)

index_approx_max_p = np.argmax(portfolio_to_maximize)
approx_max_alloc = w[index_approx_max_p]

def expression(w, m, cov):
    return np.linalg.multi_dot([w, np.transpose(m)])-0.3*np.linalg.multi_dot([w, cov, np.transpose(
w)])

def find_max_w(func, w0, num_iter, change, m, cov):
    current_w = w0
    current_exp = func(current_w, m, cov)

    for i in range(num_iter):
        w1_change = np.random.uniform(-change, change)
        next_w1 = current_w[0]+w1_change
        next_w2 = 1-next_w1
        if(next_w1<-1 or next_w1>2):
            break
        next_w = np.array([next_w1, next_w2])
        next_exp = func(next_w, m, cov)

        if(next_exp>current_exp):
            current_exp = next_exp
            current_w = next_w
    return [current_w, current_exp]

max_alloc = find_max_w(expression, approx_max_alloc, 1000, 0.0001, mu, C)[0]
print("The portfolio with the maximum value for the expression is: w  = " + str(max_alloc))
```

```
The portfolio with the maximum value for the expression is: w  = [-0.13189053  1.13189053]
```

Problem 4

In [6]:

```
# Part 1
mu = np.array([0.1, 0.12, 0.16])
C = np.array([[0.09, 0.03, 0], [-0.03, 0.16, 0.04], [0, 0.04, 0.16]])

ones = np.ones(np.size(C, 0))
w_min = np.linalg.multi_dot([np.transpose(ones),
np.linalg.inv(C)])/np.linalg.multi_dot([np.transpose(ones),
                                                                                        np.linalg.
(C), ones])
print("The minimum total variance portfolio is: " + str(w_min))
```

```
print("The minimum total variance portfolio is: " + str(w_min))
```

```
The minimum total variance portfolio is: [0.59354839 0.12903226 0.27741935]
```

In [7]:

```
# Part 2
def eff_port(target_mu, m, cov):
    ones = np.ones(np.size(cov, axis=0))

    aa = np.ones((2, 2))
    bb = np.ones((2, 2))
    cc = np.ones((2, 2))

    aa[0,1] = np.linalg.multi_dot([ones, np.linalg.inv(cov), np.transpose(m)])
    aa[1,0] = target_mu
    aa[1,1] = np.linalg.multi_dot([m, np.linalg.inv(cov), np.transpose(m)])

    bb[0,0] = np.linalg.multi_dot([ones, np.linalg.inv(cov), np.transpose(ones)])
    bb[1,0] = np.linalg.multi_dot([m, np.linalg.inv(cov), np.transpose(ones)])
    bb[1,1] = target_mu

    cc[0,0] = bb[0,0]
    cc[0,1] = aa[0,1]
    cc[1,0] = bb[1,0]
    cc[1,1] = aa[1,1]

    myW = (np.linalg.multi_dot([np.linalg.det(aa)*ones, np.linalg.inv(cov)])+np.linalg.multi_dot([n
p.linalg.det(bb)*m,
                                                                                                      np.linalg.inv(c
]))/np.linalg.det(cc)
    return myW

target_mu = 0.15
min_w = eff_port(target_mu, mu, C)
sigma_min_w = np.sqrt(np.linalg.multi_dot([min_w, C, np.transpose(min_w)]))

print("With target return " + str(target_mu) + ", the minimum variance portfolio is: " + str(min_w
))
print("The associated portfolio standard deviation is: " + str(sigma_min_w))
```

```
With target return 0.15, the minimum variance portfolio is: [0.15152616 0.0912064  0.79914608]
The associated portfolio standard deviation is: 0.33378119138281354
```

In [8]:

