Indian Institute of Technology Guwahati
Department of Physics

End-semester Examination, November 23rd 2023

Subject: Heat and Thermodynamics (PH 207)

Full Marks: 50

Time: 3 hours

1. Consider two blocks of Copper, each having heat capacity $C_v = 200$ J K^{-1}. Initially one of the blocks has a uniform temperature 300 K while the other has a uniform temperature of 310 K. The blocks are placed in thermal contact with one another and are surrounded with perfect thermal insulation.

   (a) What are the final equilibrium temperatures of the blocks?

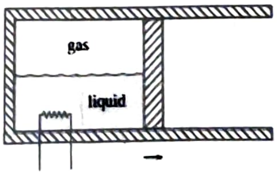
   (b) Calculate changes in entropy of each block in this process.

   (c) Is the total change in entropy of the system consistent with second law of thermodynamics? Explain. [1+2+2]

2. The Figure shows a hypothetical liquid in equilibrium with its vapour. The liquid and the gas are confined in a cylinder by a piston. An electrical resistor of resistance 50 Ω is immersed in the liquid. The liquid, the gas and the resistor together constitute the thermodynamic system. The initial thermodynamic state of the system is described by pressure $P_1 = 2.5 \times 10^5$ Pa, volume $V_1 = 0.22$ m^3 and temperature T_1=300 K. A constant current 0.5 A is passed through the resistor for 1600 s. The piston moves slowly to the right against a constant external pressure equal to the vapour pressure of the liquid, P_1. Some of the liquid now vaporises. Assume the process is adiabatic where T and P remain constant so that thermodynamic parameters of the final state are $T_2 = T_1$, $P_2 = P_1$ and V_2=0.24 m^3.

   (a) Calculate the change in Enthalpy in the process.

   (b) Is the process reversible? Explain. [3+2]



3. When an ordinary rubber band is hung from a clamp and stretched with constant downward tension τ, gentle heating is observed to cause the rubber band to contract in length. To keep the length L of the rubber band constant during heating, τ must be increased. From this information,

   (a) show that $\left(\frac{\partial T}{\partial L}\right)_S = \left(\frac{\partial \tau}{\partial S}\right)_L$.

   (b) Using result of (a), predict whether adiabatic stretching of the rubber band will cause a heating or cooling effect. [2+3]

4. Consider a simple magnetic system at temperature T in a magnetic field H. If the magnetic field is changed to $H + \Delta H$ isothermally, the change in the entropy is found to be $\Delta S = -\frac{CH\Delta H}{T^2}$, C is a constant, characteristic of the system.

   (a) Find out the relation between magnetization M and temperature T.

   (b) Plot M versus T for small H, that is $H \to 0$. [3+2]

5. (a) Prove that $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$ for a system whose thermodynamic state is described by Pressure P, Volume V, Temperature T, Entropy S.

(b) Using (a), obtain Clausius Clapeyron equation $\frac{dP}{dT} = \frac{L}{T(v_V - v_L)}$ for a vessel containing a liquid in equilibrium with its vapor; L, P, T, v_V, v_L are the Latent heat of vaporisation per unit mass, the saturated vapour pressure, the equilibrium temperature, the volume per unit mass of the vapour phase and volume per unit mass of the liquid phase, respectively. [2+3]

6. The equation for inversion curve of ^{4}He is $P = -21 + 5.44T - 0.132T^2$, T and P are in K and in atm, respectively.

(a) What is the maximum temperature above which the Joule-Thomson effect will produce heating instead of cooling?

(b)Find the pressure and the corresponding temperature above which Joule-Thomson effect will produce heating instead of cooling. [2+3]

7. How many grams of CO_2 gas is dissolved in a 1 Litre bottle of soda if the manufacturer used a pressure of 2.4 atm in the bottling process at 25^0 C? Given that the Henry-Dalton constant is 29.76 atm/(mol - Litre^{-1}), molar masses of C and O are 12 gm and 16 gm, respectively. [5]

8. A man slowly exhaled an ideal gas molecule in a room filled with same gas. The temperature and the pressure in the room are 300 K and 10^5 Pa, respectively. Calculate the time (in seconds) it will take for the molecule to diffuse to some point at a distance of 1 meter? What is the mean speed of the molecule? (Given that the collision cross section is 10^{-20} m^2, mean collision time is 1.4×10^{-8} second and Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J K^{-1}.) [4+1]

9. The Maxwell velocity distribution for a monatomic ideal gas with molecular mass m is $f(\vec{v}) = \frac{1}{(\sqrt{\pi} v_{th})^3} e^{-v^2/v_{th}^2}$, where v is the molecular speed.

(a) Express pressure P in the system in terms of m, particle density n and $\langle v^2 \rangle$.

(b) Express P and temperature T in the system in terms of v_{th}, the thermal velocity. [1+4]

Use the following:

$$\int_0^\infty x^n e^{-ax^2} dx = \frac{\Gamma\left(\frac{n+1}{2}\right)}{2a^{(n+1)/2}}$$

$$n\Gamma(n) = \Gamma(n+1); \quad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}, \quad \Gamma(1) = 1$$

10. Consider a system of charged particles (of charge e) confined to a volume V. The particles are in thermal equilibrium at temperature T in presence of an electric field of magnitude E along z-direction. If $n(z)$, the density of particles is given as $n(z) = n(0)\, exp\left(\frac{eEz}{k_BT}\right)$ then

(a) if the transport of particles along z is characterised by a diffusion constant D, then find the flux J_D arising from the particle density gradient.

(b) If the particles are also characterised by a mobility μ that connects their drift speed $\bar{v}$ along z (average speed along z) with the electric field E ($\bar{v} \propto E$) then find the flux J_μ associated with this mobility.

(c) Show that, at global equilibrium, $\mu = \frac{eD}{k_BT}$. [2+2+1]

$1 = -21 + 5.44 \frac{dT}{dP} - 0.264 \frac{dT}{dP}$

$\frac{11}{2a^2}$