INDEX

NAME: Ajay STD.: SEC.: ROLL NO.: SUB.:

S. No.	Date	Title	Page No.	Teacher's Sign / Remarks

Engineering Optics

Syllabus

⇒ Geometrical optics.

Matrix formulation for lens mirrors and combination under paraxial approx. image formation, brief introduction to primary monochromatic aberrations & chromatic aberrations

Physical Optics

* Diffraction :- Fresnel & Fraunhoffer diffraction.

→ rectangular and circular aperture Lens as a fourier transforming tool, spatial frequency filtering.

→ Image processing, working principle of holography

* Interference :-

Two & multi beam interference, Michelson & Fabry – Perot interferometer line width & coherence, multilayer thin films as anti reflection coating

* Polarisation :-

Linear & elliptically polarised light, Poincarre representation, Jones vector, polariser & retarder, production of polarised light, Polarisation by reflection, scattering & selective absorption

S. No.	Date	Title	Page No.	Teacher's Sign / Remarks

birefringens, anisotropic media, optics of liquid ~~crysta~~ crystals, optical activity, magneto optics, electro optics & accousto optics.

Books

Jenkins & White

Ajay Ghatak

Pedrotti

Born & Wolf

JW. Goodman
Introduction to fourier optics

Sayan . chakarborti@iitg.ac.in

classmate
Date Ajay
Page

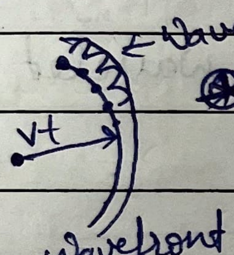
Engineering Optics PH305

Geometrical optics & Physical Optics

$$\lim_{\lambda \to 0} \{\text{Physical optics}\} = \{\text{Geometrical optics}\}$$

↑ ray

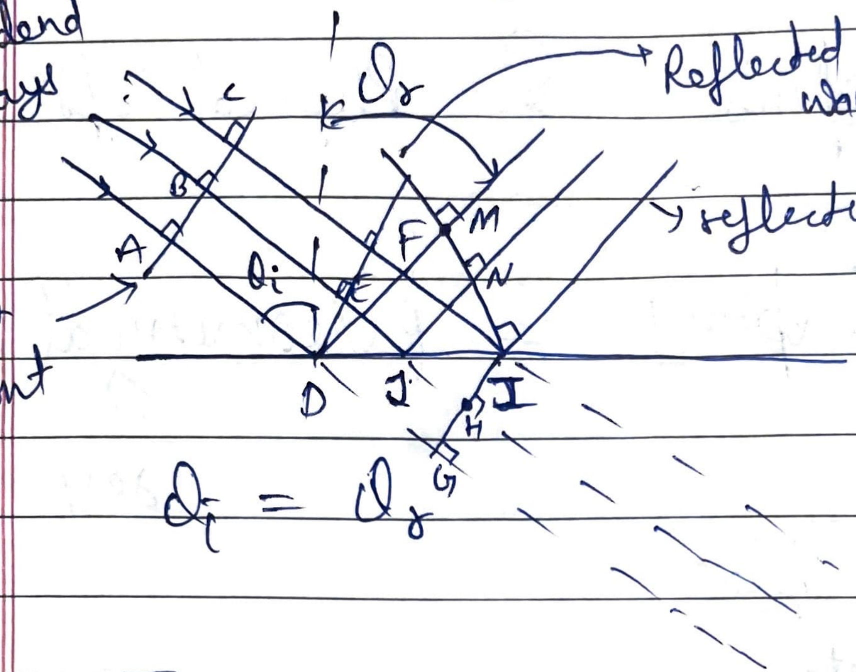
→ Haygens Principle :-



each point of a propagating disturbance is capable of originating new pulses that contribute to the ~~bi~~ disturbance an instant later.

Each point on the leading surface ~~of~~ of a wave disturbance - the wavefront, may be regarded as secondary source of spherical waves (wavelets) which themselves propagate with the speed of light in the medium & whose envelope at a later instant constitutes the new wavefront.

Reflection using Huygen's principle

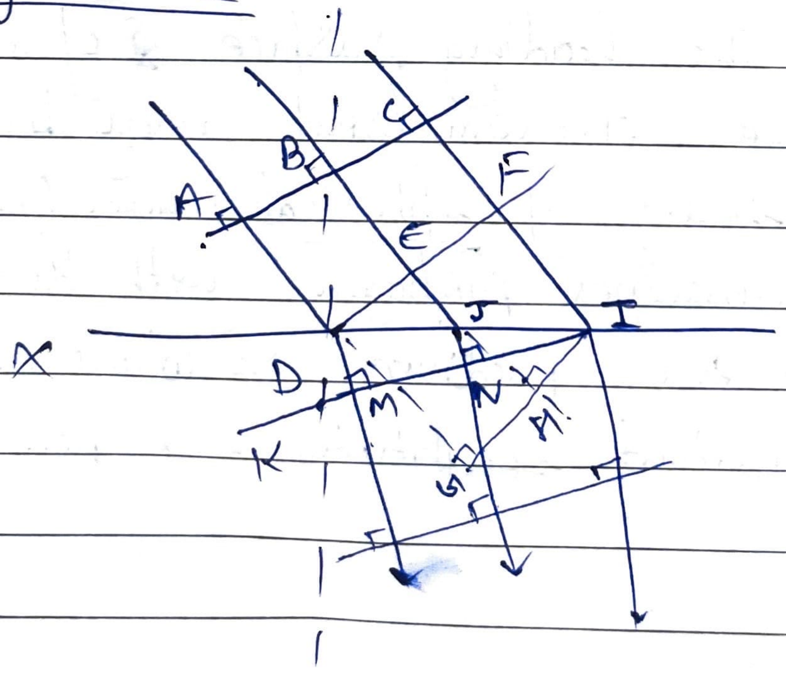


$\theta_i = \theta_r$

$JN = JH \rightarrow$ radius of the wavelet centred at J

$DM = DG =$ ______ at D

Refraction



$v_i = c/n_i$

$v_t = \frac{c}{n_t}$

n_i

n_t

$FI = v_i t = DG$

$DM = v_t t$ $= v_t\left(\frac{DG}{\quad}\right) = \left(\frac{n_i}{\quad}\right) DG$

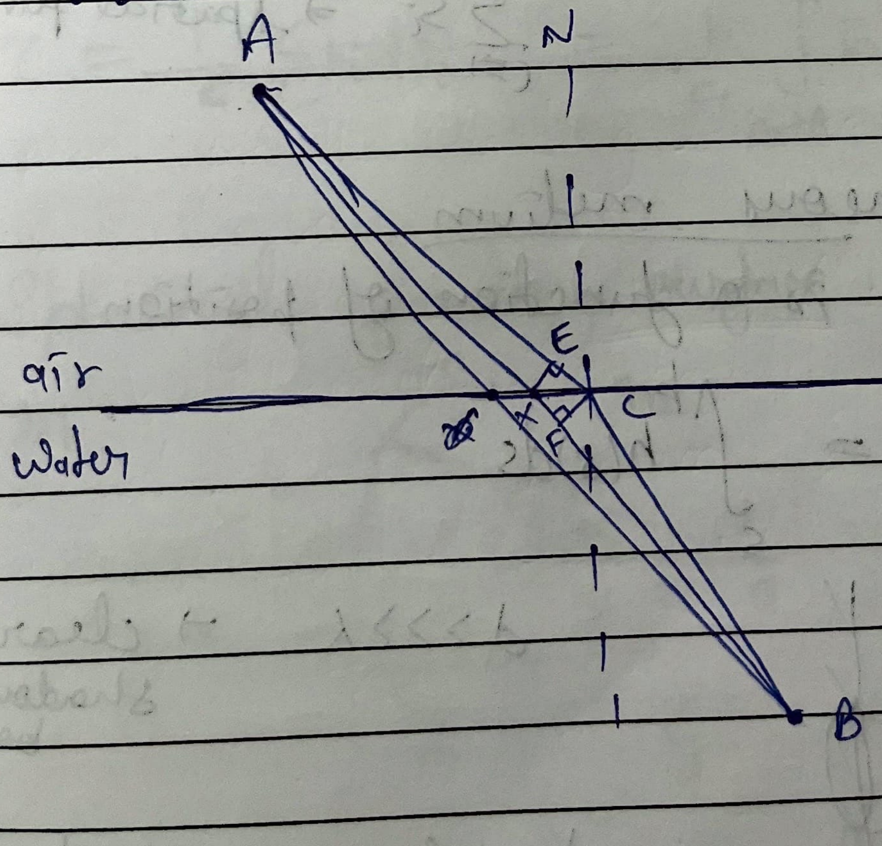
$$JN = \left(\frac{n_i}{n_t}\right) JH$$

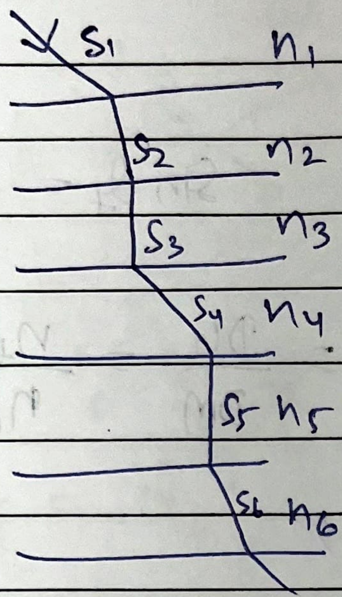
$$\sin\theta_i = \frac{PI}{DI} \qquad \sin\theta_t = \frac{DM}{DI}$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{PI}{DM} = \frac{DG}{DM} = \frac{n_t}{n_i} = \text{constant}$$

→ <u>Fermat's Principle</u>

→ Out of all possible paths that it might take to get from one point to another, light takes the path which require shortest time.





$$t = \frac{s_1}{v_1} + \frac{s_2}{v_2} + \cdots + \frac{s_m}{v_m}$$

$$= \sum_{i=1}^{m} \frac{s_i}{v_i}$$

optical path length

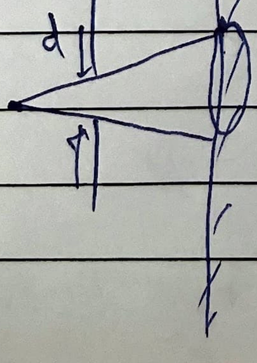
$$t = \sum_{i=1}^{m} \frac{n_i s_i}{c} = \frac{1}{c} \sum_{i=1}^{m} n_i s_i$$

