EE 220 : Signals and Systems

Department of Electronics and Electrical Engineerng

Indian Institute of Technology, Guwahati

End Semester Examination - 22 Nov 2023

Instructions : [1.] Solve all questions.

[2.] The question paper is in two parts. Part-A is subjective and can be solved between 9 AM - 12 noon. Part-B is multiple-choice and will be distributed at 11 AM and has to be submitted at 12 noon along with the Part-A answersheet.

[3.] Maximum marks: Part A = 30, Part B = 15.

PART - A

Q1: Consider a linear, time-invariant system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^{|n|}$$

Find the Fourier series of the output $y[n]$ for each of the following inputs: [8 marks]

a.

$$x[n] = \sin\left(\frac{3\pi n}{4}\right)$$

b.

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]\ ;\ k \in \mathbb{Z}$$

c. $x[n]$ is periodic with period 6, and

$$x[n] = \begin{cases} 1, & n = 0, \pm 1 \\ 0, & n = \pm 2, \pm 3, \pm 4 \end{cases}$$

d.

$$x[n] = j^n + (-1)^n$$

Q2: In the continuous-time Fourier series synthesis equation, the summation limits are from $-\infty$ to ∞:

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_o t};$$

While in the discrete-time Fourier series synthesis, the summation limits are from 0 to $N-1$, where N is the fundamental period of $x[n]$:

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\Omega_o n};$$

Explain the reasoning for this dissimilarity. [2 marks]

Q3: The synthesis equation for an aperiodic DT signal can be written as [6 marks]

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(\Omega) e^{j\Omega n} d\Omega$$

a. Consider another frequency variable Ω_1 and show that

$$\sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega_1 n} = \frac{1}{2\pi}\int_{2\pi} X(\Omega) \sum_{n=-\infty}^{\infty} e^{j(\Omega-\Omega_1)n} d\Omega$$

b. Consider $\sum_{n=-\infty}^{\infty} e^{j(\Omega-\Omega_1)n}$ as the Fourier series representation of some continuous-time periodic function. Show that

$$\sum_{n=-\infty}^{\infty} e^{j(\Omega-\Omega_1)n} = 2\pi \sum_{n=-\infty}^{\infty} \delta(\Omega-\Omega_1+2\pi n)$$

c. By combining the results of parts (i) and (ii), establish that

$$\sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = X(\Omega)$$

Q4: The figure below gives a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure. [2+2+1+1+1+1 = 8 marks]

a. Write the expression for $x_p(t)$ in terms of $x(t)$ and impulse functions.

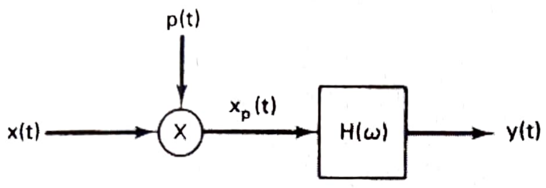
b. Write the simplified expression for $X_p(\omega)$.

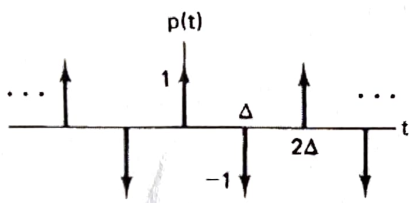
c. For $\Delta < \pi/2\omega_M$, sketch and label the Fourier transform of $x_p(t)$

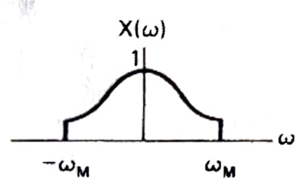
d. For $\Delta < \pi/2\omega_M$, sketch and label the Fourier transform of $y(t)$.

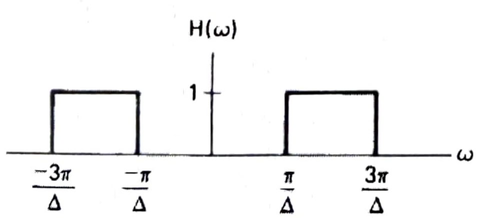
e. For $\Delta < \pi/2\omega_M$, sketch and label the Fourier transform of the system that will recover $x(t)$ from $x_p(t)$.

f. What is the maximum value of Δ in relation to ω_M for which $x(t)$ can be recovered from $x_p(t)$.

