Tut - 3

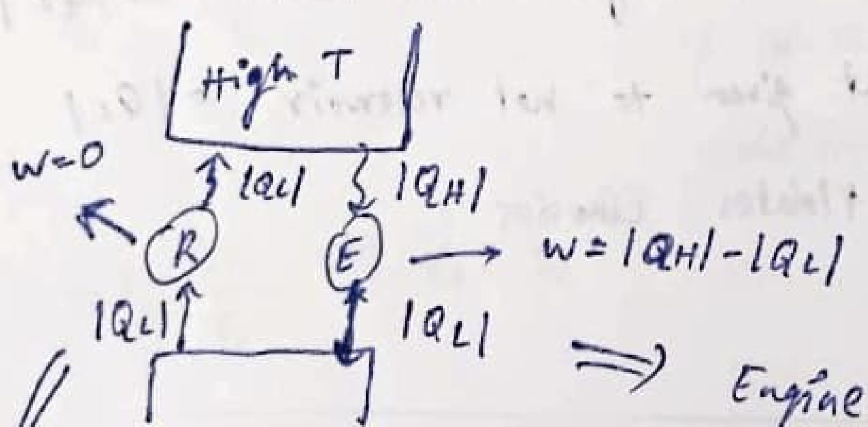
1) Let K = Truth of Kelvin statement

$-K$ = Falsity of " "

C = Truth of Clausius "

$-C$ = Falsity of " "

$\Rightarrow$ If $-K > -C$, $-C > -K$, then two statements are equivalent -



$\Longrightarrow$ Engine is not violating Kelvin

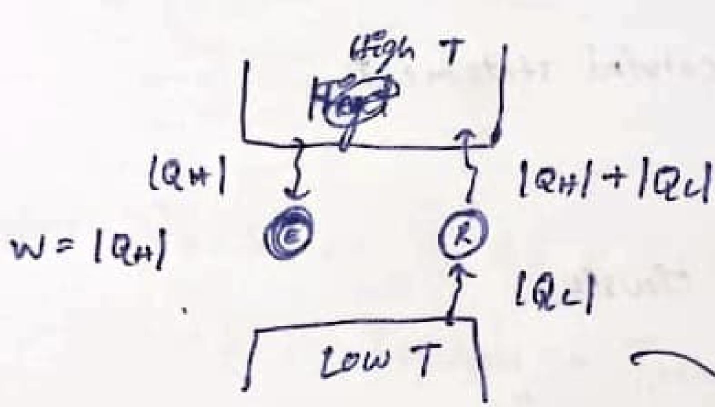
Violates Clausius

R+E :→ Heat extracted from hot reservoir $= |Q_H| - |Q_L|$

Work $= |Q_H| - |Q_L|$

⇒ R+E violates Kelvin

⇒ $-C > -K$



→ Does not violate Clausius

E violates Kelvin

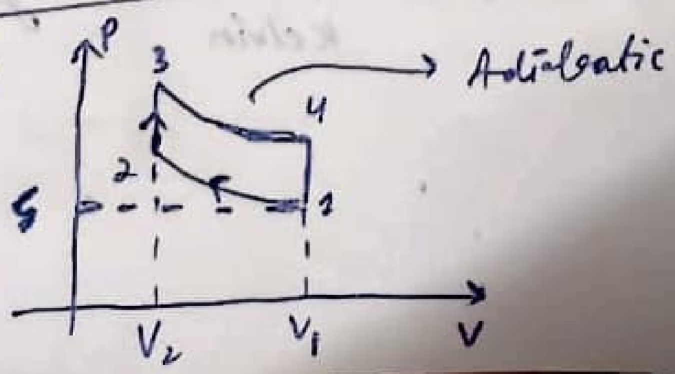
E+R :→ Heat extracted from cold reservoir $: |Q_L|$

Net heat given to hot reservoir $= |Q_L|$

⇒ violates Clausius

⇒ $-K > -C$

Q2)



$$\eta = 1 - \frac{T_1}{T_2}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

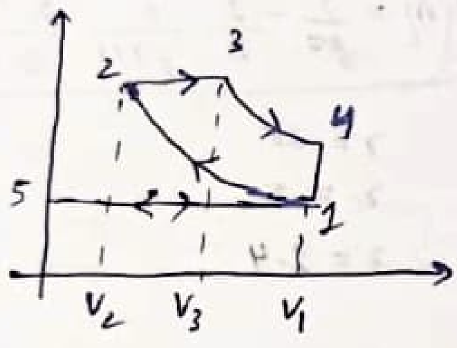
$\eta = 1 - \frac{1}{r^{\gamma - 1}}$

Gasoline engine : $r \leq 10$

$r = 9$, $\gamma = 1.3$

$\eta = 0.48$

3 -



$$\eta = 1 - \frac{1}{\gamma}\left(\frac{T_4 - T_1}{T_3 - T_2}\right)$$

$$= 1 - \frac{1}{\gamma}\frac{(T_4/T_1 - 1)}{(T_3/T_2 - 1)} \times \frac{T_1}{T_2}$$

$\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2}$, $P_3 = P_2$

$\Rightarrow \frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c \longrightarrow$ cut off ratio, measure of duration of heat addition at constant p.