$\sum_{i=1}^{m} s_i \Rightarrow$ spatial path length

<u>Inhomogeneous medium</u>

n as a function of position

$$OPL = \int_{s}^{p} n(s)\,ds$$



$d >>> \lambda \to$ clear shadow boundary

$d \approx \lambda \to$ no sharp shadow boundary

diffraction

Ray

$\lambda \to 0$ limit : infinitesimally thin pencil of light

→ The ray will corresponds to that path for which the time taken is an extremum in comparison to nearby paths

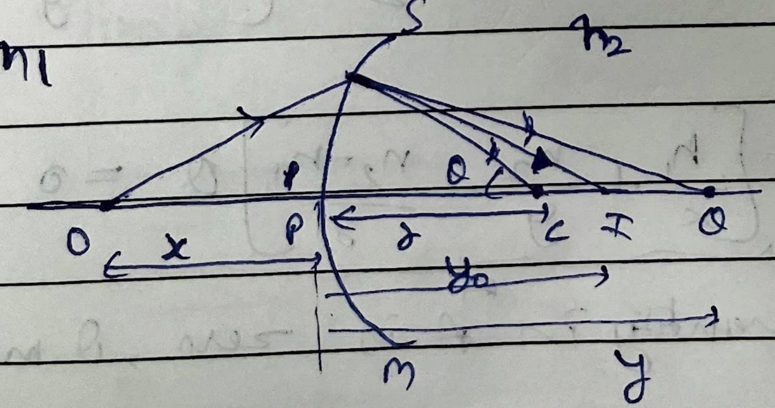
$n(x, y, z) \to$ position dependent Refractive index

$$\frac{ds}{c/n} = \frac{n\,ds}{c} \Rightarrow$$ time taken to traverse the geometric path ds in a medium of R.I. n

$$\tau = \frac{1}{c}\sum n_i\,ds_i = \frac{1}{c}\int_{A \to B} n\,ds$$

Eg. Spherical refracting surface SPM

θ to be small



ΔSOC

Calculate optical path length

$\theta \to$ small

$\cos\theta \simeq 1 - \frac{\theta^2}{2} + \dots$

$$OS = \left[(x+r)^2 + r^2 - 2(x+r)r\cos\theta\right]^{1/2}$$

$$\simeq \left[x^2 + 2rx + 2r^2 - 2(xr + r^2)\left(1 - \frac{\theta^2}{2}\right)\right]^{1/2}$$

$$\simeq \left[x^2\left\{1 + \frac{2(\cancel{xr} + r^2)}{x^2} - 2\left(\frac{\cancel{xr} + r^2}{x^2}\right) + \frac{\cancel{2}(xr+r^2)}{x^2}\frac{\theta^2}{\cancel{2}}\right\}\right]^{1/2}$$

$$\simeq x\left[1 + \frac{(xr+r^2)\theta^2}{x^2}\right]^{1/2} \simeq x + \frac{1}{2}\frac{(rx+r^2)\theta^2}{x^2}$$

$$\simeq x + \frac{1}{2}r^2\left(\frac{1}{r} + \frac{1}{x}\right)\theta^2$$

$$\simeq y - \frac{1}{2}r^2\left(\frac{1}{r} - \frac{1}{y}\right)\theta^2$$

$$OPL = n_1(OS) + n_2(SQ)$$

$$= (n_1x + n_2y) + \frac{1}{2}r^2\left[\frac{n_1}{x} + \frac{n_2}{y} - \frac{n_2 - n_1}{r}\right]$$

$$= 0 \Rightarrow r^2\underbrace{\left[\frac{n_1}{x} + \frac{n_2}{y} - \frac{n_2 - n_1}{r}\right]}_{A}\theta = 0$$

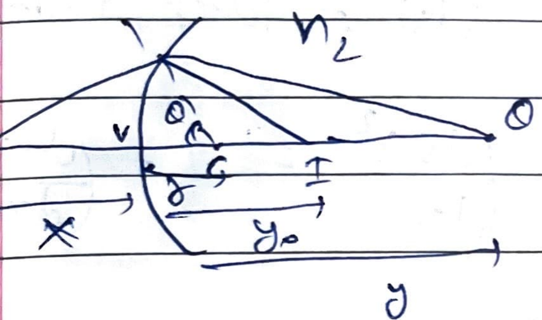
unless the quantity in A is zero, θ must be zero

If the value of y is such that the quantity in A was zero, is for some $y = y_0$

$$\boxed{\frac{n_2}{y_0} + \frac{n_1}{x} = \frac{n_2 - n_1}{r}}$$

Then $\frac{dL_{op}}{d\theta}$ ~~would~~ would be zero for all θ

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$ any point / dist meas to the right of P — +ve



$$L_{op} = n_1 OS + n_2 SO$$

$$\frac{dL_{op}}{d\theta} = 0 = r^2\left[\frac{n_1}{x} + \frac{n_2}{y} - \frac{n_2 - n_1}{r}\right]\theta$$

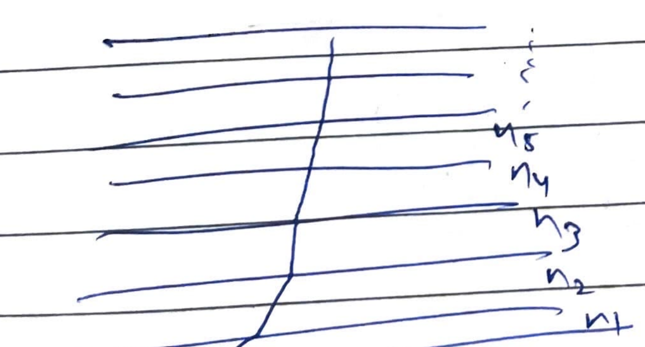
$$\frac{n_1}{x} + \frac{n_2}{y_0} - \frac{n_2 - n_1}{r} = 0$$

$u = -x$, $v = y$, $r = +R$

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

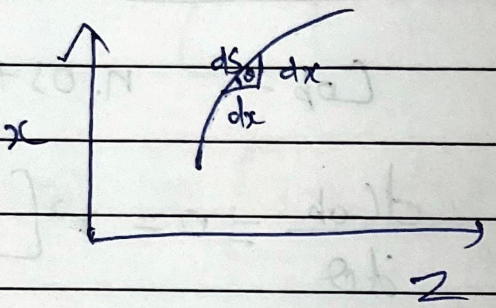
Mirage :-

$$n_1 \sin\phi_1 = n_2 \sin\phi_2 = n_3 \sin\phi_3 =$$



product $n(x)\sin\phi(x) = n(x)\cos\theta(x)$ is the invariant of the ray path.

$$n(x)\cos\theta(x) = n_1\cos\theta_1 = \tilde{\beta}$$



$$(ds)^2 = (dx)^2 + (dz)^2$$

$$\left(\frac{ds}{dz}\right)^2 = 1 + \left(\frac{dx}{dz}\right)^2$$

$$\frac{dz}{ds} = \cos\theta = \frac{\tilde{\beta}^2}{n(x)}$$

$$\left(\frac{dx}{dz}\right)^2 = \frac{n^2(x)}{\tilde{\beta}^2} - 1$$

$$2\left(\frac{dx}{dz}\right)\frac{d^2x}{dz^2} = \frac{1}{\tilde{\beta}^2}\frac{d(n^2)}{dx}\frac{dx}{dz}$$

$$\Rightarrow \boxed{\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2}\frac{d}{dx}(n^2)}$$

$n(x) =$ constant

$$\frac{d^2x}{dz^2} = 0 \Rightarrow x = Az + B$$

Date
Page

$n(x) = n_0 + kx$

$n_0, k \rightarrow$ constant

$$\frac{d^2 x(z)}{dz^2} = \frac{1}{2\tilde{\beta}^2} \frac{d(n_0 + kx)^2}{dx}$$

$$\frac{d(n_0^2 + k^2x^2 + 2nkx)}{dx}$$

$$\frac{1}{2\tilde{\beta}^2}(2k^2x + 2nk)$$

$$\frac{d^2x}{(2k^2x + 2nk)} = \frac{1}{2\tilde{\beta}^2} dz^2$$

$2k^2x + 2nk = l$ $\qquad \frac{d^2 l}{l\, 4k^4}$

$\cancel{dx} \cdot 2k^2 = \frac{dl}{dx}$

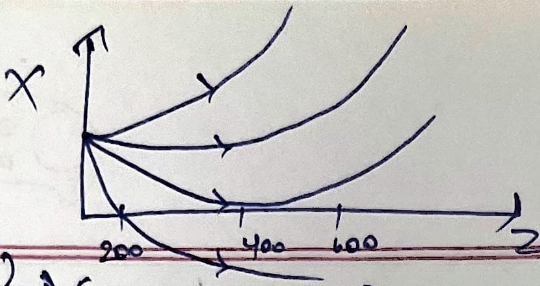
$2\cancel{k^2}\, dx = \frac{dl}{2k^2}$

$$\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2} \frac{d(n_0 + kx)^2}{dx} = \frac{1}{\tilde{\beta}^2}(n_0 + kx)k$$

$$\frac{d^2x}{dz^2} = \frac{k^2}{\tilde{\beta}^2}\left(\frac{n_0}{k} + x\right)$$