```
# Part 3
rf = 0.04
uVec = np.ones(np.size(C, axis=0))
market_w = np.linalg.multi_dot([(mu-rf*uVec), np.linalg.inv(C)])
market_w /= np.sum(market_w)
mu_market = np.linalg.multi_dot([market_w, np.transpose(mu)])
sigma_market = np.sqrt(np.linalg.multi_dot([market_w, np.transpose(C), np.transpose(market_w)]))

print("The market portfolio is: " + str(market_w) + " which has expected return: " + str(mu_market
) + " and standard "
      "deviation: " + str(sigma_market))
```

```
The market portfolio is: [0.45016077 0.11575563 0.4340836 ] which has expected return:
0.12836012861736334 and standard deviation: 0.2335600661158311
```

Problem 5

In [48]:

```
# Part a)
mu = [0.06, 0.08, 0.12]
C = np.array([[0.04, 0.01, -0.01], [0.01, 0.09, 0], [-0.01, 0, 0.25]])

def find_w(target_sigma, w0, num_iter, tolerance, change, cov):
    current_w = w0
```

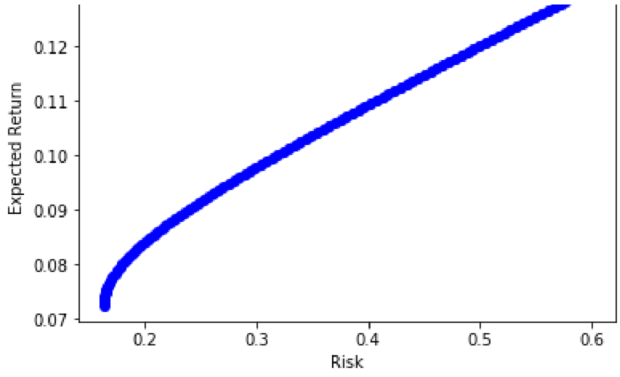
```
    current_sigma = np.sqrt(np.linalg.multi_dot([current_w, cov, np.transpose(current_w)]))

    for i in range(num_iter):
        w_changes = []
        next_w = []

        for i in range(np.size(cov, axis=0)-1):
            w_changes.append(np.random.uniform(-change, change))
            next_w.append(current_w[i]+w_changes[i])

        next_w.append(1-np.sum(next_w))
        next_sigma = np.sqrt(np.linalg.multi_dot([np.array(next_w), cov, np.transpose(np.array(next
_w))]))

        if(abs(current_sigma-target_sigma)>abs(next_sigma-target_sigma)):
            current_sigma = next_sigma
            current_w = next_w

        if(target_sigma-tolerance <= current_sigma <= target_sigma+tolerance):
            break
    return [current_w, current_sigma]

w0=[1/3, 1/3, 1/3]
num_iter = 100000
tolerance = 0.0001
change = 0.0001

target_sigma_0 = 0
w_0 = find_w(target_sigma_0, w0, num_iter, tolerance, change, C)[0]

target_sigma_0_6 = 0.6
w_06 = find_w(target_sigma_0_6, w0, num_iter, tolerance, change, C)[0]

print("The portfolio with variance closest to zero is: "+ str(w_0))
print("The portfolio with variance closest to 0.6 is: "+ str(w_06))
```

```
The portfolio with variance closest to zero is: [0.643076719436744, 0.2246158135018204,
0.1323074670614356]
The portfolio with variance closest to 0.6 is: [-0.14287469367242225, -0.04933417706538431, 1.1922
088707378067]
```

In [49]:

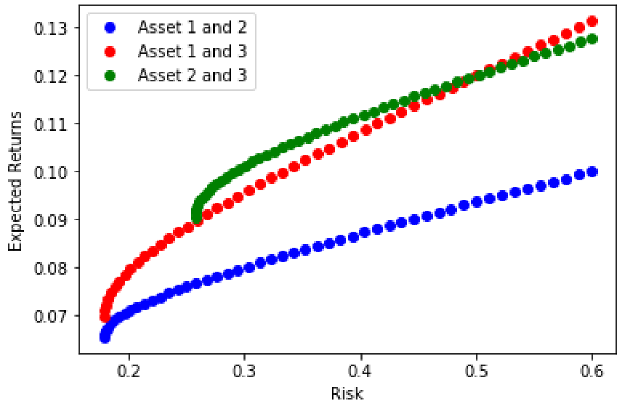
```
mu_vals = []
sig_vals = []