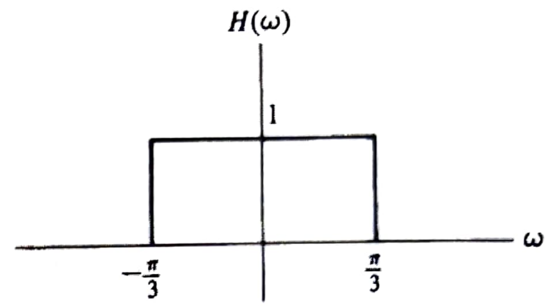








Q5: Consider a lowpass filter with real frequency response H(w) as shown in Figure [2+1+1+1+1 =6 marks]



a. Show that the filter impulse response $h(t)$ is real-valued.

b. Determine whether the filter impulse response $h(t)$ is even or odd.

c. Determine whether the filter impulse response $h(t)$ is causal or non-causal.

d. Consider the filter input

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - 9n)$$

Sketch and label the Fourier transform of the filter output $y(t)$.

e. Write the expression for the filter output $y(t)$ for the input considered in part (d).

PH 203 – Classical Mechanics, Dr. M. K. Nandy, Endsem, 21 Nov. 2023

(Show labelled diagrams and details of calculations)

1. A comet of mass m is moving due to the gravitational attraction of the sun of mass M. Assume both m and M to be point particles. m moves in the XY plane with M sitting sationary at the origin. Let (x, y) be the instantaneous co-ordinate of m.

   (a) Write the expression for the kinetic energy T and potential energy V in the Cartesian (x, y) system.

   (b) Transform the above Cartesian expressions for T and V into the plain polar (r, θ) system. .

   (c) Find the equations of motion of the comet employing Lagrange's equation.

   (d) (i) What quantities are conserved?

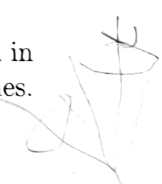
   (ii) What property of the Lagrangian L make them conserved?

   (iii) Express the conserved quantities in the plain polar system.

2. (a) Write the Euler's equations (for $\dot{\omega}_1$, $\dot{\omega}_2$, $\dot{\omega}_3$) for a rigid body.

   (b) Consider the earth as a rigid body (oblate spheroid, with x_3 as symmetry axis). Solve the Euler's equations for the earth. Obtain the expression for precession frequency. $\omega_1 = \omega_2$ 300 days

   (c) Draw the body cone and space cone diagrams and identify them in the diagram. Indicate the direction of rotation by arrows on the cones. Describe (in 2-3 lines) the motion(s) of the cones.



3. For a one-dimensional UNDAMPED simple harmonic oscillator

   (a) Write the Hamiltonian.

   (b) Write the Hamilton-Jacobi equation. $\delta S/\delta t - H$

   (c) Implement separation-of-variables and solve the Hamilton-Jacobi equation.

4. For a single particle with instantaneous position (x, y, z), find the following Possion brackets

   (a) $[x, p_x^2]$, (b) $[x, L_y]$, (c) $[L_x, L_y]$

   where p_x, p_y, p_z and L_x, L_y, L_z are Cartesian components of the linear momentum and angular momentum respectively.

5. Consider a one-dimensional DAMPED simple harmonic oscillator (of natural frequency ω and damping constant γ).

   (a) Write the equation-of-motion.

   (b) Find the solution of the equation-of-motion.

   (c) Write the conditions for the three cases (underdamped, overdamped, critically damped). $\lambda^2 > \omega^2/4$

   (d) Find the solution for the critically damped case. $\lambda^2 = \omega^2/4$

Indian Institute of Technology

PH205 (Nov 2023)

End-semester Examination

Full marks: 50 ***Time: 3 hours***

***Section 1**: Write the answer(s) for each question in your answer script. Each question carry one mark.*

1. In a doped semiconductor sample of cross-sectional area A and length L, the doping density, N_D, varies with distance (x) as $N_D(x) = N_0 \exp\left(-\frac{x}{L}\right)$, where N_0 is a constant. Assuming that the mobility μ of the majority carrier remains constant, the electrical conductivity (σ) of the sample is given by (expression) $\sigma =$ ....

2. The energy E(**k**) of electrons of wave vector **k** in a solid is given by $E(\mathbf{k})=Ak^2+Bk^4$, where A and B are constants. The effective mass (m*) of the electron at $|\mathbf{k}| = k_0$ is given by

m* = ...