Let : $X = x + \frac{n_0}{k}$ $\qquad \alpha = \frac{k}{n}$

$$\frac{d^2(x + \frac{n_0}{k})}{dz^2} = \alpha^2 X = \frac{d^2}{dz^2} X$$



$$\frac{d^2X}{dz^2} = k^2 X$$

$$X = C_1 e^{kz} + C_2 e^{-kz} \qquad k = \frac{K}{\beta^2}$$

$$x = -\frac{n_0}{K} + C_1 e^{kz} + C_2 e^{-kz}$$

⇒ say, $X(z=0) = x_1$

$$\left.\frac{dx}{dz}\right|_{z=0} = \tan\theta_1 \quad \Big\} \text{ given}$$

⇒ $$\left.\frac{dx}{dz}\right|_{z=0} = k(C_1 - C_2) = \tan\theta_1$$

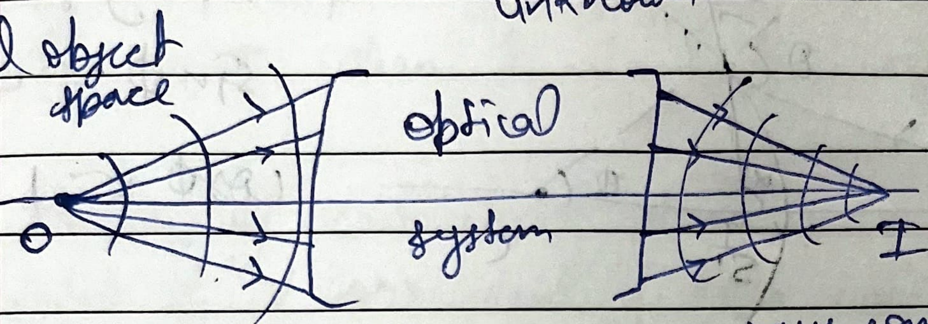
$$x(z=0) = -\frac{n_0}{K} + C_1 + C_2 = x_1$$

$$C_1 + C_2 = x_1 + \frac{n_0}{K} \qquad C_1 - C_2 = \frac{\tan\theta_1}{k}$$

⇒ $$C_1 = \frac{1}{2}\left[x_1 + \frac{1}{K}(n_0 + n_1\sin\theta_1)\right]$$

$$C_2 = \frac{1}{2}\left[x_1 + \frac{1}{K}(n_0 - n_1\sin\theta_1)\right]$$

Imaging by optical system



rays diverge
waveforms expand

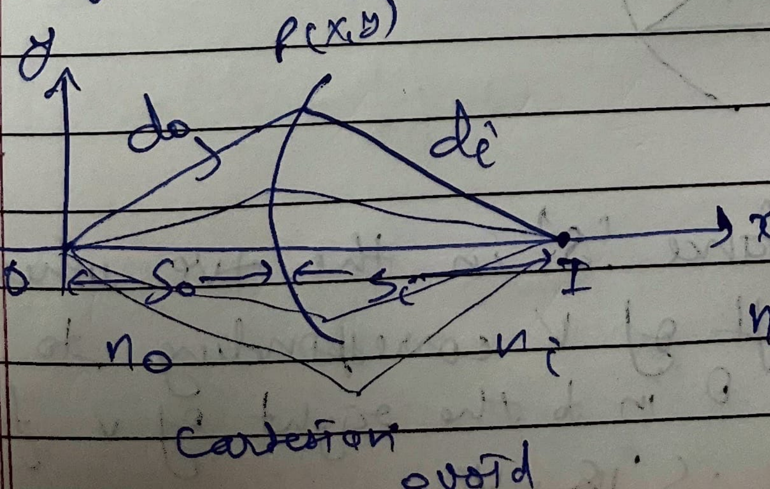
rays converge
waveforms contract

Isochronous rays Fermat's principle
paraxial approximation
principle of reversibility

$O \to I$, $I \to O$ } conjugate points

Non ideal image → scattering → Aberrations → diffraction

Surfaces that produces ideal image → cartesian surface

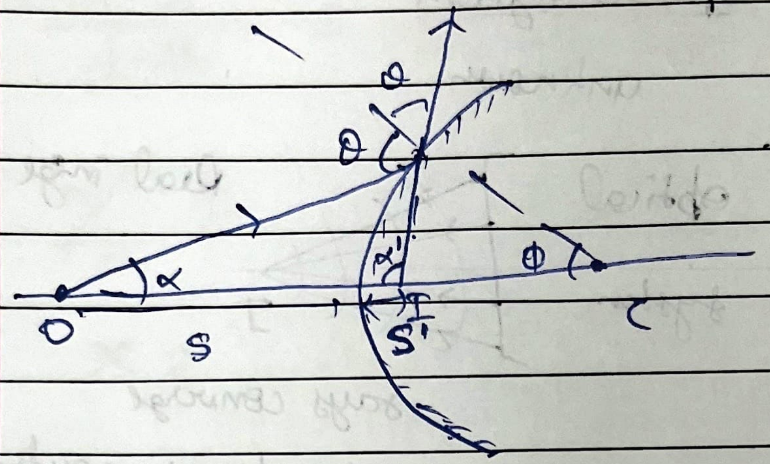


$n_o d_o + n_i d_i = n_o s_o + n_i s_i$
$= \text{constant}$

$n_o(x^2+y^2)^{1/2} + n_i\left[y^2 + (s_o + s_i - x)^2\right]^{1/2}$

$= \text{constant}$

$\Rightarrow$ Reflection at a spherical surface



$\phi \rightarrow$ very small

$\sin\phi \simeq \phi$

$\cos\phi \simeq 1$

$2\theta = \alpha + \alpha'$

$\theta = \phi + \alpha$

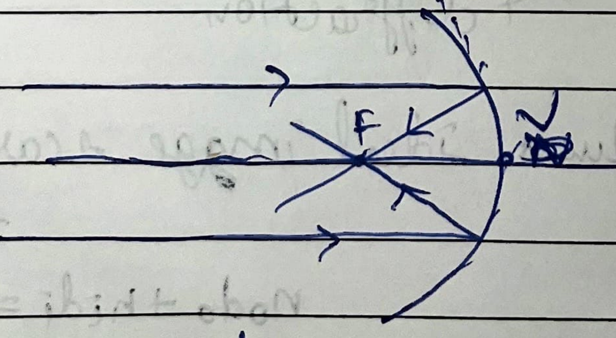
$2\phi + 2\alpha = \alpha + \alpha'$

$\alpha - \alpha' = -2\phi$

$$\boxed{\frac{h}{S} - \frac{h}{S'} = -\frac{2h}{R}}$$

neglecting $V\theta$

$$\boxed{\frac{1}{S} - \frac{1}{S'} = \frac{-2}{R}}$$



$\frac{1}{S} + \frac{1}{S'} = \frac{-2}{R}$

Sign convention

① Object distance 'S' in the +ve when O is to the left of V corresponding to a real object when O is to the right of V for a virtual object. S is -ve

Image distance s' is +ve when I is to the left of V, corresponding to a real ~~image~~ image & −ve when I is to the right of V ⇒ virtual image

Radius of curvature R is +ve when C to the right of V corresponding to a convex mirror & −ve when C is to the left of V → concave mirror.

$R \to \infty$

$s = -s'$

② $s \to \infty$

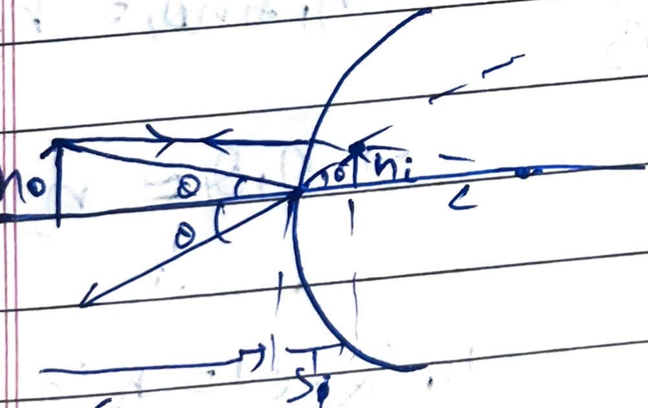
$s' = -R/2$

$f = -\frac{R}{2}\begin{cases} >0 \text{ for concave} \\ <0 \text{ for convex} \end{cases}$

$$\left(\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}\right)$$

Magnification :-

$$\frac{h_o}{s} = \frac{h_i}{s'}$$



$$|m| = \frac{h_i}{}$$

4.) An object of 3cm height is placed 20 cm from a (a) convex and (b) concave mirror, each of 10cm focal length. Determine the position and nature of image.

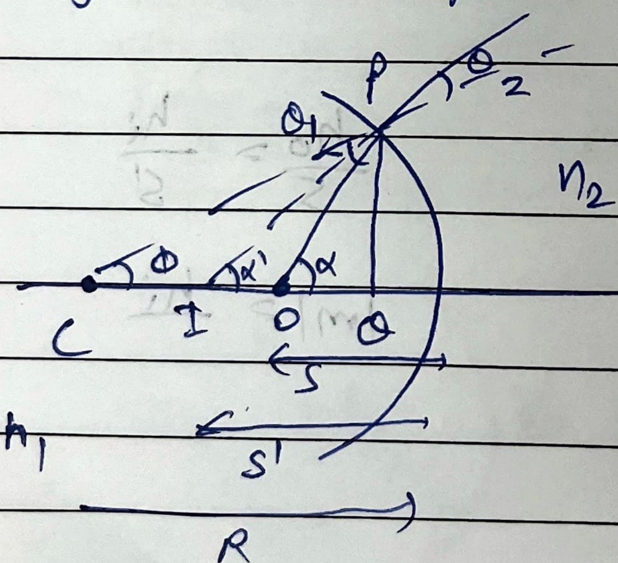
(a) $f = -10$ cm $s = +20$ cm

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = -6.67$ cm

virtual image

$m = \frac{-s'}{s} = \frac{-(-6.67)}{20} = 0.33$ ∴ erect image

⇒ Refraction at spherical surface



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\rightarrow n_1\theta_1 \simeq n_2\theta_2$

$\Downarrow$

$n_1(\alpha - \phi) = n_2(\alpha' - \phi)$

$n_1\left(\frac{h}{s} - \frac{h}{R}\right) = n_2\left(\frac{h}{s'} - \frac{h}{R}\right)$

$$\boxed{\frac{n_1}{s} - \frac{n_2}{s'} = \frac{n_1 - n_2}{R}}$$

Sign convention:
+ for real object
− for virtual

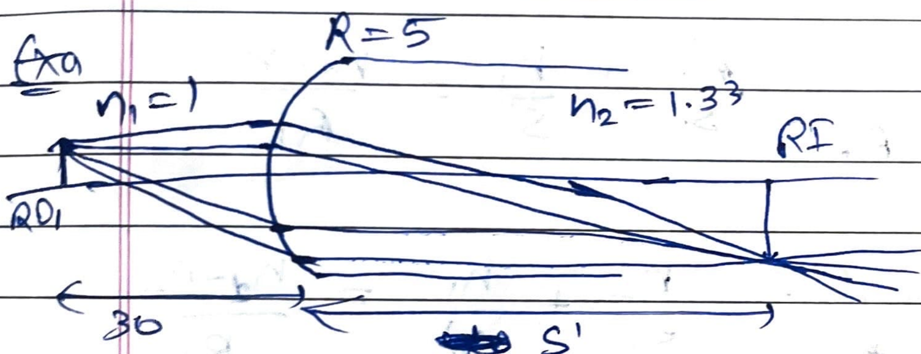
$R \to \infty \Rightarrow s' = -\left(\frac{n_2}{n_1}\right) s$

apparent depth

+ For real object, $s > 0$ the $-$ve sign in $s' = -\left(\frac{n_2}{n_1}\right) s$ implies a virtual image.

