EE 220 : Signals and Systems

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Indian Institute of Technology, Guwahati
End Semester Examination - Nov 2022

Instructions : Q1 is for 3 marks only. Q2 to Q8 carry 6 marks each.

Q1. Four different Fourier representations were introduced in this course. In the following table, fill in the blanks depending on which representation(s) can be used to represent the signal described on the left. Finite duration means that the signal is guaranteed to be nonzero over only a finite interval.

Signal description			Fourier representation(s)
Continuous time	Infinite duration	Periodic	
Continuous time	Infinite duration	Aperiodic	
Continuous time	Finite duration	Aperiodic	
Discrete time	Infinite duration	Periodic	
Discrete time	Infinite duration	Aperiodic	
Discrete time	Finite duration	Aperiodic	

Q2. [a.] For continuous-time signals, we saw that if $x(t) \overset{FT}{\longleftrightarrow} X(\omega)$ then

$$x(at) \overset{FT}{\longleftrightarrow} \frac{1}{|a|} X(\omega/a)$$

Is there a similar property for discrete-time signals? If so, what is it? If not, why not?

[b.] Compute the convolution $y[n] = x[n] * h[n]$; when $x[n] = \alpha^n u[n]$, $0 < \alpha < 1$, and $h[n] = \beta^n u[n]$, $0 < \beta < 1$. Assume that α and β are not equal.

Q3. Suppose we have an LTI system characterized by an impulse response

$$h[n] = \frac{\sin \frac{\pi n}{3}}{\pi n}$$

a. Sketch the magnitude of the system transfer function.

b. Evaluate $y[n] = x[n] * h[n]$ when

$$x[n] = (-1)^n \cos\left(\frac{3\pi}{4} n\right)$$

Q4. A particular discrete-time system has input $x[n]$ and output $y[n]$. The Fourier transforms of these signals are related by the following equation:

$$Y(\Omega) = 2X(\Omega) + e^{-j\Omega} X(\Omega) - \frac{dX(\Omega)}{d\Omega}$$

a. Is the system linear? Clearly justify your answer.

b. Is the system time-invariant? Clearly justify your answer.

c. What is $y[n]$ if $x[n] = \delta[n]$?

Q5. Consider the following two periodic sequences

$$x_1[n] = 1 + \sin(2\pi n/10)$$

$$x_2[n] = 1 + \sin\left(\frac{20\pi n}{12} + \frac{\pi}{2}\right)$$

a. Determine the period of $x_1[n]$ and of $x_2[n]$.

b. Determine the sequence of Fourier series coefficients $X_1[k]$ for $x_1[n]$ and $X_2[k]$ for $x_2[n]$.

c. In each case, the sequence of Fourier series coefficients is periodic. Clearly determine the period of the sequence $X_1[k]$, and the sequence $X_2[k]$.

Q6. Consider two specific periodic sequences $x[n]$ and $y[n]$. $x[n]$ has period N and $y[n]$ has period M. The sequence $w[n]$ is defined as $w[n] = x[n] + y[n]$.

a. Show that $w[n]$ is periodic with period MN.

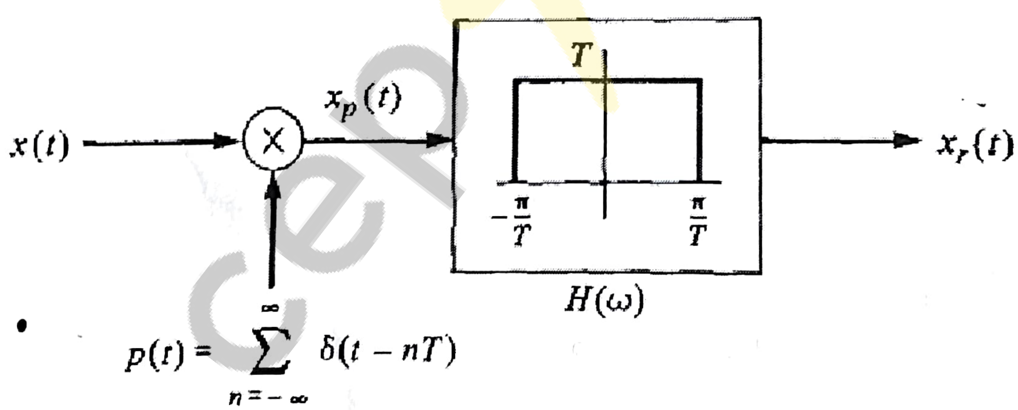
b. If $x[n]$ has its discrete Fourier series coefficients as $X[k]$, $y[n]$ has its discrete Fourier series coefficients as $Y[k]$, and the discrete Fourier series coefficients of $w[n]$ are $W[k]$. Determine $W[k]$ in terms of $X[k]$ and $Y[k]$.

Q7. In the system shown in Figure below, $x(t)$ is sampled with a periodic impulse train, and a reconstructed signal $x_r(t)$ is obtained from the samples by lowpass filtering. The sampling period T is 1 ms, and $x(t)$ is a sinusoidal signal of the form $x(t) = \cos(2\pi f_o t + \theta)$. For each of the following choices of f_o and θ, determine $x_r(t)$.

a. $f_o = 250Hz, \theta = \pi/4$

b. $f_o = 750Hz, \theta = \pi/2$

c. $f_o = 500Hz, \theta = \pi/2$



Q8. [a.] Consider the often-used alternative definition of the continuous time Fourier transform, widely known as $X_a(f)$. The forward transform is written as

$$X_a(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

where f is the frequency variable in Hertz. Derive the inverse transform formula for this definition.

[b.] A second, alternative definition is

$$X_b(v) = \frac{1}{\sqrt{(2\pi)}}\int_{-\infty}^{\infty} x(t)e^{-jvt}dt$$

Find the inverse transform formula for this definition.