CYK/2023/PH201 Mathematical Physics

Mid-Semester Exam

Total Marks: 30; **Duration:** 2 Hours (9AM-11AM); **Date:** 18 Sept 2023, Monday

1. [6×2 **Marks**] Answer the following questions: (Write steps and reasons in **brief**. Writing just final answers will not be awarded marks.)

   (a) Find and sketch the image of the straight line $y = x + 1$ under the transformation $w = \frac{1}{z}$.

   (b) The branch cut of $\ln z$ is chosen along the radial line making an angle of 120 degrees with the positive x axis. If $\ln 1 = 0$, then what are the values of $\ln(i)$, $\ln(1)$ and $\ln(-1)$?

   (c) Show that if a function $f(z) = u + iv$ is analytic at z, level curves of u and v passing through z are orthogonal.

   (d) At which points the function $f(z) = \bar{z}^2$, analytic?

   (e) If $C : |z| = R$ is a positively oriented circular contour, compute

$$\oint_C \frac{e^z}{\left(z - \frac{\pi i}{4} a\right)^2} dz \qquad (R > a).$$

   write in terms of sin & cos.

   (f) Discuss and classify the singularities of $\frac{1}{\sin(\pi/z)}$. essential / isolated case $\left(\frac{\pi}{z}\right) \to \infty$ $m\pi$

2. [2×3 **Marks**] Answer the following questions:

   (a) [3] The Euler numbers E_n are defined by the power series $\frac{1}{\cosh z} = \sum_{n=0}^{\infty} \frac{E_n}{n!} z^n$. What is the radius of convergence for this series? Compute E_0 to E_4. 1/2 each 1

   v. easy (b) [3] Given the function $f(z) = \frac{z}{(z-2)(z+i)}$, expand the function in a series about $z_0 = 0$, in the regions (i) $|z| < 1$ (ii) $1 < |z| < 2$ and (iii) $|z| > 2$.

3. [3×4 **Marks**] Using the method of residues, answer the following questions: (**sketch contours and show contributions from each segment of contours explicitly**)

   Tutorial Question Type-2 (a) Compute 14

$$\text{pv} \int_{-\infty}^{\infty} \frac{dx}{x^2 + 2x + 2}$$

   type3 (b) Compute

$$\int_{-\infty}^{\infty} \frac{\cos kx}{(x+b)^2 + a^2} dx$$

   where $a > 0$, $k > 0$ and b is a real number.

   v. difficult (c) Compute

$$\int_0^{\pi} \frac{\cos 2\theta \, d\theta}{a^2 - 2a\cos\theta + 1}; \qquad -1 < a < 1.$$

Indian Institute of Technology Guwahati
Department of Physics

Mid-semester Examination, September 22nd 2023

Subject: Heat and Thermodynamics (PH 207)

Full Marks: 30 **Time:** 2 hours

Easy 1. Consider n moles of an ideal gas whose thermodynamic state is described by parameters pressure, volume and temperature P, V and T, respectively. Show that (a) coefficient of volume expansion $\beta = \frac{1}{T}$ and (b) isothermal compressibiity $\kappa = \frac{1}{P}$. [2+2] 4

2. A compressor designed to compress air is used instead to compress Helium. It is found that the compressor overheats when Helium is used. Explain why this happened. Consider both as ideal gas, initial pressure is same for both, the compression process is adiabatic and $\gamma_{He} = 1.6, \gamma_{air} = 1.4$. [3] 3

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Think 3. Consider a process that increases internal energy of a system. How can one tell if it is due to work done or due to flow of heat- by measuring temperature of the system before and after the process or by measuring temperature of the surroundings before or after the process? Explain. [2] 2

4. A combustion experiment is performed by burning a mixture of fuel and oxygen in a constant-volume container surrounded by a water bath. During the experiment, the temperature of water rises. Considering mixture of fuel and oxygen as the system: (a) has heat been transferred? If yes, what is the direction of heat transfer? (b) has work been done by the system? Explain. [2+1] 3

Easy 5. Consider a Carnot engine operating between two heat reservoirs at temperatures T_H and T_L ($T_H > T_L$). In process A, T_H is increased keeping T_L constant and in process B, T_L is decreased keeping T_H constant. Explain which process increases the efficiency of the engine more. [3] 3

Solve. Think 6. A freezer operating in a Carnot cycle has efficiency 0.1. It is kept in a room with temperature 32^0 C. A tray of ice cube is kept inside the freezer. Determine whether the ice cubes will remain frozen when put inside the freezer. [3] 3

