

Transport processes: No scattering of Bloch electrons in ideal periodic potential.
Real semiconductor crystals: many sources of scattering:

Sources of Scattering	Imperfections
Ionized Impurity	Due to dopant in semiconductors
Phonon	Due to lattice vibrations
Alloy	Random potential function
Interface roughness	Impurity in heterostructure devices
Chemical Impurity	Unintentional impurities with short range potential
Photons	electrons

Relaxation time (τ_r)- Average time it takes to loose coherence or memory of initial state properties.

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Velocity-Electric Field Relations in Semiconductors

When an electron distribution is subjected to an electric field, the electrons tend to move in the field direction (opposite to the field \mathbf{F}) and gain velocity from the field. However, because of imperfections, they scatter in random directions. A steady state is established in which the electrons have **some net drift velocity** in the field direction. [The response of the electrons to the field can be represented by a velocity-field relation.](#)

Low Field Response: Mobility

Three assumptions:

i) The electrons in the semiconductor do not interact with each other. This approximation is called the [independent electron approximation](#).

ii) Electrons suffer collisions from various scattering sources and the time τ_{sc} describes the [mean time between successive collisions](#).

iii) The electrons move according to the free electron equation $\frac{\hbar d\mathbf{k}}{dt} = \mathbf{F}_{ext}$

The average velocity gain is then that of an electron with mass m^* traveling in a field F for a time τ_{sc} :

$$v_{avg} = -\frac{eF\tau_{sc}}{m^*} = v_d$$

where V_d is the drift velocity.

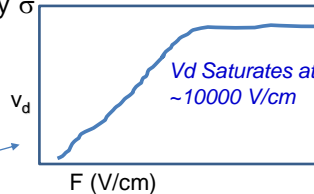
The current density is now
$$\mathbf{J} = -nev_d = \frac{ne^2\tau_{sc}}{m^*}\mathbf{F}$$

Comparing this with the Ohm's law result for conductivity σ

$\mathbf{J} = \sigma\mathbf{F}$ we get
$$\sigma = \frac{ne^2\tau_{sc}}{m^*}$$

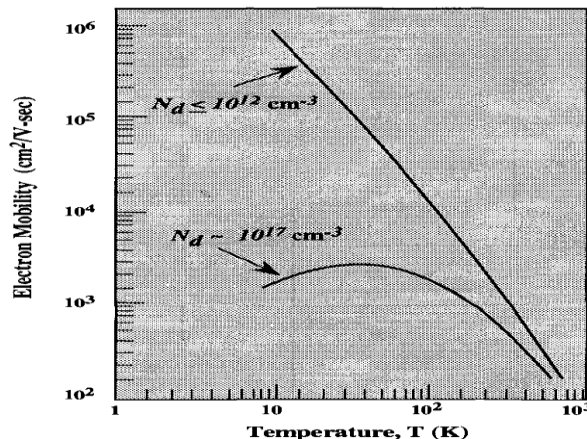
The resistivity of the semiconductor is simply the inverse of the conductivity. From the definition of mobility μ , for electrons

$\mathbf{v}_d = \mu\mathbf{F}$ we have
$$\mu = \frac{e\tau_{sc}}{m^*}$$



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Notice that the **mobility has an explicit $1/m^*$ dependence** in it. Additionally, τ_{sc} **decreases with increasing m^*** due to the dependence of scattering rates on the density of states. Thus, the mobility has a strong dependence on the carrier mass. The effective mass to be used in these transport parameters is the **conductivity effective mass**.



$$m_o^* = 3 \left(\frac{2}{m_i^*} + \frac{1}{m_t^*} \right)^{-1}$$

μ for Si $\sim 1500 \text{ cm}^2/\text{V-s}$
 μ GaAs $\sim 8500 \text{ "}$
 AlGaAs $\sim 11000 \text{ "}$

Fig. 23: Electron Mobility as a function of temperature for low doping and high doping cases in Silicon. 87

For ionized impurity scattering:

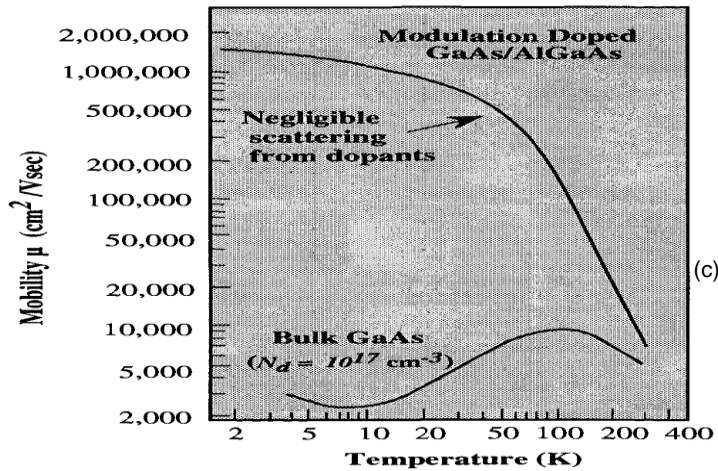
$$\mu_i \propto T^{3/2}$$

Acoustic phonon scattering:

$$\mu_{AP} \propto T^{-3/2}$$

Alloy scattering:

$$\mu_A \propto T^{-1/2}$$



Mobility vs temperature for modulation doped and uniform doped GaAs.