$$\boxed{m = \frac{h_i}{h_o} = -\frac{n_1 s'}{n_2 s}}$$

Ex

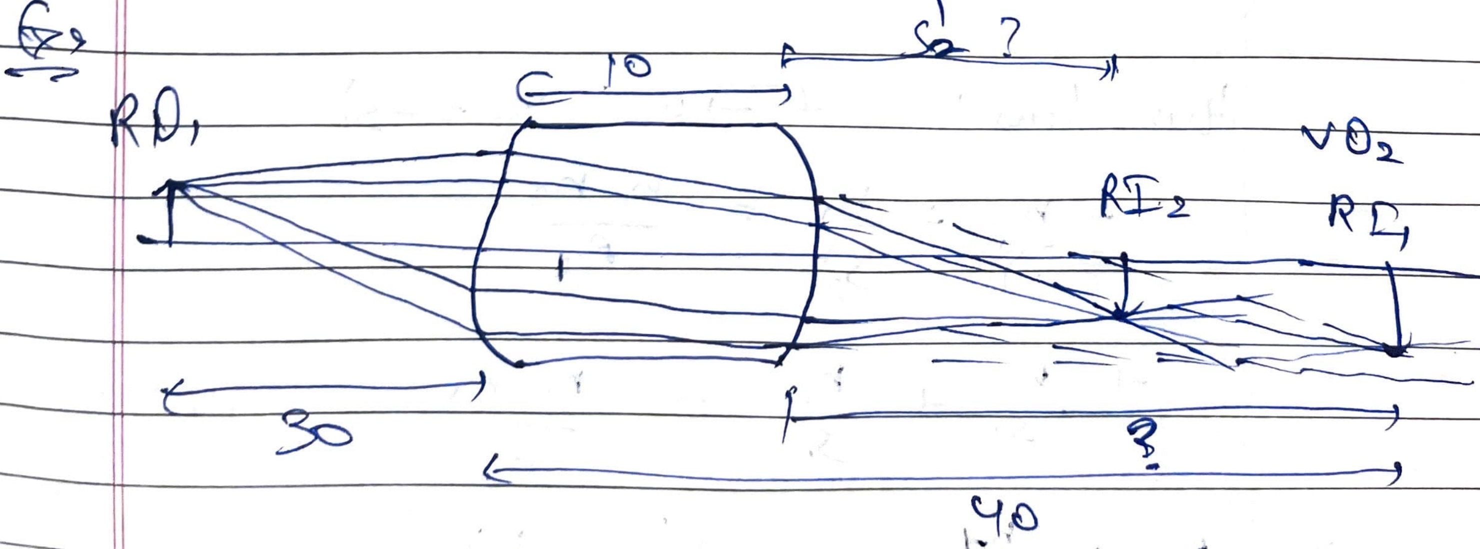


$s' = 40$

$$m = \frac{h_i}{h_o} = -\frac{n_1 s'}{n_2 s}$$

$$= -\frac{(1)(40)}{(1.33)(30)} \Rightarrow -1$$

Ex



$S_2 = -$ve $= -30$

classmate

Date
Page

$$\frac{1.33}{-30} + \frac{1}{S_2'} = \frac{1-1.33}{5} \qquad S_2 = +9\text{ cm}$$

$$\text{magnification} = \frac{(-1.33)(9)}{(1)(-30)} = \frac{2}{5}$$

For lens – 2 surfaces

Surface 1 with $RC = R_1$ $\quad \frac{n_1}{S_1} + \frac{n_2}{S_1'} = \frac{n_2 - n_1}{R_1}$

Surface 2 with $RC = R_2$ $\quad = \frac{n_2}{S_2} + \frac{n_1}{S_2'} = \frac{n_1 - n_2}{R_2}$

2nd object distance

$S_2 = t - S_1'$. (assuming thickness of lens is t)

thin lens: $\quad t \to 0 \quad S_2 = -S_1'$

$$-\frac{n_2}{S_1'} + \frac{n_1}{S_2'} = \frac{n_1 - n_2}{R_2}$$

$$+\frac{n_1}{S_1} + \frac{n_2}{S_1'} = \frac{n_2 - n_1}{R_1}$$

$$\frac{n_1}{S_1} + \frac{n_1}{S_2'} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

lens maker's formula

$$\boxed{\frac{1}{s}+\frac{1}{s'}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

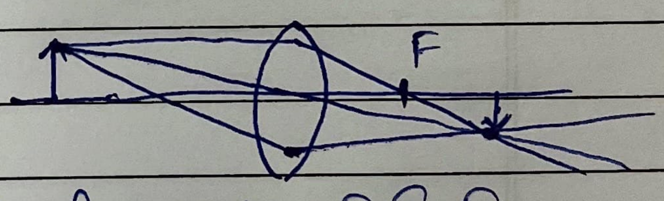
→ Focal length is defined as the image distance for an object at ∞

$$\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

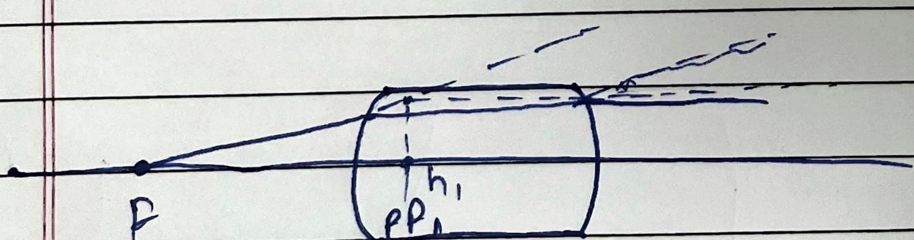
→ $\left(\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\right)$

Matrix method for Paraxial optics:

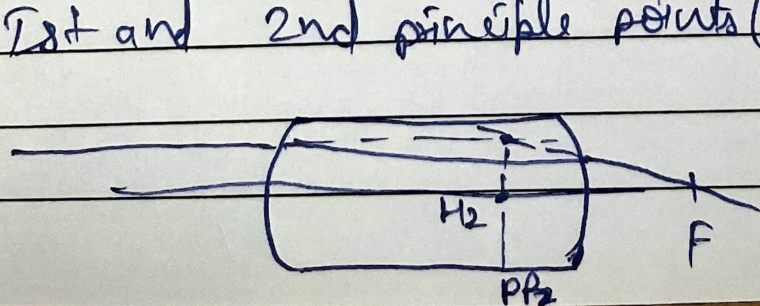
Thick lens
↓
6 Cardinal points



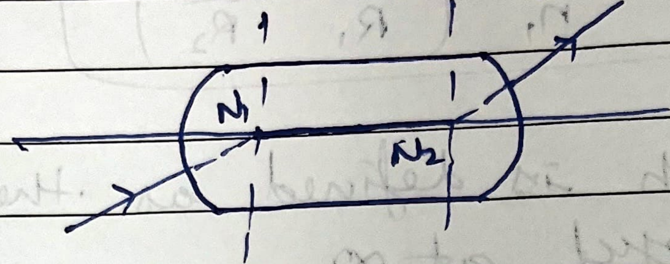
* First and second focal point F_1 & F_2



* Ist and 2nd principle points (H_1 & H_2)

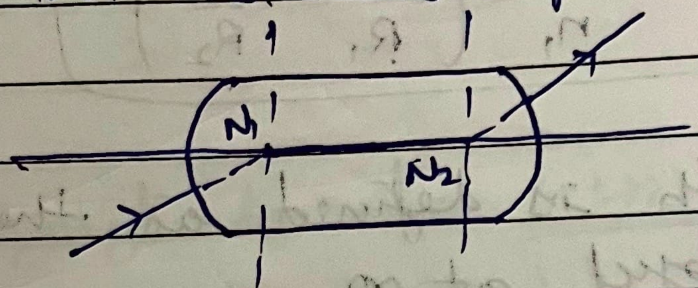


* 1st and 2nd nodal points (N_1 & N_2).

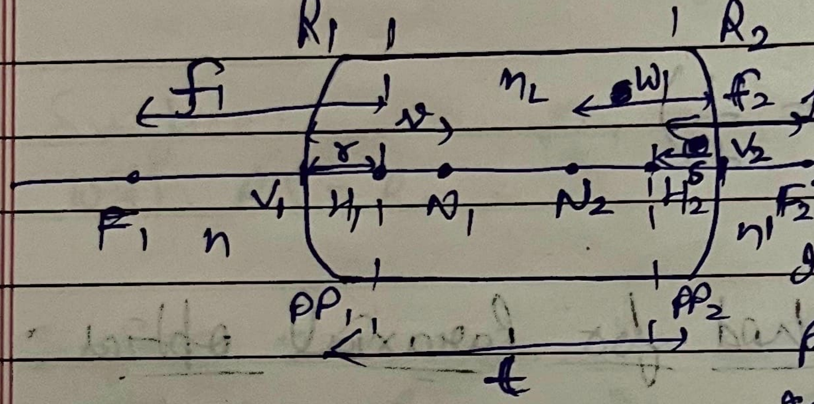


Date
Page

* 1st and 2nd nodal point (N_1 & N_2)



Matrix method in paraxial approximation



6 cardinal points

1st and 2nd system focal point (F_1 & F_2)

1st and 2nd principal points (H_1 & H_2)

1st and 2nd nodal point (N_1 & N_2)

H.W

* $\frac{1}{f_1} = \frac{n_L - n'}{n R_2} - \frac{n_L - n}{n R_1} - \frac{(n_L - n)(n_L - n')}{n n_L}\frac{t}{R_1 R_2}$

