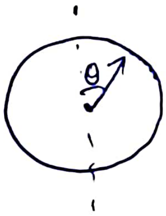
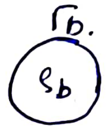
Tutorial-7

(1)



$\equiv$



$\nabla^2 V = 0.$

$\sigma_b = \vec{P}\cdot(\hat{n}=\hat{k})$

$\sigma_b = P\cos\theta.$

$\rho_b = \vec{\nabla}\cdot\vec{P} = 0.$

$$V_{in}(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$$

$$V_{out}(r,\theta) = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta)$$

$$A_l r^l = \frac{B_l}{r^{l+1}}$$

$$\boxed{B_l = A_l r^{2l+1}}$$

$$E_{out}^{\perp} - E_{in}^{\perp} = \frac{\sigma_\perp}{\varepsilon_0}.$$

$$-\frac{\partial V_{out}}{\partial r}\Big|_{r=R} = -\frac{\partial V_{\cancel{out}\,in}}{\partial r}\Big|_{r=R} = \frac{\sigma_0}{\varepsilon_0}$$

2) a) $E_{out}^{\parallel} - E_{in}^{\parallel} = 0.$

$$\Rightarrow \frac{PR^3}{3\cancel{r^3}\varepsilon_0}\cancel{\cos\theta}\sin\theta\,\hat{\theta} - \frac{P\sin\theta}{3\varepsilon_0}\hat{\theta} = 0.$$

$r=R$

$\Rightarrow 0.$

b)

~~$D_{out}^{\perp}$ $D_{in}^{\perp}$~~

sol

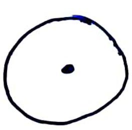
$V_{in} = Pr\cos\theta/3\varepsilon_0$

$V_{out} = \frac{P}{3\varepsilon_0}\frac{R^3}{r^2}\cos\theta$

$E_{in} = \frac{-P}{3\varepsilon_0}\cos\theta\,\hat{r} + \frac{P\sin\theta}{3\varepsilon_0}$

$E_{out} = \frac{2PR^3\cos\theta}{3\varepsilon_0 r^3}\hat{r} + \frac{P\ldots}{3\ldots}$

(3)



$\chi_e, R.$

$$\oint E\cdot ds = \frac{Q_{in}}{\varepsilon_0}.$$

$$\vec{\nabla}\cdot E = \frac{\rho}{\varepsilon_0} \leftarrow \text{vol charge density}$$

$$\vec{\nabla}\cdot D = \rho_{free}.$$

$\vec{D} = \vec{P} + \varepsilon\vec{E_0}$

$\nabla P = p_0\hat{n}$

$\rho_b = -\vec{\nabla}\cdot\vec{P}$

$\sigma_b = \vec{P}\cdot\hat{n}$

$$\oint D \cdot dA = \int \rho_{free}\, d\tau$$

$$D \cdot (4\pi r^2) = Q.$$

$$\boxed{D = \frac{Q}{4\pi r^2}\hat{r}.}$$

$$\vec{D} = \vec{P} + \varepsilon_0 \vec{E}$$

$$\Rightarrow \quad \vec{P} = \chi_e \varepsilon_0 \vec{E}.$$

$$\vec{D} = \varepsilon_0 \vec{E}(1+\chi_e).$$

$$\frac{Q}{4\pi r^2}\hat{r} = \varepsilon_0 \vec{E}(1+\chi_e)$$

$$\Rightarrow \quad \vec{E} = \frac{Q\,\hat{r}.}{4\pi r^2 (1+\chi_e)\varepsilon_0}$$

$$\vec{P} = \chi_e \cancel{\varepsilon_0} \frac{Q\,\hat{r}}{4\pi r^2 (1+\chi_e)\cancel{\varepsilon_0}.} = \frac{\chi_e Q\,\hat{r}}{4\pi r^2(1+\chi_e)}$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} \Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cdot \vec{P}) \qquad \sigma_b = \vec{P}\cdot\hat{n}$$

$$=$$

$$\cancel{\rho_b \Rightarrow \frac{(-2)\chi_e Q}{4\pi r^3 (1+\chi_e)}}.$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2 \chi_e Q}{4\pi r^2(1+\chi_e)}\right) = 0 \qquad r \neq 0.$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} \quad = \nabla\cdot\left(\frac{\hat{r}}{r^2}\right) = -4\pi\,\delta(r).$$

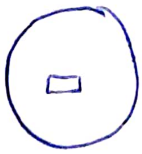
$$\vec{\nabla}\cdot\vec{P} = \frac{\chi_e}{1+\chi_e}\frac{Q}{4\pi}\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) - \frac{\chi_e}{1+\chi_e}\frac{Q}{\cancel{4\pi}}\left(\cancel{4\pi}\,\delta(\vec{r})\right)$$

$$\nabla\cdot E = \frac{\rho}{\varepsilon_0}$$

$$\int E \cdot dA = \int \frac{\rho}{\varepsilon_0}\, d\tau.$$

$$E\, 4\pi r^2 = \frac{Q}{\varepsilon_0}.$$

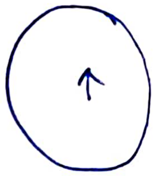
4).



R, K.