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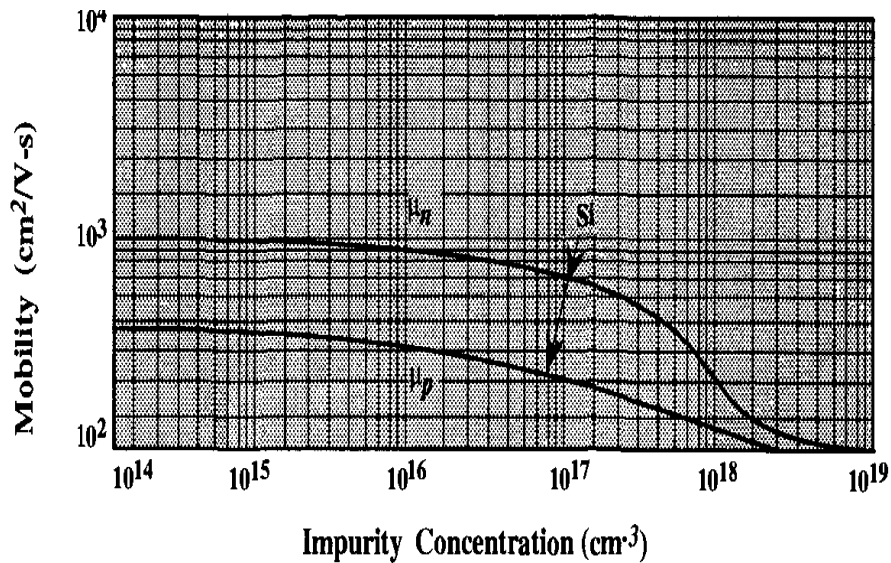


Fig. 24:: Mobility of electrons and holes in Si as a function of doping conc at 300K.

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The response of the free carriers to an external electric field is given by v & F relationship.

- (i) For low field, $v = \mu F$, μ = mobility = velocity/ unit field
- (ii) Higher field ($F \geq 1$ KV/cm), **v - F relationship non-linear**, negative resistance in direct gap materials.
- (iii) Externally high field, ($F > 10^5$ V/cm). **Breakdown of semiconductor due to impact ionization or tunneling**. Avalanche detectors exploit this phenomena.

$$\mu = \frac{e \langle \tau \rangle}{m^*}$$

For ionized impurity scattering :

mobility increases with temperature.

$$\mu_i \propto T^{\frac{3}{2}}$$

For acoustic phonon scattering

Mobility decreases with temperature

$$\mu_{AP} \propto T^{-\frac{3}{2}}$$

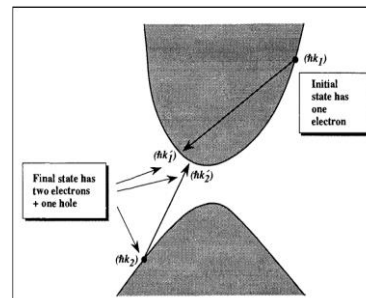
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High field transport-Breakdown phenomena

For $F \geq 100$ KV/cm, breakdown due to carrier multiplication.

(A) Impact Ionization (Avalanche Breakdown)

- (i) For low field, electron remains in the same band.
- (ii) High field, hot electron scatters with VB electron via Coulombic interaction.
- (iii) Initial electron must have higher energy than Bandgap.
- (iv) Carrier multiplication- avalanching.



impact ionization process

Energy conservation:
$$\frac{\hbar^2 K_1^2}{2m_c^*} = \frac{\hbar^2 K_2^2}{2m_v^*} + \frac{\hbar^2 K_1'^2}{2m_c^*} + \frac{\hbar^2 K_2'^2}{2m_c^*}$$

$$K_1 = K_2 + K_1' + K_2'$$

Because of simultaneous condition, minimum value of E_{K1} : $E_{K1} = E_g \left(1 + \frac{m_c^*}{m_{av}^*}\right)$

Current depends on bandgap.

$$\frac{dI}{dt} = \alpha_t I$$

In narrow gap material, it happens for low electric field.

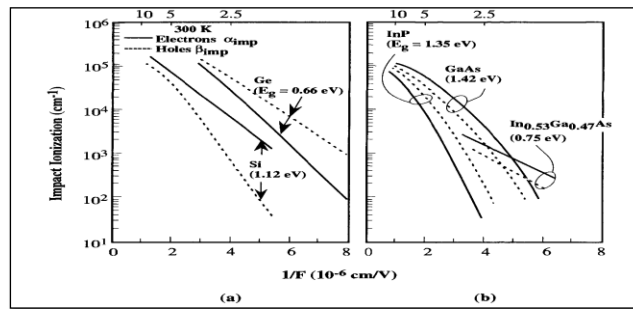
$$\frac{dI(z)}{dz} = \alpha_z I$$

Rate of ionization per unit time.

$$N(x) = \frac{I(x)}{I(0)} = \exp(\alpha_{imp} x)$$

No of times initial electrons will suffer impact ionization after travelling a distance x

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Ionization rate of electrons/holes vs $1/F$.

Fig. 25

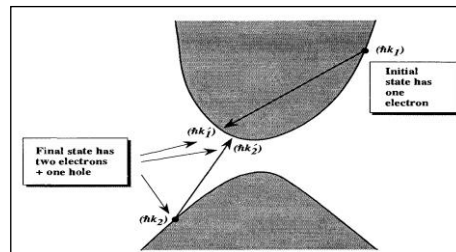


Fig. 26: The impact ionization process where a high energy electron scatters from a valence band electron producing two conduction band electrons and a hole. Hot holes can undergo a similar process. Both energy and momentum conservation are required in such a process. Thus the initiating electron must have an energy above a minimum threshold.

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Critical field is defined as $\alpha_{\text{imp}} = 10^4 \text{ V/cm}$. One impact ionization for travel of $1 \mu\text{m}$.

Disadvantage: Limitation of power output of a device. (control of device due to Breakdown).

Advantage: Avalanche photodetector - High gain detection. Also applied in Microwave devices.

