CYK/2023/PH201 Mathematical Physics

QUIZ 3

Total Marks: 10 Marks, Duration: 1 Hour
Date: 30 Oct 2023, Monday

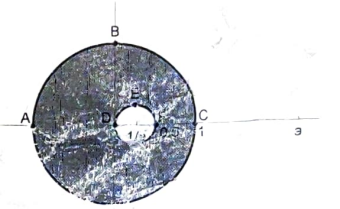
B

1. **[5 Marks]** A circular metallic (thermally conducting) disc of radius a is subjected to the boundary conditions

$$T(a,\phi) = \begin{cases} \sin\phi, & 0 < \phi < \pi \\ 0, & \text{otherwise.} \end{cases}$$

Find the steady state temperature $T(\rho,\phi)$ in the disc. Sketch isotherms.

2. **[5 Marks]** Find the potential V in the gray region shown in the figure by completing the following steps. The outer circle has a unit radius and is kept at potential $V = 0$. Inner circle has a radius of $1/4$ and has a center at $(\frac{1}{4}, 0)$ and is kept at $V = 1$.



(a) Find a (> 1) such that the points $(a, 0)$ and $(1/a, 0)$ are symmetric wrt the inner circle.

(b) Consider the conformal transformation

$$w = \frac{z - a}{az - 1}.$$

Find the images of points A, B, C, D, E and F. Find the image of the gray region.

(c) Obtain the expression for the potential in w-*plane* with given boundary conditions.

(d) Obtain the expression for the potential in z-plane.

$$\int \sin\varphi \cos m\varphi \, d\varphi$$