$\bar{V} = ?$

$\bar{E} = ?$



$\vec{P}$ e

k

$$V_{in}(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta) + \frac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_0 r^2}$$

$$V_{out}(r,\theta) = \sum_{l=0}^{\infty} \frac{C_l}{r^{l+1}} P_l(\cos\theta)$$

$\Rightarrow$

$\cancel{A_l r^l P_l(c}$

Boundary conditions

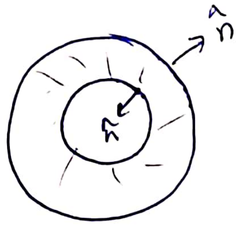
i) $V_{in} = V_{out}$ $\quad r = R$.

ii)

probm 4.15 griffith

a) $\vec{P} = \varepsilon_0 \chi_e \vec{E}$

$\vec{P} = \frac{k}{r}\hat{r}$



$\rho_b = -\vec{\nabla}\cdot\vec{P}$

$= -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 P_r)$

$= \frac{-1}{r^2}\frac{\partial}{\partial r}(kr)$

$\boxed{\rho_b \Rightarrow \frac{-1}{r^2}(k)}$

$\varepsilon = \varepsilon_0 \cancel{\chi}$

$\varepsilon = \varepsilon_0(1+\chi_e)$

$\varepsilon\vec{E} = \varepsilon_0(1+\chi_e)\vec{E}$

$= \vec{D}$

$\sigma_b = \vec{p}\cdot\hat{n}$

$\Rightarrow$

$r=a$ $\quad \hat{n} = -\hat{r}$

$\sigma_b = \frac{k}{r}\hat{r}\cdot(-\hat{r})$

$\boxed{\sigma_b = \frac{-k}{a}}$

$r=b$ $\quad \hat{n}=\hat{r}$

$\sigma_b = \frac{k}{r}\hat{r}\cdot\hat{r}$

$\boxed{\sigma_b = \frac{k}{b}}$

b) $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$

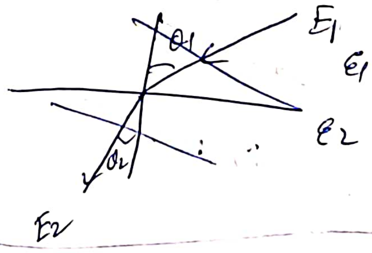
$\int \vec{\nabla}\cdot\vec{E}\,dV = \int\frac{\rho\,dV}{\varepsilon_0}$

$\int \vec{E}\cdot d\vec{A} = \int_a^r \frac{\rho\,dV}{\varepsilon_0} + \int\frac{\sigma_a\,dA}{\varepsilon_0}$

$E\cdot 4\pi r^2 = \frac{\int \frac{-k}{r^2}4\pi r^2 dr}{\varepsilon_0} + \frac{-k}{a}\cdot\frac{4\pi a^2}{\varepsilon_0}$

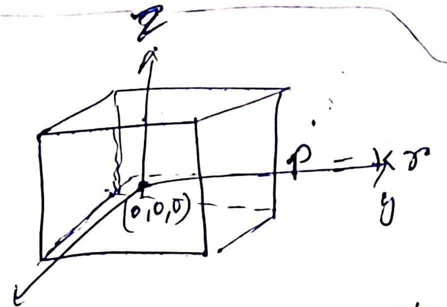
4.31

$\cancel{\frac{\tan\theta_2}{\tan\theta_1} = \frac{\varepsilon_2}{\varepsilon_1}}$



$E_1\cos\theta_1 = E_2\cos\theta_2$

4.33



$\vec{P} = k\vec{r}$ $\quad$ k is const.

$\sigma_b = \vec{P}\cdot\hat{n}$

$\sigma_1 = ka/2$

$\sigma_2 = ka/2$

$\sigma_5 = \sigma_6 = ka/2$

$A_1 \Rightarrow x = a/2\ \hat{x}$

$A_2 \Rightarrow x = -a/2\ (-\hat{x})$

$A_3 = z = a/2\ \hat{z}$

$A_4 = z = a/2(-\hat{z})$

$A_5 = y = a/2\ \hat{y}$

$A_6 = y = -a/2\ (-\hat{y})$

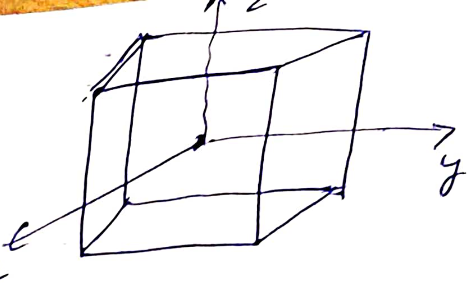
$\sigma_b = \cancel{k\hat{x}(a/2)}$

$\cancel{k(a/2\ \hat{z})}$. for A_3.

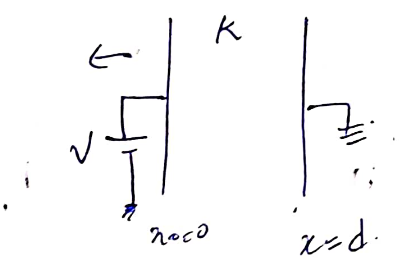
$\sigma_b = k\left(x\hat{x}+y\hat{y}+a/2\,\hat{z}\right)\cdot\hat{z}$

$\sigma_{b_3} = \left(\frac{a}{2}\right)$, $\sigma_{b_4} = \frac{-a}{2}\hat{z}\cdot-\hat{z} = (a/2)$

