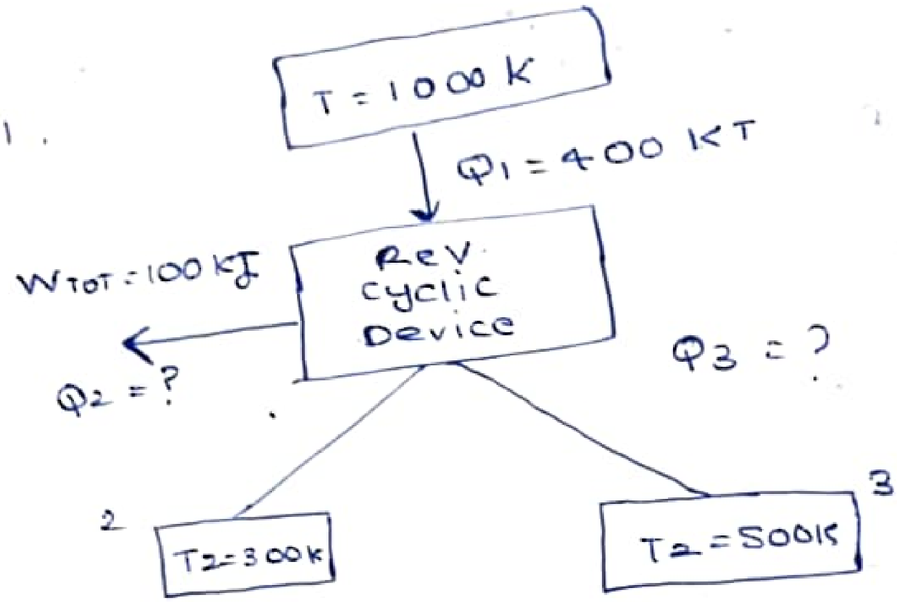
Heat and Thermodynamics

Problems- 4

1.



Rev. cyclic process $\Rightarrow \Delta U = 0$

$\Delta Q = W_{Tot}$

$Q + Q_2 + Q_3 = W_{total}$

$400 + Q_2 + Q_3 = 100 \longrightarrow (1)$

$$\oint \frac{dQ_{rev}}{T} = 0 \Rightarrow \frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} = 0$$

$$\frac{400}{100\cancel{0}} + \frac{Q_2}{30\cancel{0}} + \frac{Q_3}{50\cancel{0}} = 0$$

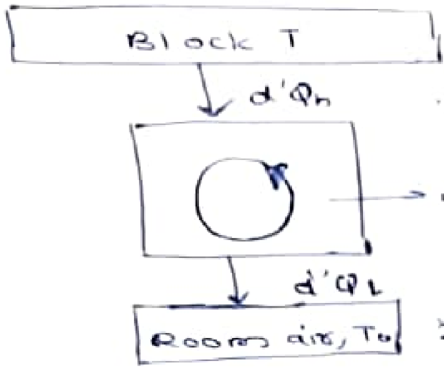
$$0.4 + \frac{Q_2}{300} + \frac{Q_3}{500} = 0 \longrightarrow$$

$Q_2 = -300 - Q_3$

$Q_3 = -450\ kJ$

$Q_2 = 150\ kJ$

2.



$5\,kg,\ 40^\circ C,\ c_p = 400\ J/kg\,K^{-1}$

$20^\circ C$

$dS = \frac{dQ_{rev}}{T}$

$dQ_{rev} = mc_p dT$

$\int dS = mc_p \int_{T_1}^{T_2} \frac{dT}{T}$ $\quad T_1 = 40^\circ C$, $T_2 = 20^\circ C$