w1_vals = np.linspace(w_0[0], w_06[0], 100)
w2_vals = np.linspace(w_0[1], w_06[1], 100)
w3_vals = 1-w1_vals-w2_vals
w_vals =np.transpose(np.array([w1_vals, w2_vals, w3_vals]))

ones = np.ones(np.size(C, 0))
w_min = np.linalg.multi_dot([np.transpose(ones),
np.linalg.inv(C)])/np.linalg.multi_dot([np.transpose(ones),
                                                                                                np.linalg.
(C), ones])
min_var = np.sqrt(np.linalg.multi_dot([w_min, C, np.transpose(w_min)]))
mu_cutoff = np.linalg.multi_dot([w_min, np.transpose(mu)])
print("The minimum portfolio variance possible is: " + str(min_var))

for i in range(np.size(w_vals, axis=0)):
    mu_vals.append(np.linalg.multi_dot([w_vals[i], np.transpose(mu)]))
    sig_vals.append(np.sqrt(np.linalg.multi_dot([w_vals[i], C, np.transpose(w_vals[i])])))

plt.plot(sig_vals_efficient, mu_vals_efficient, 'bo')
plt.plot(sig_vals_inefficient, mu_vals_inefficient, 'ro')
plt.xlabel("Risk")
plt.ylabel("Expected Return")
plt.show()
```

```
The minimum portfolio variance possible is: 0.16323649667324353
```



In [50]:

```
# Part b)
mu_12 = np.array([0.06, 0.08])
mu_13 = np.array([0.06, 0.12])
mu_23 = np.array([0.08, 0.12])
C_12 = np.array([[0.04, 0.01], [0.01, 0.09]])
C_13 = np.array([[0.04, -0.01], [-0.01, 0.25]])
C_23 = np.array([[0.09, 0], [0, 0.25]])

w0=[1/2, 1/2]
num_iter = 100000
tolerance = 0.0001
change = 0.0001

w_0_12 = find_w(target_sigma_0, w0, num_iter, tolerance, change, C_12)[0]
w_06_12 = find_w(target_sigma_0_6, w0, num_iter, tolerance, change, C_12)[0]
w1_vals_12 = np.linspace(w_0_12[0], w_06_12[0], 50)
w2_vals_12 = 1-w1_vals_12
w_vals_12 = np.transpose(np.array([w1_vals_12, w2_vals_12]))
muV_vals_12 = []
risk_vals_12 = []

w_0_13 = find_w(target_sigma_0, w0, num_iter, tolerance, change, C_13)[0]
w_06_13 = find_w(target_sigma_0_6, w0, num_iter, tolerance, change, C_13)[0]
w1_vals_13 = np.linspace(w_0_13[0], w_06_13[0], 50)
w2_vals_13 = 1-w1_vals_13
w_vals_13 = np.transpose(np.array([w1_vals_13, w2_vals_13]))
muV_vals_13 = []
risk_vals_13 = []

w_0_23 = find_w(target_sigma_0, w0, num_iter, tolerance, change, C_23)[0]
w_06_23 = find_w(target_sigma_0_6, w0, num_iter, tolerance, change, C_23)[0]
w1_vals_23 = np.linspace(w_0_23[0], w_06_23[0], 50)
w2_vals_23 = 1-w1_vals_23
w_vals_23 = np.transpose(np.array([w1_vals_23, w2_vals_23]))
muV_vals_23 = []
risk_vals_23 = []