$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$



$$\vec{P} = K(x\hat{x} + y\hat{y} + z\hat{z})$$
$$= K\vec{r}$$
$$\rho_b = -K\frac{1}{r}\left(\frac{\partial}{\partial r} r \vec{P}_r\right)$$
$$\Rightarrow K\frac{1}{r}\frac{\partial r^2}{\partial r} = \frac{2K\cancel{r}}{\cancel{r}} = 2K$$



$$D_1^+ - \cancel{D_2}^{\,0} = \sigma_f$$
$$D_1^+ = \sigma_f$$
$$\varepsilon E_1 = \sigma_f$$
$$E_1 = \frac{\sigma_f}{\varepsilon_0(1+x/d)}\hat{x}$$

$$V = -\int_d^0 \vec{E}_1\cdot d\vec{l}$$
$$= -\int_d^0 \frac{\sigma_f}{\varepsilon_0\left(1+\frac{x}{d}\right)}\cdot dx$$

$$1+\frac{x}{d} = t$$
$$\frac{x}{d} = t-1$$
$$dx = (d)\,dt$$

$$\Rightarrow \int \frac{d\,\sigma_f\,(dt)}{\varepsilon_0 t}$$
$$\int_0^d d\frac{\sigma_f}{\varepsilon_0}\ln\left(1+\frac{x}{d}\right)$$
$$\Rightarrow d\frac{\sigma_f}{\varepsilon_0}\ln\left(1+\frac{d}{d}\right) - d\frac{\sigma_f}{\varepsilon_0}\cancel{\ln(1)}^{\,0}$$
$$V \Rightarrow d\frac{\sigma_f}{\varepsilon_0}\ln(2)$$

$$\sigma_f = \frac{\varepsilon_0 V}{d\ln 2}$$
$$\Rightarrow \vec{E} = \frac{\sigma_f}{\varepsilon_0\left(1+\frac{x}{d}\right)}\hat{x}$$

$\Rightarrow \bar{E} = \cancel{\dfrac{\varepsilon_0 V}{d\,\ldots}}$

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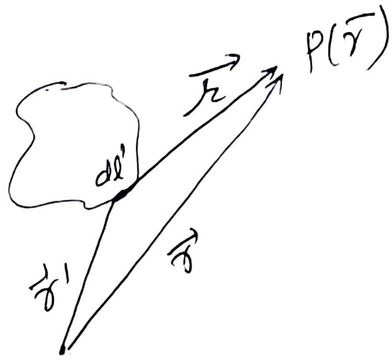
Magnetism



① $\bar{\nabla}.\bar{B} = 0$.

② Lorentz force = $\vec{F} = q\vec{E} + q(\bar{v}\times\bar{B})$

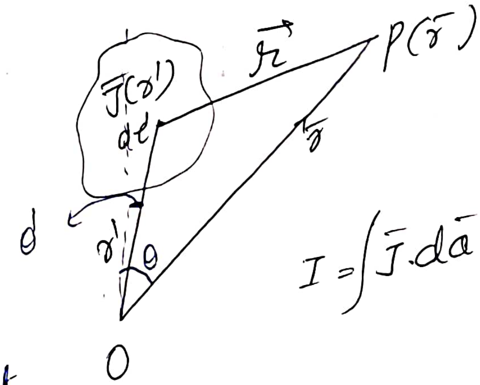
③ $\vec{F}_{mag} = I\int d\bar{l}\times\bar{B}$

④ Biot savart's law

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}'$$

$$\vec{B}(\bar{r}) = \frac{\mu_0}{4\pi}\int \frac{I\,d\bar{l}'\times\hat{\mathscr{r}}}{\mathscr{r}^2} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}'\times\hat{\mathscr{r}}}{\mathscr{r}^2} = \frac{\mu_0 I}{4\pi}\int\frac{\vec{J}(r')\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'$$

Amphere's law



$$I = \int \bar{J}.d\bar{a}$$

$$\oint \bar{B}.d\bar{l} = \mu_0 I_{en}$$

$\Downarrow$

$$\int(\bar{\nabla}\times\bar{B})da = \mu_0 I_{enclosed}$$

$$= \mu_0\int \bar{J}.d\bar{a}$$

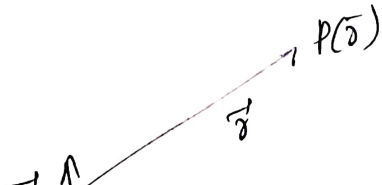
$\rightarrow \boxed{\bar{\nabla}\times\bar{B} = \mu_0\vec{J}}$

$\bar{J} \rightarrow$ vol current density.

⑥ vector potential

$$\bar{B} = \bar{\nabla}\times\bar{A}$$

$\bar{A}$ potential $= \dfrac{\mu_0 I}{4\pi}\displaystyle\int\frac{d\bar{l}'}{\mathscr{r}} = \frac{\mu_0 I}{4\pi}\int\frac{dl'}{|r-r'|}$



⑦ magnetic dipole moment

$$\vec{m} = I\int d\bar{a} = I\bar{A}\text{ (Area)}$$

$$\vec{B}_{dip} = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\vec{m}.\bar{r})\hat{r} - \vec{m}\right]$$

$$\frac{1}{4\pi\varepsilon_0 r^3}\left[3\,\bar{p}.\hat{r} - \bar{p}\right] = \bar{E}_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

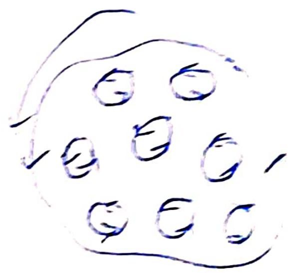
(8) magnetic dipole ~~moment~~ placed in a magnetic field

$$\vec{N} = \bar{m} \times \bar{B}$$

(9) magnetization $\vec{M}$ = magnetic moment per unit vol.

