(5) Wave guide and Cavity
- metallic wave guide
- optical fiber
- Power loss

(6) Radiation & scattering
- Oscillating Dipole
- center fed liner antenna
- Radiation emitted by the charge particle

Books
(1) David Griffiths
Introduction to electrodynamics.

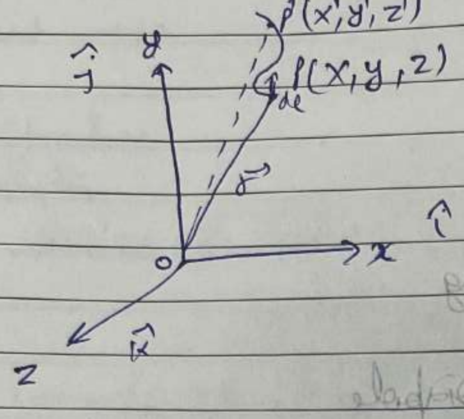
(2) J D Jackson
classical electrodynamics

Ref:
→ E C Jordan & G. Bal main
Electromagnetics and waves radiating systems

→ J D Krous
Antennas.

① → Cartesian co-ordinate system
② → Spherical Polar Co-ordinates system
③ → cylindrical Co-ordinate

①



$\vec{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ $\quad\quad$ $\vec{OP'} = x'\hat{i} + y'\hat{j} + z'\hat{k}$

$d\vec{l} = \vec{OP'} - \vec{OP}$ $\quad\quad$ $x' = x + dx$

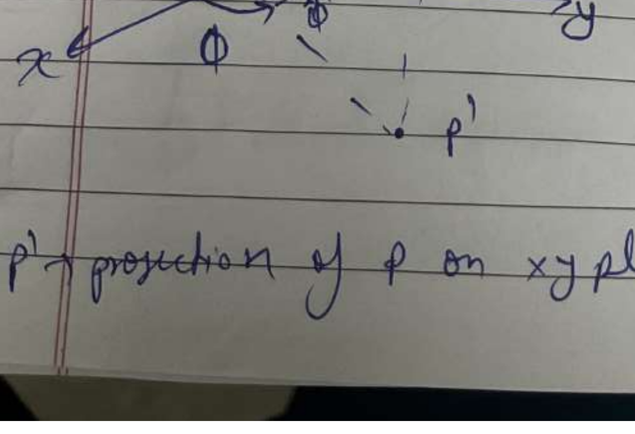
$= dx\hat{i} + dy\hat{j} + dz\hat{k}$ $\quad\quad$ $y' = y + dy$

$z' = z + dz$

$d\vec{A}_{xy} = dx\,dy\,\hat{k}$ $\quad\quad$ $dA_{yz} = dy\,dz\,\hat{i}$

$dV = dx\,dy\,dz$ $\quad\quad$ $dA_{zx} = dz\,dx\,\hat{j}$

②



$\lim \theta \Rightarrow 0 \text{ to } \pi$

$\lim \phi \rightarrow 0 \text{ to } 2\pi$

$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$

$P' \rightarrow$ projection of P on xy plane

$d\tau = dr \cdot r\,d\theta \cdot r\sin\theta\,d\phi$

$= r^2 \sin\theta\,dr\,d\theta\,d\phi$

→ PH202 Electrodynamics

Surprise quiz → 5
Announced quiz → 10
Assignment → 10

mid sem → 30
end sem → 45

→ Course Structure

① Electrostatic
- → solution of laplace eqn — spherical polar co-ordinate sys / ~~cylindrical co-ordination~~
  - variable separation
- → Green's function
- → Multipole expansion

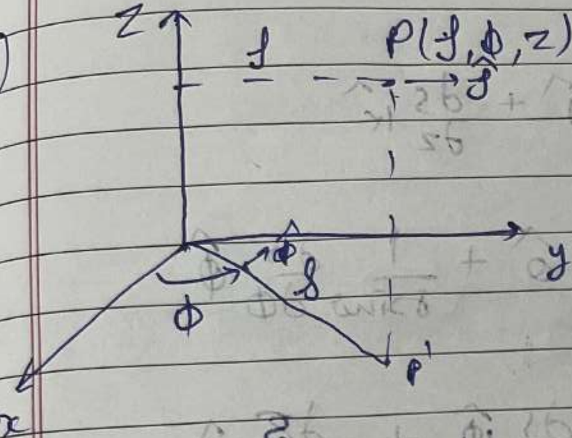
② Dielectric
- → Boundary condition
- → Clausius Mossotti equation

