PH 203 – Classical Mechanics, Dr. M. K. Nandy, Endsem, 21 Nov. 2023

(Show labelled diagrams and details of calculations)

1. A comet of mass $m$ is moving due to the gravitational attraction of the sun of mass $M$. Assume both $m$ and $M$ to be point particles. $m$ moves in the XY plane with $M$ sitting sationary at the origin. Let $(x, y)$ be the instantaneous co-ordinate of $m$.

   (a) Write the expression for the kinetic energy $T$ and potential energy $V$ in the Cartesian $(x, y)$ system.

   (b) Transform the above Cartesian expressions for $T$ and $V$ into the plain polar $(r, \theta)$ system. .

   (c) Find the equations of motion of the comet employing Lagrange's equation.

   (d) (i) What quantities are conserved?

   (ii) What property of the Lagrangian $L$ make them conserved?

   (iii) Express the conserved quantities in the plain polar system.

2. (a) Write the Euler's equations (for $\dot{\omega}_1$, $\dot{\omega}_2$, $\dot{\omega}_3$) for a rigid body.

   (b) Consider the earth as a rigid body (oblate spheroid, with $x_3$ as symmetry axis). Solve the Euler's equations for the earth. Obtain the expression for precession frequency.

   $\omega_1 = \omega_2$ 300 days

   (c) Draw the body cone and space cone diagrams and identify them in the diagram. Indicate the direction of rotation by arrows on the cones. Describe (in 2-3 lines) the motion(s) of the cones.

3. For a one-dimensional UNDAMPED simple harmonic oscillator

   (a) Write the Hamiltonian.

   (b) Write the Hamilton-Jacobi equation. $\delta S/\delta t - H$

   (c) Implement separation-of-variables and solve the Hamilton-Jacobi equation.

4. For a single particle with instantaneous position $(x, y, z)$, find the following Possion brackets

   (a) $[x, p_x^2]$, (b) $[x, L_y]$, (c) $[L_x, L_y]$

   where $p_x$, $p_y$, $p_z$ and $L_x$, $L_y$, $L_z$ are Cartesian components of the linear momentum and angular momentum respectively.

5. Consider a one-dimensional DAMPED simple harmonic oscillator (of natural frequency $\omega$ and damping constant $\gamma$).

   (a) Write the equation-of-motion.

   (b) Find the solution of the equation-of-motion.

   (c) Write the conditions for the three cases (underdamped, overdamped, critically damped).

   (d) Find the solution for the critically damped case.

   $\lambda^2 = \frac{\omega^2}{4}$