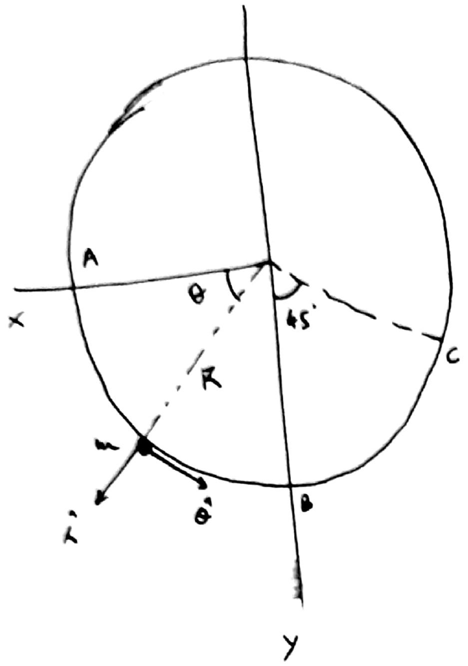
PH - 203

Tutorial - 1

1.




a) $\vec{F} = \dfrac{d\vec{p}}{dt}$

$$m\vec{g} + \vec{N} = \frac{d\vec{p}}{dt}$$

$$\vec{N} = -N\hat{r}$$

$$m\vec{g} = \hat{r}g\sin\theta + \hat{\theta}g\cos\theta$$

$$\vec{p} = p\hat{\theta}$$

$$m(\hat{r}g\sin\theta + \hat{\theta}g\cos\theta) - N\hat{r} = \frac{d}{dt}(p\hat{\theta})$$

$$= \frac{dp}{dt}\hat{\theta} + p\frac{d\hat{\theta}}{dt}$$

$$= \frac{dp}{dt}\hat{\theta} + p(-\hat{r}\dot{\theta})$$

$$mg\sin\theta - N = -p\dot{\theta}$$

$$mg\cos\theta = \frac{dp}{dt}$$

$$mg\cos\theta = \frac{m\,dv}{dt}$$

$$g\cos\theta = \frac{dv}{d\theta}\frac{d\theta}{dt}$$

$$g\cos\theta = \frac{dv}{d\theta}\cdot\dot{\theta} = \frac{dv}{d\theta}\cdot\frac{v}{r}$$

$$\int_0^{\theta} rg\cos\theta\, d\theta = \int_0^{v} v\, dv$$

$$\Rightarrow rg\sin\theta = \frac{v^2}{2}$$

$$v^2 = 2gr\sin\theta$$

$$\cancel{v = \sqrt{2g\ldots}}$$

$$v = \sqrt{2gr\sin\theta} \qquad \vec{p} = \hat{\theta}m\sqrt{2gR\sin\theta}$$

c) $W_{AC} = mgR(\sin\theta_C - \sin\theta_A)$

b) $\vec{\tau} = \frac{d\vec{L}}{dt}$

$\vec{r} = R\hat{r}$

$\vec{F} = m\vec{g} + \vec{N}$

$\vec{\tau} = \vec{r} \times \vec{F}$

$= R\hat{r} \times (m\vec{g} + \vec{N})$

$= mgR\cos\theta\,\hat{z}$

$\vec{\tau} = \frac{d\vec{L}}{dt}$

$\vec{L} = \vec{r} \times \vec{p}$

$= R\hat{r} \times p\hat{\theta}$

$= Rp\,\hat{z}$

$= mvR\,\hat{z}$

$mgR\cos\theta = mR\frac{dv}{dt}$

$\frac{dv}{dt} = g\cos\theta$

$mgR\cos\theta = \frac{dL}{d\theta}\frac{d\theta}{dt} = \frac{dL}{d\theta}\cdot\dot{\theta}$

$mgR\cos\theta = \frac{dL}{d\theta}\frac{v}{R}$

$mgR\cos\theta = \frac{dL}{d\theta}\frac{\sqrt{2gR\sin\theta}}{R}$

$\int dL = \int \frac{mgR^2\cos\theta}{\sqrt{2gR\sin\theta}}\,d\theta$

$L = \frac{mgR^2}{\sqrt{2gR}}\cdot 2\sqrt{\sin\theta}$

$\vec{L} = \frac{2mgR^2}{\sqrt{2gR}}\sqrt{\sin\theta}\,\hat{z}$

c) $\vec{L} = \vec{r} \times \vec{p}$

$= R\hat{r} \times \hat{\theta}\, m\sqrt{2gR\sin\theta}$

$= mR\sqrt{2gR\sin\theta}\,\hat{z}$

d) $W_{AB} = \int_A^B \vec{F}\cdot d\vec{r}$

$= \int (m\vec{g} + \vec{N})\cdot(R\,d\theta\,\hat{\theta})$

$W_{AB} = \int_A^B mg\cos\theta R\,d\theta$

$= mgR\int_0^{\pi/2}\cos\theta\,d\theta = mgR\sin\theta\Big|_A^B = mgR(\sin\theta_B - \sin\theta_A)$

