Simulation of Electroweak Phase Transitions:
Standard, Singlet and Doublet Models

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Introduction

Aim

The main aim of this mini-project is to describe the phenomenon of electroweak phase transition and perform simulations for different models that aim to explain this phenomenon which occurred within 10^{-11} s since the Big Bang. Simulation of models like the Standard Model, Singlet Model and Doublet Model have been performed using code written in Python along with plots and observations, which serve as results that show the findings and drawbacks of each model. Let us begin by understanding what an Electroweak Phase Transition is.

Electroweak Phase Transitions?

As we move back in time towards the dawn of creation, prior to one-hundredth of a second, the Universe starts to become hotter and denser until matter begins to change its phase, i.e., there is a change in its form and properties. A parallel to everyday life familiar to all of us is simply water [11].

Water undergoes a succession of phase transitions on increasing the temperature. In these transitions, there is a significant change in its physical properties. Ice, the solid-state of water, melts into the liquid phase, water, and then to steam, the gaseous phase, through boiling. One shall observe that steam is 'more symmetric' than water, which is, in turn, more symmetric than ice. Here symmetric refers to there being more randomness and freedom. The matter in our Universe is a similar case. It starts in a unified or 'symmetric' phase, after which it goes through a sequence of phase transitions until we finally obtain the matter particles with which physicists are familiar today: electrons, protons, neutrons, photons and others, at lower temperatures.

Phase transitions may have had a significant impact on the evolution of our Universe. Moreover, some direct 'remnants' of these transitions are still around today! In an abstract sense, phase transitions are a hallmark of gauge field theories, for example, the Standard Model, which is based on elementary particle mass generation by sudden symmetry-breaking [7]. While the Standard Model, as we shall see, has a "crossover" or a pseudo phase transition, many extensions of the Standard Model with an increased number of scalar fields lead to first-order phase transitions at the electroweak scale, which indicate temperatures in the range of 100 – 1000 GeV. In this electroweak scale, gravitational wave signals are produced that may end up lying in the frequency range of

LISA (Laser Interferometer Space Antenna), the forthcoming space-based gravitational wave detector [10]. These gravitational waves have become such a widely discussed topic with the recent confirmation of their existence in 2015 [4] and the approval of the LISA mission, which in turn has led to an increased interest in phase transitions at the dawn of the Universe [3].

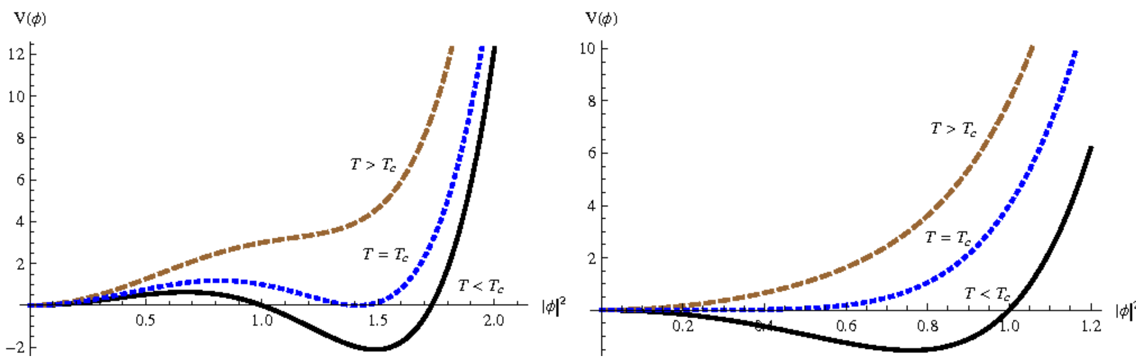

Figure 1: 1^{st} and 2^{nd}-order Phase Transitions [2]

Types Of Phase Transitions

In general, we observe two different types of phase transitions - the dramatic, first-order phase transition and the smooth, second-order phase transition, as shown in Figure 1.

There are three striking features in a standard first-order phase transition: latent heat, critical temperature, and surface energy at the phase boundaries of the mixed-phase. When systems are cooled below their critical temperature in the laboratory, they proceed by nucleating little bubbles or droplets of the new phase around impurities. Until the complete disappearance of the old phase, the bubbles expand and collide, leading to the completion of a phase transition. Figure 2 is a sketch of this process. If a first-order phase transition occurs at the dawn of the Universe, we expect this mechanism to generate gravitational waves (GW).

Pictorially, a first-order phase transition is when the function, in our case the effective potential, after the zero-point value starts increasing, reaches a maximum, starts decreasing, reaches a minimum and then continues to rise. Thus we obtain two minima and a maximum for this particular phase transition, depicted by the black curve on the left side of Figure 1. As the temperature increases slowly, this phase transition disappears at a specific temperature as the second minimum takes a value greater than the zero-point minima. The temperature at which these two minima take on the same value of effective potential is called the critical temperature. Beyond the critical temperature, the minimum starts to disappear, and for sufficiently higher temperatures, the curve takes the shape of a parabola. In our case, we plot the effective potential with the Higgs Field, ϕ, which will be explained later. For a stable first-order phase transition, we have the condition,

$$\frac{\phi_C}{T_C} \geq 1,$$

where ϕ_C is the broken phase minimum at the critical temperature, T_C. Figure 3 clearly shows how a phase transition occurs as we vary the effective potential with temperature.

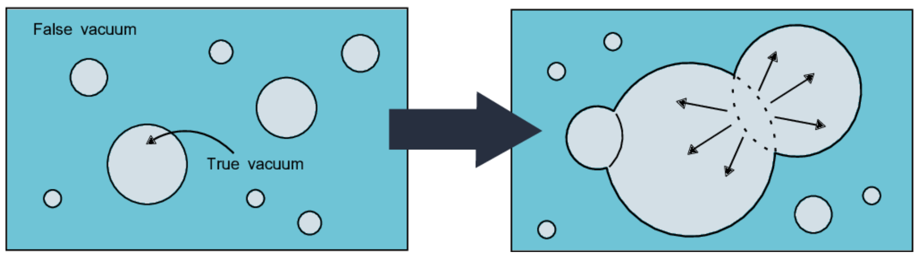

Figure 2: Bubbling Mechanism in 1^{st} order Phase Transitions [6]

The second-order phase transition is where the function starts at a zero-point value, decreases, reaches a minimum and starts increasing for higher values. Second-order phase transitions proceed smoothly, as shown by the black curve on the right side of Figure 1. The old phase transforms itself into the new phase in a continuous manner. In an ideal scenario, the model that we have utilized for simulation should be able to explain both the first-order and second-order phase transitions. Now let us take a deeper look into the models that are being simulated.

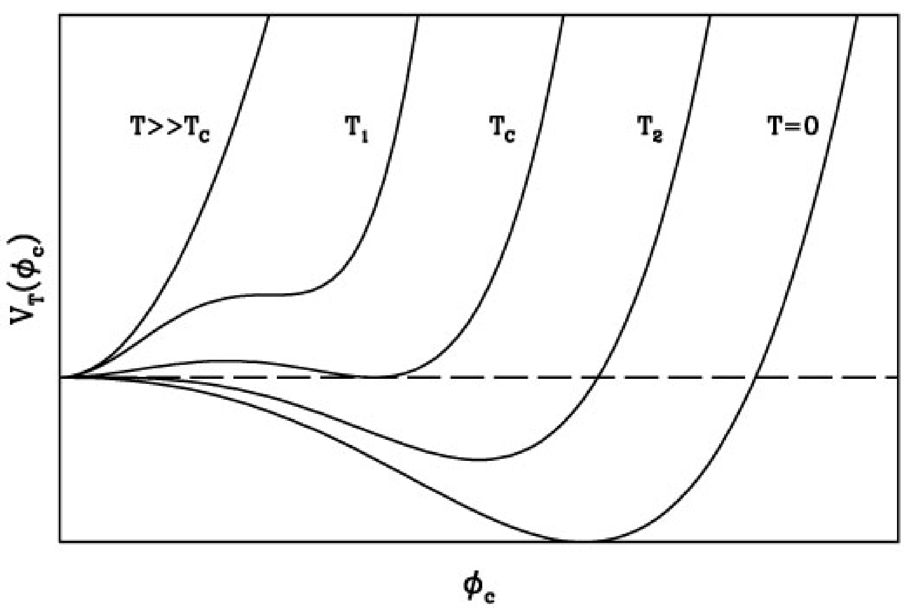

Figure 3: Phase Transition at different temperatures including Critical Temperature [1]

The Standard Model

Matter is the most abundant constituent of our known Universe. There is an asymmetry between the amount of antimatter and matter in the Universe, which contradicts thermal equilibrium and is unavoidable in a first-order phase transition. We can express this asymmetry by a term called the baryon to photon ratio,

$$\rho = \frac{n_b - n_{\bar{b}}}{n_\gamma}$$

whose value is around 10^{-9}. where n_b, $n_{\bar{b}}$, and n_γ are the number densities of baryons, anti-baryons, and photons. In a universe with symmetry, one expects $\rho = 0$, but experiments reveal that ρ has a relatively small but non-zero value. The phenomenon that leads to this observed baryon asymmetry is called Baryogenesis, abbreviated as BAU. In Electroweak Baryogenesis, one assumes that a strong first-order electroweak phase transition takes place(EWPT) [8].

