PH-207

Tutorial - 2

1. $U = U(L,T)$

$$dU = \left(\frac{\partial U}{\partial L}\right)_T dL + \left(\frac{\partial U}{\partial T}\right)_L dT$$

$$\left.\frac{dU}{dT}\right|_L = \left(\frac{\partial U}{\partial L}\right)\bigg|_T \cdot \cancel{\left.\frac{dL}{dT}\right|_L}^{\,0} + \left(\frac{\partial U}{\partial T}\right)_L$$

$$\left.\frac{dU}{dT}\right|_L = \left.\frac{\partial U}{\partial T}\right|_L$$

$$= \left.\frac{dQ}{dT}\right|_L + \tau \cancel{\left.\frac{dL}{dT}\right|_L}^{\,0} = \left.\frac{dQ}{dT}\right|_L$$

$$\left.\frac{dQ}{dT}\right|_L = C_L = \left.\frac{\partial U}{\partial T}\right|_L$$

$U = U(T, \tau)$

$$dU = \left.\frac{\partial U}{\partial \tau}\right|_T d\tau + \left.\frac{\partial U}{\partial T}\right|_\tau dT$$

$$\left.\frac{dU}{dT}\right|_\tau = \left.\frac{\partial U}{\partial \tau}\right|_T \cancel{\left.\frac{d\tau}{dT}\right|_\tau}^{\,0} + \left.\frac{\partial U}{\partial T}\right|_\tau$$

$$= \left.\frac{\partial U}{\partial T}\right|_\tau$$

$$\left.\frac{dU}{dT}\right|_\tau = \left.\frac{dQ}{dT}\right|_\tau + \tau \left.\frac{dL}{dT}\right|_\tau$$

$$\left.\frac{dU}{dT}\right|_\tau = C_\tau + \tau L \alpha \qquad \left(\alpha = \frac{1}{L}\left.\frac{dL}{dT}\right|_\tau\right)$$

$$C_\tau = \left.\frac{dU}{dT}\right|_\tau - \tau L \alpha$$

2. $U = U(T, P)$

$$dU = \left.\frac{\partial U}{\partial T}\right|_P dT + \left.\frac{\partial U}{\partial P}\right|_T dP$$

$$dU = dQ - P\,dV$$

$$dU = dQ - P\left(\left(\frac{\partial V}{\partial T}\right)_P dT + \left.\frac{\partial V}{\partial P}\right|_T dP\right) = \left.\frac{\partial U}{\partial T}\right|_P dT + \left.\frac{\partial U}{\partial P}\right|_T dP$$

$$dQ = dP\left[\left.\frac{\partial U}{\partial P}\right|_T + P\left.\frac{\partial V}{\partial P}\right|_T\right] + dT\left[\left.\frac{\partial U}{\partial T}\right|_P + P\left.\frac{\partial V}{\partial T}\right|_P\right]$$

$$\left.\frac{dQ}{dT}\right|_P = \left.\frac{\partial U}{\partial T}\right|_P + P\left.\frac{\partial V}{\partial T}\right|_P$$

$$C_p = \left.\frac{\partial U}{\partial T}\right|_P + P(+V\beta) \qquad \left(\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P\right)$$

$$C_p - PV\beta = \left.\frac{\partial U}{\partial T}\right|_P$$

$$P = P(V, T)$$

$$dP = \left.\frac{\partial P}{\partial V}\right|_T dV + \left.\frac{\partial P}{\partial T}\right|_V dT$$

$$\left.\frac{dP}{dT}\right|_V = \cancel{V}\left.\frac{\partial P}{\partial T}\right|_V$$

$$\left.\frac{dQ}{dT}\right|_V = \left.\frac{dP}{dT}\right|_V \cdot \left.\frac{\partial U}{\partial P}\right|_T + \left.\frac{\partial U}{\partial T}\right|_P$$

$$C_v = \left.\frac{\partial P}{\partial T}\right|_V \left.\frac{\partial U}{\partial P}\right|_T + \left.\frac{\partial U}{\partial T}\right|_P$$

$$C_v = \frac{\beta}{k}\left.\frac{\partial U}{\partial P}\right|_T + (C_p - PV\beta) \qquad \left(\left.\frac{\partial P}{\partial T}\right|_V = \frac{\beta}{k}\right)$$

$$\left.\frac{\partial U}{\partial P}\right|_T = PVk - (C_p - C_v)\frac{k}{\beta}$$

3.