Band-to-Band Tunneling: Zener Tunneling

From quantum mechanics, Tunneling probability through rectangular barrier:

$$T = \exp\left(-\frac{4\sqrt{2m^*}}{3e\hbar F} E_g^{\frac{3}{2}}\right) \quad m^* - \text{e-h reduced mass.}$$

In narrow gap materials, Zener tunneling is very important.

It is the basis of Zener diode, where current is 0 until b-b tunneling starts & current increases sharply. $T \geq 10^{-6}$ necessary for breakdown to start.

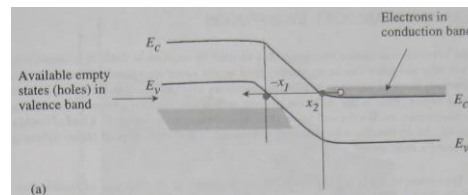


Fig.27: E-x and E-k diagram of p-n junction

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Carrier transport by Diffusion:

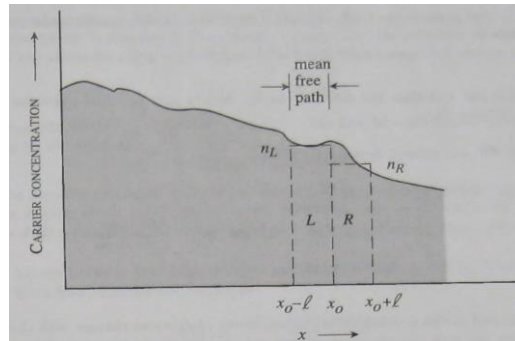


Fig.28: Concentration profile of electrons as a function of space

Diffusion due to thermodynamic reasons. From high concentration region to low concentration region- due to concentration gradient. Disorder/entropy decreases in this process.

Flux to the right through x_0

$$\phi_n(x, t) = \left(\frac{n_L - n_R}{2} \right) \cdot \frac{l}{\tau_{sc}} \quad l \text{ is the mean free path}$$

$$n_L - n_R = -\frac{dn}{dx} \cdot l$$

$$\phi_n(x, t) = -\left(\frac{l^2}{2\tau_{sc}} \right) \frac{dn}{dx} = -D_n \frac{dn}{dx}$$

diffusion coefficient D_n depends on temperature, as $l = v_{th}\tau_{sc}$

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Similarly for the holes $\phi_p = -D_p \frac{dp}{dx}(x, t)$

$$\begin{aligned} J_{\text{total}}(\text{diff.}) &= J_n(\text{diff.}) + J_p(\text{diff.}) \\ &= eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx} \end{aligned}$$

Transport by drift and diffusion (combined): Einstein's relation

$$\begin{aligned} J_n(x) &= -e\mu_n n F + eD_n \frac{dn}{dx} \\ J_p &= e\mu_p p F - eD_p \frac{dp}{dx} \end{aligned} \quad \mu F \text{ is const.} = v_d \text{ for high field.}$$

Force on electron in Field $F = -eF(x)$

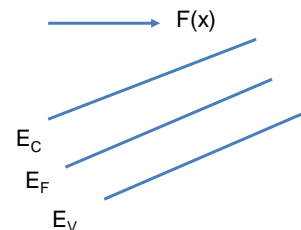
Force = $-\nabla U(x)$

Pot. Energy $U = eF(x) \cdot x$ for uniform electric field.

As electron charge is negative, **the energy band bends**

$$E_c(x) = E_c(F=0) + eF(x) \cdot x$$

$$\text{At equilibrium, } J_n = 0 = J_p \quad F(x) = \frac{D_n}{\mu_n} \cdot \frac{1}{n(x)} \cdot \frac{dn}{dx}$$



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$$n(x) = n_i \exp\left(-\frac{E_{Fi} - E_F(x)}{kT}\right) \leftarrow \text{Boltzman distribution for electron}$$

$$\frac{dn}{dx} = \frac{n(x)}{kT} \cdot \left(\frac{dE_{Fi}}{dx} - \frac{dE_F}{dx}\right)$$

As, Fermi level cannot vary spatially at equilibrium $\frac{dE_F}{dx} = 0$

As, $F(x) = \frac{1}{e} \frac{dE_{Fi}}{dx}$ at equilibrium

$$\frac{D_n}{\mu_n} \cdot \frac{1}{n(x)} \cdot \frac{dn}{dx} = \frac{1}{e} \frac{dE_{Fi}}{dx} = \frac{1}{e} \cdot \frac{dn}{dx} \cdot \frac{kT}{n(x)}$$

$\frac{D_n}{\mu_n} = \frac{kT}{e}$

→ Einstein Relation

Charge injection and Quasi-Fermi Level:

For non-equilibrium electron and hole concentrations, the occupation of the carriers is given by an electron and hole Fermi function independent of each other.

- Note that in equilibrium, the same Fermi function describes both electrons and holes.

$$\begin{aligned} n &= N_c \exp\left[\frac{(E_{Fn} - E_c)}{k_B T}\right] \\ p &= N_v \exp\left[\frac{(E_v - E_{Fp})}{k_B T}\right] \end{aligned}$$

with Boltzman approx.