3. When the PN junction is forward biased, the sequence of events (among diffusion, drift, injection and recombination) that take place are:

(a) ...., (b) ...., (c) ....

4. The junction capacitance (C) of linearly graded p-n junction varies with the applied reverse bias, V_R, as $C \propto$ ....

5. A GaAs LED emits light in the .... region and GaN emits light in the ....region of spectrum.

6. The sensitivity of a photo diode depends on parameters, such as : ...., ...

7. **Match** items in Group I with items in Group II, most suitably.

Group I

P. LED Q. Avalanche Photodiode R. tunnel diode S. LASER

Group II

1. Heavy doping, 2. Coherent radiation, 3. Spontaneous emission, 4. Current gain

P - ...., Q - ...., R - ...., S -.................................

8. Given that for GaAs, bandgap E_g=1.43 eV, and for AlAs, E_g=2.75 eV, using virtual crystal approximation, the bandgap of $Al_{0.3}Ga_{0.7}As$ alloy will be

$E_{g,\ alloy} =$ .... eV

9. Construction wise, MOSFETs can be categorized into four types as follows:

(a) .... (b) ....

(c) .... (d) ....

10. Transconductance (g) of a JFET is defined as (formula): g = ..., and its unit is ....

Section 2

11. A semiconductor sample is n-type doped. Briefly describe an experimental technique with schematic to determine its bandgap energy (E_g) and the donor energy level (E_D). Use a graphical method for the determination of E_g and E_D. [3]

12. Draw the schematic diagram of a quantum well laser for low threshold operation and briefly explain its working principle. [2+2]

13. The absorption coefficient near the band-edge of Si is ~10^3 cm^{-1}. What is the minimum thickness (in cm unit) of a sample that can absorb 90% of the incident light? [3]

14. In a p-type GaAs sample, electrons are injected from a contact. If the electron mobility is 4000 cm^2/V-s at 300K, calculate the diffusion length for electrons. Take recombination time as 0.6 ns. [3]

15. A Si diode is being used as a thermometer by operating it at a fixed forward (bias) current. The voltage is then a measure of the temperature. At 300K, the diode voltage is found to be 0.6V. How much will be the *voltage change* if the temperature changes by 1 K? [4]

16. Consider a semiconductor laser having an optical confinement factor of unity. If the threshold carrier density is 1.32×10^{18} cm^{-3} and the active layer thickness is 20 μm, calculate the threshold current density (J_{th}) for the lasing action to start. Assume a radiative recombination time of 2.4 *ns*. [3]

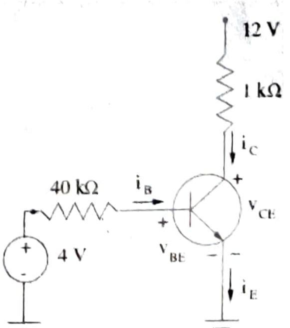
17. A Schottky barrier is formed between a metal having a work function of 4.3 eV and p-type Si (electron affinity χ=4 eV). The acceptor doping in the Si is 10^{17}/cm^3. (a) Calculate the work function of the semiconductor (φ_s). (b) Draw the equilibrium band diagram, showing numerical value of qV_0. [3+2]

18. Consider a Si solar cell with area=1 cm^2, acceptor doping density=5×10^{17} cm^{-3}, donor doping density=10^{16} cm^{-3} and photocurrent of 25 mA. If $D_n = 20$ cm^2/s, $D_p = 10$ cm^2/s, $\tau_n = 3\times10^{-7}\,s$, $\tau_p = 10^{-7}\,s$, calculate the open circuit voltage (V_{oc}) of the solar cell operated at 300K. [4]

19. Consider a GaAs p-n LED with the p-region on the top to emit light, and D_n=30 cm^2/S, D_p=15 cm^2/s, N_a=5x10^{16} cm^{-3} and N_d=5x10^{17} cm^{-3}, τ_n=10 ns, τ_p=100 ns. Calculate the diode injection efficiency (γ_{inj}). [3]

20. A Si p-n junction with cross-sectional area, A=0.001 cm^2 is formed with $N_a = 10^{15}$ cm^{-3}, $N_d = 10^{17}$ cm^{-3}. Calculate (a) built-in potential, V_0, (b) current I with a forward bias of 0.5 V. Assume that the current is diffusion dominated. Assume $\mu_n = 1500$ cm^2/V-s, $\mu_p = 450$ cm^2/V-s, $\tau_n = \tau_p = 2.5\,\mu s$. [2+3]