& $f_2 = -\frac{n'}{n} f_1$

* Principal plane ⇒

$r = \frac{n_L - n'}{n_L R_2} f_1 t$ & $s = \frac{n_L - n}{n_L R_1} f_2 t$

* Nodal points

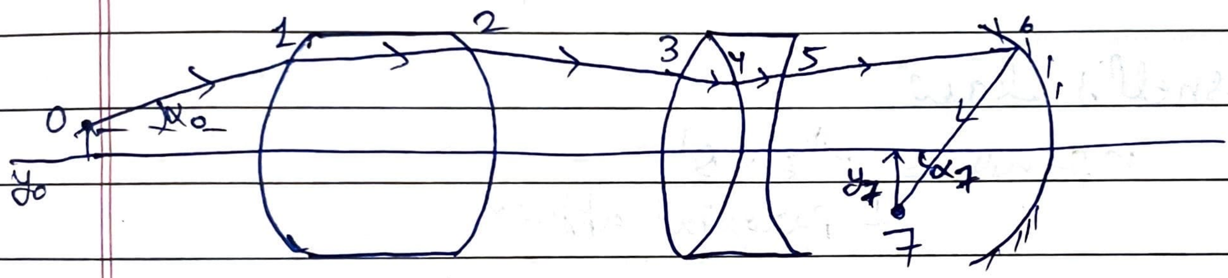
$v = \left(1 - \frac{n'}{n} + \frac{n_L - n'}{n_L R_2} t\right) f_1$ & $w = \left(1 - \frac{n}{n'} - \frac{n_L - n}{R_1 n_L}\right.$

* Image & object dist:

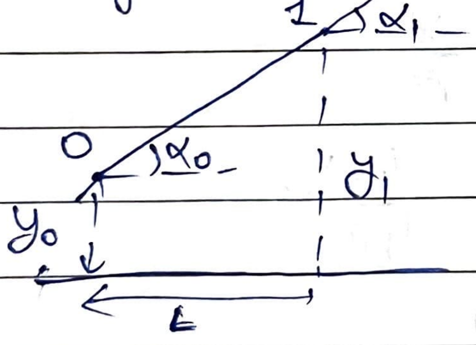
$$-\frac{f_1}{s_o} + \frac{f_2}{s_i} = 1 \qquad\qquad m = \frac{-n s_i}{n' s_o}$$

$s_i, s_o, f_1, f_2 \to$ measured from principle planes.

$\Rightarrow \quad y_0, \alpha_0 \longleftrightarrow y_7, \alpha_7$



Transformation matrix



$$\alpha_0 = \alpha_1$$

$$y_1 = y_0 + L\tan\alpha_0$$
$$\simeq y_0 + L\alpha_0$$

$$y_1 = (1)y_0 + (L)\alpha_0$$

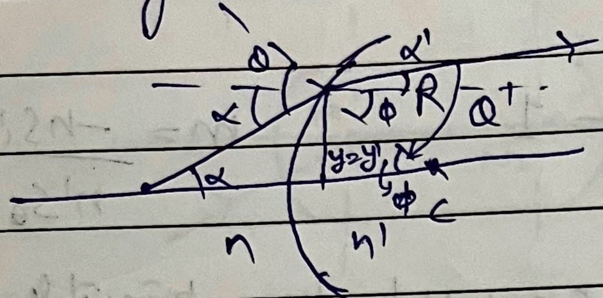
$$\alpha_1 = (0)y_0 + (1)\alpha_0$$

$$\begin{pmatrix} y_1 \\ \alpha_1 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}}_{m} \begin{pmatrix} y_0 \\ \alpha_0 \end{pmatrix}$$

2X2 ray transfer matrix

for ~~conv~~ concave surface $R \rightarrow$ negative

→ The refraction matrix



$y', \alpha' \leftrightarrow y, \alpha$

$$\alpha' = \theta' - \phi = \theta' - \frac{y}{R}$$ (paraxial approx.)

$$\alpha = \theta - \phi = \theta - \frac{y}{R}$$

→ snell's law

$$n \sin\theta = n' \sin\theta'$$

↓ paraxial approx

$$n\theta = n'\theta' \qquad \alpha' = \frac{n}{n'}\theta - \frac{y}{R}$$

$$\alpha' = \frac{n}{n'}\left(\alpha + \frac{y}{R}\right) - \frac{y}{R}$$

$$\alpha' = \left(\frac{1}{R}\right)\left(\frac{n}{n'} - 1\right)y + \left(\frac{n}{n'}\right)\alpha$$

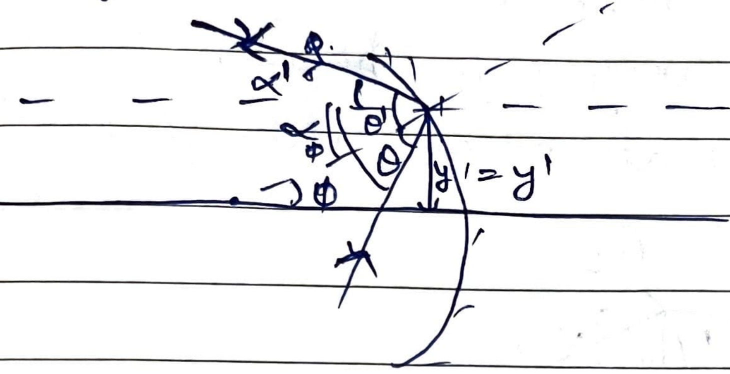
$$y' = (1)y + 0(\alpha)$$

$$\begin{pmatrix} y' \\ \alpha' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n}{n'} - 1\right) & \left(\frac{n}{n'}\right) \end{pmatrix}}_{} \begin{pmatrix} y \\ \alpha \end{pmatrix}$$

$$M = R$$

The refraction matrix

→ The reflection matrix.



$$\alpha = \theta + \phi = \theta + \frac{y}{-R}$$

$$\alpha' = \theta' - \phi = \theta' - \frac{y}{-R} = \theta' + \frac{y}{R}$$

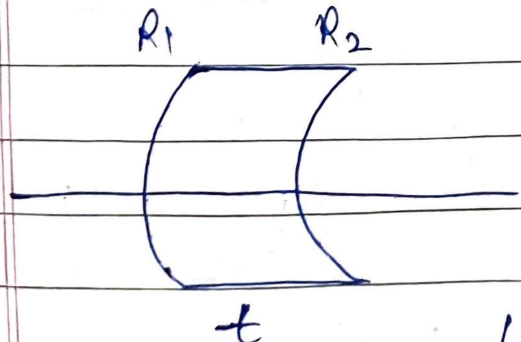
$$= \alpha + \frac{2y}{R}$$

$$y' = 1(y) + 0(\alpha)$$

$$\alpha' = \left(\frac{2}{R}\right) y + 1(\alpha)$$

$$\begin{pmatrix} y' \\ \alpha' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{pmatrix} \begin{pmatrix} y \\ \alpha \end{pmatrix}$$

→ Thick lens



$$\begin{pmatrix} y_1 \\ \alpha_1 \end{pmatrix} = M_1 \begin{pmatrix} y_0 \\ \alpha_0 \end{pmatrix}$$: first refraction

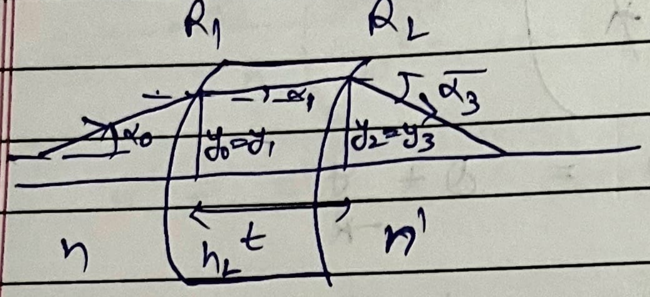
$$\begin{pmatrix} y_1 \\ \alpha_1 \end{pmatrix} = M_2 \begin{pmatrix} y_2 \\ \alpha_2 \end{pmatrix}$$: for translation of the ray inside lens

$$\begin{pmatrix} y_3 \\ \alpha_3 \end{pmatrix} = M_3 \begin{pmatrix} y_2 \\ \alpha_2 \end{pmatrix}$$: for 2nd refraction

$$\begin{pmatrix} y_3 \\ \alpha_3 \end{pmatrix} = \underbrace{M_3 \cdot M_2 M_1}_{\text{order is}} \begin{pmatrix} y_0 \\ \alpha_0 \end{pmatrix}$$

M = $R_2 \, \tau \, R_1$

order is very imp. ~~because~~ in general. matrices are non-commutative



$$M = \begin{pmatrix} 1 & 0 \\ \frac{1}{R_2}\left(\frac{n_L}{n'} - 1\right) & \frac{n_L}{n'} \end{pmatrix} \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{R_1}\left(\frac{n}{n_L} - 1\right) & \frac{n}{n_L} \end{pmatrix}$$

$$= \begin{pmatrix} 1 + t\left(\frac{n - n_L}{n_L R_1}\right) & t\frac{n}{n_L} \\ \frac{n_L - n'}{n' R_2}\left\{1 + t\left(\frac{n - n_L}{n_L R_1}\right)\right\} & \frac{n_L - n'}{n' R_2} t \frac{n}{n_L} + \frac{n}{n'} \end{pmatrix}$$

thin lens $t = 0$ & $n = n'$

n lens

$$M = \begin{pmatrix} 1 & 0 \\ \frac{n_L - n}{n}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

Translation

$$\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} = \tau$$

Refraction

$$M = \begin{pmatrix} 1 & 0 \\ \frac{n - n'}{R n'} & \frac{n}{n'} \end{pmatrix} = R \text{ curved}$$

$$M = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n}{n'} \end{pmatrix} = R_P \text{ (Plane surface)}$$