Bound current density

$$\vec{J}_b = \bar{\nabla} \times \vec{M} \longrightarrow \text{Bound volume current density.}$$



Bound surface current density

$$\vec{K}_b = \bar{M} \times \hat{n}$$

$$\bar{J} = \bar{J}_b + \bar{J}_f$$

$$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J} = \mu_0 (\bar{J}_b + \bar{J}_f)$$

$$\frac{\bar{\nabla} \times \bar{B}}{\mu_0} = \bar{\nabla} \times \bar{M} + J_f \Rightarrow \nabla \times \left(\frac{\bar{B}}{\mu_0} - \bar{M}\right) = J_f$$

$$\bar{\nabla} \times \bar{H} = J_f$$

(10) Boundary condition at the interface.

$$\bar{\nabla} \cdot \bar{B} = 0.$$

$$\bar{B}_1^{\perp} = \bar{B}_2^{\perp}$$

$$\bar{B}_1^{\parallel} - \bar{B}_2^{\parallel} = \mu_0 (\vec{K} \times \hat{n})$$

surface current density

current / ~~length~~

$$\bar{H}_1^{\parallel} - \bar{H}_2^{\parallel} = K_f \times \hat{n}$$

Electro Magnetic Waves

11/3/24

→ E, B and direction of propagation ⊥ to each other.
→ oscillating → $E \times B$.
→ velocity ∝ $\frac{c}{n}$ ← refractive index

$\omega = 2\pi\nu$

$$\boxed{V = \frac{c}{n}}$$

$\lambda = \frac{V}{\nu}$

Application
1. LASER
2. Wireless communication
3. Optical fibre
4. MRI, X Ray & crystal structure
5. CT scan database

→ spectrum of electromagnetic waves

Gamma rays	X-ray	UV → sterilization	Visible	IR → thermometer
γ - 10^{20}–10^{22} Hz	10^{20}–10^{16} Hz	10^{16}–10^{15} Hz	10^{15}–10^{14} Hz	10^{13}–10^{12} Hz

Microwave - Radiowave → communication
~10^{6} Hz

$\frac{3 \times 10^{8}}{10^{15}} \Rightarrow 3 \times 10^{-7}$

- Amplitude of electric field
- direction of electric field.
- Frequency -
- direction of propagation of wave

Should obey Maxwell's Eqn

~~Electron~~

Maxwell's Eqn

$$\left.\begin{aligned} \bar{\nabla} \cdot \bar{D} &= \rho_f \\ \bar{\nabla} \cdot \bar{B} &= 0 \end{aligned}\right\} \text{Gauss's law.}$$

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow \text{faraday's law.}$$

$$\bar{\nabla} \times \bar{B} = \mu \bar{J}_f + \mu \frac{\partial D}{\partial t}$$

Dielectric medium

$\vec{\nabla}\cdot\vec{D} = 0.$ $\qquad \vec{\nabla}\cdot\vec{E} = 0.$

$\vec{\nabla}\cdot\vec{B} = 0.$

$\vec{\nabla}\times\vec{E} = \frac{\partial \vec{B}}{\partial t}$

$\vec{\nabla}\times\vec{B} = \mu \frac{\partial D}{\partial t} = \mu\varepsilon \frac{\partial \vec{E}}{\partial t}$ $\qquad \nabla^2$

$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\vec{\nabla}\times\left(\frac{\partial \vec{B}}{\partial t}\right)$

$\vec{\nabla}(\cancel{\vec{\nabla}\cdot\vec{E}})^{0} - \nabla^2\vec{E} = \frac{-\partial}{\partial t}(\vec{\nabla}\times\vec{B})$

$\nabla^2\vec{A} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\vec{A}$

$\nabla^2\vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial^2 t}$ wave eqn.

$c = \frac{1}{\sqrt{\mu\epsilon}}$

$\vec{E}(z,t) = \vec{E_0}\, e^{[i(kz-\omega t)]} = \vec{E_0}\sin[kz-\omega t].$

↳ Amplitude $\vec{E_0}$

→ direc^n in the Electric field → polarisation of the wave.

↓

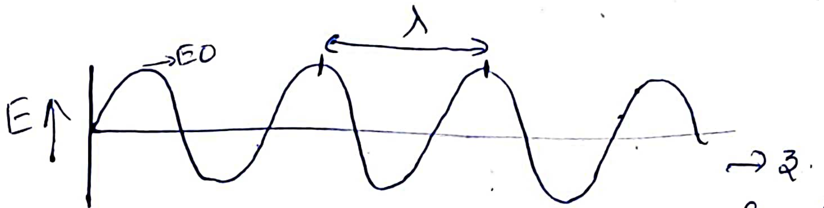
(this is different from polarisation in dielectric here it is just direction of electric field)

z or r → direction of propagation of wave

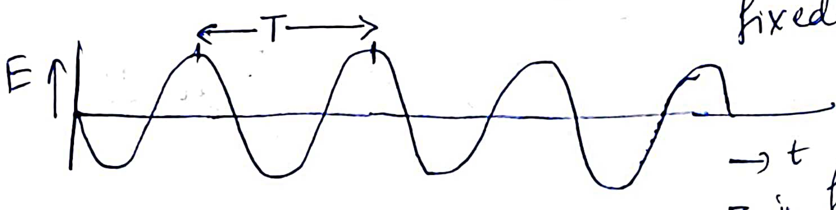
$K = \frac{2\pi}{\lambda}$ → magnitude of the propagation

ω → freq of the wave.

$\phi = kz - \omega t$



fixed t.



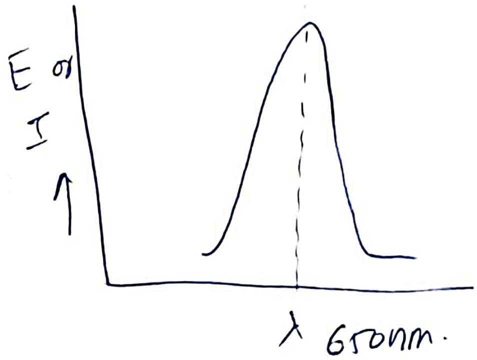
z is fixed

$k\frac{dz}{dt} - \omega = 0.$

$c = \left|\frac{dz}{dt}\right| = \frac{\omega}{k}$

$c = \frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda}$

$c = \nu\lambda$



$\frac{\omega}{k}$

$\frac{2\pi f\lambda}{2\pi}$

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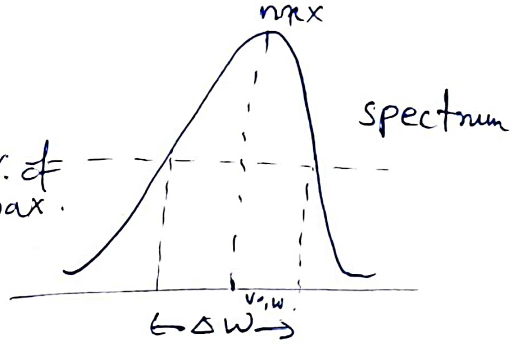
when moving

consider 2 electromagnetic components:



$\vec{E}_1 = \vec{E}_0 \sin(k_1 z - \omega_1 t)$

$\vec{E}_2 = \vec{E}_0 \sin(k_2 z - \omega_2 t)$

travel simultaneously.