21. For the given circuit below for a Si NPN transistor with β=100, calculate the I_B, I_C and V_{CE}. [3]



Useful Formula:

$\gamma_{inj} = \frac{J_n}{J_n+J_p}$, $L_n=\sqrt{D_n\tau_n}$, $J_n=\frac{eD_nn_p}{L_n}$

$V_{oc}=\frac{mkT}{e}\ln(1+\frac{I_L}{I_0})$, $I_0 = A\left[\frac{eD_nn_p}{L_n}+\frac{eD_pP_n}{L_p}\right]$

$\alpha_{loss} = -\frac{1}{L}lnR$, $R=\frac{(n_2-n_1)^2}{(n_2+n_1)^2}$, $n_{th} = \frac{J_{th}\tau_r}{e\,d_{laser}}$

$I = I_0[\exp\left(\frac{eV}{kT}\right)-1]$

n_i=1.5×10^{10} /cc for Si

$V_0 = \frac{kT}{e}\ln(\frac{N_aN_d}{n_i^2})$

Indian Institute of Technology Guwahati
Department of Physics

End-semester Examination, November 23rd 2023

Subject: Heat and Thermodynamics (PH 207)

Full Marks: 50

Time: 3 hours

1. Consider two blocks of Copper, each having heat capacity $C_v = 200$ J K^{-1}. Initially one of the blocks has a uniform temperature 300 K while the other has a uniform temperature of 310 K. The blocks are placed in thermal contact with one another and are surrounded with perfect thermal insulation.

   (a) What are the final equilibrium temperatures of the blocks?

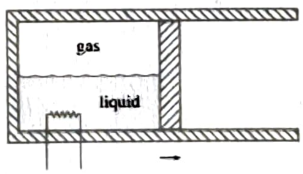
   (b) Calculate changes in entropy of each block in this process.

   (c) Is the total change in entropy of the system consistent with second law of thermodynamics? Explain. [1+2+2]

2. The Figure shows a hypothetical liquid in equilibrium with its vapour. The liquid and the gas are confined in a cylinder by a piston. An electrical resistor of resistance 50 Ω is immersed in the liquid. The liquid, the gas and the resistor together constitute the thermodynamic system. The initial thermodynamic state of the system is described by pressure $P_1 = 2.5 \times 10^5$ Pa, volume $V_1 = 0.22$ m^3 and temperature T_1=300 K. A constant current 0.5 A is passed through the resistor for 1600 s. The piston moves slowly to the right against a constant external pressure equal to the vapour pressure of the liquid, P_1. Some of the liquid now vaporises. Assume the process is adiabatic where T and P remain constant so that thermodynamic parameters of the final state are $T_2 = T_1$, $P_2 = P_1$ and V_2=0.24 m^3.

   (a) Calculate the change in Enthalpy in the process. ΔH

   (b) Is the process reversible? Explain. [3+2]



3. When an ordinary rubber band is hung from a clamp and stretched with constant downward tension τ, gentle heating is observed to cause the rubber band to contract in length. To keep the length L of the rubber band constant during heating, τ must be increased. From this information,

   (a) show that $\left(\frac{\partial T}{\partial L}\right)_S = \left(\frac{\partial \tau}{\partial S}\right)_L$.

   (b) Using result of (a), predict whether adiabatic stretching of the rubber band will cause a heating or cooling effect. [2+3]

4. Consider a simple magnetic system at temperature T in a magnetic field H. If the magnetic field is changed to $H + \Delta H$ isothermally, the change in the entropy is found to be $\Delta S = -\frac{CH\Delta H}{T^2}$, C is a constant, characteristic of the system.

   (a) Find out the relation between magnetization M and temperature T.

   (b) Plot M versus T for small H, that is $H \to 0$. [3+2]

5. (a) Prove that $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$ for a system whose thermodynamic state is described by Pressure P, Volume V, Temperature T, Entropy S.

