PH 203 – Classical Mechanics, Dr. M. K. Nandy, Endsem, 21 Nov. 2023

(Show labelled diagrams and details of calculations)

1. A comet of mass $m$ is moving due to the gravitational attraction of the sun of mass $M$. Assume both $m$ and $M$ to be point particles, $m$ moves in the XY plane with $M$ sitting sationary at the origin. Let $(x, y)$ be the instantaneous co-ordinate of $m$.

   (a) Write the expression for the kinetic energy $T$ and potential energy $V$ in the Cartesian $(x, y)$ system.

   (b) Transform the above Cartesian expressions for $T$ and $V$ into the plain polar $(r, \theta)$ system. .

   (c) Find the equations of motion of the comet employing Lagrange's equation.

   (d) (i) What quantities are conserved?

   (ii) What property of the Lagrangian $L$ make them conserved?

   (iii) Express the conserved quantities in the plain polar system.

2. (a) Write the Euler's equations (for $\dot{\omega}_1$, $\dot{\omega}_2$, $\dot{\omega}_3$) for a rigid body.

   (b) Consider the earth as a rigid body (oblate spheroid, with $x_3$ as symmetry axis). Solve the Euler's equations for the earth. Obtain the expression for precession frequency.

   (c) Draw the body cone and space cone diagrams and identify them in the diagram. Indicate the direction of rotation by arrows on the cones. Describe (in 2-3 lines) the motion(s) of the cones.

3. For a one-dimensional UNDAMPED simple harmonic oscillator

   (a) Write the Hamiltonian.

   (b) Write the Hamilton-Jacobi equation.

   (c) Implement separation-of-variables and solve the Hamilton-Jacobi equation.

4. For a single particle with instantaneous position $(x, y, z)$, find the following Possion brackets

   (a) $[x, p_x^2]$, (b) $[x, L_y]$, (c) $[L_x, L_y]$

   where $p_x$, $p_y$, $p_z$ and $L_x$, $L_y$, $L_z$ are Cartesian components of the linear momentum and angular momentum respectively.

5. Consider a one-dimensional DAMPED simple harmonic oscillator (of natural frequency $\omega$ and damping constant $\gamma$).

   (a) Write the equation-of-motion.

   (b) Find the solution of the equation-of-motion.

   (c) Write the conditions for the three cases (underdamped, overdamped, critically damped).

   (d) Find the solution for the critically damped case.