PH207: Year 2023

Problem Set-VII

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1. Maxwell Boltzmann velocity distribution of molecular velocities in an ideal gas at rest is given by

$$f\left(v_x, v_y, v_z\right) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} exp\left(-\frac{mv^2}{2k_B T}\right)$$

   (a) Obtain the average and r.m.s. speeds. (b) Obtain the average values of the 3 velocity components. What can be concluded from this result? (c) Obtain the average kinetic energy

2. (a) Consider the emission or absorption of visible light by the molecules of hot gas. Derive an expression for frequency distribution $f\left(\nu\right)$ expected for a spectral line of central frequency ν_0 due to Doppler broadening.

   Assume an ideal gas at temperature T with molecular mass M. Consider a vessel filled with Argon gas at a pressure of 10 Torr and a temperature of 200 ^{0} C. Inside the vessel is a small piece of sodium which is heated so that the vessel will contain some sodium vapor. The sodium absorption line at 5896 $\AA$ in light from a tungsten filament is observed to pass through the vessel.

   (b) Estimate the magnitude of the Doppler broadening of the line and the collision broadening of the line.

3. An insulated box of volume $2V$ is divided into two equal parts by a thin, heat conducting portion. One side contains an ideal gas of hard sphere molecules at atmospheric pressure and $T = 293$K.

   (a) Calculate the number of molecules striking the partition per unit area and time. Express the result in terms of average velocity and number density.

   (b) A small round hole of radius r is opened in the partition (small enough to maintain thermal equilibrium between two sides through heat conduction via the partition). Calculate the pressure and temperature as functions of time in both halves of the box.

   (c) Suppose the partition is a non conductor of heat. Discuss qualitatively the deviations in results of (b).

4. (a) What fraction of H_2 gas at sea level and $T = 300$K has sufficient speed to escape from the earth's gravitational field? (Consider ideal gas and earth's escape velocity $v_e = \sqrt{2GM/R}$, $M_{earth}=6\times10^{24}$ kg, $R_{earth}=6.4\times10^3$ km,$G = 6.67\times10^{-11} Nm^2kg^{-2}$)

   (b) Imagine an H_2 molecule in the upper atmosphere with a speed equal to earth's escape velocity. Assume that the remaining atmosphere above the molecule has thickness $d = 100$ km, and that the earth's entire atmosphere is isothermal and homogeneous with mean number density $n = 2.5\times10^{25}/m^3$. Using simple arguments, estimate the average time needed for the molecule to escape. Assume all collisions are elastic and that the total atmospheric height is small compared with earth's radius.