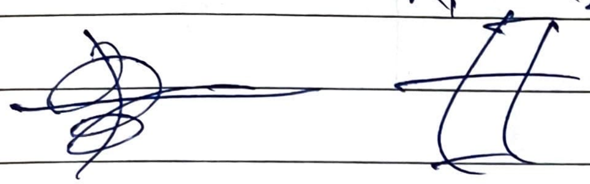
Thin lens

$$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

Reflection

$$M = \begin{pmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{pmatrix}$$

Eg. Find system matrix for thick lens



$R_1 = 45\,cm$

$R_2 = 30\,cm$

$t = 5\,cm$ $\quad n_L = 1.6$

$n = n' = 1$

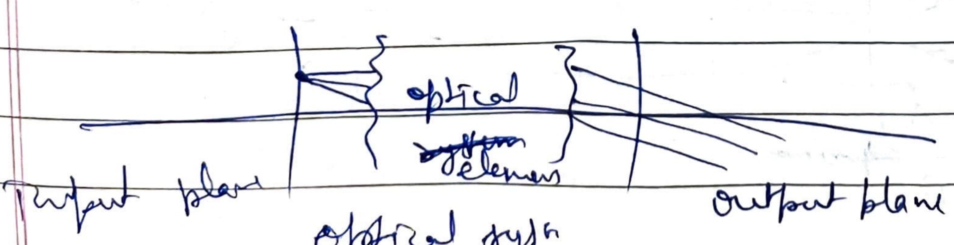
$$= \begin{pmatrix} 1 + 5\left(\frac{1-1.6}{1.6\times 45}\right) & \frac{5\cdot 1}{1.6} \\ \frac{1.6-1}{1\,30}\left\{1 + 5\left(\frac{1-1.6}{1.6\times 45}\right)\right\} & \frac{1.6-1}{1\cdot 30} + \frac{1}{1.6} + \frac{1}{1} \end{pmatrix}$$

Significance of the system transfer matrix

$$\begin{pmatrix} y_f \\ \alpha_f \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_0 \\ \alpha_0 \end{pmatrix}$$

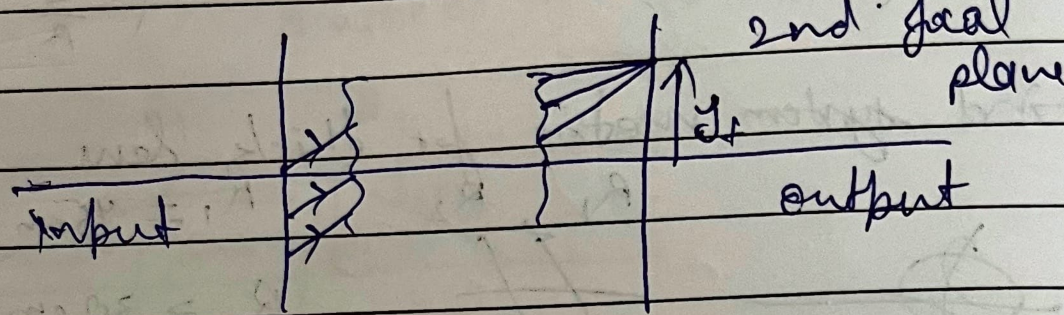
* $D = 0 \Rightarrow \alpha_f = C y_0$

y_0 is fixed → all rays ~~a~~ leaving from the same height in the input plane must have same α_f

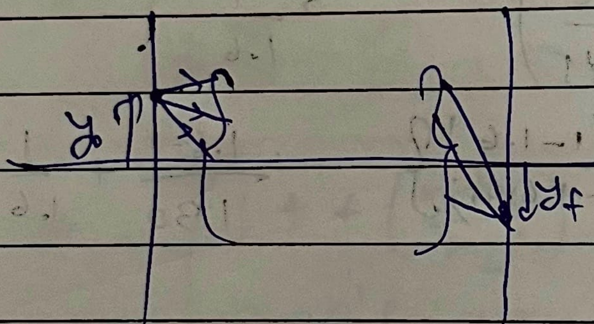


If the input plane is focal plane

* $A = 0 \Rightarrow y_f = B\alpha_0$



* $B = 0 \Rightarrow y_f = A y_0$

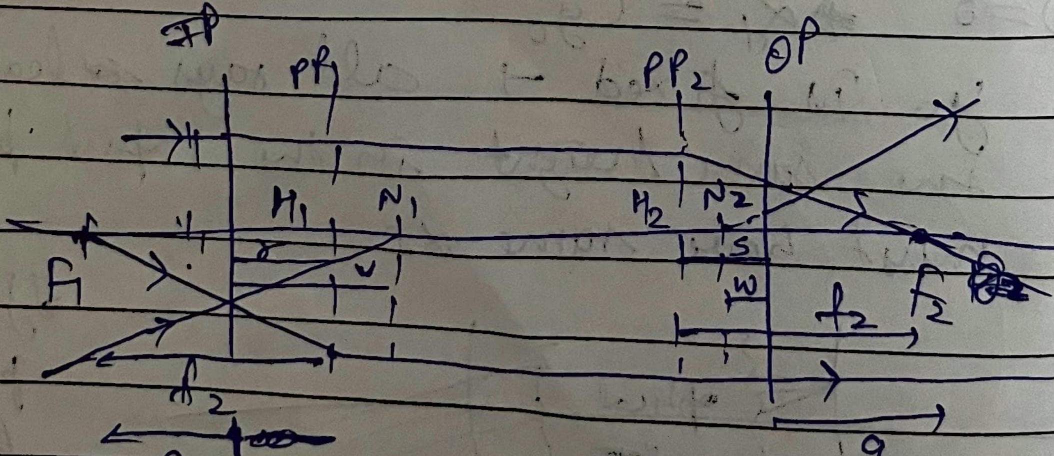


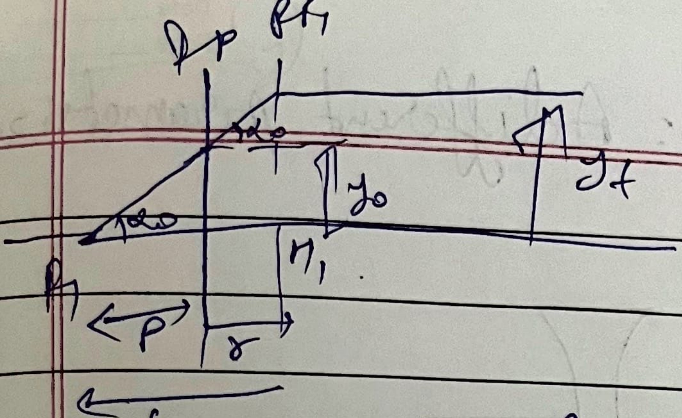
$A = \frac{y_f}{y_0}$ lateral magnification

* $C = 0 \Rightarrow \alpha_f = D\alpha_0$

$D = \frac{\alpha_f}{\alpha_0} \Rightarrow$ ~~angle~~ angular magnification

#





$$\alpha_0 = \frac{y_0}{-p}$$

y_0	α_0
y_f	0

$$y_f = Ay_0 + B\alpha_0$$

$$0 = Cy_0 + D\alpha_0$$

$$\Rightarrow y_0 = -\left(\frac{D}{C}\right)\alpha_0$$

$$p = -\frac{y_0}{\alpha_0} = \frac{D}{C}$$

$$f = \frac{-y_f}{\alpha_0} = \frac{-(Ay_0 + B\alpha_0)}{\alpha_0}$$

$$= \frac{AD}{C} - B$$

$$\frac{AD - BC}{C} = \frac{\det(M)}{C} = \left(\frac{n_0}{n_f}\right)\frac{1}{C}$$

$$r = p - f_1 = \frac{1}{C}\left(D - \frac{n_0}{n_f}\right)$$

d) $p = \frac{D}{C}$ F_1

$r = \frac{D - \frac{n_0}{n_f}}{C}$ H_1

$q = -\frac{A}{C}$ F_2

$s = \frac{1 - A}{C}$ H_2

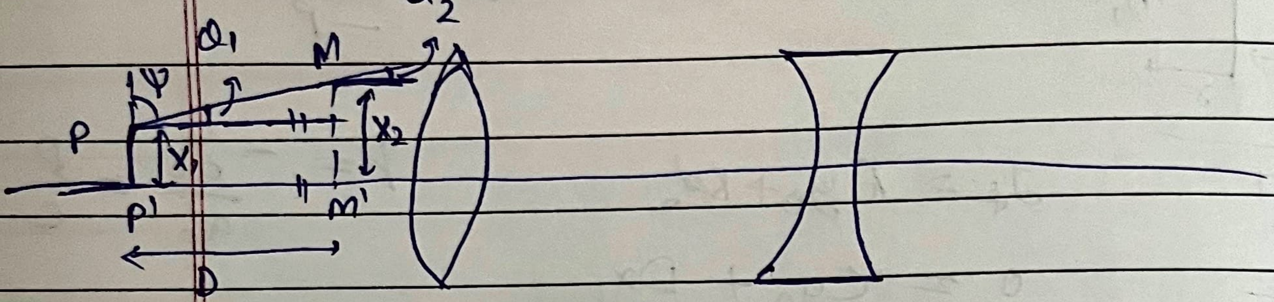
$v = \frac{D-1}{C}$ N_1

$w = \frac{\frac{n_0}{n_f} - A}{C}$ N_2

$$f_1 = p - r = \left(\frac{n_0}{n_f}\right)\frac{1}{C}$$

$$f_2 = q - s = -\frac{1}{C}$$

Matrix Method : A different parametrization

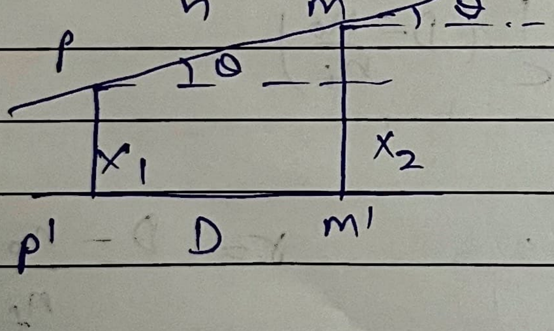


$(x_1, \theta_1) \rightarrow \alpha_1 = n\cos\varphi = n\sin\theta_1$

new parametrization. optical direction cosine

(x_1, λ_1)

→ Translation matrix



$\theta_1 = \theta_2 = \theta$

$X_2 = X_1 + D\tan\theta$

$\simeq X_1 + D\theta$

(paraxial approx.)