(b) Using (a), obtain Clausius Clapeyron equation $\frac{dP}{dT} = \frac{L}{T(v_V - v_L)}$ for a vessel containing a liquid in equilibrium with its vapor; L, P, T, v_V, v_L are the Latent heat of vaporisation per unit mass, the saturated vapour pressure, the equilibrium temperature, the volume per unit mass of the vapour phase and volume per unit mass of the liquid phase, respectively. [2+3]

6. The equation for inversion curve of ^{4}He is $P = -21 + 5.44T - 0.132T^2$, T and P are in K and in atm, respectively.

$T \geq T_{inv}$ $\mu < 0$

(a) What is the maximum temperature above which the Joule-Thomson effect will produce heating instead of cooling?

(b)Find the pressure and the corresponding temperature above which Joule-Thomson effect will produce heating instead of cooling. [2+3]

7. How many grams of CO_2 gas is dissolved in a 1 Litre bottle of soda if the manufacturer used a pressure of 2.4 atm in the bottling process at 25^0 C? Given that the Henry-Dalton constant is 29.76 atm/(mol - Litre^{-1}), molar masses of C and O are 12 gm and 16 gm, respectively. [5]

8. A man slowly exhaled an ideal gas molecule in a room filled with same gas. The temperature and the pressure in the room are 300 K and 10^5 Pa, respectively. Calculate the time (in seconds) it will take for the molecule to diffuse to some point at a distance of 1 meter? What is the mean speed of the molecule? (Given that the collision cross section is 10^{-20} m^2, mean collision time is 1.4×10^{-8} second and Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J K^{-1}.) [4+1]

9. The Maxwell velocity distribution for a monatomic ideal gas with molecular mass m is $f(\vec{v}) = \frac{1}{(\sqrt{\pi} v_{th})^3} e^{-v^2/v_{th}^2}$, where v is the molecular speed.

(a) Express pressure P in the system in terms of m, particle density n and $\langle v^2 \rangle$.

(b) Express P and temperature T in the system in terms of v_{th}, the thermal velocity. [1+4]

Use the following:

$$\int_0^\infty x^n e^{-ax^2} dx = \frac{\Gamma\left(\frac{n+1}{2}\right)}{2a^{(n+1)/2}}$$

$$n\Gamma(n) = \Gamma(n+1); \quad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}, \quad \Gamma(1) = 1$$

10. Consider a system of charged particles (of charge e) confined to a volume V. The particles are in thermal equilibrium at temperature T in presence of an electric field of magnitude E along z-direction. If $n(z)$, the density of particles is given as $n(z) = n(0)\, exp\left(\frac{eEz}{k_B T}\right)$ then

(a) if the transport of particles along z is characterised by a diffusion constant D, then find the flux J_D arising from the particle density gradient.

(b) If the particles are also characterised by a mobility μ that connects their drift speed $\bar{v}$ along z (average speed along z) with the electric field E ($\bar{v} \propto E$) then find the flux J_μ associated with this mobility.

(c) Show that, at global equilibrium, $\mu = \frac{eD}{k_B T}$. [2+2+1]

Semiconductor

Final Semester

Course: PH209

Full marks: 50

1. a) Find the *CC h* parameters in terms of the *CE h* parameters. [Marks 3]

b) For the circuit shown below (Figure 1), find *small signal voltage gain* (A_{vs}) *and input impedance* (in terms of A_{vs}) as seen from the source terminals. Analyze the problem ***without using Feedback concept*** **and using Millar's theorem** applied to the resistance R_1. Use h-parameter model and assume $h_{oe} = h_{re} = 0$. [Marks 5+2]

$R' = \frac{R_1}{1-A_v}$ $R'' = \frac{R_1}{1-\frac{1}{A_v}}$

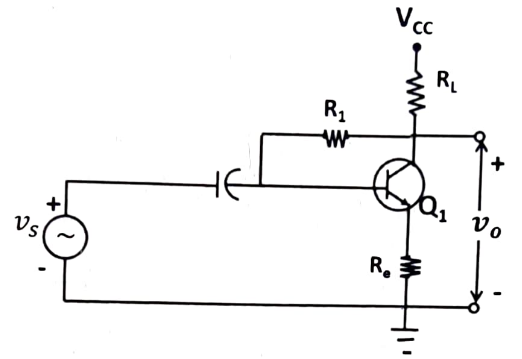


Figure 1

2. a) How negative feedback influence *voltage gain, input impedance* and *output impedance* of a voltage series amplifier? [Marks 1]

b) For the circuit shown below (figure 2), find the voltage gain, and input impedance seen from the source voltage terminals ***using the feedback concept*** *(without using the direct formula of voltage gain and input impedance).*

