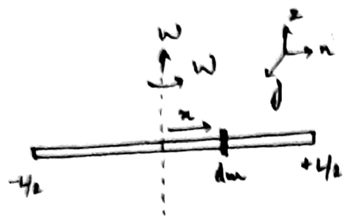
PH - 203

Tutorial - 2

1.



$$dm = \cancel{}\ \lambda\, dx$$

$$\vec{r} = x\hat{i}$$

$$\vec{v} = \omega x \sin 90^\circ\ \hat{u}$$

$$L = \int \vec{r} \times \vec{dp}$$

$$\vec{dL} = \vec{r_i} \times \vec{dp_i}$$

$$= \cancel{}\ x\, dm\, v\ \cancel{}\ \hat{z}$$

$$= x\, \lambda\, dx\, (\omega x)\ \hat{z}$$

$$L = \cancel{}\ \lambda \omega \int_{-L/2}^{+L/2} x^2\, dx$$

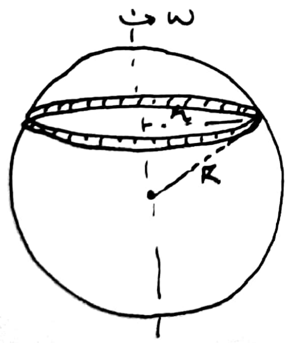
$$= \lambda \omega \left[\frac{x^3}{3}\right]_{-L/2}^{L/2} \hat{z}$$

$$= \lambda \omega \left[\frac{L^3}{24} + \frac{L^3}{24}\right]$$

$$= \frac{\lambda \omega L^3}{12}$$

$$\vec{L} = \frac{ML^2}{12}\vec{\omega}$$

2.



Hollow sphere

$$\vec{L} = \Sigma \vec{L}_{rings}$$

$$\vec{dL} = dM\, r^2 \vec{\omega}$$

$$\vec{L} = 2\pi\sigma R\, \vec{\omega} \int r^3\, d\theta$$

$$\vec{L} = 2\pi\sigma R^4 \int_0^\pi \sin^3\theta\, d\theta$$

$$\vec{L} = \frac{2}{3} MR^2 \vec{\omega}$$

$$\vec{dL} = \vec{r} \times \vec{dp}$$

$$= r\, dp\, \hat{\omega}$$

$$= dm\, r\, v\, \hat{\omega}$$

$$= dm\, r\, r\frac{d\theta}{dt}\, \hat{\omega}$$

$$= dm\, r^2\, \vec{\omega}$$

$$L = \vec{dL} = \dot{m} r^2 \vec{\omega}$$

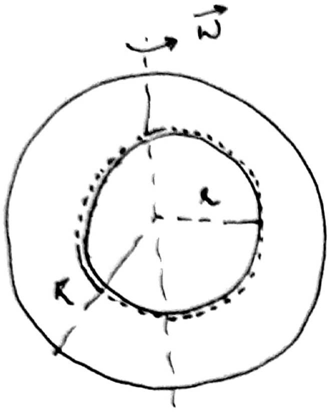
$$dM = \sigma\, dA$$

$$= \sigma(2\pi r\, R d\theta)$$

$$= \frac{M}{4\pi R^2} \cdot 2\pi r R\, d\theta$$

$$= \frac{Mr}{2R}\, d\theta$$

3.



Solid sphere

$$\vec{L} = \Sigma L_{\text{hollow sphere}}$$

$$d\vec{L} = \frac{2}{3} dM\, r^2 \vec{\omega}$$

$$\vec{L} = \cancel{\int} \frac{2}{3}\vec{\omega} \int dM\, r^2$$

$$= \frac{2}{3}\vec{\omega} \int_0^R (\rho \cdot 4\pi r^2 dr)\, r^2$$

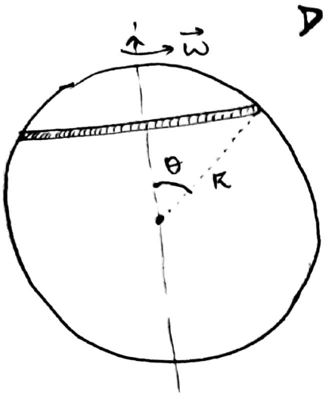
$$\vec{L} = \frac{2}{5} M R^2 \vec{\omega}$$

$$dM = \rho\, dV$$

$$= \rho\,(4\pi r^2 dr)$$

$$\rho = \frac{M}{4/3\, \pi R^3}$$

5.



Disc

$$\vec{dL}_{rod} = \frac{1}{12} dm \left(2R\sin\theta\right)^2 \vec{\omega}$$

$$\vec{L}_{disc} = \int \vec{dL}_{rod}$$

$$= \frac{1}{12} (2R)^2 \vec{\omega} \int \sin^2\theta \, dm$$

$$= \frac{1}{12} (4R^2) \vec{\omega} \int \sin^2\theta \quad \sigma . 2R \sin\theta \, dz$$

$$= \cancel{\frac{R^2 \vec{\omega}}{3} \int \sigma}$$

$$= \frac{-2R^4}{3} \vec{\omega} \, \sigma \int_0^\pi \sin^4\theta \quad d\theta$$

$$= \frac{-2R^4 \sigma}{3} \vec{\omega}$$

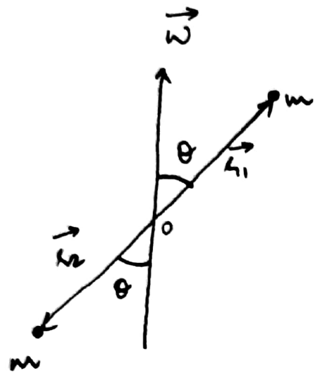
$$\sigma = \frac{M}{\pi R^2}$$

$$dM = \sigma (2R\sin\theta) \, dz$$

$$z = R\cos\theta$$

$$dz = -R\sin\theta \, d\theta$$

6.



$$\vec{L} = \vec{r_1} \times \vec{p_1} + \vec{r_2} \times \vec{p_2}$$

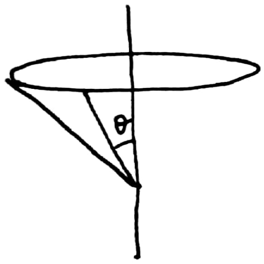
$$\vec{p_1} = m\vec{v_1} = m(\vec{\omega} \times \vec{r_1}) = m\omega \frac{l}{2} \sin\theta$$

$$\vec{L_1} = \vec{r_1} \times \vec{p_1} = \frac{l}{2} p_1 \sin 90^\circ \hat{n}$$

$$= \frac{1}{4} m\omega l^2 \sin\theta \hat{n}$$

$$\vec{L_2} = \vec{r_2} \times \vec{p_2} = \vec{L_1}$$

$$\vec{L} = 2\vec{L_1} = \frac{1}{2} m\omega l^2 \sin\theta \hat{n}$$



$$|\vec{dL}| = L\cos\theta \, d\phi$$

$$\frac{|\vec{dL}|}{dt} = L\cos\theta \frac{d\phi}{dt} = L\omega\cos\theta$$

$$\frac{\vec{dL}}{dt} = L\omega\cos\theta \, \hat{\phi}$$

$$\vec{\tau} = \frac{1}{2} m\omega l^2 \sin\theta\cos\theta \, \hat{\phi}$$