Solve + Think 7. A steam turbine is operated with an intake temperature of 400^0C and an exhaust temperature of 150^0C. What is the maximum amount of work the turbine can do for a given heat input Q. Under what condition can it achieve the maximum work? [3+1] 4

Solve 8. An inventor proposes an engine that operates between 27^0C warm surface layer of the ocean and 10^0C layer a few metres down. The inventor claims that the engine produces 100 kW by pumping seawater at a rate of 20 kg/ sec. Is this engine possible? (Consider specific heat per unit mass at constant pressure $C_p = 4.18$ kJ/kg-K) [4] 4

Think 9. The conversion of white tin into grey tin occurs at 13^0C with a release of 2.1 kJ/mol of heat. Which one among the two allotropes of tin is more ordered? [2] 2

10. Plot T(Temperature) versus S (Entropy) relations for an adiabatic, and an isothermal reversible process of transformation. [2] 2

Indian Institute of Technology

PH205

Mid-semester Examination (2023)

Full marks: 30 **Time: 120 minutes**

1. (a) Draw a schematic of the energy band diagram for GaAs showing the different valleys in the conduction band and the bandgap value. [2]

   (b) Derive the energy dependence of the electronic density of states *N(E)* for a one-dimensional semiconductor, taking the effective mass of electron as m*. [3]

2. The absorption coefficient (α) of GaAs is given as $\alpha(\omega) = 5.6 \times 10^4 \frac{\sqrt{(\hbar\omega - E_g)}}{\hbar\omega}$ cm^{-1}. If α is measured as a function of wavelength/frequency of incident light, briefly describe a graphical method (you may look for linear plot) to find the bandgap (E_g) of GaAs from the frequency dependent absorption data. Note that the $\alpha(\omega)$ data is available only for $\hbar\omega > E_g$. [2]

3. In an n-type Si, the Fermi level is 0.3 eV below the conduction band edge. Find the electron (n) and hole (p) concentrations in the same at room temperature (300K). (For Si, E_g=1.1 eV, intrinsic carrier density $n_i = 1.5 \times 10^{10}$ cm^{-3} and kT =0.026 eV). [3]

4. For a hypothetical semiconductor, $\mu_n = \mu_p = 1000$ cm^2/V-s and $N_c = N_v = 10^{19}$ cm^{-3}. If the electrical conductivity (σ) of the intrinsic semiconductor at 300 K is 4×10^{-6} $(\Omega\text{-cm})^{-1}$, what is the conductivity at 600 K. Neglect temperature dependent changes in N_c and N_v. Note that $n = N_c \exp\left[\frac{E_F - E_C}{kT}\right]$ and $p = N_V \exp\left[\frac{E_V - E_F}{kT}\right]$ [4]

5. A Si sample with $10^{15}/cm^3$ donors is uniformly optically excited at room temperature such that $10^{19}/cm^3$ electron-hole pairs are generated (g_{op}) *per second*. Find the separation of the quasi-Fermi levels (E_{Fn}, E_{Fp}) in eV. Electron and hole lifetimes (τ) are both 10^{-6} s. Take μ_n=1300 cm^2/V-s, μ_p=463 cm^2/V-s. Note that excess carrier density $\delta n = \delta p = g_{op}.\tau$ and $E_{Fn} - E_{Fp} = kT \ln (np / n_i^2)$ [2]

6. (a) How long time (in sec) does it take an average electron to drift 1 μm length in a pure Si at an electric field of 100 V/cm? Take mobility μ_n=1500 cm^2/V-s. [2]

   (b) An abrupt Si p-n junction has $N_a = 10^{18}$ cm^{-3} on one side and $N_d = 5 \times 10^{15}$ cm^{-3} on the other. Calculate the contact potential V_0 and draw an equilibrium band diagram for the junction showing the value of V_0. Note that $V_0 = kT \ln \frac{N_a N_d}{n_i^2}$. [2]

7. Consider a *linearly graded* p-n junction, with doping density varying linearly with distance (x) from the junction in each side and is described by $N_d = N_a = Gx$, G is a constant. The doping is symmetrical such that width of the depletion region extends to *W/2* on each side. Using Poisson's equation, find the electric field as a function of x (distance from the junction), and show that the total width of the depletion region, $W = \left[\frac{12\epsilon(V_0 - V)}{qG}\right]^{1/3}$, V and V_0 are the applied bias and built-in potential of the junction, respectively, and ε is the dielectric permittivity of the material. [2+2]

8. Write *short notes* (with diagram, if applicable) on **any three** of the following: [2x3]

   (a) Band structure modification, (b) Modulation doping, (c) I-V characteristics of non-ideal diode, (d) Optical gain in semiconductors, (v) Varactor diode.