③ Magnetism
- → Boundary value problems
- → Behaviour of Metal and dielectric

④ Maxwell's equation
- → time varying fields
- → Plane wave → propagation — conductors / dielectric (dispersion relation)
- → Fresnel equation

$$dA_{(r,\theta)} = dr \cdot r\,d\theta\,\hat{\phi} \qquad dA_{(\phi,r)} = r\sin\theta\,d\phi\,dr\,\hat{\theta}$$

(3)



$$\vec{dl} = d\rho\,\hat{\rho} + \rho\,d\phi\,\hat{\phi} + dz\,\hat{k}$$

$$dA_{(\rho,\phi)} = d\rho \cdot \rho\,d\phi\,\hat{k}$$
$$= \rho\,d\rho\,d\phi\,\hat{k}$$

$$dA_{(k,\rho)} = dz \cdot d\rho\,\hat{\phi}$$

$$dV = d\rho \cdot \rho\,d\phi \cdot dz$$
$$= \rho\,d\rho\,d\phi\,dz$$

$\Rightarrow$

$$\vec{E} = -\vec{\nabla}\vec{V} \qquad \vec{\nabla}\times\vec{E} = 0$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$ → charge density

$$\int_V \vec{\nabla}\cdot\vec{E}\,dz = \oint \vec{E}\cdot\vec{da}$$
volume

$$E_1 A - E_2 A = \frac{Q}{\varepsilon_0}$$

vector operator $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

(i) Gradient of a scalar

$$\vec{\nabla} S = \frac{\partial S}{\partial x}\hat{i} + \frac{\partial S}{\partial y}\hat{j} + \frac{\partial S}{\partial z}\hat{k}$$

$$= \frac{\partial S}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial S}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial S}{\partial \phi}\hat{\phi}$$

$$= \frac{\partial S}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\cdot\frac{\partial S}{\partial \phi}\hat{\phi} + \frac{\partial S}{\partial z}\hat{k}$$

(ii) Divergence of a vector

$\vec{\nabla}\cdot\vec{V}$ $\qquad$ $\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$

$$\vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(V_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial \phi}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho V_\rho) + \frac{1}{\rho}\cdot\frac{\partial V_\phi}{\partial \phi} + \frac{\partial V_z}{\partial z}$$

(3) Curl of a vector.

$$\vec{\nabla}\times\vec{V} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\hat{i} + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\hat{j} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{k}$$

$$= \frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta\, V_\phi)}{\partial\theta} - \frac{\partial V_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial V_r}{\partial\phi} - \frac{\partial(rV_\phi)}{\partial r}\right]\hat{\theta}$$

$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial\theta}\right]\hat{\phi}$$

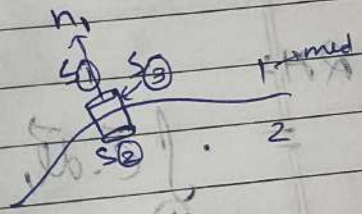
$$= \left[\frac{1}{\rho}\frac{\partial V_z}{\partial\phi} - \frac{\partial V_\phi}{\partial z}\right]\hat{\rho} + \left[\frac{\partial V_\rho}{\partial z} - \frac{\partial V_z}{\partial\rho}\right]\hat{\phi} + \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho V_\phi) - \frac{\partial V_\rho}{\partial\phi}\right]\hat{k}$$

$\Rightarrow$ (i) $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$

(ii) Lorentz force $\vec{F} = q\vec{E} + q(\vec{v}\times\vec{B})$

(iii) Continuity eqn = $\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$

$\Rightarrow \int_V \vec{\nabla}\cdot\vec{E}\, d\tau = \oint_S \vec{E}\, d\vec{a}$