The effective potential is a helpful tool for the investigation of EWPT. It is the potential to the lowest order in perturbation theory when we replace the quantum field, in a quantum field theory, with its expectation value in a vacuum. It represents the energy density of the quantum field in use. This effective potential, after normalization, has been plotted with the Higgs Field, ϕ. The Higgs field, thought to be present in every part of the Universe, is an energy field. It is accompanied by a fundamental particle, the Higgs boson, using which the field interacts with other particles, such as the electron, continuously. Particles which interact with the field are said to be "given" mass. Giving mass to an object/particle is termed the Higgs effect. Hence, in simple terms, we are relating energy density with mass but on a quantum scale, where these terms have different meanings. We shall not be getting deeper into the details of the Standard Model in this report.

Model Equation

The equation that represents the effective potential at a finite temperature under the Standard Model is given by,

$$V_T(\phi) = \frac{D}{2!}(T^2 - T_0^2)\phi^2 - \frac{A}{3!}T\phi^3 + \frac{\lambda_T}{4!}\phi^4 + ...$$

$$A = \frac{1}{12\pi\phi^3}(M_H^3 + 6M_W^3 + 3M_Z^3)$$

$$D = \frac{1}{12\phi^2}(M_H^2 + 6M_W^2 + 3M_Z^2 + 6M_t^2)$$

$$\lambda_T = \lambda$$

$$T_0 = \sqrt{\frac{1}{2D}}M_H$$

Here A, D, T_0, and λ_T are as shown above. The logarithmic dependence of λ_T on T has been abandoned, and it is made a free parameter. The subscripts W, Z, H and t, denote the W and Z bosons, the Higgs-boson, and the Standard Model's top quark, respectively. We also note that the bosons are the sole contributors to the potential's cubic term. For the Standard Model the masses of the respective fields take constant values as follows, $M_H = 125$ GeV, $M_Z = 81$ GeV, $M_W = 91$ GeV, $M_t = 173$ GeV.

Simulation Strategy

The code used to simulate this effective potential function is attached in the appendix. This simulation is relatively simple, as we need just a single function in Python where we apply the formulas mentioned above. This potential function has been plotted for different temperatures using the code attached in this report and the obtained plots at T = 0, 75 and 150 GeV, respectively, for ϕ values from 0.01 to 250 GeV. We have not used the exact zero value of ϕ to start, both here and in the following models, since there are occurrences of division by ϕ, which would lead to errors. Numpy, a python library, is used for arrays that serve as places of storage of data and then this data is plotted using Matplotlib, another library using which all the plots shown below have been obtained.

Results

In Figure 4, we can see that there seems to be some type of phase transition which is observed, but we realize that it is not exactly a phase transition. As the temperature rises, the shape of the curve looks the same instead of the curve slowly moving from having a minimum to a parabolic shape. The starting point also moves up with temperature instead of staying at a constant value as in our model; the zero-point value depends on temperature. Thus, there is an absence of both first and second-order phase transitions in this model. This is due to the fact that we only have one free parameter in λ in the model, and the rest of the values are fixed, with the masses of the scalar fields taking constant values instead of varying with ϕ. Figures 5 and 6 show two different plots for the Standard Model where the free parameter, λ, takes the values 0.5 and 2, respectively. We observe that with the increase of the value of λ, the ϕ value at which the minimum occurs decreases.

Despite their impressive successes, the Standard Models (SMs) of particle physics and cosmology remain incomplete. Several theoretical and observational problems cannot be explained within this framework [9]. Thus, we need a model that improves on the Standard Model and can clearly show us the necessary first and second-order phase transitions. So, we move on to Scalar Extended Models.

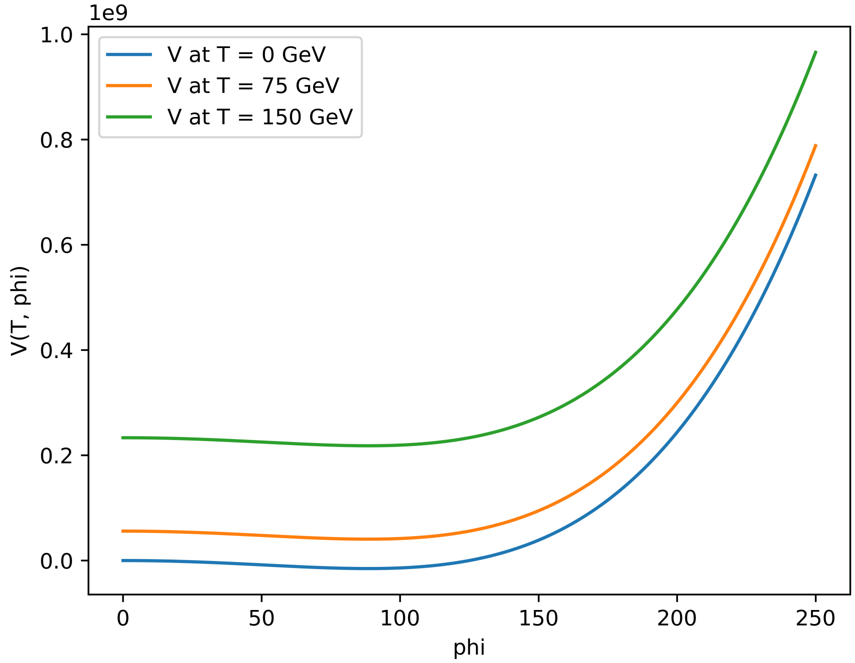

Figure 4: Pseudo-Phase Transition in Standard Model with parameter $\lambda = 1$

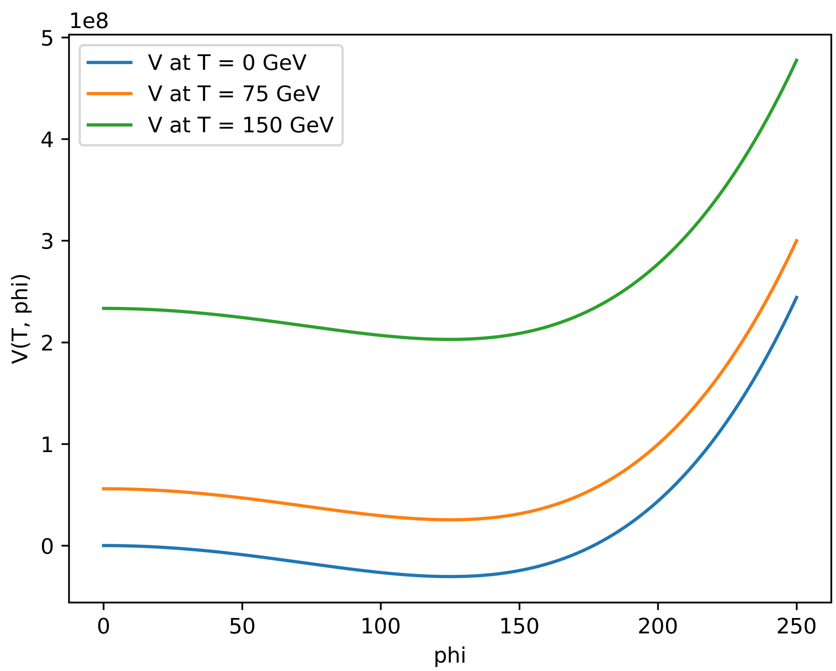

Figure 5: Pseudo-Phase Transition in Standard Model with parameter $\lambda = 0.5$

