EE 220 : Signals and Systems

Department of Electronics and Electrical Engineerng
Indian Institute of Technology, Guwahati
Sample question paper

Instructions: Solve all questions. **[Total:30 marks]**

Q1. Find out whether following signals are even/odd signals [1 mark]

a.

$$x(t) = t^2$$

b.

$$x[n] = 2n + 1$$

Answer: (a) Even. (b) Neither even nor odd.

Q2. Is $x(t) = e^{-|t|}$ absolutely integrable? Draw the graph. [1 mark]
Answer: No

Q3. What is the fundamental period of $x[n] = \sin[3n]$? Explain your answer? [1 mark]
Answer: No. A discrete-time sinusoidal sequence is periodic if its $2\pi/\Omega$ is a rational number. Otherwise, it is non-periodic.

Q4. The input $x(t)$ - output $y(t)$ relationships are described below for two systems. Determine if these are stable, causal, and time invariant systems?

a. $y(t) = \int_{-\infty}^{t/2} x(\tau) d\tau$

b. $y(t) = \frac{d}{dt} x(t)$ [2 marks]

Answer: a. No, No, No. Unstable: Check when $x(t) = 1$ for all t. Not causal: check for -ve time instants.

b. No, Yes, yes. To check causality: use the definition

$$x^{'}(t) = \lim_{h \to 0} \frac{x(t) - x(t-h)}{h}$$

This definition is sufficient to compute the derivative. Hence, it's a causal system. To check stability : Try x(t) = u(t). It will result in bounded input, unbounded output.

Q5: Find the convolution between following sequences. [3 marks]

a. $[2 + j, 0, 1]$ and $[2 - j, 0, -1]$

b. $[1, 2, 3]$ and $[4, 5, -1, -2]$

Answer: [5,0,-2j,0,-1] and [4,13,21,11,-7,-6]

Q6: Find the solution for the system described by the following equation [3 marks]

$$y''(t) + 5y'(t) + 6y(t) = x(t)$$

given the input is $x(t) = e^{-t}u(t)$; and initial conditions are $y(0) = -1/2$ and $y'(0) = 1/2$.

Answer: Homogeneous solution:
Characteristic equation is $r^2 + 5r + 6 = 0$. Roots = -2, -3.
Thus homogeneous solution : $y_h(t) = c_1 e^{-2t} + c_2 e^{-3t}$

Particular solution:
Let's assume the particular solution is of the form ce^{-t} for $t \geq 0$. Substituting this in the given differential equation will result in:

$$y_p(t) = (1/2)e^{-t} \text{ for } t \geq 0$$

So the complete solution becomes:

$$y(t) = y_h(t) + y_p(t) = c_1 e^{-2t} + c_2 e^{-3t} + (1/2)e^{-t} \text{ for } t \geq 0$$

It's derivative: $y'(t) = -2c_1 e^{-2t} - 3c_2 e^{-3t} - (1/2)e^{-t}$
Susbstituting initial conditions will result in: $c_1 = -2; c_2 = 1$
Hence, the complete solution:

$$y(t) = y_h(t) + y_p(t) = -2e^{-2t} + e^{-3t} + (1/2)e^{-t} \text{ for } t \geq 0$$

Q7: Write and plot the continuous time Fourier series coefficients for the signal [3 marks]

$$x(t) = \sum_{m=-\infty}^{\infty} \delta(t - 4m)$$

Solution : The fundamental period is T = 4. Hence, fundamental frequency $\omega_0 = \pi/2$. Note that x(t) is even symmetric, so it is easy to integrate over a period that is symmetric around the origin (at m=0). Let's choose the interval of integration from -2 to 2 rather than 0 to 4 for fun. [It's not going to change the answer as mentioned in the note above.] This results in

$$\begin{aligned} X[k] &= \frac{1}{4}\int_{-2}^{2} \delta(t) e^{-jk\pi t/2} dt \\ &= \frac{1}{4}e^{0} = \frac{1}{4}. \end{aligned}$$

We could have also integrated over 2 to 6 (for m= 1). That would lead to

$$\begin{aligned} X[k] &= \frac{1}{4}\int_{2}^{6} \delta(t-4) e^{-jk\pi t/2} dt \\ &= \frac{1}{4}e^{-jk2\pi} = \frac{1}{4}. \end{aligned}$$

Q8: What are the convergence conditions for writing continuous time Fourier transform representation of a function x(t)? Can we write the Fourier transform of $u(t)$? If yes, explain the steps. If No, why not? [4 marks]

1. $x(t)$ is **absolutely integrable**, i.e.

$$\int_{-\infty}^{\infty} |x(t)| dt \leq B < \infty$$

2. $x(t)$ has a finite number of maxima and minima in any finite interval

3. $x(t)$ has a finite number of finite discontinuities in any finite interval.

Although, $u[t]$ is not absolutely integrable, we have the following trick:
Replace $u(t) = \frac{1}{2}[1 + sgn(t)]$. which leads to $U(\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$

Q9: Find the Laplace transform of $x(t) = e^{-at}\cos(\omega t)u(t)$ and write its region of convergence? [4 marks]
Answer: $\frac{s+a}{(s+a)^2+\omega^2}$; ROC is $\sigma > -a$

Q10: Using the bilateral time-shift and differentiation properties, find the Laplace transform of

$$x(t) = \frac{d^2}{dt^2}(e^{-3(t-2)}u(t-2))$$

[4 marks]

Answer: $\frac{s^2}{s+3}e^{-2s}$ with ROC $\sigma > -3$

Q11: Prove the following identity [4 marks]

$$\delta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t} d\omega$$

Solution: CTFT properties can be used to define impulse function in a new way. Let us start with the definitions first

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega, \tag{1}$$

where

$$X(\omega) = \int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau} d\tau, \tag{2}$$

Replace $X(\omega)$ in Eq. 1:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau} d\tau e^{j\omega t} d\omega$$

Rearranging the integrals, we get:

$$\begin{aligned} x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} x(\tau)\Big[\int_{-\infty}^{\infty} e^{j\omega(t-\tau)} d\omega\Big] d\tau \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} x(\tau)g(t-\tau) d\tau \end{aligned} \tag{3}$$

where

$$g(t) = \int_{-\infty}^{\infty} e^{j\omega t} d\omega$$

Now once again Eq. (3) can be observed to be equivalent to:

$$x(t) = \frac{1}{2\pi} x(t) * g(t)$$

We know from the impulse function property that this expression can be true only if

$$\frac{1}{2\pi} g(t) = \delta(t)$$

Thus, we have a new definition of the impulse function $\delta(t)$:

$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} d\omega$$

But since we know that $\delta(t)$ is an even function, and hence,

$$\delta(t) = \delta(-t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j\omega t} d\omega$$

PS. Note that different texts may have different signs of the exponential within the integral in the above definition, which is alright. It is also intuitive to observe that both integrals reprsent area and it is going to be same in both cases.