With Joyce-Dixon approx.

$$(E_{Fn} - E_c) = k_B T \left[\ln \frac{n}{N_c} + \frac{n}{\sqrt{8} N_c} \right]$$

Hall effect

Force on hole $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

$$F_y = q(E_y - v_x B_z)$$

$$E_y = v_x B_z$$

$$v_e = \mu_e E = \mu_e F/e, F_y = 0$$

$$R_H = \frac{E_y}{J_x B} = \frac{v_x B}{nq v_x B} = \frac{1}{nq}$$

$$J_x = nq v_x$$

$$\begin{aligned} m \mathbf{v}_x &= -e \mathbf{E}_x - e v_y \mathbf{B} \\ &= -e \mathbf{E}_x - e \mu_e E_y \mathbf{B} \end{aligned}$$

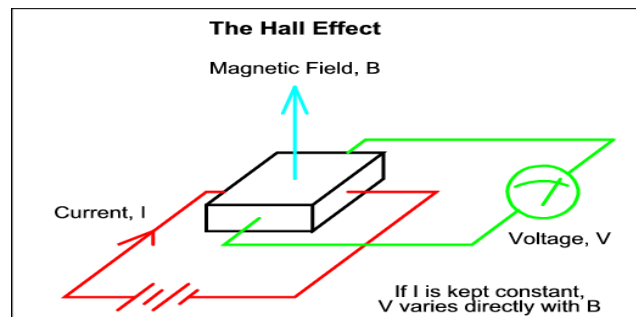


Fig. 29: Hall effect set up

If n and p are comparable, then the following treatment applies:

$$m\vec{v}_y = -e\epsilon_y - eB\mu_e\epsilon_x$$

using $\vec{v}_x = -\mu\epsilon_x$

$$\begin{aligned} \overline{v_{ye}} &= -\mu_e\epsilon_y - B\mu_e^2\epsilon_x \\ \overline{v_{yh}} &= \mu_h\epsilon_y - B\mu_h^2\epsilon_x \end{aligned}$$

$$J_y = -ne\vec{v}_{ye} + pe\vec{v}_{yh}$$

$$= ne\mu_e\epsilon_y + ne\mu_e^2B\epsilon_x + pe\mu_h\epsilon_y - pe\mu_h^2B\epsilon_x = 0$$

$$\frac{\epsilon_y}{\epsilon_x} = \frac{p\mu_h^2 - n\mu_e^2}{n\mu_e + p\mu_h} \quad \text{net}$$

$$R = \frac{\epsilon_y}{J_B} = \frac{\epsilon_y}{\sigma E_x B} = \frac{p\mu_h^2 - n\mu_e^2}{e(n\mu_e + p\mu_h)^2}$$

$$\sigma = ne\mu_e + pe\mu_h$$

$$b = \frac{\mu_e}{\mu_h}$$

Hall coefficient:
$$R = \frac{1}{e} \frac{p - b^2 n}{(p + nb)^2}$$

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OPTICAL PROPERTIES OF SEMICONDUCTORS

Scattering caused by light or photon

Electro-magnetic wave eqn.

$$\psi = A \exp \{i(kx - \omega t)\}$$

Light as electromagnetic wave travelling through semiconductor

Electric field vector

$$\mathbf{E} = \mathbf{E}_0 \exp \left\{ i\omega \left(\frac{n_r z}{c} - t \right) \right\} \exp \left(-\frac{\alpha z}{2} \right) \quad n = \frac{ck}{\omega}$$

$\alpha \rightarrow$ absorption coefficient

$z \rightarrow$ propagation direction

For nonzero α , photon flux $\mathbf{I} = (\mathbf{F}^* \mathbf{F}) = \mathbf{I}_0 \exp(-\alpha z)$

Absorption \rightarrow By impurities, multiband absorption (C-B electron)

Most important in semiconductors is Band to band transition.

Momentum conservation ensures vertical transition.

$$\text{Absorption: } \mathbf{E}_f = \mathbf{E}_i + \hbar\omega$$

$$\text{Emission: } \mathbf{E}_f = \mathbf{E}_i - \hbar\omega$$

$$\text{Photon: } \mathbf{K}_{ph} = \frac{2\pi}{\lambda}$$

Since 1 eV photons correspond to a wavelength of $1.24 \mu\text{m}$, the k -values of relevance are $\sim 10^{-4} \text{ \AA}^{-1}$, which is essentially zero compared to the k -values for electrons. Thus k -conservation ensures that the initial and final electrons should have the same k -value.

$$k_e = 2\pi/a$$

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Scattering involves the following issues :

Conservation of energy : Absorption $E_f = E_i + \hbar\omega$
 Emission $E_f = E_i - \hbar\omega$
Conservation of momentum : $\mathbf{k}_{ph} = \frac{2\pi}{\lambda}$

k conservation needs that **initial and final electron should have same k-value.**, i.e., **only vertical k-transitions** are allowed.

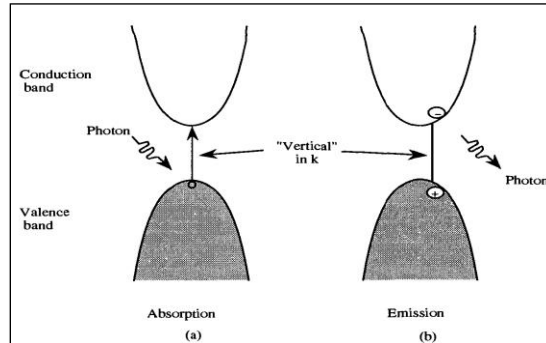


Fig. 32: Band to band absorption and emission in semiconductors. In (a) an electron in the valence band absorbs a photon and moves into the conduction band; (b) In the reverse process an electron in the conduction band emits a photon and moves "vertically" down into the valence band.

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- ❖ Direct bandgap optical transitions are strong.
- ❖ Indirect bandgap optical transitions are weak.
- ❖ Indirect gap optical transitions require the help of lattice vibrations to satisfy k-conservation.

The k-values for the electron & holes are same in vertical transition.

$$\hbar\omega = E_g + \frac{\hbar^2 k^2}{2} \left(\frac{1}{m_e^*} + \frac{1}{m_h^*} \right) = E_g + \frac{\hbar^2 k^2}{2m_r^*}$$

m_r^* is the reduced mass.

So, relevant density of states here in the scattering process is the reduced mass.

This DOS is called Joint Density of States. (J-DOS).