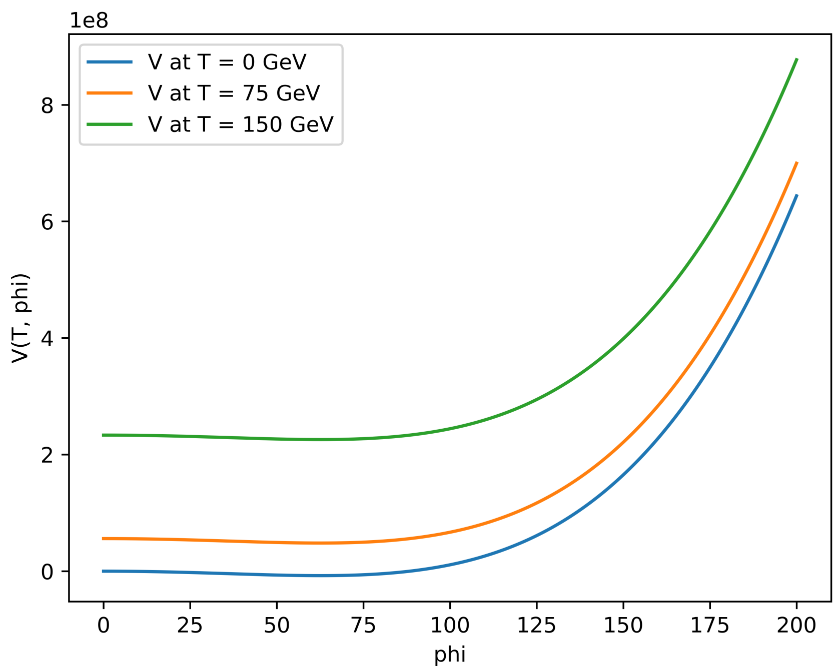

Figure 6: Pseudo-Phase Transition in Standard Model with parameter $\lambda = 2$

Scalar Extended Models

The Singlet Model

As the Universe cooled down, the effective potential at a temperature, T, above the critical temperature, T_C, had an absolute minimum located at the origin. Two degenerate minima, separated by an energy barrier, were present at $T = T_C$. Below the critical temperature, the second minimum became the global one, as it took a value less than the value at the origin. There is also an absence of the energy barrier [8]. In our first attempt at achieving this shape for the effective potential, we use the Singlet model, a modified form of the Standard Model. Here, we have expanded the field-dependent masses to use their logarithmic forms instead of the earlier constant values and added a scalar field as well. The number of free parameters has also thus been increased to two from the one in the Standard model.

Model Equation

The equation that represents the effective potential at a finite temperature under the Singlet Model is given by,

$$V_{eff}(\phi, T) = V_{tree}(\phi) + V_{CW}(\phi) + V_T(\phi, T)$$

where $V_{tree}(\phi)$, $V_{CW}(\phi)$, $V_T(\phi, T)$ represents the Tree potential, Coleman-Weinberg potential (a correction to the tree-level potential) and the temperature-dependent potential respectively given by,

$$V_{tree}(\phi) = \frac{1}{4}\lambda_H(\phi^2 - v^2)^2$$

$$V_{CW}(\phi) = \frac{1}{64\pi^2} \sum_{i=H,s,W,Z,t} n_i M_i^4(\phi)[log(\frac{M_i^2(\phi)}{v^2}) - C_i]$$

$$V_T(\phi, T) = n_t T^4 J_F(\frac{M_t^2(\phi)}{T^2}) + \sum_{i=H,s,W,Z} n_i T^4 J_B(\frac{M_i^2(\phi)}{T^2})$$

Subtraction by $V_T(0.01, T)$ and division by v^4 are used for normalizing the curve and for ease of visualization, as can be seen in the axes of the plots shown below. J_F and J_B denote the contributions from fermions and bosons, which are derived by the study of free fermionic and bosonic fields and their thermodynamic properties. The derivation is quite mathematically heavy and not the main focus of this project and hence shall not be included. The expressions are given by,

$$J_F(x^2) = \int_0^\infty y^2 log(1 + e^{-\sqrt{y^2+x^2}})\, dy$$
$$J_B(x^2) = \int_0^\infty y^2 log(1 - e^{-\sqrt{y^2+x^2}})\, dy$$

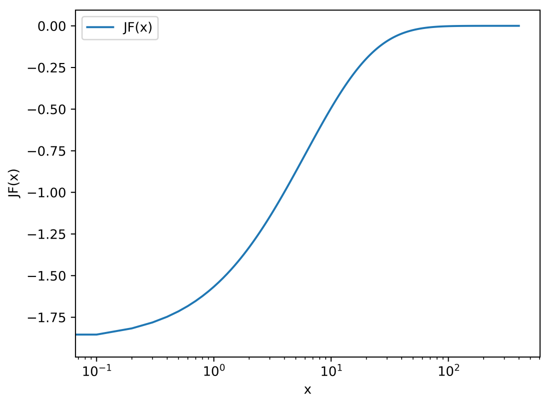

Figure 7: $J_F(x)$

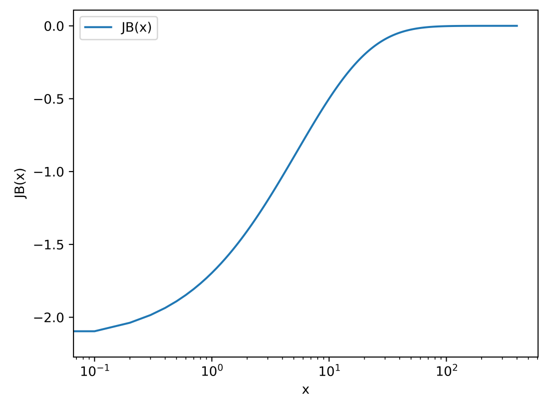

Figure 8: $J_B(x)$

Figures 7 and 8 show the behaviour of the J_F and J_B functions. The M_i's are the scalar field-dependent masses where i = H(Higgs Field),s(Scalar Singlet), W,Z(Gauge Bosons) and t(contribution due to fermions where only the top quark is considered), where now they have a ϕ dependence in their expressions instead of being constant as shown below,

$$M_H^2(\phi) = |3\lambda_H\phi^2 - \lambda_H v^2|$$
$$M_s^2(\phi) = \mu_s^2 + \frac{\lambda_{HS}^2\phi^2}{2}$$
$$M_W^2(\phi) = \frac{g^2\phi^2}{4}$$
$$M_Z^2(\phi) = \frac{(g^2 + g^{'2})\phi^2}{4}$$
$$M_t^2(\phi) = \frac{y_t^2\phi^2}{2}$$

The constant values used in this potential function are as follows, expectation value of the Higgs Field at vacuum, v = 246 GeV, which is a determining factor for the particle masses that we observe in our experiments, n_H = 1, n_s = 1, n_W = 6, n_Z = 3, n_t = -12, $C_{W,Z} = \frac{5}{6}$, $C_{h,s,t} = \frac{3}{2}$, g = 0.652, $g^{'}$ = 0.352, y_t = 0.995, μ_H = 125 GeV ($\mu_H^2 = \lambda_H v^2$) and λ_{HS} and μ_S are free parameters.

Simulation Strategy

The code used to simulate this effective potential function has been attached in the appendix. Two functions have been created named potential and JF, which are used for the effective potential, and the J_F, J_B functions according to the formulas mentioned above. For integration, the integrate function from the Scipy library of Python has been used. Effective potential by v^4 versus ϕ by v is plotted. Plotting and arrays are yet again done using Matplotlib and Numpy. For the T = 0 GeV case, the $V_T(\phi, T)$ function has been assigned zero as the function involves division by $M_i(\phi) \propto \phi$, which can lead to errors. Other than that, the function uses the formulas shown before and the variation of ϕ is from 0.01 to 350 GeV.

Results

The Singlet Model clearly improves the Standard Model as it clearly shows the second-order phase transitions at low temperatures. In Figure 9, the free parameters $\mu_S = 10$ GeV and $\lambda_{HS} = 5$ have been used and plotted at temperatures 0, 50 and 75 GeV respectively. We observe that with the rise in temperature, the minima present initially starts to disappear. The point where it shifts is the critical temperature. This temperature seems to be a meagre value, at around 50-60 GeV. The point at which the curve has its minima has its ϕ value close to v.

Thus the addition of the singlet scalar has improved the model. However, this model is not complete as the first-order phase transitions are not observed.