[Use h-parameter model and assume, $R_s = 0, h_{fe} = 50$, $hie{=}1.1k$, $hre{=}hoe{=}0$, and current through 4.7k resistance in the emitter of Q1 is negligible as compared to current through R_2, and both the transistors are identical [Marks 10]

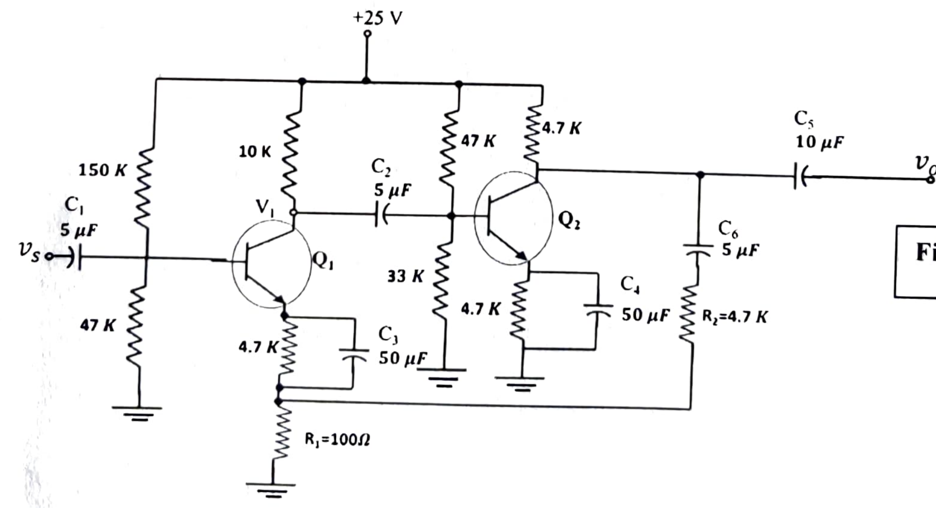


Figure 2

3. a) Briefly explain how a differential amplifier with very large CMRR can be used for amplification with minimal effect from noise signal. easy [Marks 3]

$\rho = \frac{|A_d|}{|A_c|}$

b) What is the *virtual ground* of an ideal OPAMP? [Marks 2]

c) Design Log and Anti-log amplifier using OPAMP (ideal), and find expression of their outputs. Using these Log and Anti-log amplifiers, draw a multiplier circuit, and show the outputs at every stage.

[Marks 2+2+1+2] -7 -6

d) Show that the circuit of the accompanying figure 3 can simulate a grounded inductor if $R_1 > R_2$. In other words, show that the reactive part of the input impedance of the circuit is positive if $R_1 > R_2$. The OPAMP is an ideal one.

[Marks 6]

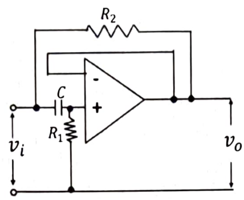


Figure 3

4. An OPAMP-based oscillator circuit (Figure 4). The signal v_s is given once and then the key switched from B point to A point

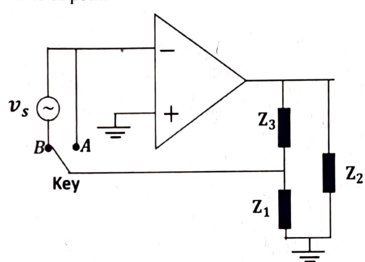


Figure 4

Considering a **non-ideal** OPAMP having *non-zero* output impedance (R_O) and *infinite input impedance* , and finite gain without load, Show that
either (i) Z_1 and Z_2 are capacitor and Z_3 is an inductor ,
or (ii) Z_1, Z_2 are inductor and Z_3 is a capacitor [Marks 6]
[Note: In a practical oscillator circuit no external signal voltage (v_s) is given, the signal is taken from the noise while switching on the device]

5. If $|I_{DSS}| = 4\ mA$, $V_P = 4\ V$, calculate the quiescent values of I_D, V_{GS} and V_{DS} for the circuit given below [Marks 5]

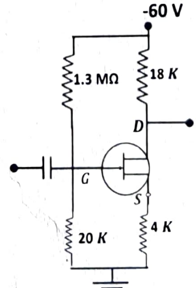


Figure 5