For light polarized along arbitrary direction , **absorption coefficient is**

Dipole matrix element between VB and CB.

$$\alpha = \frac{\pi e^2 \hbar}{m_0^2 c n_r \epsilon_0} \cdot \frac{1}{\hbar\omega} |(\mathbf{a} \cdot \mathbf{p})_{if}|^2 N_{ev}(\hbar\omega)$$

$$\langle |(\mathbf{a} \cdot \mathbf{p})_{if}|^2 \rangle \simeq \frac{2}{3} \mathbf{p}_{cv}^2$$

$$N_{ev}(\hbar\omega) = \frac{\sqrt{2}}{\pi^2 \hbar^3} (m_r^*)^{\frac{3}{2}} (\hbar\omega - E_g)^{\frac{1}{2}}$$

Joint density of states

$$\alpha = \frac{\pi e^2 \hbar}{2 n_r c \epsilon_0 m_0} \left(\frac{2 \mathbf{p}_{cv}^2}{m_0} \right) \cdot \frac{N_{cv}(\hbar\omega)}{\hbar\omega} \cdot \frac{2}{3}$$

$$2 \frac{\mathbf{p}_{cv}^2}{m_0} = 23 \text{ eV}$$

for most semiconductors.

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EXAMPLE 5.4 Calculate the absorption coefficient of GaAs as a function of photon frequency.

The joint density of states for GaAs is (using a reduced mass of $0.058m_0$)

$$\begin{aligned}
 N_{cv}(E) &= \frac{\sqrt{2}(m_r^*)^{3/2}(E - E_g)^{1/2}}{\pi^2 \hbar^3} \\
 &= \frac{1.414 \times (0.058 \times 0.91 \times 10^{-30} \text{ kg})^{3/2} (E - E_g)^{1/2}}{9.87 \times (1.05 \times 10^{-34})^3} \\
 &= 1.5 \times 10^{54} (E - \hbar\omega)^{1/2} \text{ J}^{-1} \text{ m}^{-3}
 \end{aligned}$$

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The absorption coefficient for unpolarized light is

$$\alpha(\hbar\omega) = \frac{\pi e^2 \hbar}{2n_r c \epsilon_0 m_0} \left(\frac{2p_{cv}^2}{m_0} \right) \frac{N_{cv}(\hbar\omega)}{\hbar\omega} \cdot \frac{2}{3}$$

The term $\frac{2p_{cv}^2}{m_0}$ is ~ 23.0 eV for GaAs. This gives

$$\begin{aligned}
 \alpha(\hbar\omega) &= \frac{3.1416 \times (1.6 \times 10^{-19} \text{ C})^2 (1.05 \times 10^{-34} \text{ J.s})}{2 \times 3.4 \times (3 \times 10^8 \text{ m/s}) (8.84 \times 10^{-12} \text{ F/m}^2)} \\
 &\quad \cdot \frac{(23.0 \times 1.6 \times 10^{-19} \text{ J})}{(0.91 \times 10^{-30} \text{ kg})} \frac{(\hbar\omega - E_g)^{1/2}}{\hbar\omega} \times 1.5 \times 10^{54} \times \frac{2}{3} \\
 \alpha(\hbar\omega) &= 1.9 \times 10^{-3} \frac{(\hbar\omega - E_g)^{1/2}}{\hbar\omega} \text{ m}^{-1}
 \end{aligned}$$

Here the energy and $\hbar\omega$ are in units of Joules. It is usual to express the energy in eV, and the absorption coefficient in cm^{-1} . This is obtained by multiplying the result by

$$\begin{aligned}
 &\left[\frac{1}{(1.6 \times 10^{-19})^{1/2}} \times \frac{1}{100} \right] \\
 \alpha(\hbar\omega) &= 4.7 \times 10^4 \frac{(\hbar\omega - E_g)^{1/2}}{\hbar\omega} \text{ cm}^{-1}
 \end{aligned}$$

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For GaAs,
$$\alpha(\hbar\omega) = 5.6 \times 10^4 \frac{(\hbar\omega - E_g)^{1/2}}{\hbar\omega} \text{ cm}^{-1} \quad (\text{With } m^* = 0.065 m_0)$$

So, a few μm of GaAs is enough to absorb a significant fraction of light above bandgap.

α (Si) is low near bandedge. k-conservation rule makes transition weak.

Spontaneous emission (no photon before): *Recombination of electrons with holes*

Emission rate:
$$W_{\text{em}} = \frac{e^2 n_r \hbar \omega}{3 \pi \epsilon_0 m_0^2 c^3 \hbar^2} |\mathbf{p}_{cv}|^2 \quad \text{Output: Incoherent photons}$$

Stimulated emission rate:
$$W_{\text{st}} = \frac{e^2 n_r \hbar \omega}{3 \pi \epsilon_0 m_0^2 c^3 \hbar^2} |\mathbf{p}_{cv}|^2 \cdot n_{\text{ph}}(\hbar\omega) \quad \text{Coherent photons}$$

Radiative recombination rate of an electron having momentum $\hbar\mathbf{k}$ with a hole of moment $\hbar\mathbf{k}$ or energy $\hbar\omega$:

$$\tau_0 \rightarrow \frac{1}{W_{\text{em}}} = \text{ns} - \mu\text{s} \quad (\text{direct} - \text{indirect})$$

In indirect gap material, τ_0 is large ($\sim \mu\text{s}$)

In direct gap material, τ_0 is small. ($\sim \text{ns}$)