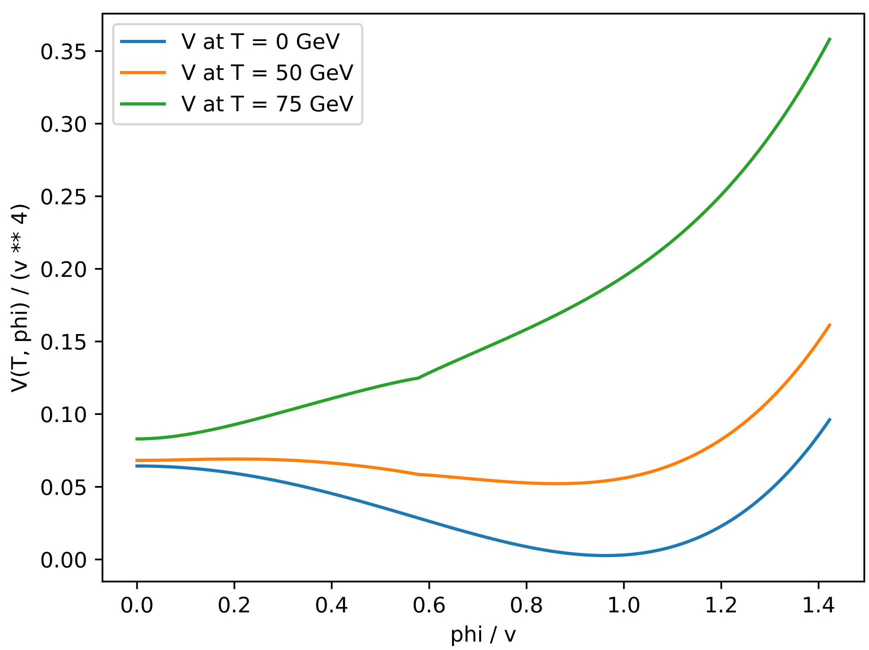

Figure 9: Second-Order Phase Transition using Singlet Model at different temperatures

The Doublet Model

In our second attempt at achieving the expected shape for the effective potential, we have used the Doublet Model, a modified form of the Standard Model. We have kept the field-dependent masses using their logarithmic forms as we used earlier in the Singlet Model. We have added another term in the expression, V_{daisy}, along with an extended Coleman-Weinberg term which includes the contributions of the field-dependent masses of the charged and neutral scalar fields, $S_{\pm}$, H and A. The number of free parameters has increased to four from the one in the Standard model, providing us more freedom than both the earlier models.

Model Equation

The equation that represents the effective potential at a finite temperature under the Doublet Model is given by,

$$V_{eff}(\phi,T) = V_{tree}(\phi) + V_{CW}(\phi) + V_T(\phi,T) + V_{daisy}(\phi,T)$$

where $V_{tree}(\phi)$, $V_{CW}(\phi)$, $V_T(\phi,T)$ and $V_{daisy}(\phi,T)$ represent the Tree potential, Coleman-Weinberg potential (a correction to the tree-level potential), the temperature-dependent potential and the Daisy potential respectively given by,

$$V_{tree}(\phi) = \frac{1}{4}\lambda_H(\phi^2 - v^2)^2$$

$$V_{CW}(\phi) = \frac{1}{64\pi^2}\sum_{i=h,\pm,H,A,W,Z,t} n_i M_i^4(\phi)[log(\frac{M_i^2(\phi)}{v^2}) - C_i]$$

$$V_T(\phi,T) = n_t T^4 J_F(\frac{M_t^2(\phi)}{T^2}) + \sum_{i=h,\pm,H,A,W,Z} n_i T^4 J_B(\frac{M_i^2(\phi)}{T^2})$$

$$V_{daisy}(\phi,T) = -\frac{T}{2\pi^2}\sum_{i=\pm,H,A,W_L,Z_L,\gamma_L} n_i[M_i^3(\phi,T) - M_i^3(\phi)]$$

Subtraction by $V_T(0.01,T)$ and division by v^4 are used for normalizing the curve and for ease of visualization, as can be seen in the axes of the plots shown below, similar to how we did in the Singlet model. J_F and J_B denote the contributions from fermions and bosons as mentioned in the Singlet model, given by,

$$J_F(x^2) = \int_0^\infty y^2 log(1 + e^{-\sqrt{y^2+x^2}})\, dy$$

$$J_B(x^2) = \int_0^\infty y^2 log(1 - e^{-\sqrt{y^2+x^2}})\, dy$$

The M_i's are the scalar field-dependent masses where i = ±(charged scalar field, S±), H(neutral scalar field, H), A(neutral scalar field A),h(Higgs Field), W,Z(Gauge Bosons) and t(contribution due to fermions where only the top quark is considered), where now they have a ϕ dependence in their expressions instead of being constant as shown below, given by,

$$M_h^2(\phi) = |3\lambda_h\phi^2 - \lambda_h v^2|$$
$$M_\pm^2(\phi) = m_1^2 + \frac{\lambda_1\phi^2}{2}$$
$$M_{H,A}^2(\phi) = m_1^2 + \frac{\lambda_{H,A}\phi^2}{2}$$
$$M_W^2(\phi) = \frac{g^2\phi^2}{4}$$
$$M_Z^2(\phi) = \frac{(g^2 + g^{'2})\phi^2}{4}$$
$$M_t^2(\phi) = \frac{y_t^2\phi^2}{2}$$

Here, $\lambda_{H,A} = \lambda_1 + \lambda_2 \pm 2\lambda_3$. Now we add temperature dependent terms to these scalar field masses for the Daisy potential term and the resultant expressions are as follows,

$$M_{\pm,H,A}^2(\phi, T) = M_{\pm,H,A}^2(\phi) + \Pi_S(T)$$
$$M_{W_L}^2(\phi, T) = M_W^2(\phi) + \Pi_W(T)$$
$$M_{Z_L}^2(\phi, T) = \frac{1}{2}(M_Z^2(\phi) + \Pi_W(T) + \Delta(\phi, T))$$
$$M_{\gamma_L}^2(\phi, T) = \frac{1}{2}(M_Z^2(\phi) + \Pi_W(T) - \Delta(\phi, T) + \Pi_Y(T))$$
$$\Delta^2(\phi, T) = (\frac{1}{4}g^2\phi^2 - \frac{1}{4}g^{'2}\phi^2 + \Pi_W(T) - \Pi_Y(T))^2 + \frac{1}{4}g^2g^{'2}\phi^4$$
$$\Pi_S(T) = \frac{1}{8}g^{'2} + \frac{1}{16}(g^2 + g^{'2}) + \frac{1}{2}\lambda_S + \frac{1}{12}\lambda_1 + \frac{1}{24}\lambda_A + \frac{1}{24}\lambda_H$$
$$\Pi_W(T) = 2g^2T^2$$
$$\Pi_Y(T) = 2g^{'2}T^2$$

Here, Π_S, Π_W and Π_Y are the thermal masses for the inert scalar doublets, expressions as shown above. The constant values used in this potential function are as follows, expectation value of the Higgs Field at vacuum, v = 246 GeV, $n_h = 1$, $n_\pm = 2$, $n_H = 1$, $n_A = 1$, $n_W = 6$, $n_{W_L} = 2$, $n_{Z_L} = 1$, $n_{\gamma_L} = 1$, $n_Z = 3$, n_t = -12, $C_{W,Z} = \frac{5}{6}$, $C_{h,\pm,H,A,t} = \frac{3}{2}$, g = 0.652, $g^{'}$ = 0.352, y_t = 0.995, λ_h = 0.129, λ_s = 0.1 and λ_1, λ_2, λ_3 and m_1 are the free parameters.

Simulation Strategy

The code used to simulate this effective potential function has been attached in the appendix. Similar to the code for the Singlet model, the Numpy, Matplotlib and Scipy libraries have been used for arrays, plotting and integration. Effective potential by v^4 versus ϕ by v is plotted. Two functions, pot and JF, have also been created for the effective potential and the J_F, J_B functions, respectively, using the previous formulae. Initially, plotting of the doublet model for different temperatures was performed, and it was observed that both the second-order and first-order phase transitions were observed with critical temperatures in the range of 120 - 140 GeV. In this model, finding the critical temperature for a random set of the four free variables, generated using the "random" module of Numpy, has been automated.

A loop is used to slowly increase the temperature for a particular set of free variables to check for a maximum, followed by a minimum. These points can be found by checking the potential values as they will increase and then start decreasing, indicating a maximum and vice versa for the minima, leading to us concluding that the maxima/minima is close to that turning point (For simplicity of calculations, we assume that the turning point is itself the maxima/minima). This method is helpful because, in the first-order phase transition, a minimum is obtained after a maximum, as mentioned earlier. For finding the critical temperature, we have the minima value being equal to the zero-point value, and hence the closeness is checked at every pass of the loop and when the difference between the values becomes less than or equal to 1e-3 and 1e-4 $(GeV)^4$, the step size of temperature becomes 0.12 and 0.012 GeV respectively to get a more accurate measurement of T_C. When the difference first goes below 1e-5, the value of temperature at that point is taken as T_C and printed along with the ϕ_C and the ratio, $\frac{\phi_C}{T_C}$, to check if the phase transition is stable or not. Figure 11 shows this working along with the values of the free parameters and the field-dependent masses of the scalar fields in an example.