$\frac{P_4 V_4}{T_4} = \frac{P_1 V_1}{T_1}$, $V_4 = V_1$

$\Rightarrow \frac{T_4}{T_1} = \frac{P_4}{P_1}$.

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$, $P_4 V_4^{\gamma} = P_3 V_3^{\gamma}$

$\Rightarrow \frac{P_4}{P_1} = \left(\frac{V_3}{V_2}\right)^{\gamma} = r_c^{\gamma}$

$\Rightarrow \eta = 1 - \frac{1}{\gamma} \frac{r_c^{\gamma} - 1}{r_c - 1} \times \frac{T_1}{T_2}$

$= T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$

$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma - 1} = \left(\frac{1}{r}\right)^{\gamma - 1}$ $\Rightarrow$ $\boxed{\eta = 1 - \frac{1}{\gamma} \cdot \frac{1}{r^{\gamma - 1}} \frac{r_c^{\gamma} - 1}{r_c - 1}}$

$r = 20$

$r_c = 5$

$\gamma = 1.4$

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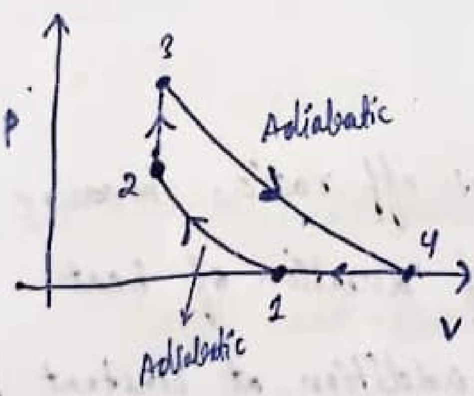
4)



$Q_H = C_V(T_3 - T_2)$

$Q_L = C_P(T_4 - T_1)$

$\eta = 1 - \frac{|Q_L|}{|Q_H|}$

$= 1 - \frac{C_P}{C_V} \left(\frac{T_4 - T_1}{T_3 - T_2}\right)$

$= 1 - \gamma \frac{T_4 - T_1}{T_3 - T_2}$

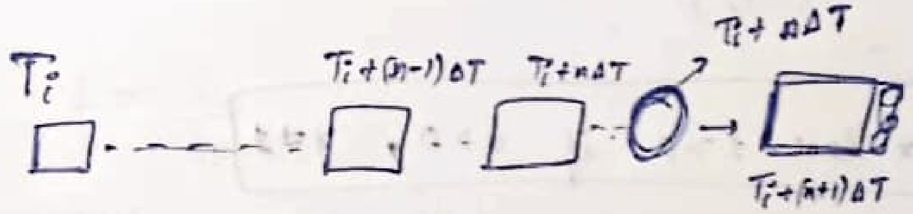
5) (a) $\Delta S_R = 0$ [no change in thermodynamic parameter]

(b) $\Delta Q = i^2 R T = \underline{3 \times 10^6 \text{ J}}$

$$\Delta S_U = \frac{3\times 10^6}{300} = 10^4 \text{ J/K}$$

(c) $\Delta U_U = 3 \times 10^6$ J

6)



$$\Delta S_{mat} = \int_{T_i+n\Delta T}^{T_i+(n+1)\Delta T} C \cdot \frac{dT}{T} = C \ln\left[\frac{T_i + (n+1)\Delta T}{T_i + n\Delta T}\right]$$

$$\Delta S_{res} = \frac{-C\Delta T}{T_i + (n+1)\Delta T}$$

$$\Rightarrow \Delta S_{tot} = C\left[\ln\left(\frac{T_i+(n+1)\Delta T}{T_i+n\Delta T}\right) - \frac{\Delta T}{T_i+(n+1)\Delta T}\right]$$

$$\Delta S = C \ln \frac{T_f}{T_i} - C \sum_{n=0}^{N-1} \frac{\Delta T}{T_i + (n+1)\Delta T}$$

$$N \to \infty \quad \sum_{n=0}^{N-1} \to \int_{T_i}^{T_f} \to \int_{T_i}^{T_f} \frac{dT}{T}$$

$\Delta S = 0$

$$\Delta S = \int_{T_i}^{T_f} C \cdot \frac{dT}{T} = C \ln \frac{T_f}{T_i}$$

$$\Delta S_{rev} = -\frac{Q}{T_f} = C \frac{(T_i - T_f)}{T_f}$$

$$\Delta S = C \left(\ln \frac{T_f}{T_i} + \frac{T_i - T_f}{T_f} \right)$$

$$\Rightarrow \quad \Delta S = C f\left(\frac{T_i}{T_f} \right) > 0$$

$$\boxed{f(x) = x - \ln x - 1 > 0 \quad \text{if} \quad x > 0, \; x \neq 1}$$