FWHN

full width half maximum

Here $\Delta\omega$ is FWHM.

superposition $= \bar{E}_1 + \bar{E}_2 = E_0(\sin(k_1 z - \omega_1 t) + \sin(k_2 z - \omega_2 t))$

$\bar{E} = \bar{E}_0\, 2\sin\left(\frac{k_1+k_2}{2} z - \frac{(\omega_1+\omega_2)}{2} t\right) \cos\left(z\left(\frac{k_1-k_2}{2}\right) + t\left(\frac{\omega_2-\omega_1}{2}\right)\right)$

let $k = \frac{k_1+k_2}{2}$, $\omega = \frac{\omega_1+\omega_2}{2}$

let $\Delta k = k_1 - k_2$; $\Delta\omega = \omega_1 - \omega_2$

so, $\bar{E} = \bar{E}_0\, 2\sin(kz - \omega t)\cos\left(\frac{z\Delta k - \Delta\omega t}{2}\right)$

Here $k \gg \Delta k$, $\omega \gg \Delta\omega$

Here the amplitude is $2E_0\cos\left(\frac{\Delta k z}{2} - \frac{\Delta\omega t}{2}\right)$

group velocity $\Rightarrow v_g = \frac{\Delta\omega/2}{\Delta k/2} = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$

phase velocity $\Rightarrow v_p = \omega/k$.

always k is a function of $\omega \to k(\omega)$

In a conductor, $\rho_f = 0$

i) $\bar{\nabla}\cdot\bar{D} = 0$

ii) $\bar{\nabla}\cdot\bar{B} = 0$ $\qquad\qquad \nabla \bar{J}_f = -\dfrac{\partial \rho_f}{\partial t}$

iii) $\nabla\times E = -\dfrac{\partial \bar{B}}{\partial t}$

iv) $\bar{\nabla}\times\bar{B} = \mu\bar{J}_F + \mu\epsilon\dfrac{\partial E}{\partial t}$ $\qquad \bar{J}_f = \sigma\vec{E}$

v). $\bar{\nabla}\times(\bar{\nabla}\times\bar{E}) = -\dfrac{\partial}{\partial t}(\bar{\nabla}\times\bar{B})$ $\Rightarrow \nabla^2\bar{E} = \mu\sigma\dfrac{\partial E}{\partial t} + \mu\epsilon\dfrac{\partial^2 E}{\partial t^2}$

Here $E = E_0\hat{1}\exp(i(kz-\omega t))$

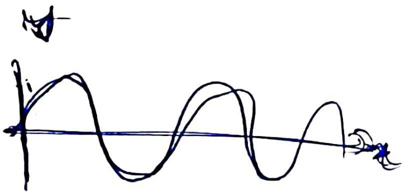
$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \mu\sigma\frac{\partial E}{\partial t} + \mu\epsilon\frac{\partial^2 E}{\partial t^2}$$

$$k^2 = \mu\epsilon\omega^2 + i\mu\sigma\omega.$$

$$\boxed{\vec{K} = k.\hat{n}}$$

$$K = k_r + i\,k_{img}.$$

So, $k_r = \omega\sqrt{\dfrac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\dfrac{\sigma}{\epsilon\omega}\right)^2}+1\right]^{1/2}$

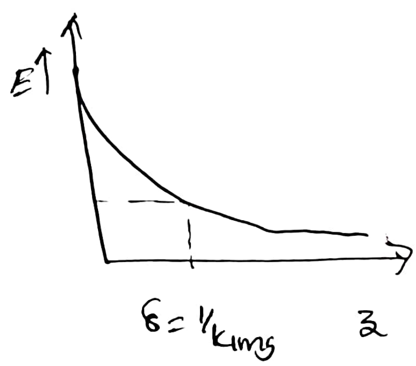


$k_{img} = \omega\sqrt{\dfrac{\epsilon\mu}{2}}\left(\sqrt{1+\left(\dfrac{\sigma}{\epsilon\omega}\right)^2}-1\right)^{1/2}$

$$\bar{E} = E_0\hat{1}\exp[i k_r z + i k_{img} z) - \omega t]$$

$$= E_0\hat{1}\, e^{-k_{img} z}\, e^{i(k_r z - \omega t)}$$

Scale length :- $\dfrac{1}{k_{img}}$



$\cancel{E_0} \rightarrow E$

Here $k_r = k e^{i\beta}$

$K = \sqrt{k_r^2 + k_{img}^2}$

$\tan\phi = \frac{K_{img}}{K_r}$

$\cos 4\phi$

$2\cos^2 2\phi - 1$

so, group velocity $(v_g) = \frac{\partial \omega}{\partial K_r}$.

K_{img} is responsible for decipation of wave & K_r is responsible for propagation of wave (means velocity)

$2[2\cos^2\phi - 1]^2 - 1$

$\Rightarrow 8\cos^4\phi - 8\cos^2\phi - 2 - 1$

C-1 when $\frac{\sigma}{\epsilon\omega} << 1$ → at this condition is considered as perfect dielectric

$K_r = \omega\sqrt{\epsilon\mu}$ $K_{img} = 0$.