Results

The Doublet Model is a clear improvement on the Standard Model as it clearly shows both the first order and second-order phase transitions. The presence of four free parameters because of the increased number of scalar fields is certainly useful in this case, which leads us to conclude that these scalar fields have an extremely significant contribution to the bump (maxima) in the curve, important in a first-order phase transition. Following are several plots showing effective potential at different temperatures using the values of free parameters, $\lambda_1 = 2.330495$, $\lambda_2 = -0.239875$, $\lambda_3 = 0.346325$ and $m_1 = 39$GeV. Figures 9 and 10 show the second-order phase transition and the first-order phase transition at temperatures of 100 GeV and 122 GeV, respectively, both being less than T_C.

After finding the critical temperature, as shown in Figure 11, the way the potential looks at that temperature is plotted as shown in Figure 12 and a zoomed-in version of the two minima in Figure 13. A T_C of 124.268 GeV is obtained using the simulation, and we can also see that it is a stable phase transition as the ratio is greater than 1. Figures 14 and 15 show the effective potential at temperatures 125 and 135 GeV, which are greater than T_C. We can see that the second minimum assumes a higher value than the zero-point value in Figure 14 and the parabolic shape of the curve in Figure 15. After several tests, a range was also obtained for the free parameter values where a stable first-order phase transition is obtained. They are, $\lambda_1 \rightarrow 2.3$ to 5, $\lambda_2 \rightarrow -0.5$ to 0.1, $\lambda_3 \rightarrow -2$ to 1.5 and $m_1 \rightarrow 39$ to 80 GeV. The range of the field-dependent masses of the scalar fields using this parameter space is, $M_\pm \rightarrow 266.67$ to 397.10 GeV, $M_H \rightarrow 255.04$ to 501.49 GeV and $M_A \rightarrow 264.46$ to 420.73 GeV. Note that the exact values could be a broader range of values, and this is just a subset.

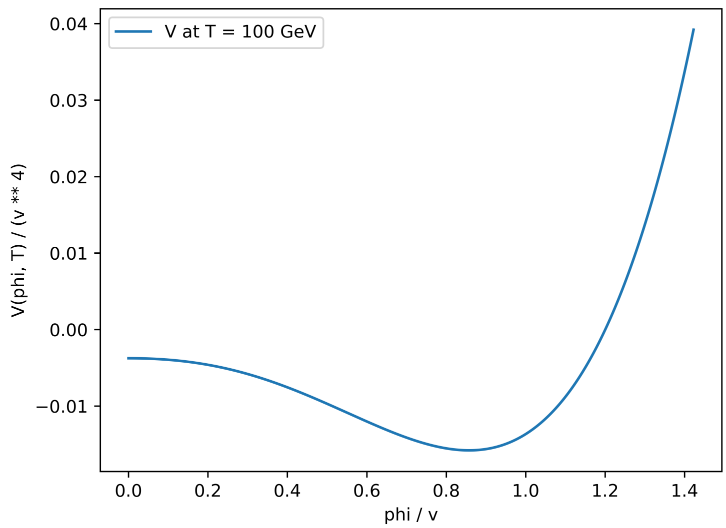

Figure 10: Second-Order Phase Transition using Doublet Model at $T < T_C$

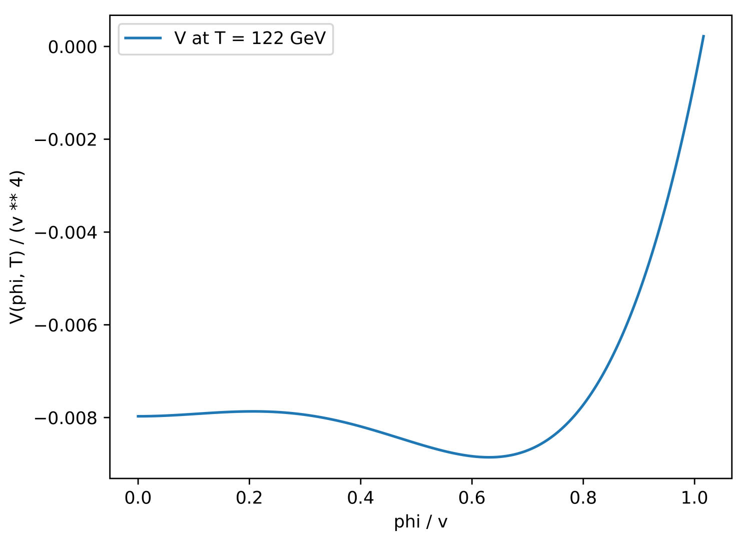

Figure 11: First-Order Phase Transition using Doublet Model at $T < T_C$

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Parameters used:
Lambda 1 = 2.330495
Lambda 2 = -0.239875
Lambda 3 = 0.346325
M1 = 39 GeV


Physical masses:
Field Dependent Mass of Charged Scalar Field S+- = 268.397308 GeV
Field Dependent Mass of Neutral Scalar Field H = 292.809101 GeV
Field Dependent Mass of Neutral Scalar Field A = 209.334139 GeV