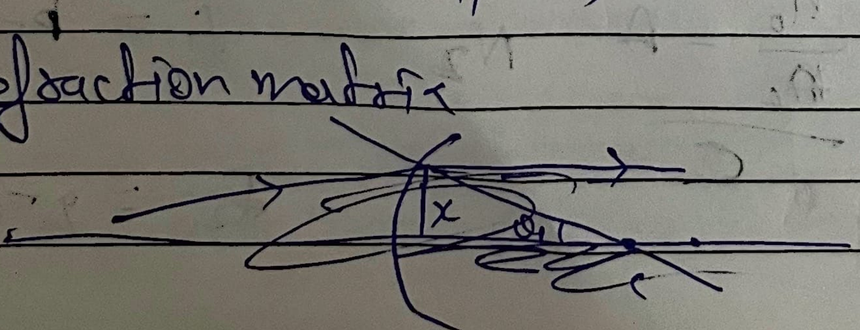
$X_2 \simeq X_1 + D\dfrac{\lambda_1}{n_1}$

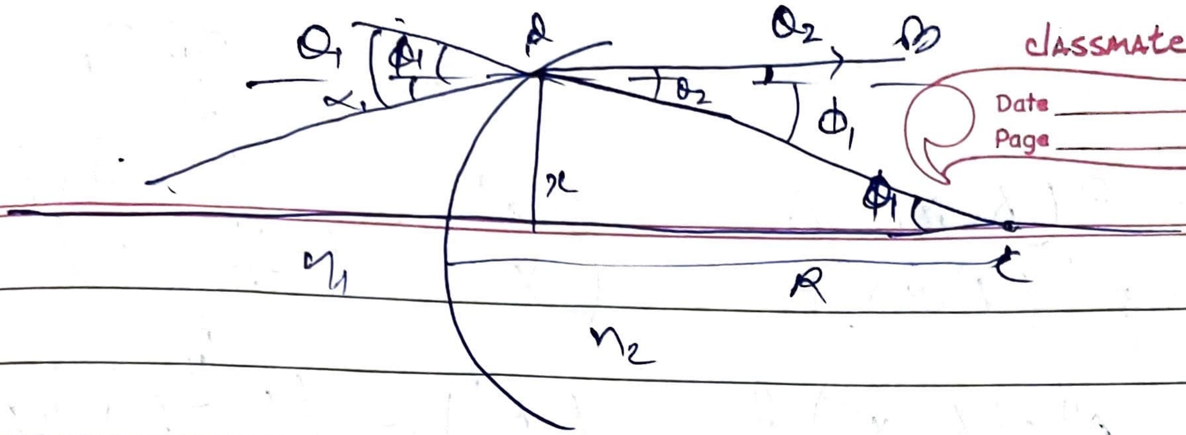
$\lambda_2 = \lambda_1$

$[\lambda_1 \simeq n_1\theta_1]$

$$\begin{pmatrix} \lambda_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{D}{n_1} & 1 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ x_1 \end{pmatrix}$$

Refraction matrix





$n_1\theta_1 \simeq n_2\theta_2$ (paraxial approx.)

$\theta_1 = \phi_1 + \alpha_1$; $\theta_2 = \phi_1 + \alpha_2$.

$$n_2\alpha_2 = n_1\alpha_1 - (n_2 - n_1)\frac{x}{R}$$

$\boxed{\phi_1 = \frac{x}{R}}$ (for small ϕ_1). $(\lambda_2 = \lambda_1 - Px)$

$$n_1(\phi_1+\alpha_1) = n_2(\phi_1+\alpha_2) \qquad P = \frac{n_2-n_1}{R}$$

$$n_2\alpha_2 \simeq n_1\phi_1 + n_1\alpha_1 - n_2\phi_1$$

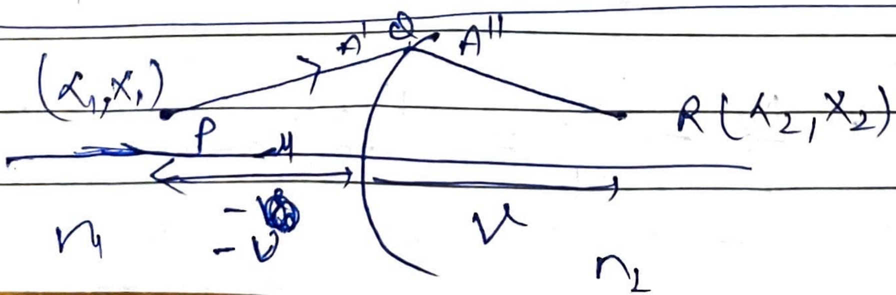
$$= (n_1-n_2)\phi_1 + n_1\alpha_1 \qquad x_2 = x_1 = x$$

$$\begin{pmatrix}\lambda_2 \\ x_2\end{pmatrix} = \begin{pmatrix}1 & -P \\ 0 & 1\end{pmatrix}\begin{pmatrix}\lambda_1 \\ x_1\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}\lambda_2 \\ x_2\end{pmatrix} = \begin{pmatrix}b & -a \\ -d & c\end{pmatrix}\begin{pmatrix}\lambda_1 \\ x_1\end{pmatrix}$$

$\Downarrow$

det S = 1   S : system transformation matrix



classmate
Date
Page

$$\left.\begin{matrix} P(\lambda_1, x_1) \\ A'(\lambda', x') \end{matrix}\right\} \quad \begin{pmatrix} \lambda' \\ x' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{u}{n_1} & 1 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ x_1 \end{pmatrix}$$

$$\left.\begin{matrix} A'':(\lambda'', x'') \\ R:(\lambda_2, x_2) \end{matrix}\right\} \quad \begin{pmatrix} \lambda'' \\ x'' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & -P \\ 0 & 1 \end{pmatrix}}^{R} \begin{pmatrix} \lambda' \\ x' \end{pmatrix} \quad P = \frac{n_2 - n_1}{R}$$

$$\begin{pmatrix} \lambda_2 \\ x_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ \frac{v}{n_2} & 1 \end{pmatrix}}_{\Downarrow \, \tau_2} \begin{pmatrix} \lambda'' \\ x'' \end{pmatrix}$$

$$\begin{pmatrix} \lambda_2 \\ x_2 \end{pmatrix} = \tau_2 R \tau_1 \begin{pmatrix} \lambda_1 \\ x_1 \end{pmatrix}$$

$$\begin{pmatrix} \lambda_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 + P\frac{u}{n_1} & -P \\ \frac{v}{n_2}\left(1 + P\frac{u}{n_1}\right) - \frac{u}{n_1} & 1 - P\frac{v}{n_2} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ x_1 \end{pmatrix}$$

$$x_2 = \left[\frac{v}{n_2}\left(1 + P\frac{u}{n_1}\right) - \frac{u}{n_1}\right]\lambda_1 + \left(1 - \frac{vP}{n_2}\right)x_1$$

For $x_2 = 0$ & $x_1 = 0$ (axial object & axial image)

$$\frac{u}{n_1} = \frac{v}{n_2}\left(1 + \frac{Pu}{n_1}\right)$$

$$\frac{v}{n_2} + \frac{uv}{n_1 n_2}\left(\frac{n_2 - n_1}{R}\right) = \frac{u}{n_1}$$

$$\Rightarrow \left[\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} = P\right]$$

→ On the Image plane :-

$$\begin{pmatrix}\lambda_2\\ x_2\end{pmatrix} = \begin{pmatrix}1+\frac{Pu}{n_1} & -P\\ 0 & 1-P\frac{v}{n_2}\end{pmatrix}\begin{pmatrix}\lambda_1\\ x_1\end{pmatrix}$$

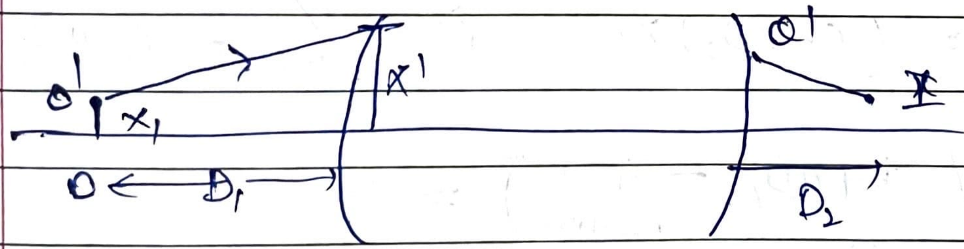
$$x_2 = \left(1-\frac{vP}{n_2}\right)x_1$$

→ magnification

$$m : \quad \frac{x_2}{x_1} = 1-\frac{vP}{n_2}$$

$$= 1-\frac{v}{n_2}\left(\frac{n_2}{v}-\frac{n_1}{u}\right) = \frac{n_1 v}{n_2 u}$$

→ Imaging by coaxial system :



O' : (λ_1, x_1)

O : (λ', x')

O'' : (λ'', x'')

I : (λ_2, X_2)

$$\begin{pmatrix}\lambda'\\ x'\end{pmatrix} = \begin{pmatrix}1 & 0\\ -D_1 & 1\end{pmatrix}\begin{pmatrix}\lambda_1\\ x_1\end{pmatrix}$$

$$\begin{pmatrix}\lambda''\\ x''\end{pmatrix} = \begin{pmatrix}b & -a\\ -d & c\end{pmatrix}\begin{pmatrix}\lambda'\\ x'\end{pmatrix}$$

$$\begin{pmatrix}\lambda_2\\ X_2\end{pmatrix} = \begin{pmatrix}1 & 0\\ D_2 & 1\end{pmatrix}\begin{pmatrix}\lambda''\\ x''\end{pmatrix}$$

$$\begin{pmatrix}\lambda_2\\x_2\end{pmatrix} = \underbrace{\begin{pmatrix}1 & 0\\D_2 & 1\end{pmatrix}}_{\text{Translation}}\begin{pmatrix}b & -a\\-d & c\end{pmatrix}\underbrace{\begin{pmatrix}1 & 0\\-D_1 & 1\end{pmatrix}}_{\text{Translation}}\begin{pmatrix}\lambda_1\\x_1\end{pmatrix}$$

$$\begin{pmatrix}\lambda_2\\x_2\end{pmatrix} = \begin{pmatrix}b+aD_1 & -a\\ bD_2+aD_1D_2-d-cD_1 & c-aD_2\end{pmatrix}\begin{pmatrix}\lambda_1\\x_1\end{pmatrix}$$