$8\cos^4\phi - 8\cos^2\phi - 3$

C-2 $\frac{\sigma}{\epsilon\omega} >> 1$

$K_r = \omega\sqrt{\frac{\epsilon\mu}{2} \cdot \frac{\sigma}{\epsilon\omega}} = \omega\sqrt{\frac{\sigma\mu}{2\omega}} = \sqrt{\frac{\mu\sigma\omega}{2}}$

$8\cos^2\phi(\cos^2\phi - 1)$

$-8\cos^2\phi\sin^2\phi$

$\Rightarrow 8\left(\frac{\sin 2\phi}{2}\right)^2$

$K_{img} = \omega\sqrt{\frac{\mu\sigma}{2\omega}}$

Here $K_r = K_{img}$

$2\sin^2 2\phi$

$\phi \to \left(\frac{\phi}{2}\right)$

$(1-\sin^2)\sin^2$

$\sin^2\phi - \sin^4\phi$

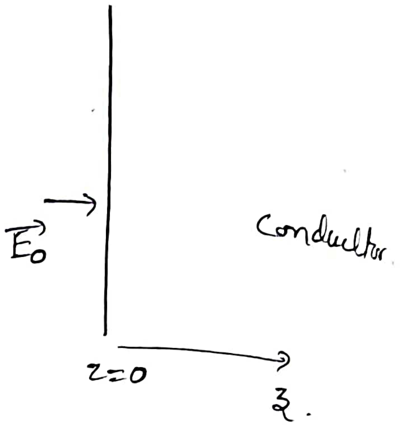
$\cos 4\phi$

$\Rightarrow \frac{e^{4\phi i} + e^{-i4\phi}}{2}$

EM wave inside the conductor

$$\vec{E} = E_0 e^{-k_{img} z} e^{i(k_r z - \omega t)}$$

$$K_{img} = \omega \sqrt{\frac{\epsilon\mu}{2}} \left[\left\{ 1 + \left(\frac{\sigma}{\epsilon\omega} \right)^2 \right\}^{1/2} - 1 \right]^{1/2}.$$



$$K_r = \omega \sqrt{\frac{\epsilon\mu}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega} \right)^2}^{1/2} + 1 \right]^{1/2}.$$

$k = |K| e^{-i\phi}$, $|K| = \sqrt{k_r^2 + K_{img}^2}$ $\quad \tan\phi = \frac{K_{img}}{K_{real}}$

$$\bar{B} = K(\hat{k} \times E) = \frac{|k| e^{i\phi} (\hat{k} \times \hat{E})}{\omega}$$

$$= \frac{|k|}{\omega} e^{i\phi} E_0 e^{-K_{img} z} \exp(i(k_z z - \omega t)) (\hat{k} \times \hat{E}).$$

conductor $\frac{\sigma}{\epsilon\omega} \gg 1 \Rightarrow K_{img} = k_r \Rightarrow \tan\phi = 1$

$\phi = \pi/4.$

↳ phase diff b/w E & B inside conductor.

conductor $\sigma \sim 10^7 (\Omega m)^{-1}$

$\omega \to 10^{10}$ rad/s.

$\quad\to 10^{15}$ rad/sec

$\mu = \mu_0 = 4\pi \times 10^{-7}$ N/A

$$K_{img} = \sqrt{\frac{\omega\sigma\mu}{2}} = \sqrt{\frac{10^{10} \times 10^7 \times 4\pi \times 10^{-7}}{2}}$$

Dielectric donot have free electron thus no phase diff b/w E and B.

for $\omega = 10^{10}$ rad/s. $\quad K_{img} \sim (2\pi \times 10^{10})^{1/2}.$

$\sim 10^5 m^{-1}.$

$\delta = \frac{1}{K_{img}} = 10^{-5} \approx 10\mu m$, $\quad 1\mu m = 10^{-6} m.$

for $\omega = 10^5$ rad/s

$\delta = 10^{-7} - 10^{-8}$ m.

$\varepsilon = 1 + \chi_e$

$\simeq$ 10 - 100 nm , $1 nm = 10^{-9}$ m.

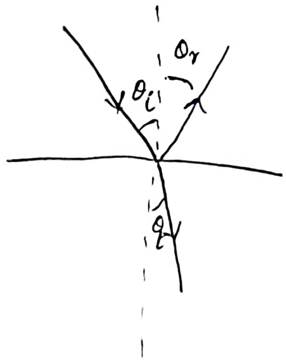
skin depth $\delta = \frac{1}{K_{img}}$.

EM wave at dielectric - conductor Interface.

dielectric - dielectric Interface & snells law.

i) $\theta_i = \theta_r$

ii) snell's law $n_1 \sin\theta_i = n_2 \sin\theta_t$ — always valid.



dielectric - conductor Interface

$\vec{E}_i = \vec{E}_{0i}\, e^{[i(k_1 z - \omega t)]}$

$\vec{B}_i = \vec{B}_{0i}\, e^{[i(k_1 z - \omega t)]}$

$\vec{E}_r = \vec{E}_{0r}\, e^{i(-k_1 z - \omega t)}$

$\bar{B}_r = \bar{B}_{0r}\, e^{i(-k_1 z - \omega t)}$

$\vec{E}_t = \vec{E}_{0t}\, e^{i(k_t z - \omega t)}$

$\bar{B}_t = \bar{B}_{0t}\, e^{i(k_t z - \omega t)}$.

K_i

$\bar{E}_r, \bar{B}_r$

$\bar{E}_i, \bar{B}_i$ — ① dielectric

$\bar{E}_t, \bar{B}_t$ — ② conductor.