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

$$C_v = \left(\frac{\partial U}{\partial T}\right)_V \qquad C_p = \left.\frac{dQ}{dT}\right|_P$$

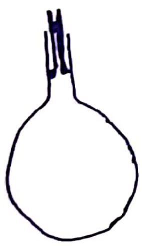
$$= \left.\frac{\partial U}{\partial V}\right|_T \left.\frac{dV}{dT}\right|_P + \left.\frac{\partial U}{\partial T}\right|_V + P\left.\frac{dV}{dT}\right|_P$$

$$C_p - C_v = \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right]\left.\frac{dV}{dT}\right|_P$$

$$C_p - C_v = \frac{a/V^2 + P}{P - \frac{a}{V^2} + \frac{2ab}{V^3}} \cdot R$$

$a, b \to 0 \Rightarrow C_p - C_v = R$

4.



$$PV^{\gamma} = C$$

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0$$

Force on ball $F = A\,dP$

Displacement of the ball $dx = \frac{dV}{A}$

$$dF = -k\,dx$$

$$\cancel{A\,dP = -k\frac{dV}{A}}$$

$$A\,dP = -k\frac{dV}{A}$$

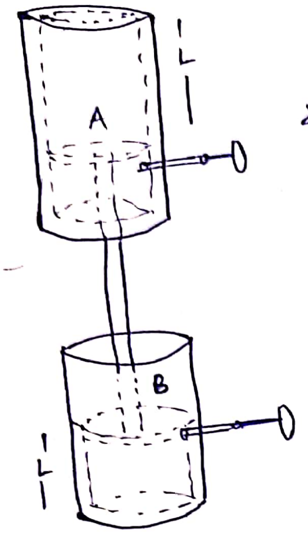
$$dP = \frac{-k}{A^2}\,dV$$

$\frac{dP}{P} + \gamma\frac{dV}{V} = 0 \Rightarrow$ can be satisfied if $k = \frac{\gamma A^2 P}{V}$

$$P = P_0 + \frac{mg}{A}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\gamma A^2 (P_0 + mg/A)}{mV}}$$

5.



2 atm, 0°c

1 atm, 0°c

a) $A + B \Rightarrow$ composite system

$$\Delta U_A + \Delta U_B = 0$$

$$\Delta U_A = -\Delta U_B$$

$$T_A^f = T_B^f = T^f$$

$$PV = nRT \qquad U = \int c_v \, dT$$

$$n_A = \frac{P_A^i V_A^i}{R T_A^i} \qquad n_B = \frac{P_B^i V_B^i}{R T_B^i}$$

$$T_A^f = T_A^i + \frac{(U_A^f - U_A^i)}{n_A C_v}$$

$$T_B^f = T_B^i + \frac{(U_B^f - U_B^i)}{n_B C_v}$$

$$T_f = \frac{n_A T_A^i + n_B T_B^i}{n_A + n_B} \Rightarrow T_f = 0°c$$

b) A + B has an internal adiabatic wall.

$$A \rightarrow dU_A = dW_A = -P_A \, dV_A$$

$$n_A C_v dT_A = -n_A R \left(dT_A - \frac{T_A}{P_A} dP_A \right)$$

$$\left(\frac{C_v + R}{R} \right) \int_{T_A^i}^{T_A^f} \frac{dT_A}{T_A} = \int_{P_A^i}^{P_A^f} \frac{dP_A}{P_A},$$

$$\frac{T_A^f}{T_A^i} = \left(\frac{P_A^f}{P_A^i} \right)^{R/(R + C_v)}$$

$$V_A^f + V_B^f = V_A^i + V_B^i = V_T \qquad V_T = \frac{(n_A T_A^f + n_B T_B^f) R}{P_A^f}$$

$$P_A^f = \frac{P_A^i V_A^i + P_B^i V_B^i}{V_A^i + V_B^i} = 1.5 \text{ atm}$$

$$T_A^f = \left(\frac{1.5}{2} \right)^{0.4} \Rightarrow T_A^f = 243 \text{ K}$$

$$n_A C_v (T_A^f - T_A^i) + n_B C_v (T_B^f - T_B^i) = 0$$

$$T_B^f = 334 \text{ K}$$