Temperatures checked:
120 GeV
120.5 GeV
121.0 GeV
121.5 GeV
122.0 GeV
122.12 GeV
122.24 GeV
122.36 GeV
122.48 GeV
122.6 GeV
122.72 GeV
122.84 GeV
122.96 GeV
123.08 GeV
123.2 GeV
123.32 GeV
123.44 GeV
123.56 GeV
123.68 GeV
123.8 GeV
123.92 GeV
124.04 GeV
124.052 GeV
124.064 GeV
124.076 GeV
124.088 GeV
124.1 GeV
124.112 GeV
124.124 GeV
124.136 GeV
124.148 GeV
124.16 GeV
124.172 GeV
124.184 GeV
124.196 GeV
124.208 GeV
124.22 GeV
124.232 GeV
124.244 GeV
124.256 GeV
PHIC =  139.760312 GeV
TC =  124.268 GeV
PHIC / TC =  1.124669
```

Figure 12: Finding Critical Temperature using Automated Code for Doublet Model

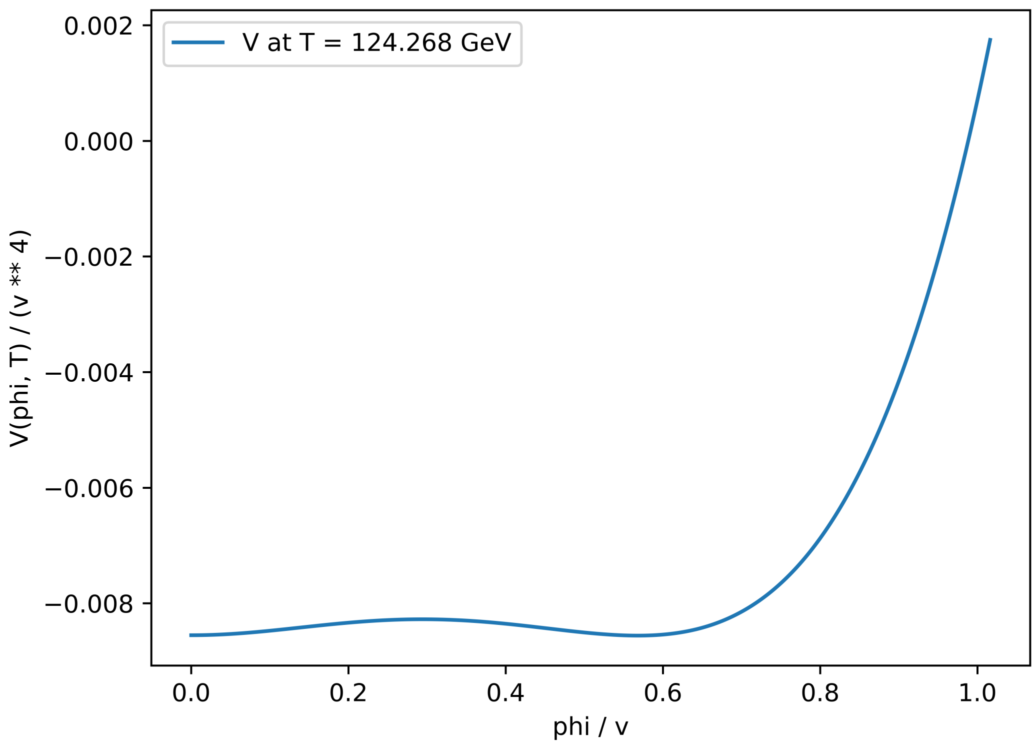

Figure 13: First-Order Phase Transition using Doublet Model at $T = T_C$

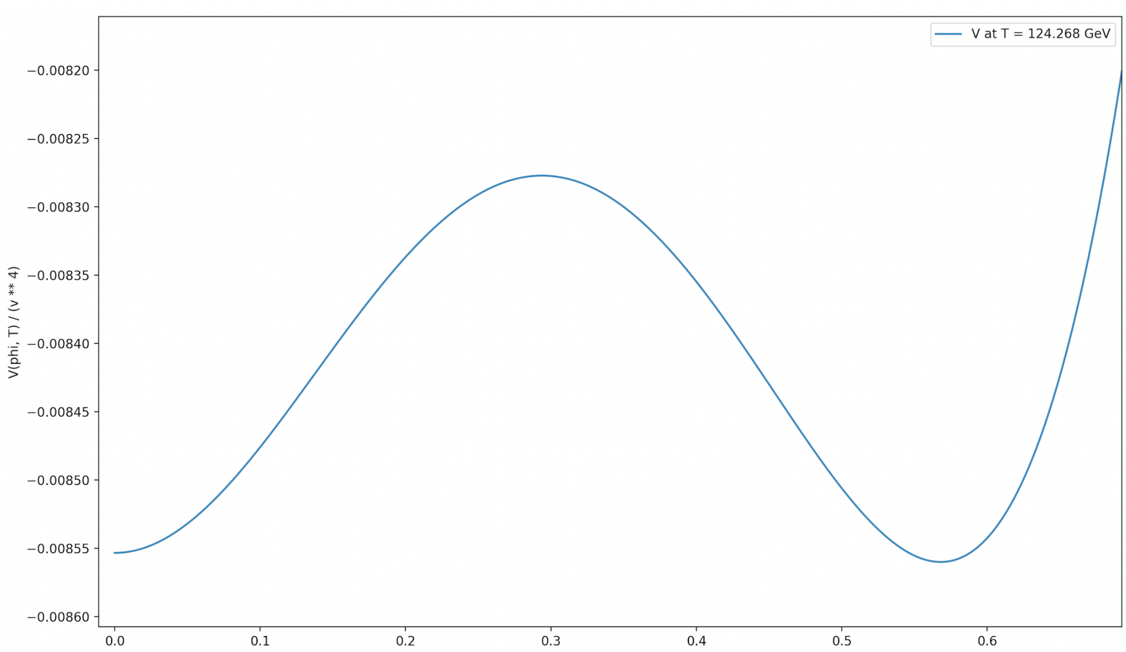

Figure 14: First-Order Phase Transition using Doublet Model at $T = T_C$ (Zoom)

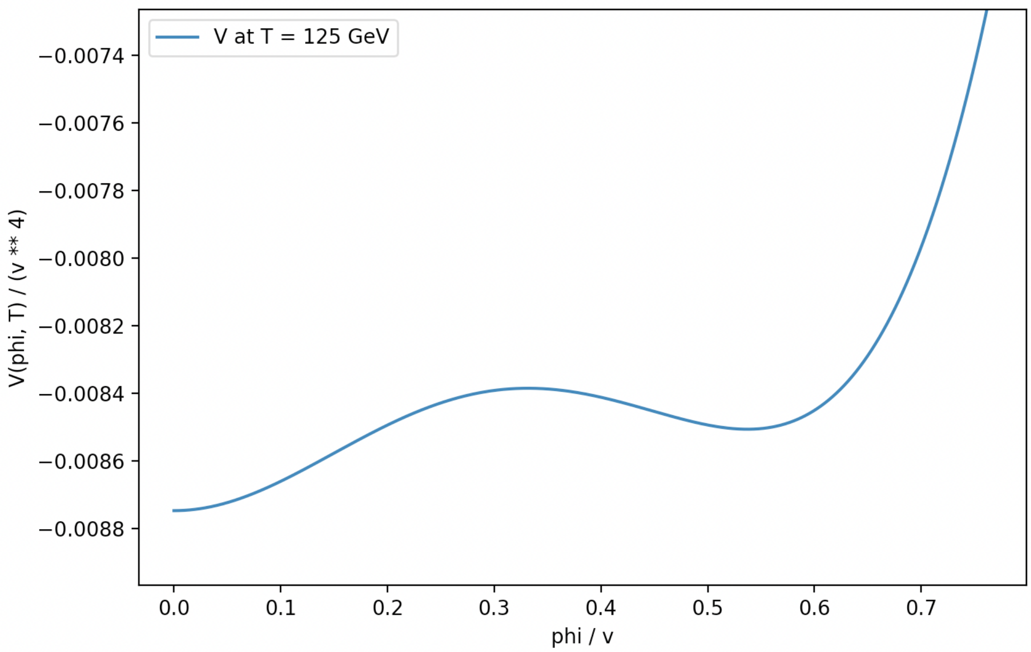

Figure 15: First-Order Phase Transition using Doublet Model at $T > T_C$

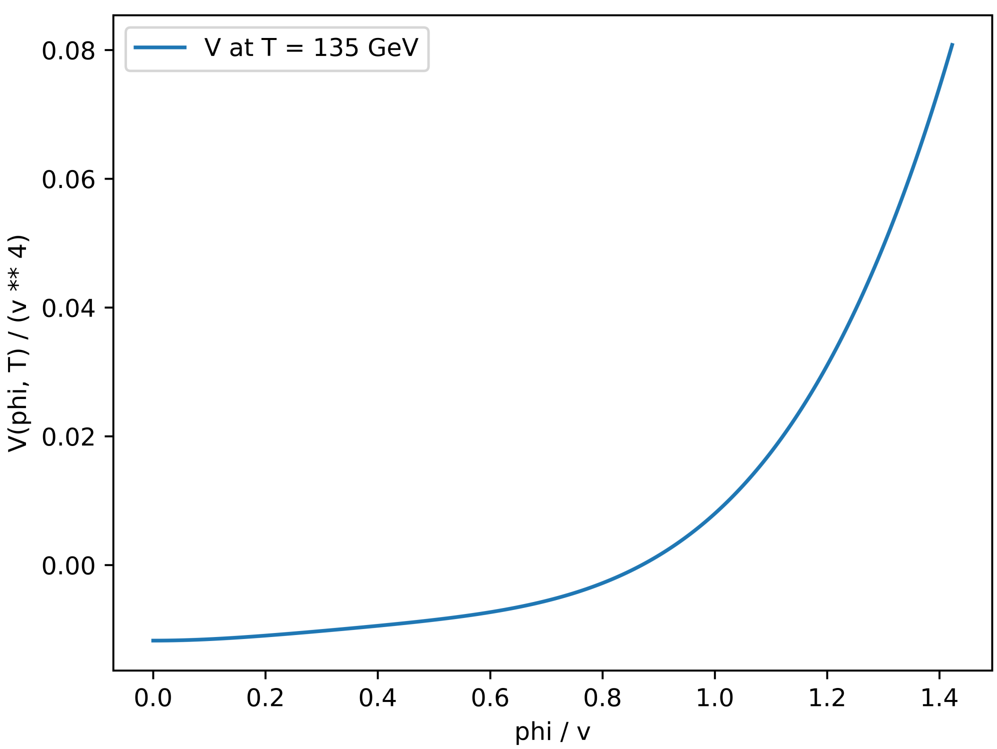

Figure 16: Parabolic Shape at $T > T_C$ using Doublet Model

Conclusion and Future Outlook

We have seen the expressions for three different models used to explain Electroweak Phase Transitions and the different inferences we can conclude from each. Although the Standard Model is one of the most influential models in quantum mechanics, there are some phenomena which it cannot fully explain. The Singlet Model showed us a clear second-order phase transition, while the Doublet Model exhibited both first and second-order phase transitions. The nature of electroweak phase transition (EWPT) is of excellent importance [12]. The main future prospective for these EWPTs are gravitational waves and their detection.

Gravitational waves are a handy tool, particularly for cosmology and astronomy, for directly observing phenomena that happened at the dawn of the Universe. Gravitational waves with characteristic power spectra are produced when a first-order phase transition occurs in the primaeval Universe. Shear stresses set up in the fluid of the asymmetric Universe due to the collision and expansion of bubbles at a low temperature are the source of these gravitational waves. Initially, sound or overlapping pressure, these shear stresses may be powerful enough to generate turbulence when the bubbles collide. We require four properties to describe the dynamics of phase transitions: the strength parameter α_n, the bubble nucleation temperature T_n, the transition rate parameter β, and the bubble wall speed v_w. In principle, all of these properties can be computed using a particle physics model, but these calculations are beyond our scope.

It is known to us that the power spectrum of gravitational waves, produced as a result of a phase transition that occurred at the dawn of the universe, when the range of temperature was between 100 GeV to 1 TeV, contains information regarding the parameters mentioned above [5]. Thus we have LISA being both an astrophysical observatory and a particle-physics experiment. As we concluded from the scalar extended models, we require physics beyond the Standard Model for a strong first-order phase transition, and thus LISA may discover new and extremely important physics in the future. There is also the theory that dark matter, a hypothetical form of matter assumed to account for nearly 85% of the matter in the Universe, is the trigger for strong electroweak phase transitions, which has been proposed with explanations from strong first-order phase transitions using the Doublet Model.

This is a dynamic and developing field. Therefore, there is much fascinating research to be done in the near future, up until LISA's successful launch in 2034, to realize the mission's potential scientific reward.

Appendix

1. Code for the Standard Model