$= mgR$

e) $W_{AC} = mgR(\sin\theta_c - \sin\theta_A)$
$= \frac{mgR}{\sqrt{2}}$

$W_{AB} = T_B - T_A$
$mgR = \frac{1}{2} m v_B^2$
$v_B = \sqrt{2gR}$

$W_{AC} = T_C - T_A$
$\frac{mgR}{\sqrt{2}} = \frac{1}{2} m v_C^2$
$v_C = \sqrt{\sqrt{2}gR}$

f) $\cancel{V(\theta) = 2mgR\sin\theta}$
$\cancel{mgR\sin\theta = \frac{1}{2} m v_\theta^2}$
$\cancel{v_\theta = 2\sqrt{2gR\sin\theta}}$

$\frac{d\theta}{dt} = \sqrt{\frac{2g}{R}\sin\theta}$

$\int dt = \frac{1}{\sqrt{\frac{2g}{R}}} \int \frac{d\theta}{\sqrt{\sin\theta}}$

$\sin\theta = z^2$
$\cos\theta \, d\theta = 2z\,dz$
$d\theta = \frac{2z\,dz}{\sqrt{1-z^4}}$

$\int \sqrt{\frac{2g}{R}}\, dt = \int \frac{2z\,dz}{z\sqrt{1-z^4}}$

$\int_0^{T/4} \sqrt{\frac{g}{2R}}\, dt = \int_0^1 \frac{dz}{\sqrt{1-z^4}}$

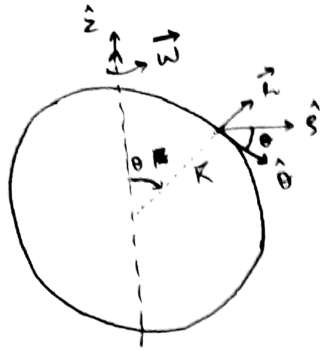
$\sqrt{\frac{g}{2R}} \cdot \frac{T}{4} = \frac{\sqrt{\pi}}{4} \frac{\Gamma(1/4)}{\Gamma(3/4)}$

$\frac{T}{\cancel{4}} \sqrt{\frac{g}{2R}} = \frac{\sqrt{\pi}}{\cancel{4}} \cdot \frac{0.62561}{1.22542}$

$\cancel{T = \sqrt{\frac{\pi g}{2R}} \frac{0.62561}{1.22542}}$

$T = \sqrt{\frac{2\pi R}{g}} \frac{0.62561}{1.22542}$

2.




$L = \sum \vec{r_i} \times \vec{p_i}$

$$\cancel{\vec{dL} = \vec{r} \times \vec{dp}}$$

$$\vec{dL} = \vec{r} \times \vec{dp} = -\hat{\theta}\, dL$$

$$L = \int \vec{dL} = -\int \hat{\theta}\, dL$$

$$= -\int (\hat{s}\cos\theta - \hat{z}\sin\theta)\, dL$$

$\downarrow$ does not contribute

$$\vec{L} = +2\hat{z}\int_0^{\pi R} \sin\theta \; dL$$

$$\vec{dL} = \vec{r} \times \vec{dp} = dm(\vec{r} \times \vec{v})$$

$$= dm\, Rv \sin 90^\circ (-\hat{\theta})$$

$$= dm\, \omega R^2 \sin\theta(-\hat{\theta})$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$= \omega R \sin\theta\, \hat{n}$$

$$\lambda = \frac{M}{2\pi R} = \frac{dm}{R\,d\theta}$$

$$\vec{L} = +2\hat{z}\int dm\, \omega R^2 \sin^2\theta$$

$$= +2\hat{z}\int \lambda R\, d\theta\; \omega R^2 \sin^2\theta$$

$$= +2\lambda\omega R^3\hat{z}\int_0^{\pi} \sin^2\theta\; d\theta$$

$$= +2\lambda\omega R^3 \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi} \hat{z}$$

$$= +2\lambda\omega R^3\left(\frac{\pi}{2}\right)\hat{z}$$

$$= \lambda\omega R^3\pi\; \hat{z}$$

$$= \omega R^3\pi \cdot \frac{M}{2\pi R}\; \hat{z}$$

$$\vec{L} = \frac{1}{2} M R^2\, \vec{\omega}$$

$$\vec{L} \parallel \vec{\omega}$$