$\Rightarrow \boxed{E_1^\perp A - E_2^\perp A = \frac{Q}{\varepsilon_0}}$

LHS

$\int_V \vec{\nabla}\cdot\vec{E}\, d\tau = \int \frac{\rho}{\varepsilon_0}\, d\tau = \frac{Q}{\varepsilon_0}$

because of $(-\hat{n}_1)$

$-E_2^\perp A$

tends to 0 because $l \to 0$ for interface

RHS

$\oint_S \vec{E}\cdot d\vec{a} = \int_{S_1}\vec{E}\cdot d\vec{a} + \int_{S_2}\vec{E}\cdot d\vec{a} + \int_{S_3}\vec{E}\cdot d\vec{a}$

$= \int \vec{E}_1\cdot d\vec{a} = \int \vec{E}_1\cdot\hat{n}_1\, da = E_1^\perp A$

$\vec{E} = -\vec{\nabla} V$ $\quad$ V → potential

$$\vec{\nabla}\cdot(-\vec{\nabla} V) = \frac{\rho}{\varepsilon_0}$$

$$\boxed{\nabla^2 V = \frac{-\rho}{\varepsilon_0}}$$ → poisions equation

for $\rho = 0$

$$\boxed{\nabla^2 V = 0}$$ → laplace equation

solution of the laplace's equation

$\nabla^2 V = 0$

① separation of variable

② Green's function

variable separation

① $$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\cancel{\nabla^2 V = \frac{1}{r^2}\frac{\partial (r^2)}{\partial r} + \frac{1}{r^2 \sin\theta}}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial^2 V}{\partial \phi}$$

$$V = (r, \theta, \phi) = R(r)\,\Theta(\theta)\,\Phi(\phi)$$

⇒ Divergence theorem

$$\int \vec{\nabla}\cdot\vec{V}\, d\tau = \oint_S \vec{V}\cdot\vec{da}$$

$$\int_a^b \vec{\nabla}s\cdot\vec{dl} = s(b) - s(a)$$

$$\int (\vec{\nabla}\times\vec{V})\, da = \oint_c \vec{V}\cdot\vec{dl}$$

⇒ Boundary Condition

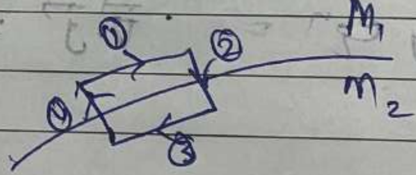
Gauss law

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}\times\vec{E} = 0$$

$$\int (\vec{\nabla}\times\vec{E})\,\vec{da} = \oint_L \vec{E}\cdot\vec{dl}$$

LHS = 0



RHS

$$= \int_1 \vec{E}\cdot\vec{dl}_1 + \underbrace{\int_2 \vec{E}\cdot\vec{dl}_2}_{\substack{0 \\ \Delta=0}} + \int_3 \vec{E}\cdot\vec{dl}_3 + \underbrace{\int_4 \vec{E}\cdot\vec{dl}_4}_{\substack{0 \\ \Delta\to 0}}$$

$$= \int_1 \vec{E}\cdot\vec{dl}_1 + \int_3 \vec{E}\cdot\vec{dl}_3$$

$$= E_1^{\parallel}\,\Delta L - E_2^{\parallel}\,\Delta L$$

$$E_1^{\parallel} - E_2^{\parallel} = 0$$

Case 1
V → dependent on r and independent of θ, ϕ

$V = V(r)$

$$\nabla V^2 = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = 0$$

$$= \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = 0$$

$$= \frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = 0 \Rightarrow r^2\frac{dV}{dr} = A$$

$$\frac{dV}{dr} = \frac{A}{r^2}$$

A & B
↳ constants

$$V(r) = -\frac{A}{r} + B$$

② Azimuthal symmetry
→ V is independent of ϕ.

$$V(r,\theta) = R\,\Theta$$

$$\nabla^2 V = \frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{1}{\Theta\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) = 0$$

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = -\frac{1}{\Theta\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) = l(l+1)$$