$K_t = K_r + iK_{img}$

$= |K| e^{i\phi}$

Boundary condition:-

i) $D_1^\perp - D_2^\perp = \sigma_f = 0$.

ii) $E_1^{\parallel} - E_2^{\parallel} = 0$

iii) $\vec{B}_1^\perp = \vec{B}_2^\perp$

iv) $\frac{1}{\mu_1}\bar{B}_1^{\parallel} - \frac{\bar{B}_2^{\parallel}}{\mu_2} = \vec{K}_f \times \hat{n} = 0$.

$E_{0i} + E_{0r} = E_{0t}$ — ①

$\vec{B} = \frac{k_t E_{0t}}{\omega}\hat{j}$

$\frac{1}{\mu_1}(B_{0i} + B_{0r}) = \frac{1}{\mu_2} B_{0t}$

$K_t = k_{2r} + ik_{2img}$.

$= ke^{i\phi}$.

$K = \sqrt{k_{2r}^2 + k_{2img}^2}$

$\frac{1}{\mu_1 v_1}(E_{0i} - E_{0r}) = \frac{k_t}{\mu_2 \omega} E_{0t}$ — ②

$\tan\phi = \frac{k_{2img}}{k_{2r}}$

$$\frac{E_{0r}}{E_{0i}} = \frac{1 - \frac{\mu_1 v_1}{\mu_2 \omega} K_t}{1 + \frac{\mu_1 v_1}{\mu_2 \omega} K_t} \simeq -1$$

$$\frac{E_{0t}}{E_{0i}} = \frac{2}{1 + \frac{\mu_1 v_1}{\mu_2 \omega} K_t} \longrightarrow 0.$$

① electric field of the reflected wave at dielectric conductor interface undergoes a phase change of π w.r.t incident wave

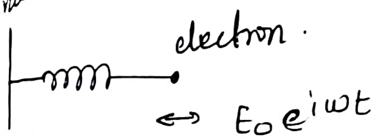
② coeff of reflection ⇒ perfect reflect r. $\quad r = \left| \frac{E_{0r}}{E_{01}} \right| = 1$

③ coeff of Transmitance $\quad t = \frac{E_{0t}}{E_{0i}} \longrightarrow 0.$

$T \to 0$

$I = \frac{1}{2} \epsilon |E|^2$ power dissipation.

nucleus



force = $F_{binding}$ + $F_{damping}$ + $F_{driving}$

$$m\frac{d^2x}{dt^2} = -m\omega_0^2 x - m\gamma \frac{dx}{dt} + E_0 e^{-i\omega t}$$

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + m\omega_0^2 x = \frac{qE_0}{m} e^{-i\omega t}$$

$$x(t) = x_0 e^{-i\omega t}$$

$$x(t) = \frac{q/m}{\omega_0^2 - \omega^2 - i\gamma\omega} e^{-i\omega t}$$

oscillating induced dipole

$$\vec{P}(t) = q x(t)$$

$$= \frac{q^2/m}{\omega_0^2 - \omega^2 - i\gamma\omega} e^{-i\omega t}$$

$$\vec{P} = N\vec{p}(t)$$

$N \uparrow$ number density

$$= \frac{N(q^2/m)\vec{E}_0 e^{-i\omega t}}{\omega_0^2-\omega^2-i\gamma\omega} = \epsilon_0 \chi_e \vec{E}_0 e^{-i\omega t}$$

$$\chi_e = \frac{N}{\epsilon_0}\frac{q^2/m}{\omega_0^2-\omega^2-i\gamma\omega}$$

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$$\chi_e = \frac{Nq^2}{\epsilon_0 m_e}\sum \frac{f_i}{\omega_i^2-\omega^2-i\gamma_f\omega}$$

dielectric const

$$K = \frac{\epsilon}{\epsilon_0} = 1+\chi_e$$

$\omega_0 \to \omega_i$

$\gamma \to \gamma_i$

$$1+\frac{Nq^2}{m_e}\sum\frac{f_i}{\omega_i^2-\omega^2-i\gamma_i\omega}.$$ K is complex number.

Refractive index

$$n = \frac{c}{v} = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}},$$ For non magnetic material $\mu=\mu_0$.

$$n = \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{1+\chi_e}, \quad \chi_e \ll 1$$

$$= 1+\frac{1}{2}\chi_e$$

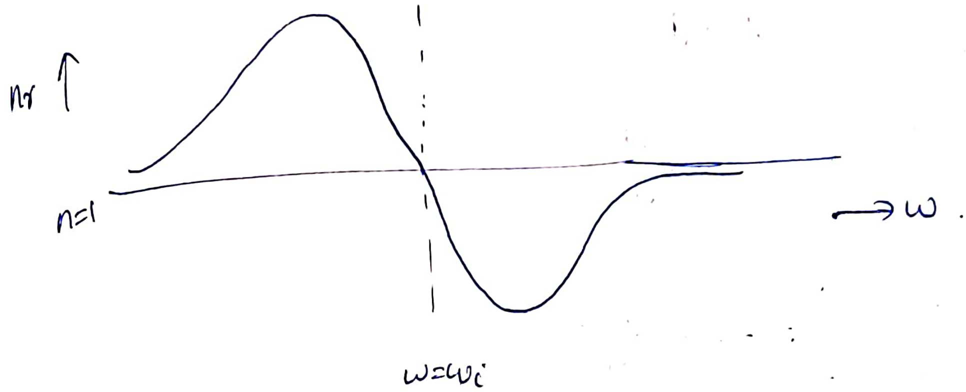
$$= 1+\frac{1}{2}\frac{Nq^2}{\epsilon_0 m_e}\sum_i\frac{f_i}{\omega_i^2-\omega^2-i\gamma_i\omega}$$

$$n = n_r + i n_{img}$$

$$\frac{f_i}{(\omega_i^2-\omega^2)-i\gamma_i\omega}\times\frac{(\omega_i^2-\omega^2)+i\gamma_i\omega}{(\omega_i^2-\omega^2)+i\gamma_i\omega} = \frac{f_i(\omega_i^2-\omega^2)+i\gamma_i\omega}{(\omega_i^2-\omega^2)^2+(\gamma_i\omega)^2}$$

$$\Rightarrow n_r = 1+\frac{1}{2}\frac{Nq^2}{\epsilon_0 m_e}\sum_i\frac{f_i(\omega_i^2-\omega^2)}{(\omega_i^2-\omega^2)^2+(\gamma_i\omega)^2}$$