for i in range(50):
    muV_vals_12.append(np.linalg.multi_dot([w_vals_12[i], np.transpose(mu_12)]))
    risk_vals_12.append(np.sqrt(np.linalg.multi_dot([w_vals_12[i], C_12,
np.transpose(w_vals_12[i])])))

    muV_vals_13.append(np.linalg.multi_dot([w_vals_13[i], np.transpose(mu_13)]))
    risk_vals_13.append(np.sqrt(np.linalg.multi_dot([w_vals_13[i], C_13,
np.transpose(w_vals_13[i])])))

    muV_vals_23.append(np.linalg.multi_dot([w_vals_23[i], np.transpose(mu_23)]))
    risk_vals_23.append(np.sqrt(np.linalg.multi_dot([w_vals_23[i], C_23,
np.transpose(w_vals_23[i])])))

plt.plot(risk_vals_12, muV_vals_12, 'bo')
plt.plot(risk_vals_13, muV_vals_13, 'ro')
plt.plot(risk_vals_23, muV_vals_23, 'go')
plt.xlabel("Risk")
plt.ylabel("Expected Returns")
plt.legend(['Asset 1 and 2', 'Asset 1 and 3', 'Asset 2 and 3'])
plt.show()
```



In [51]:

```
ones = np.ones(np.size(C_12, 0))
w_min_12 = np.linalg.multi_dot([np.transpose(ones), np.linalg.inv(C_12)])/np.linalg.multi_dot([np.
transpose(ones),
                                                                                        np.linalg.
 (C_12), ones])
min_var_12 = np.sqrt(np.linalg.multi_dot([w_min_12, C_12, np.transpose(w_min_12)]))
muV_min_var_12 = np.linalg.multi_dot([w_min_12, np.transpose(mu_12)])

w_min_13 = np.linalg.multi_dot([np.transpose(ones), np.linalg.inv(C_13)])/np.linalg.multi_dot([np.
transpose(ones),
                                                                                        np.linalg.
 (C_13), ones])
min_var_13 = np.sqrt(np.linalg.multi_dot([w_min_13, C_13, np.transpose(w_min_13)]))
muV_min_var_13 = np.linalg.multi_dot([w_min_13, np.transpose(mu_13)])

w_min_23 = np.linalg.multi_dot([np.transpose(ones), np.linalg.inv(C_23)])/np.linalg.multi_dot([np.
transpose(ones),
                                                                                        np.linalg.
 (C_23), ones])
min_var_23 = np.sqrt(np.linalg.multi_dot([w_min_23, C_23, np.transpose(w_min_23)]))
muV_min_var_23 = np.linalg.multi_dot([w_min_23, np.transpose(mu_23)])

print("The minimum variance portfolio for the asset 1 and 2 combination has weights: " + str(w_min
_12) + " with sigma and"
     " mu values of: " + str((min_var_12, muV_min_var_12)))
print("The minimum variance portfolio for the asset 1 and 3 combination has weights: " + str(w_min
_13) + " with sigma and"
     " mu values of: " + str((min_var_13, muV_min_var_13)))
print("The minimum variance portfolio for the asset 2 and 3 combination has weights: " + str(w_min
_23) + " with sigma and"
     " mu values of: " + str((min_var_23, muV_min_var_23)))
```

```
The minimum variance portfolio for the asset 1 and 2 combination has weights: [0.72727273 0.272727
27] with sigma and mu values of: (0.17837651700316892, 0.06545454545454546)
The minimum variance portfolio for the asset 1 and 3 combination has weights: [0.83870968 0.161290
32] with sigma and mu values of: (0.17870501915438117, 0.0696774193548387)
The minimum variance portfolio for the asset 2 and 3 combination has weights: [0.73529412 0.264705
88] with sigma and mu values of: (0.25724787771376323, 0.09058823529411764)
```

Part c)

> All the 4 curves start and end at the same risk value, but the expected returns are different. This is because, the different combinations of assets yields different portfolio weights at each (σ, μ) coordinate. This results in the curves having different slopes.