$$x_2 = (bD_2 + aD_1D_2 - d - cD_1)\lambda_1 + (c-aD_2)x_1$$

for $x_1 = 0$, $x_2 = 0$ $\quad bD_2 + aD_1D_2 - d - cD_1 = 0$

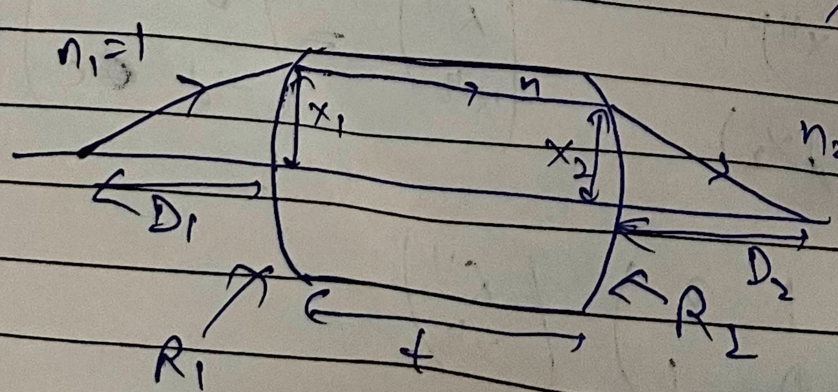
$$\begin{pmatrix}\lambda_2\\x_2\end{pmatrix} = \begin{pmatrix}b+aD_1 & -a\\ 0 & c-aD_2\end{pmatrix}\begin{pmatrix}\lambda_1\\x_1\end{pmatrix}$$

$$x_2 = (c-aD_2)x_1 \qquad M = \frac{x_2}{x_1} = c-aD_2$$

$$\det\begin{pmatrix}b+aD_1 & -a\\ 0 & c-aD_2\end{pmatrix} = 1$$

$$b+aD_1 = \frac{1}{c-aD_2} = \frac{1}{M} \rightarrow \begin{pmatrix}\lambda_2\\x_2\end{pmatrix} = \begin{pmatrix}\frac{1}{M} & -a\\ 0 & M\end{pmatrix}\begin{pmatrix}\lambda_1\\x_1\end{pmatrix}$$

Thick lens



$$P_1 = \frac{n-1}{R_1}$$

$$P_2 = \frac{1-n}{R_2} = -\left(\frac{n-1}{R_2}\right)$$

$$S = \begin{pmatrix} 1 - P_2 \frac{t}{n} & -P_1 - P_2\left(1 - \frac{t}{n} P_1\right) \\ \frac{t}{n} & 1 - \frac{t}{n} P_1 \end{pmatrix}$$

$\lim t \to 0$

Thin lens

$$S = \begin{pmatrix} \overset{b}{1} & \overset{a}{-P_1 - P_2} \\ \underset{d}{0} & \underset{c}{1} \end{pmatrix}$$

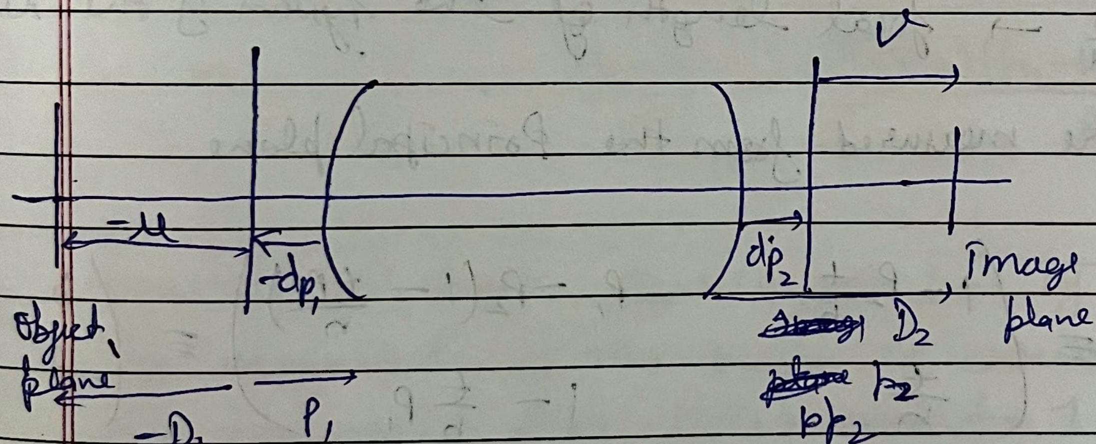
$a = P_1 + P_2$

$b = 1$

$c = 1 \quad d = 0$

$$P_1 + P_2 = \frac{1}{D_2} - \frac{1}{D_1}$$

→ <u>Principle plane</u> : The principal planes/unit planes one each in the object and image space, between which the magnification is unity



$$b + a d_{P_1} = \frac{1}{c - a d_{P_2}} = 1 \qquad b + a d_{P_1} = 1 \Rightarrow d_{P_1} = \frac{1-b}{a}$$

$$c - a d_{P_2} = 1 \Rightarrow d_{P_2} = \frac{c-1}{a}$$

$$D_1 = \cancel{u} + d_{P_1} = u + \frac{1-b}{a}$$

$$D_2 = v + d_{P_2} = v + \frac{c-1}{a}$$

$$bD_2 + aD_1D_2 - cD_1 - d = 0$$

$$D_2(b + aD_1) = d + cD_1$$

$$D_2 = \frac{d + cD_1}{b + aD_1} \quad \therefore \quad v + \frac{c-1}{a} = \frac{d + c\left(u + \frac{1-b}{a}\right)}{b + a\left(u + \frac{1-b}{a}\right)}$$

$$v = \frac{u}{au + 1} \Leftarrow \text{usin} \quad \det(S) = bc - ad = 1$$

$$\frac{v}{u} = \frac{1}{au+1} \quad = \frac{u}{v} = au + 1 \Rightarrow \frac{1}{v} - \frac{1}{u} = a$$

$\frac{1}{a} \rightarrow$ focal length of the system if the dist. are measured from the Principal plane.

$$S \equiv \begin{pmatrix} 1 - P_2\frac{t}{n} & -P_1 - P_2\left(1 - \frac{tP_1}{n}\right) \\ \frac{t}{n} & 1 - \frac{t}{n}P_1 \end{pmatrix} \equiv \begin{pmatrix} b & -a \\ -d & c \end{pmatrix}$$

$$dP_1 = \frac{1-b}{a} = \frac{1 - 1 + P_2 t/n}{P_1 + P_2\left(1 - \frac{t}{n}P_1\right)} = P_2\frac{t}{n}\frac{1}{P_1 + P_2(1-$$

$$dP_2 = -\frac{t}{n} \cdot \frac{P_1}{\left[P_1 + P_2\left(1 - \frac{t}{n}P_1\right)\right.}$$

Ans

Double side convex

$|R_1| = |R_2| = R$

Find the location of the principal plane in terms of $\rightarrow$ n, R, t of the lens.

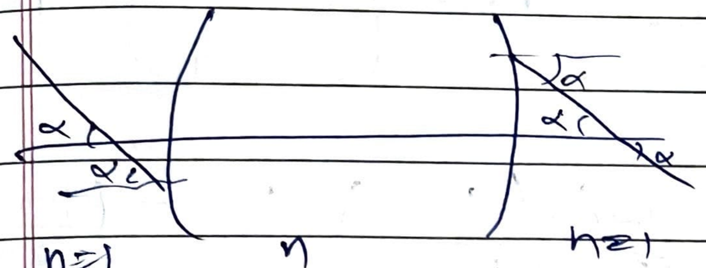
Assume $t <<<<<<<<<<<<< R$

classmate

Date ______
Page ______

$$\begin{pmatrix} \lambda_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} b+aD_2 & -a \\ 0 & c-aD_2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ x_1 \end{pmatrix}$$

$$\lambda_2 = (b+aD_1)\lambda_1 - ax_1 \qquad \alpha_1 = \lambda_2$$



using axial object point

$x_T = 0$

$$\boxed{\lambda_2 = (b+aD_1)\lambda_1 = \lambda}$$

$$(b+ad_{n_1}) = 1$$

$$D = d_{n_1}$$

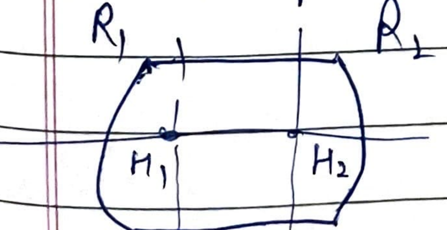
$$d_{n_1} = \frac{1-b}{a}$$

$$d_{n_2} = \frac{c-1}{a}$$

① $$\frac{1}{f_1} = \frac{n_2-n'}{nR_2} - \frac{n_2-n}{nR_1} - \frac{(n_2-n)(n_2-n')\cdot t}{n\,L_2\,R_1R_2}$$

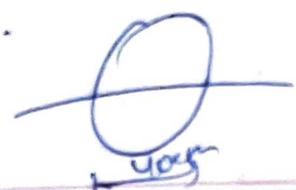
② Consider a thick equiconvex lens of R.I 1.5. The magnitude of the radii of curvature of the two surfaces is 4cm, thickness $t = 1$ cm. Lens is kept in the air. Find the system matrix, focal length and the location of principal planes.

$R_1 = +4$ cm $\quad R_2 = -4$ cm



$$S = \begin{pmatrix} 1-\frac{P_2 t}{n} & -P_1-P_2\left(1-\frac{t}{n}P_1\right) \\ \frac{t}{n} & 1-\frac{t}{n}P_1 \end{pmatrix}$$

$$P_1 = 0.125\ \text{cm}^{-1} \qquad = \begin{pmatrix} 0.9167 & -0.24 \\ 0.667 & 0.9167 \end{pmatrix}$$

③ 

sphere of radius 20 cm & R.I = 1.6 Find paraxial focal point and the location principal plane.

$$T = \begin{pmatrix} 1 & 0 \\ \frac{D}{n_1} & 1 \end{pmatrix} \; : \; R = \begin{pmatrix} 1 & -P \\ 0 & 1 \end{pmatrix}, \quad P = \frac{n_2 - n_1}{R}$$

$$\underbrace{\begin{pmatrix} 1 & 0 \\ \frac{u}{1} & 1 \end{pmatrix}}_{\text{translation in air}} \underbrace{\begin{pmatrix} 1 & \frac{1-1.6}{20} \\ 0 & 1 \end{pmatrix}}_{\text{Refraction}} \underbrace{\begin{pmatrix} 1 & 0 \\ \frac{40}{1.6} & 1 \end{pmatrix}}_{\text{translation}} \underbrace{\begin{pmatrix} 1 & -\left(\frac{1.6-1}{20}\right) \\ 0 & 1 \end{pmatrix}}_{\text{Refraction at front}}$$