```
import numpy as np
import matplotlib.pyplot as plt
import warnings
warnings.filterwarnings("ignore")

MH = 125
MW = 81
MZ = 91
Mt = 173
lambda_ = 0.5

def potential(phi, T):
        A = (((MH ** 3) + 6 * (MW ** 3) + 3 * (MZ ** 3))
         / (12 * np.pi * (phi ** 3)))

        D = ((MH ** 2 + 6 * MW ** 2 + 3 * MZ ** 2 + 6
                * Mt ** 2) / (12 * (phi ** 2)))

        T0 = (MH / ((2 * D) ** 0.5))

        V = ((((D / 2) * (T * T - T0 * T0) * phi * phi)
        - ((A / 3) * T * phi * phi * phi)
        + ((lambda_ / 4) * (phi ** 4))))

        return V

phi = np.linspace(0.01, 250, 2500)
for temp in [0, 75, 150]:
    potx = potential(phi, temp)
    plt.plot(phi, potx, label = ("V at T = " +
     str(temp) + " GeV"))

plt.xlabel('phi')
plt.ylabel('V(T, phi)')
plt.legend()
```

```
plt.savefig('standard.png', dpi=800)
plt.show()
```

2. Code for the Singlet Model

```
import numpy as np
from scipy import integrate
import matplotlib.pyplot as plt
import warnings
warnings.filterwarnings("ignore")

lambdaH = (125 * 125) / (246 * 246)
v = 246
nh = 1
nt = -12
ns = 1
nw = 6
nz = 3
g = 0.652
gprime = 0.352
yt = 0.995
muS = 10
lambdaHS = 5

def JF(x2, plus_minus):
    arr = []
    if plus_minus == 1:
        for x in x2:
            arr.append((integrate.quad(lambda y: (y * y *
             np.log(1 + np.exp(-((y * y + x) ** 0.5)))),
              0, np.inf))[0])

    elif plus_minus == -1:
        for x in x2:
            arr.append((integrate.quad(lambda y: (y * y *
             np.log(1 - np.exp(-((y * y + x) ** 0.5)))), 0,
              np.inf))[0])

    return np.array(arr)
```

```python
def potential(T, phi):

    Vtree = ((lambdaH / 4) * (phi ** 2 - v ** 2)
     * (phi ** 2 - v ** 2))

    Mt2 = yt * yt * phi * phi / 2
    Mh2 = abs(3 * lambdaH * phi * phi - lambdaH * v * v)
    Ms2 = muS * muS + lambdaHS * phi * phi / 2
    Mw2 = g * g * phi * phi / 4
    Mz2 = (g * g + gprime * gprime) * phi * phi / 4

    Vcw1 = ((1 / (64 * np.pi * np.pi)) * (Mh2 ** 2)
     * (np.log(Mh2 / (v * v)) - (3 / 2)))

    Vcw2 = ((1 / (64 * np.pi * np.pi)) * (Ms2 ** 2)
     * (np.log(Ms2 / (v * v)) - (3 / 2)))

    Vcw3 = (6 * (1 / (64 * np.pi * np.pi)) * (Mw2 ** 2)
     * (np.log(Mw2 / (v * v)) - (5 / 6))
      + 3 * (1 / (64 * np.pi * np.pi)) * (Mz2 ** 2)
       * (np.log(Mz2 / (v * v)) - (5 / 6)))

    Vcw4 = (-12 * (1 / (64 * np.pi * np.pi)) * (Mt2 ** 2)
     * (np.log(Mt2 / (v * v)) - (3 / 2)))

    if T == 0:
        VT = np.zeros(Vtree.shape)
        VT0 = np.zeros(Vtree.shape)
    else:
        VT = (nt * (T ** 4) * JF(Mt2 / (T * T), 1)
         + nh * (T ** 4) * JF(Mh2 / (T * T), -1)
          + ns * (T ** 4) * JF(Ms2 / (T * T), -1)
           + nw * (T ** 4) * JF(Mw2 / (T * T), -1)
            + nz * (T ** 4) * JF(Mz2 / (T * T), -1))

        VT0 = (nt * (T ** 4)
         * JF([(yt * yt * 0.01 * 0.01 / 2) / (T * T)], 1)
         + nh * (T ** 4) * JF([(3 * lambdaH * 0.01 * 0.01
          -  lambdaH * v * v) / (T * T)], -1) + ns * (T ** 4)
           * JF([(muS * muS + lambdaHS * 0.01 * 0.01 / 2)
            / (T * T)], -1) + nw * (T ** 4)
             * JF([(g * g * 0.01 * 0.01 / 4)
             / (T * T)], -1) + nz * (T ** 4)
              * JF([((g * g + gprime * gprime)
              * 0.01 * 0.01 / 4) / (T * T)], -1))

    return ((Vtree + Vcw1 + Vcw2 + Vcw3 + Vcw4 + VT - VT0)
```

```
    / (v ** 4))

phi = np.linspace(0.01, 350, 3500)
for temp in [0, 50, 75]:
    potx = potential(temp, phi)
    plt.plot(phi / v, potx, label = ("V at T = " + str(temp) + " GeV"))
plt.xlabel('phi / v')
plt.ylabel('V(T, phi) / (v ** 4)')
plt.legend()
plt.savefig('singlet.png', dpi=800)
plt.show()
```

3. Code for the Doublet Model

```
import numpy as np
from scipy import integrate
import matplotlib.pyplot as plt
import warnings
from math import sqrt
warnings.filterwarnings("ignore")

v = 246
lamb_h = 0.129
lamb_s = 0.1
nh = 1
n_plus_minus = 2
n_H = 1
n_A = 1
nw = 6
nz = 3
nt = -12
n_WL = 2
n_ZL = 1
n_gammaL = 1
g = 0.652
gprime = 0.352
yt = 0.995
```

```
def JF(x2, plus_minus):
    arr = []
    if plus_minus == 1:
        for x in x2:
            arr.append((integrate.quad(lambda y: (y * y *
                np.log(1 + np.exp(-((y * y + x) ** 0.5)))),
                 0, np.inf))[0])

    elif plus_minus == -1:
        for x in x2:
            arr.append((integrate.quad(lambda y: (y * y *
                np.log(1 - np.exp(-((y * y + x) ** 0.5)))),
                 0, np.inf))[0])

    return np.array(arr)

def pot(phi, T, lamb_1, lamb_2, lamb_3, m1):

        PI_S = (((gprime ** 2) / 8) + ((g ** 2 + gprime ** 2) / 16)
         + (lamb_s / 2) + (lamb_1 / 12) + ((lamb_1 + lamb_2 +
          2 * lamb_3) / 24) +  ((lamb_1 + lamb_2 - 2 * lamb_3) / 24))

        PI_W = 2 * g * g * T * T
        PI_Y = 2 * gprime * gprime * T * T
        DelT2 = ((((g * g * phi * phi / 4) + PI_W -
         (gprime * gprime * phi * phi / 4)
         - PI_Y) ** 2) + (g * g * gprime * gprime
          * phi * phi * phi * phi / 4)

