

**Note: All the problems in this tutorial are to be solved via multipole expansion. From the practice point of view, please try some of the problems of tut 3-5 also via multipole expansion.**

1. A circular ring of radius  $R$  (in x-y plane) carries a line charge density on its rim given by  $\lambda(\varphi) = K \cos \varphi$ . Obtain the potential due to this at any point outside the ring ( $r \gg R$ ). What is the equivalent dipole moment and corresponding electric field in this case?
2. A spherical shell of radius  $R$  (origin as the centre) carries a surface charge density  $\sigma(\theta) = K \cos(\theta)$ .
  - (a) Calculate the potential far away from this sphere using multipole expansion
  - (b) Calculate the dipole moment due to this charge distribution.
  - (c) Corresponding electric field
  - (d) Compare the solutions with that of worked out earlier by solving the Laplace's equation.
3. Consider a sphere of radius  $R$  (centred at origin) filled with a uniform volume charge density  $\rho_0$  on the northern hemisphere and that of  $-\rho_0$  on southern hemisphere.
  - a. Find the potential in the region  $r > R$  by multipole expansion.
  - b. From this obtain the very first non zero term.
  - c. From part (a) above, find the expression for dipole moment.
  - d. Work out the approximate expression for electric field due to the very first non zero term in the potential).
4. A spherical shell of radius  $R$  carries a surface charge density  $\sigma(\theta) = K \cos(\theta)$ .
  - (e) Calculate the dipole moment due to this charge distribution.
  - (f) Calculate the potential far away from this sphere and
  - (g) Corresponding electric field.
  - (h) What is the potential in the limit  $r \rightarrow \infty$
5. A sphere of radius  $R$  carries a charge density  $\rho(r, \theta) = K \frac{R}{r^2} (R - 2r) \sin \theta$ . Calculate the very first non zero term in the potential far from the sphere along the z axis.
6. A charge distribution is specified by  $\rho(r, \theta) = \frac{1}{64\pi} r^2 e^{-r} \sin^2 \theta$ . Using multipole approximation, obtain the expression for potential.