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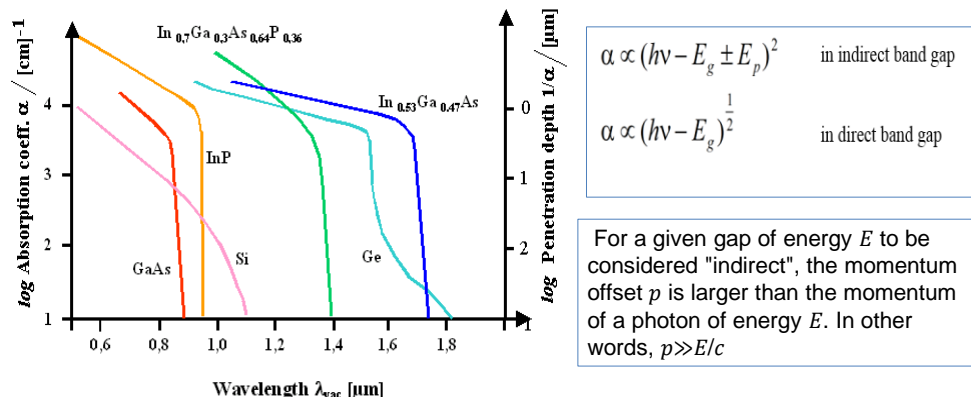


Fig. 33: Absorption coefficient for several direct and indirect semiconductors. For the direct gap material, the absorption coefficient is higher.

Excess carrier Recombination Mechanisms:

- (i) Direct \rightarrow band to band
- (ii) Indirect \rightarrow via deep energy level
- (iii) Auger recombination (3 particle process) : bound to bound (trap mediated)

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Charge injection and radiative recombination

At Equilibrium
$$f(E) = \frac{1}{\exp\left(\frac{E - \mu}{kT}\right) + 1} \quad np = \text{constant}$$

Non-equilibrium , quasi-Fermi level

Two assumptions : (i) electron in thermal equilibrium with CB etc.

(ii) Recombination time is larger than time required for equilibrium.

Quasi-equilibrium

$$f^e(E) = \frac{1}{\exp\left(\frac{E - E_{Fn}}{kT}\right) + 1} \quad n = \int_{E_c}^{\infty} n(E) f^e(E) dE$$

$$f^h(E) = 1 - f^v(E) = 1 - \frac{1}{\exp\left(\frac{E - E_{Fp}}{kT}\right) + 1} = \frac{1}{\exp\left(\frac{E_{Fp} - E}{kT}\right) + 1}$$

With excess electron, E_{Fn} moves towards CB.

With excess hole, E_{Fp} moves towards VB.

With Boltzmann approx.

$$\begin{cases} n = N_c \exp\left[\left(\frac{E_{Fn} - E_c}{kT}\right)\right] \\ p = N_v \exp\left[\left(\frac{E_v - E_{Fp}}{kT}\right)\right] \end{cases}$$

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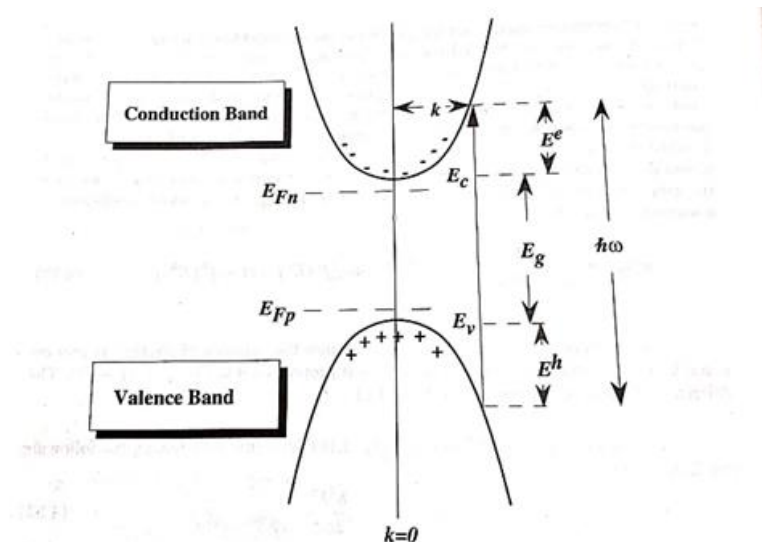


Figure 4.16: The positions of the quasi-Fermi levels, the electron and hole energies at vertical k -values. The electron and hole energies are determined by the photon energy and the carrier masses.

Scanned with CamScanner

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With Joyce Drain approx., $E_{Fn} - E_c = kT \left[\ln \frac{n}{N_c} + \frac{n}{\sqrt{8} N_c} \right]$

With more injections, $(E_{Fn} - E_{Fp})$ increases.

With hole in VB and electron in CB, if photon falls.

More emission than absorption \implies **optical gain = emission Coefficient - absorption coefficient**

$$g(\hbar\omega) = \frac{\pi e^2 \hbar}{m_n c m_0^2 (\hbar\omega)} \cdot |\mathbf{a} \cdot \mathbf{p}_{if}|^2 N_{cv}(\hbar\omega) [f^e(E^e) - (1 - f^h(E^h))]$$

Gain is positive when

$$f^e(E^e) > 1 - f^h(E^h)$$

$$I(z) = I_0 \exp(gz)$$

$$f^e f^h - (1 - f^e)(1 - f^h)$$

Optical gain \longrightarrow **basis for semiconductor laser.**

Spontaneous Recombination Rate

$$R_{\text{spont}} = \frac{1}{\tau_0} \int d(\hbar\omega) N_{ev} \{f^e(E^e) \cdot f^h(E^h)\}$$

Minority carrier injection:

$n \gg p$, sample heavily doped

$$R_{\text{spont}} = \frac{1}{\tau_0} \int d(\hbar\omega) N_{ev} f^h(E^h)$$

$$= \frac{1}{\tau_0} \left(\frac{m_r^*}{m_h^*} \right)^{\frac{3}{2}} \cdot p$$

$$f^e(E^e) \simeq 1$$

Useful for p-n diode and bipolar transistor

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Strong injection, f^e and f^h are sharp step functions,

$$R_{\text{spont}} = \frac{n}{\tau_0} = \frac{p}{\tau_0}$$

Charge injection: Non-radiative effects

Impurity \longrightarrow Bandgap states \longrightarrow Localized (in a finite space)

Defect allow recombination of e and p without emitting a photon.