        Mh2 = 2 * lamb_h * phi * phi
        M_plus_minus2 = m1 ** 2 + lamb_1 * phi * phi / 2
        M_plus_minus2T = M_plus_minus2 + PI_S
        MH2 = abs(m1 ** 2 + (lamb_1 + lamb_2 + 2 * lamb_3)
         * phi * phi / 2)

        MH2T = MH2 + PI_S
        MA2 = abs(m1 ** 2 + (lamb_1 + lamb_2 - 2 * lamb_3)
         * phi * phi / 2)

        MA2T = MA2 + PI_S
        Mw2 = g * g * phi * phi / 4
        Mw2T = Mw2 + PI_W
        Mz2 = (g * g + gprime * gprime) * phi * phi / 4
        Mz2T = (Mz2 + PI_W + (DelT2 ** 0.5)) / 2
        Mt2 = yt * yt * phi * phi / 2
        M_gamma2 = Mz2
        M_gamma2T = (Mz2 + PI_W + PI_Y - (DelT2 ** 0.5)) / 2
```

```
Vtree = np.array((lamb_h / 4) *
 (((phi ** 2 - v ** 2) ** 2) - (v ** 4)))

Vcw1 = ((1 / (64 * np.pi * np.pi)) * (Mh2 ** 2)
 * (np.log(Mh2 / (v * v)) - (3 / 2)))

Vcw2 = ((2 / (64 * np.pi * np.pi))
 * (M_plus_minus2 ** 2) * (np.log(
        M_plus_minus2 / (v * v)) - (3 / 2)))

Vcw3 = ((1 / (64 * np.pi * np.pi)) * (MH2 ** 2)
 * (np.log(MH2 / (v * v)) - (3 / 2)))

Vcw4 = ((1 / (64 * np.pi * np.pi)) * (MA2 ** 2)
 * (np.log(MA2 / (v * v)) - (3 / 2)))

Vcw5 = (6 * (1 / (64 * np.pi * np.pi))
 * (Mw2 ** 2) * (np.log(Mw2 / (v * v))
 - (5 / 6)) + 3 * (1 / (64 * np.pi * np.pi))
  * (Mz2 ** 2) * (np.log(Mz2 / (v * v)) - (5 / 6)))

Vcw6 = (-12 * (1 / (64 * np.pi * np.pi)) * (Mt2 ** 2)
 * (np.log(Mt2 / (v * v)) - (3 / 2)))

Vcw = (Vcw1 + Vcw2 + Vcw3 + Vcw4 + Vcw5 + Vcw6)

Vdaisy = ((-T / (2 * np.pi * np.pi)) *
 (n_plus_minus * ((M_plus_minus2T ** 1.5)
 - (M_plus_minus2 ** 1.5)) + n_H *
  ((MH2T ** 1.5) - (MH2 ** 1.5))
  + n_A * ((MA2T ** 1.5) - (MA2 ** 1.5))
   + n_WL * ((Mw2T ** 1.5) - (Mw2 ** 1.5))
   + n_ZL * ((Mz2T ** 1.5) - (Mz2 ** 1.5))
    + n_gammaL* ((M_gamma2T ** 1.5) - (M_gamma2 ** 1.5))))

if T == 0:
        VT = np.zeros(Vtree.shape)
        VT0 = np.zeros(Vtree.shape)

else:
        VT = (np.array((1 / (2 * np.pi * np.pi))
         * (nt * (T ** 4) * JF(Mt2 / (T * T), 1)
         + nh * (T ** 4) * JF(Mh2 / (T * T), -1)
          + n_plus_minus * (T ** 4)
           * JF(M_plus_minus2 / (T * T), -1)
           + n_H * (T ** 4) * JF(MH2 / (T * T), -1)
```

```
                    + n_A * (T ** 4) * JF(MA2 / (T * T), -1)
                     + nw * (T ** 4) * JF(Mw2 / (T * T), -1)
                      + nz * (T ** 4) * JF(Mz2 / (T * T), -1))))

            VT0 = ((1 / (2 * np.pi * np.pi)) * (nt * (T ** 4)
             * JF([(yt * yt * 0.01 * 0.01 / 2) / (T * T)], 1)
              + nh * (T ** 4)
               * JF([abs(3 * lamb_h * 0.01 * 0.01
                - lamb_h * v * v) / (T * T)], -1)
                + n_plus_minus * (T ** 4)
                 * JF([(m1 ** 2 + lamb_1 * 0.01 * 0.01 / 2)
                 / (T * T)], -1) + n_H * (T ** 4)
                  * JF([(m1 ** 2 + (lamb_1 + lamb_2 + 2 * lamb_3)
                   * 0.01 * 0.01 / 2) / (T * T)], -1)
                   + n_A * (T ** 4) * JF([(m1 ** 2 +
                    (lamb_1 + lamb_2 - 2 * lamb_3)
                     * 0.01 * 0.01 / 2) / (T * T)], -1)
                     + nw * (T ** 4)
                      * JF([(g * g * 0.01 * 0.01 / 4)
                      / (T * T)], -1) + nz * (T ** 4)
                       * JF([((g * g + gprime * gprime)
                        * 0.01 * 0.01 / 4)
                       / (T * T)], -1)))

            VT0 = VT0 * np.ones(Vtree.shape)

        return ([(Vtree + Vcw + VT - VT0 + Vdaisy) / (v ** 4),
         Vtree, Vcw, VT, VT0, Vdaisy])

phi = np.linspace(0.01, 350, 3500)
rng = np.random.default_rng()
l1s = []
l2s = []
l3s = []
ms = []
phics = []
Tcs = []
phibyTs = []
flag = 0

for test in range(1):
    print("Parameters used:")
    l1 = rng.random() * 3 + 2
    l2 = 0.5 * rng.random() - 0.5 * rng.random()
    l3 = 1.5 * rng.random() - 1.5 * rng.random()
    m = rng.integers(low=35, high=125)
    l1s.append(l1)
```

```
l2s.append(l2)
l3s.append(l3)
ms.append(m)

print("Lambda 1 =", round(l1, 6))
print("Lambda 2 =", round(l2, 6))
print("Lambda 3 =", round(l3, 6))
print("M1 =", round(m, 6), "GeV")
print("\n")
print("Physical masses:")
print("Field Dependent Mass of Charged Scalar Field S+- =",
 round(sqrt(abs(m ** 2 + l1 * v * v / 2)), 6), "GeV")

print("Field Dependent Mass of Neutral Scalar Field H =",
 round(sqrt(abs(m ** 2
  + (l1 + l2 + 2 * l3) * v * v / 2)), 6), "GeV")

print("Field Dependent Mass of Neutral Scalar Field A =",
 round(sqrt(abs(m ** 2
  + (l1 + l2 - 2 * l3) * v * v / 2)), 6), "GeV")

temp = 120
add = 0.5
print("\n")
print("Temperatures checked:")
while temp > 119.5 and temp < 140.5:
        maxi = False
        potx = pot(phi, temp, l1, l2, l3, m)
        for i in range(1, 2499):
                if (potx[0][i + 1] < potx[0][i]
                 and potx[0][i - 1] < potx[0][i]):

                        maxi = True

                if (potx[0][i + 1] > potx[0][i]
                 and potx[0][i - 1] > potx[0][i]
                 and abs(potx[0][i] - potx[0][0])
                  <= 1e-3 and maxi == True):

                        add = 0.12

                if (potx[0][i + 1] > potx[0][i]
                 and potx[0][i - 1] > potx[0][i]
                 and abs(potx[0][i] - potx[0][0])
                  <= 1e-4 and maxi == True):

                        add = 0.012
```

```
                if (potx[0][i + 1] > potx[0][i]
                 and potx[0][i - 1] > potx[0][i]
                 and abs(potx[0][i] - potx[0][0])
                  <= 1e-5 and maxi == True):

                        pot0_val = potx[0][i]
                        phic_val = phi[i]
                        phics.append(phic_val)
                        Tcs.append(temp)
                        phibyTs.append(phic_val / Tc_val[0])
                        flag = 1
                        plt.plot(phi / v, potx[0]
                                , label = "V at T = "
                         + str(round(temp, 6)) + " GeV")

                        break
            if flag == 1:
                flag = 0
                break
            print(round(temp, 6), "GeV")
            temp += add
            add = 0.5

        print("PHIC = ", round(phics[test], 6), "GeV")
        print("TC = ", round(Tcs[test], 6), "GeV")
        print("PHIC / TC = ", round(phibyTs[test], 6))

plt.xlabel('phi / v')
plt.ylabel('V(phi, T) / (v ** 4)')
plt.legend()
plt.savefig('doublet.png', dpi=800)
plt.show()
```

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