PH 202 Course coordinator/ Instructor**: Alika Khare**

Problem Set no. 1(revision) **Due date: Monday, Jan 16, 2024**

Note: The very first problem set is a sort of revision exercise. Please submit the solutions of at least two of the problems, one each from problem no.1-4 and from 5-8 on Tuesday , 16, 2024, during the class. The solutions can be submitted in a group of maximum of four students. Name and Roll no. of all the group members should be written clearly on the top. If solutions are found to be reasonably correct then six marks (three each) will be awarded and in case of non submission/wrong solution zero marks.

1. a. A vector field in "mixed" coordinate variables is given by
   $\vec{G} = \frac{xcos\varphi}{\rho}\hat{\imath} + \frac{2yz}{\rho^2}\hat{\jmath} + \left(1 - \frac{x^2}{\rho^2}\right)\hat{k}$
   Express $\vec{G}$ completely in spherical polar coordinates system.

   b. Express vector $\vec{A} = \rho z sin\varphi\hat{\rho} + 3\rho cos\varphi\hat{\varphi} + \rho cos\varphi sin\varphi\hat{k}$ in Cartesian coordinates.
2. A vector $\vec{A} = 5\rho\hat{\varphi}$, calculate the flux of this vector over the surface defined by $0<\rho<1$ and $-3<z<3$ for a constant φ..
3. Verify the divergence theorem for a vector field given by $\vec{G}(\mathrm{r}) = 10e^{-2z}\left(\rho\hat{\rho} + \hat{k}\right)$ over a cylinder of unit radius and unit height. Sketch the geometry and mark the direction of normal on all the surfaces.
4. Verify the Stoke's theorem for the vector $\hat{A} = \rho cos\varphi\hat{\varphi} + sin\varphi\hat{\varphi}$ over the contour as shown in fig 1 below:

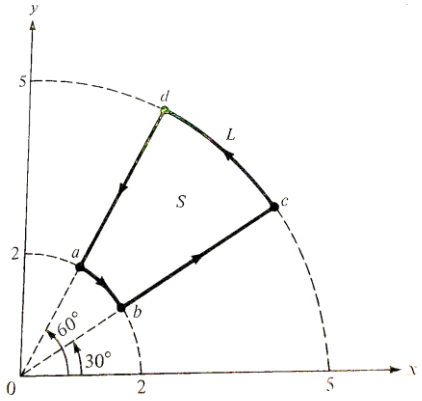

Fig 1

5. The electric field in the atmosphere at the earth's surface is 200V/m directed downward. At 1400m above the earth's surface the electric field is only 20V/m again directed downward. What is the average volume charge density in this region? Does it consist of predominantly positive or negative ions?

6. In a vacuum tube, electrons are emitted from a hot plane metal surface (emitter) and collected by another metal plate (collector) placed parallel to the emitter a distance *d* away as shown in fig 2 below. The distance *d* is small compared to the dimensions of the plates. The electric potential between the plates is given by $V(x) = Cx^{4/3}$ where *x* is the distance from the emitter plate and *C* is a constant. What is the surface charge density σ on the emitter and collector plates. What is the volume charge density $\rho_v(x)$ in the region between the two plated. Plot the charge density as a function of *x*? (neglect the space charge effect).

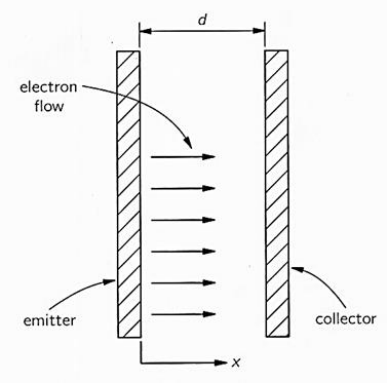

Fig 2

7. Verify which of the following scalar functions satisfy poisson's equation and Laplace's equation:
   i. $V = e^{-z}sin2xcoshy$
   ii. $U = \rho^{2z}cos2\varphi$
   iii. $W = 10rsin^2\theta cos\varphi$
8. Potential in a certain system is given by $V = \frac{qe^{-r/\lambda}}{4\pi\epsilon_0 r}$. Find corresponding electric field and charge density.