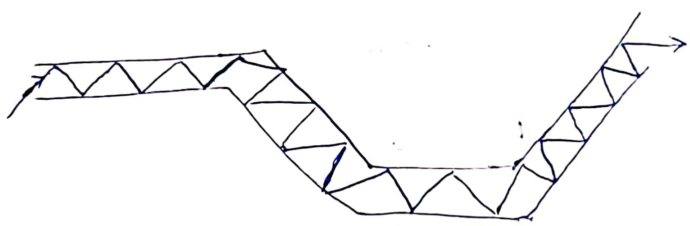
$$n_{img} = \sum_i \frac{\gamma_i \omega}{(\omega_i^2 - \omega^2)^2 + (\gamma_i \omega)^2} \frac{1}{2} \frac{N q^2}{\varepsilon_0 m_e}$$



At resonance the absorption is maximum.

Wave guide:-

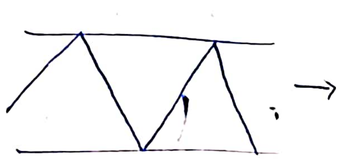
$$\vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z$$



attenuation ?

perfect dielectric - no attenuation

$$\vec{\nabla} \times \bar{E} = \frac{-\partial B}{\partial t}, \quad \vec{\nabla} \times \bar{B} = \mu\epsilon \frac{\partial \vec{E}}{\partial t} = \frac{1}{c^2}\frac{\partial \bar{E}}{\partial t}$$



$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

$$\bar{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

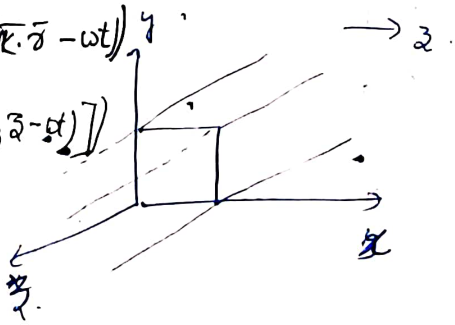
$$\bar{E} = E_0 \exp[(i(\vec{k}\cdot\vec{r} - \omega t))]$$

$$\Downarrow$$

$$\vec{E} = \vec{E_0} \exp[i(k_z z - \omega t)]$$

$$E = E_0 \exp(i)(\vec{k}\cdot\vec{r} - \omega t)$$

$$E_0 \exp(i)[(k_z z - \omega t)]$$



$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = \frac{-\partial B_z}{\partial t}$$

x, y, z component of $\vec{\nabla} \times \bar{E}$ & $\vec{\nabla} \times \bar{B}$

Tut-9

1) $v = \frac{1}{\sqrt{\mu_0 \varepsilon}}$ $\quad \omega = 10^8$ rad/sec

$\vec{E} = 20\sin(\omega t - kz)\hat{j}$ V/m.

$$\nabla \times \vec{E} = \frac{-\partial B}{\partial t} = -\mu_0 \frac{\partial H}{\partial t}.$$

$$\left(\frac{\partial E_y}{\partial x}\hat{z} - \frac{\partial E_y}{\partial z}\hat{x}\right) = -\mu_0 \frac{\partial H}{\partial t}.$$

$$-(-20K\cos(\omega t - kz)) = -\mu_0 \frac{\partial H}{\partial t}$$

2) $\vec{E} = 2e^{-az}\sin(10^8 t - kz)\hat{j}$ V/m. $\quad K=1$

$\frac{\mu}{\mu_0} = 20$, $\quad \sigma = 3$ mhos/m. $\quad \vec{B} = \frac{ke^{-i\phi}(\hat{k}\times\vec{E})}{\omega}$

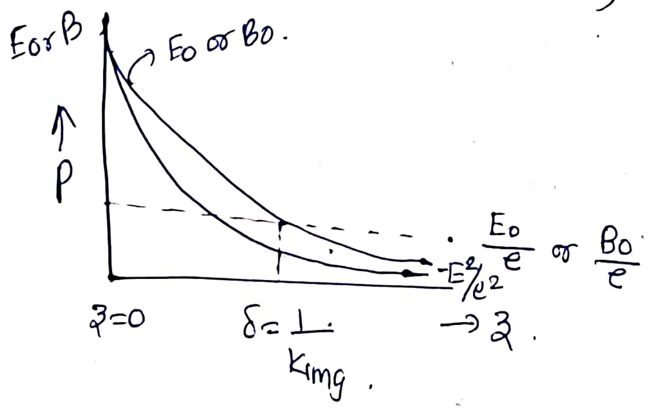
$-az = k_{img}\,z.$ $\quad K = \sqrt{k_0^2 + k_{img}^2}\cdot e^{-i\phi}$

$K_{img} = a.$

$E_0 = 2$ V/m.

$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$

$$a = K_{im} = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2} - 1\right]^{1/2}.$$



Absorption coeff $= 2K_{img}$

$$P \propto [E\times B]$$
$$\propto E^2$$

$e^{-2\alpha z} \Rightarrow$ at $z = 1/\alpha$, $P \to E^2/e^2$