Midsem exam -2023

Full marks: 30

Course: PH209

1.

a) The width of base of a transistor should not be too thick and should not be too thin: Justify this statement [Marks 2]

b) What is Early effect? How does it affect the output characteristic of a transistor? [Marks 2+3]

c) What is the main advantage and disadvantage of a class A amplifier, and how does a class B amplifier solve the main disadvantage of a class A amplifier? [Marks 2+2]

(11)

2. a) If $\alpha = 0.98$, and the transistor is operating in the active region, find the value of the resistance R_1 for which the emitter current is I_E=2 mA. Neglect the reverse saturation current.

[Marks 3]

$\alpha = \frac{\beta}{1+\beta} - 1$

$\frac{\alpha}{1-\alpha} = \beta$

$\frac{0.98}{1-0.98} = \beta = 49$.

0.02

(3)

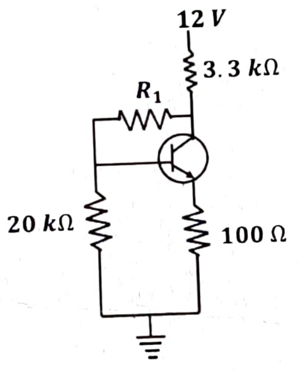


Figure 1

b) In the circuit given below both the transistor cannot be ON simultaneously: Justify this statement. [Marks 2]

(2)

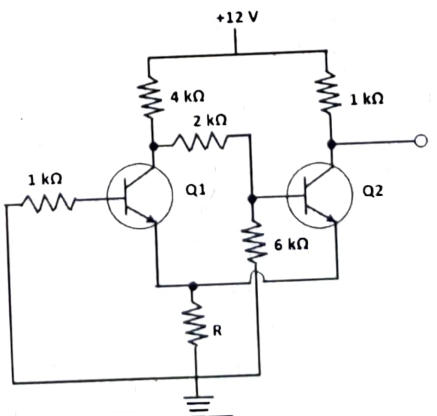


Figure 2

$\frac{1-\alpha}{\alpha} = \beta$ $\frac{\alpha}{1-\alpha} = \frac{0.98}{0.2} = 49$

|| → short because ac is not affected

Also justify

3. a) Draw a small signal equivalent circuit for the given amplifier in Figure 3, and hence find out the input impedance, current gain, and voltage gain seen from the source. [Marks 7]

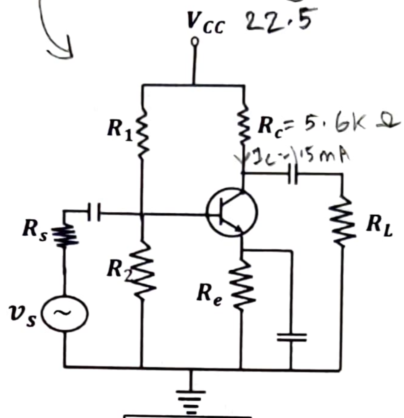


Figure 3

14

$R_B = R_1 \| R_2$

b) Assume a silicon transistor ($\beta = 50$) as shown in Figure 3. It is desired to establish a Q-point in the active region with $V_{CE} = 12\ V$, $I_C = 1.5\ mA$, and stability factor $S = 3$. If $V_{CC} = 22.5\ V$ and $R_C = 5.6k\ \Omega$, find the value of R_e, R_1 and R_2. [Marks 7]

[Ignore reverse saturation current in all the calculations, except finding the expression of $S(=\frac{\partial I_C}{\partial I_{CO}})$]

EE 220 : Signals and Systems

Department of Electronics and Electrical Engineerng

Indian Institute of Technology, Guwahati

Mid-semester examination: Sep 2023

Instructions: Solve all questions. Illegible answers and answers without proper justification may receive no marks. **[Total: 30 marks]**

Q1. Tofu and Laxmi are arguing about whether the system $y[n] = x[n^2]$ is causal or not.

Tofu: "The system is causal. We know that when the input $x[n] = \delta[n]$, the output $y[n] = h[n]$. Substituting this into the system equation gives $h[n] = \delta[n^2]$. From this, we see that $h[n]$ is one when $n^2 = 0$, which is n = 0, and otherwise, $h[n] = 0$ for all other values of n. Since $h[n]$ is zero for all $n < 0$, the system must be causal."

Laxmi: "The system is not causal. Consider the output when $n = 2$. Then, $y[2] = x[4]$. The output at this value of n depends on an input at a future time sample, so the system must not be causal."

Clearly explain the mistake in the incorrect argument. [2 marks]

a

Q2. Consider a system that performs differentiation of the input.