$\Delta S_{block} = mc_p \ln \frac{T_2}{T_1}$

$= 5 \times 400 \times \ln \frac{293}{313}$

$\Delta S_{block} = -134\ J/K$

If block + engine + room is considered system,

then $\Delta dQ_{surr} = 0$

$\Delta S_{block + engine + room} = 0$

$\Delta S_{room} = -\Delta S_{block} = 134\ J/K$

$\eta = \frac{d'W}{Q_{block}} = \frac{-d'W'}{d'Q_h} = \frac{-dW'}{mc_p dT}$

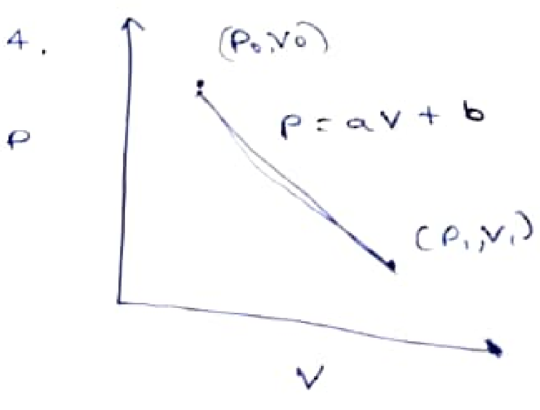
$\eta = 1 - \frac{T_L \rightarrow \text{Room } T}{T_H \rightarrow \text{Al block}}$

$\int dW' = mc_p T_0 \int_{313}^{293} \frac{dT}{T} - mc_p \int_{313}^{293} dT$

$= 5 \times 400 \times 293 \times \ln \frac{293}{313} - 5 \times 400 \times (293 - 313)$

$W' = 738\ J$

4.



$a = -\frac{31}{56}$, $b = \frac{255}{7}$

$PV = nRT$ (always)

$nRT = aV^2 + bV$

a. $T = \frac{aV^2 + bV}{nR}$

b. $\left.\frac{dT}{dV}\right|_{V_{max}} = 0$, $V_{max} = 2aV + b = 0 \Rightarrow V = \frac{-b}{2a} = 32.9\,m^3$

c. $T_0 = \frac{P_0 V_0}{nR} = 308.4\ K$ $\qquad R = 8.3\ J/K$

$T_1 = \frac{P_1 V_1}{nR} = 77.1\ K$

$T_{max} = \frac{aV_{max}^2 + bV_{max}}{nR} = 722.03\ K$

d. $Q = \int dU + \int p\,dW$

$U = \frac{3}{2} nRT = \frac{3}{2} pV = \frac{3}{2}(aV^2 + bV)$

$\int p\,dV = \int (aV+b)\,dV = \frac{aV^2}{2} + bV$

$\Rightarrow Q(V) = 2aV^2 + \frac{5}{2} bV$

(Integrate from V_0 to V)

e. $Q_{max} \Rightarrow 4aV + \frac{5}{2} b = 0$

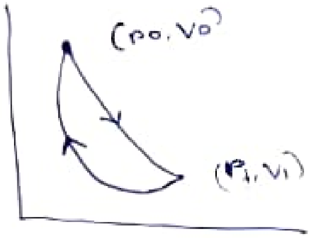
$V_m = \frac{-5b}{8a} = 41.12\ m^3$

$P_m = aV_m + b = 13.67\ Pa$

f. Heat transferred $Q(V_{max}) - Q(V_0) = 1215\ J$

g. Heat transferred $Q(V_1) - Q(V_{max}) = -579.3\ J$

5.



$P_0 V_0^{\gamma} = P_1 V_1^{\gamma} \quad \gamma_1 = \frac{5}{3}$

$32 \times 8^{\gamma} = 2^5 \times 2^{3\gamma} = 2^{5+3\gamma} = 2^{10}$

$64^{\gamma} = 2^{6\gamma} = 2^{10}$

Hence adiabatic curve

a. Work done by the gas

$W_{1\to 0} = -\Delta Q(1 \to 0)$

$= -\frac{3}{2} nR(T_0 - T_1) = -288.3\ J$

a. $W_{0\to 1} = \int_8^{64} (aV+b)\,dV = 924\ J$

b. Net $W = 635.7\ J$

c. $\Delta Q_{0\to 1} = 635.7\ J$

$\Delta Q_{1\to 0} = 0$

Net $\Delta Q = 635.7\ J$

d. $\eta = \frac{|W|}{|Q|} = \frac{635.7}{1215} = 0.52$

e. $\eta_{carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{77}{722.01} \simeq 1 - 0.11 = 0.89$