EE 220 : Signals and Systems

Department of Electronics and Electrical Engineerng

Indian Institute of Technology, Guwahati

End Semester Examination - 22 Nov 2023

Instructions : [1.] Solve all questions.
[2.] The question paper is in two parts. Part-A is subjective and can be solved between 9 AM - 12 noon. Part-B is multiple-choice and will be distributed at 11 AM and has to be submitted at 12 noon along with the Part-A answersheet.
[3.] Maximum marks: Part A = 30, Part B = 15.

PART - A

Q1: Consider a linear, time-invariant system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^{|n|}$$

Find the Fourier series of the output $y[n]$ for each of the following inputs: [8 marks]

a.

$$x[n] = \sin\left(\frac{3\pi n}{4}\right)$$

b.

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k] \; ; \; k \in \mathbb{Z}$$

c. $x[n]$ is periodic with period 6, and

$$x[n] = \begin{cases} 1, & n = 0, \pm 1 \\ 0, & n = \pm 2, \pm 3, \pm 4 \end{cases}$$

d.

$$x[n] = j^n + (-1)^n$$

Q2: In the continuous-time Fourier series synthesis equation, the summation limits are from $-\infty$ to ∞:

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_o t};$$

While in the discrete-time Fourier series synthesis, the summation limits are from 0 to $N-1$, where N is the fundamental period of $x[n]$:

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\Omega_o n};$$

Explain the reasoning for this dissimilarity. [2 marks]

Q3: The synthesis equation for an aperiodic DT signal can be written as [6 marks]

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j\Omega n} d\Omega$$

a. Consider another frequency variable Ω_1 and show that

$$\sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega_1 n} = \frac{1}{2\pi} \int_{2\pi} X(\Omega) \sum_{n=-\infty}^{\infty} e^{j(\Omega-\Omega_1)n} d\Omega$$

b. Consider $\sum_{n=-\infty}^{\infty} e^{j(\Omega-\Omega_1)n}$ as the Fourier series representation of some continuous-time periodic function. Show that

$$\sum_{n=-\infty}^{\infty} e^{j(\Omega-\Omega_1)n} = 2\pi \sum_{n=-\infty}^{\infty} \delta(\Omega - \Omega_1 + 2\pi n)$$

c. By combining the results of parts (i) and (ii), establish that

$$\sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = X(\Omega)$$

Q4: The figure below gives a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure. [2+2+1+1+1+1 = 8 marks]

a. Write the expression for $x_p(t)$ in terms of $x(t)$ and impulse functions.

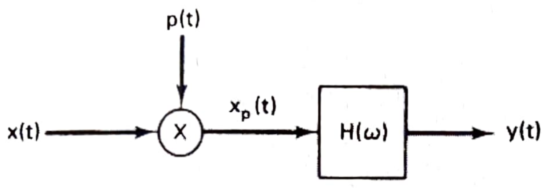
b. Write the simplified expression for $X_p(\omega)$.

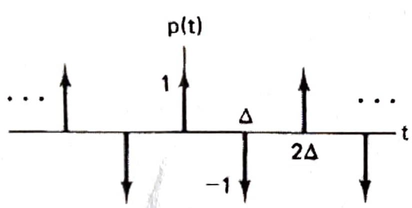
c. For $\Delta < \pi/2\omega_M$, sketch and label the Fourier transform of $x_p(t)$

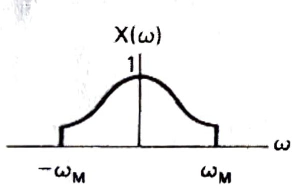
d. For $\Delta < \pi/2\omega_M$, sketch and label the Fourier transform of $y(t)$.

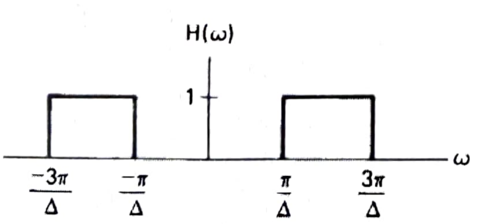
e. For $\Delta < \pi/2\omega_M$, sketch and label the Fourier transform of the system that will recover $x(t)$ from $x_p(t)$.

f. What is the maximum value of Δ in relation to ω_M for which $x(t)$ can be recovered from $x_p(t)$.

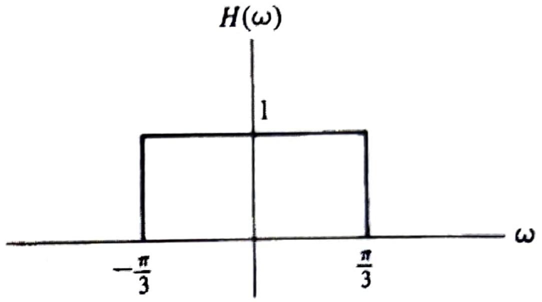








Q5: Consider a lowpass filter with real frequency response H(w) as shown in Figure [2+1+1+1+1 =6 marks]



a. Show that the filter impulse response $h(t)$ is real-valued.

b. Determine whether the filter impulse response $h(t)$ is even or odd.

c. Determine whether the filter impulse response $h(t)$ is causal or non-causal.

d. Consider the filter input

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - 9n)$$

Sketch and label the Fourier transform of the filter output $y(t)$.

e. Write the expression for the filter output $y(t)$ for the input considered in part (d).