-2

$$y(t) = \frac{dx(t)}{dt}$$

a. Is this system causal? Explain your reasoning. [1 mark]

b. What about its realizability? Can we completely (exactly and in all circumstances) realize this system? Explain your thoughts in specific points. [1 mark]

1

Graphical **Q3.** Determine the convolution of the following two signals: [2 marks]

-2

a. $x(t) = e^{2t}u(-t)$ and $h(t) = u(t-3)$. Plot the output $y(t) = x(t) * h(t)$. graph

easy b. $x[n] = [1, -1, 0, 0, -1, 1]$ and $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$. Plot the output $y[n] = x[n] * h[n]$.

0 -1 -2 Proof. 2

Q4. A continuous time signal $x(t)$ is an input to an LTI system $h(t)$. The output of the system $y(t)$ can be written by the convolution operation between $x(t)$ and $h(t)$. Mathematically show that the convolution operation can be transformed to multiplication if we express $x(t)$ as the sum of eigenfunctions. [2 marks]

Easy **Q5.** Show that the energy of a continuous-time signal in the time domain equals the energy of the transformed signal in the frequency domain. [2 marks] CTFS/FT direct apply

Q6. A system is described by the following differential equation: 4

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = 2x(t)$$

with the following input and initial conditions

$$x(t) = cos(t)u(t);\ y(0) = -\frac{4}{5};\ \frac{d}{dt}y(t)|_{t=0} = 3/5$$

Determine the output y(t) using

a. the method of differential equations. [2 marks]

b. the method of unilateral Laplace transform. [2 marks]

2

Q7. Write clear answers with proper justification: 3

a. The expression for the Laplace transform of $u(t)$ is $1/s$. The Laplace transform of $-u(-t)$ is also $1/s$. Where is the distinction between the two? [1 mark]

easy b. We have seen that the Laplace transform may also be used to conveniently write Fourier transform (by appropriately substituting $s = j\omega$). Based on the given Laplace transform expressions in part a, can we write the Fourier transform of $u(t) = \frac{1}{j\omega}$? [1 mark]

easy c. Determine the signal $x(t)$ if its unilateral Laplace transform is $X(s) = s$. [1 mark] 1

Q8. a. State the Dirichlet's conditions required for aperiodic functions to write their Fourier transform. [1 mark] 3

fin dis, fin max, $\int |x(t)| dt < \infty$

b. Determine the Fourier transform of the following [2 marks]

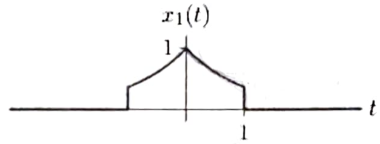
$$x(t) = \frac{2}{1 + t^2}$$

Q9. Let $X(\omega)$ represents the Fourier transform of 3

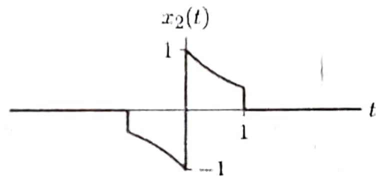
$$x(t) = \begin{cases} e^{-t} & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

Express the Fourier Transforms of the following signals (that are made from $x(t)$) in terms of $X(\omega)$: [3 marks]

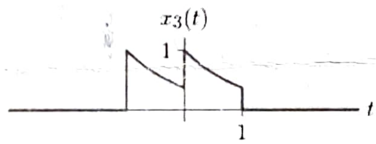
a.



b.

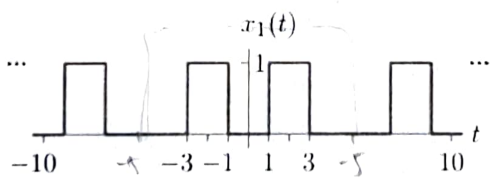


c.

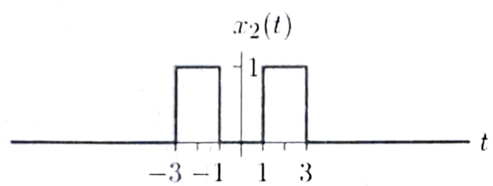


Q10. We generally say that Fourier series is written for periodic signal and Fourier transform is written for aperiodic signal. 7

a. Determine the Fourier series coefficients $X[k]$ of the following signal, which is periodic in $T = 10$. [3 marks]



b. Determine the Fourier transform $X(\omega)$ of the following signal (aperiodic version of part a), which is zero outside the indicated range. [3 marks]



c. What is the relation between the answers to parts a and b? In particular, provide an expression for $X[k]$ (the solution to part a) in terms of $X(\omega)$ (the solution to part b). [1 mark]