In laser making, \longrightarrow no non-radiative desirable

In p-n junction \longrightarrow non-radiative recombination is desirable for increasing speed.

Electron capture rate

$$r_n^c = N_t \{1 - f(E_t)\} \sigma_e v_{th} n$$

Where, σ_e is the capture cross-section, v_{th} - electron velocity, n - conc. of electron.

Total electron emission rate: $r_n^e = N_t \cdot f(E_t) P_n$ P_n - Re-emission rate for each site.

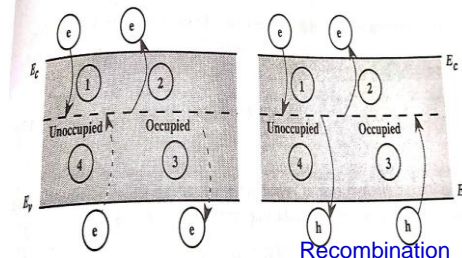
Hole capture rate: $r_p^c = N_t f(E_t) \sigma_h v_{th} p$

Hole emission rate: $r_p^e = N_t \{1 - f(E_t)\} P_p$

In steady state,

$$r_n^c = r_n^e$$

$$r_p^c = r_p^e$$



Recombination

Now, $n = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$

$$f(E_t) = \frac{1}{1 + \exp\left(\frac{E_t - E_F}{kT}\right)}$$

$$1 - f(E_t) = \frac{\exp\left(\frac{E_t - E_F}{kT}\right)}{1 + \exp\left(\frac{E_t - E_F}{kT}\right)}$$

So, solving for P_n and P_p :

$$P_n = n_i v_{th} \sigma_n \exp\left(\frac{E_t - E_{Fi}}{kT}\right)$$

$$P_p = n_i v_{th} \sigma_p \exp\left\{-\left(\frac{E_t - E_{Fi}}{kT}\right)\right\}$$

Rate of change of mobile electron concentration $-\frac{dn}{dt} = r_n^c - r_n^e$

Rate of change of hole concentration $-\frac{dp}{dt} = r_p^c - r_p^e$

If $\tau_n = \frac{1}{N_t v_{th} \sigma_n}$, $\tau_p = \frac{1}{N_t v_{th} \sigma_p}$

Then recombination rate, $R_t = -\frac{dn}{dt} = -\frac{dp}{dt}$

$$= \frac{np - n_i^2}{\tau_n \left[p + n_i \exp\left\{-\left(\frac{E_t - E_{Fi}}{kT}\right)\right\}\right] + \tau_p \left[n + n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right)\right]}$$

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This is known as **Shockley-Read-Hall (SRH) recombination rate via deep level in bandgap**.

To simplify, assume $\tau_n = \tau_p = \tau_{nr}$

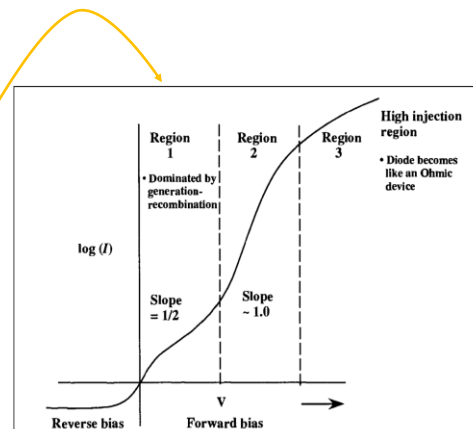
$E_t = E_{Fi}$ (midgap level)

$np \gg n_i^2$ (injection)

$$R_t = \frac{np}{\tau_{nr}(n + p)}$$

So, R_t depends on N_t , σ , v_{th} .

Non-radiative recombination gives rise to non-ideal I-V characteristic in p-n diode.



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Continuity equation

Charge transport and recombination

Rate of particle flow = Particle flow rate due to current – particle loss rate due to recombination

If excess carrier density δn , then Recombination rate $R = \frac{\delta n}{\tau_n} \cdot A \cdot \Delta x$

in volume $A \cdot \Delta x$

τ_n – electron recombination time.

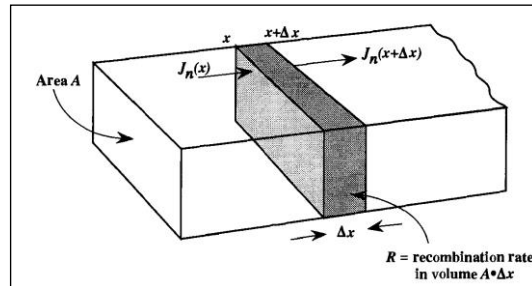


Fig. 34: A conservation of particles applied to a volume $A \cdot \Delta x$. The difference in the particle currents has to equal the recombination rate.