$l = 0, 1, 2, \ldots$

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = l(l+1)$$

$$R \rightarrow A_l r^l + \frac{B_l}{r^{l+1}}$$

→ Solution to the laplace equation

$$\nabla^2 V = 0$$

Boundary condition

(i) → V & $\vec{E}$ → finite everywhere in the space

(ii) → V → single valued

(iii) $E_1^{\perp} - E_2^{\perp} = \frac{\sigma}{\varepsilon_0}$

(iv) $E_1^{\parallel} = E_2^{\parallel}$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)$$
$$+ \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

$$V(r,\theta,\phi) = R(r)\,\Theta(\theta)\,\Phi(\phi) \equiv R\Theta\Phi$$

$$\nabla^2 V = \frac{\Theta\Phi}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{R\Phi}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right)$$
$$+ \frac{R\Theta}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2} = 0$$

multiply by $\frac{r^2}{R\Theta\Phi}$

$$\nabla^2 V = \frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right) + \frac{1}{\Phi\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2} =$$

$$R(r) = A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}$$

$$\Rightarrow \frac{1}{\Theta \sin\theta} \frac{d}{d\theta}\left(\sin\theta \frac{d\Theta}{d\theta}\right) = -\ell(\ell+1)$$

$\Rightarrow$ solution Legendre's polynomial

$P_\ell(\cos\theta)$

$\Theta(\theta) \rightarrow P_\ell(\cos\theta)$

(2) $$V(r,\theta) = \sum_{\ell=0}^{\infty}\left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right) P_\ell(\cos\theta)$$

~~(2) $V(r,\theta)$~~

$$P_\ell(x) = \frac{1}{2^\ell \ell!} \frac{d^\ell}{dx^\ell}(x^2-1)^\ell$$

$\boxed{x = \cos\theta}$

$P_0 = 1$

$P_1(x) = x \qquad P_2(x) = (3x^2-1)/2$

$P_3(x) = (5x^3 - 3x)/2$

$P_4(x) = (35x^4 - 30x^2 + 3)/8$

Orthonormality of Legendre polynomials

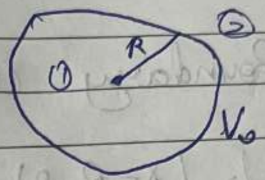
$$\int_0^{\pi} P_l(\cos\theta)\, P_n(\cos\theta) \sin\theta \, d\theta = \frac{2}{(2l+1)} \delta_{ln}$$

$$= \frac{2}{(2l+1)}, \quad l = n$$

$$= 0 \quad ; \quad l \neq n$$

Question

(1) at center.



$$\frac{B_l}{r^{l+1}} \longrightarrow \infty$$ but V must be finite. $B_l \to 0$

$$\nabla^2 V = 0$$

$$V(r,\theta) = \sum \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta) \Longrightarrow ①$$

① Ist region → inside $0 \le r \le R$

at $r = 0$ $\quad V \longrightarrow \infty$

$B_l = 0 \quad \forall l$

$$V_{in}(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta) \longrightarrow ②$$

② Outside region $\quad R \le r \le \infty$

$$V_{out}(r,\theta) = \sum \frac{B_l'}{r^{l+1}} P_l \cos\theta + A_0' P_0(\cos\theta)$$

$P_0 \cos\theta = 1$

$V_{out}(r \to \infty) = A_0'$

As A_0 is reference at ∞ so $A_0' = 0$

$$V_{in}(r,\theta) = \sum_{0}^{\infty} A_l r^l P_l(\cos\theta)$$

$$V_{out}(r,\theta) = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta)$$

⇒ Boundary condition $(r = R, V = V_0)$

$$V_{in}(r=R,\theta) = V_{out}(r=R,\theta) = V_0$$

$$\sum_{l=0}^{\infty} A_l R^l P_l(\cos\theta) = \sum_{l=0}^{\infty} \frac{B_l}{R^{l+1}} P_l(\cos\theta) = V_0$$

$$\Rightarrow A_l R^l = \frac{B_l}{R^{l+1}}$$

$$\Downarrow$$

$$B_l = A_l R^{2l+1} \quad \forall l$$

$$P_l(x) = \frac{1}{2^l l!} \frac{\partial^l (x^2-1)^l}{\partial x^l}$$

$$\int_0^{\pi} P_l(\cos\theta) P_{l'}(\cos\theta) \sin\theta d\theta = \frac{2}{2l+1} \quad l = l'$$

$$= 0 \quad l \neq l'$$