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Particle flow rate in volume $A \cdot \Delta x$ is

$$\left[\frac{J_n(x + \Delta x)}{e} - \frac{J_n(x)}{e} \right] A$$

$$= \frac{1}{e} \cdot \frac{\partial J_n}{\partial x} \cdot \Delta x \cdot A$$

The rate of electron build up in volume $A \cdot \Delta x$

$$A \cdot \Delta x \cdot \frac{\partial n}{\partial t} = A \cdot \Delta x \cdot \frac{\partial \delta n}{\partial t} = A \cdot \Delta x \left[\frac{1}{e} \frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n} \right]$$

$$\frac{\partial \delta p}{\partial t} = -\frac{1}{e} \frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$$

$$\frac{\partial \delta n}{\partial t} = \frac{1}{e} \frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$$

If current carried only by diffusion process

$$J_n(\text{diff.}) = e D_n \frac{\partial \delta n}{\partial x}, \quad J_p(\text{diff.}) = -e D_p \frac{\partial \delta p}{\partial x}$$

$$\frac{\partial \delta n}{\partial t} = D_n \frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau_n}$$

$$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

In steady state, time derivative=0;
So, LHS=0

Transient time response of p-n diode

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In steady state ,

$$\frac{d^2 \delta n}{dx^2} = \frac{\delta n}{D_n \tau_n} = \frac{\delta n}{L_n^2} \quad L_n = \sqrt{D_n \tau_n}$$

L_n is the diffusion length.

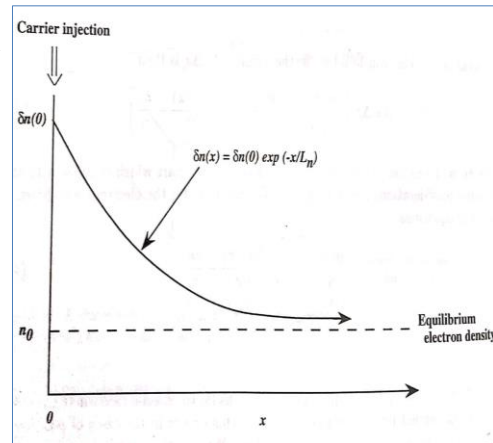
$$\frac{d^2 \delta p}{dx^2} = \frac{\delta p}{D_p \tau_p} = \frac{\delta p}{L_p^2}$$

$$\delta n(x) = A_1 e^{\frac{x}{L_n}} + A_2 e^{-\frac{x}{L_n}}$$

Due to external injection at $x=0$, $\delta n(0)$ maintained.

For large x , $\delta n = 0$, so $A_1 = 0$. $A_2 = \delta n(0)$

$$\delta n(x) = \delta n(0) e^{-\frac{x}{L_n}}$$



L_n – distance over which injected carrier density fall to 1/e of its original value.

L_n – avg. distance an electron will diffuse before it recombines with a hole.

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- Probability that an electron survive recombine up to x is $\delta n(x) / \delta n(0) = e^{-x/L_n}$
- probability that it recombines in distance δx $\frac{\delta x}{L_n}$

So, probability of moving up to a point x and then recombine .

$$P(x) \Delta x = \frac{1}{L_n} e^{-\frac{x}{L_n}} \Delta x$$

The average distance an electron can move and then recombine is

$$\langle x \rangle = \int_0^{\infty} x P(x) dx = L_n$$

The average distance $L_n = \sqrt{D_n \tau_n}$ depends upon the recombination time τ and diffusion constant D_n .

D_n = diffusion constant of electron

τ_n = recombination time for electron

$$\frac{1}{\tau_n} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}$$

Non-radiative

Junctions in Semiconductors:

- (i) p-n junction
- (ii) Metal-semiconductor junction
- (iii) Semiconductor insulator junction

To understand diode properties , the assumptions are:

- (i) Abrupt junction and uniformly doped.
- (ii) Mobile carrier density in depletion region is zero.
- (iii) Transition between bulk neutral region and depletion region is abrupt.

Drift current due to field , balances diffusion current in equilibrium.

$$J_p(x) = e \left[\mu_p p F - D_p \frac{dp}{dx} \right] = 0$$

Use Einstein relation $\frac{\mu_p}{D_p} = \frac{e}{kT}$ F is the field.

$$\text{So, } p \cdot F - \frac{kT}{e} \cdot \frac{dp}{dx} = 0 \quad \frac{dp}{p} = \frac{e}{kT} dx F$$

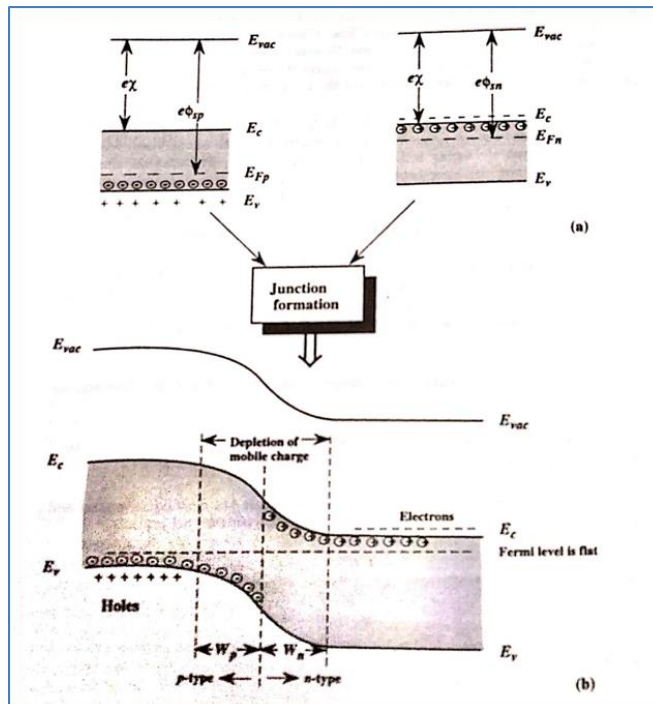
$$-\frac{e}{kT} \int_{V_p}^{V_n} dV = \int_{p_p}^{p_n} \frac{dp}{p} \quad F = -dV/dx$$

$$-\frac{e}{kT} (V_n - V_p) = \ln \frac{p_n}{p_p}$$

$$V_{bi} = \frac{kT}{e} \ln \frac{p_p}{p_n} \quad \text{where } V_{bi} = V_n - V_p$$

V_{bi} = built-in-potential.

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