

Note: Please submit the solutions of at least two of the problems, on April 12, 2024, during the class. The solutions can be submitted in a group of maximum of four students. Name and Roll no. of all the group members should be written clearly on the top. If solutions are found to be reasonably correct then ten (five marks each) marks will be awarded and in case of non submission/wrong solution zero marks.

1. A medium is characterized by μ_0 , ϵ , and $\sigma=0$. The expression for the electric field for a plane monochromatic wave propagating in this medium is given by $\vec{E} = 20 \sin(10^8 t - kz) \hat{j}$ V/m. Find the value of k and \mathbf{H} (using Maxwell's equation).
2. The electric field associated with an uniform plane wave propagating in a certain medium is given by $\vec{E} = 2e^{-\alpha z} \sin(10^8 t - kz) \hat{j}$ V/m. The medium is characterized by dielectric constant $K=1$ (real part), $\mu/\mu_0=20$ and $\sigma=3\text{mhos/m}$. Find the absorption coefficient, wavelength of electromagnetic wave and expression for \mathbf{H} .
3. Obtain the phase and group velocity of above wave (Q no. 2).
4. In the limiting case of $\frac{\sigma}{\epsilon\omega} \gg 1$, obtain the relation between the conductivity and the real part of the refractive index.
5. The permittivity of water in the visible frequency range (e.g. $\lambda \sim 500\text{nm}$) is $\sim 1.8\epsilon_0$ and that of at AC line frequency ($\sim 50\text{ Hz}$) is $\sim 80\epsilon_0$. Its conductivity (assuming independent of frequency) is $\sim 4 \times 10^{-6}\text{ S/m}$. Consider water as a nonmagnetic medium. Calculate the skin depth and speed of propagation at these frequencies. Hence conclude whether the water is conductor and or insulator at these frequencies.
6. An enclosure is to be made out of aluminium to protect it from the entering of radio frequency signal (rf) up to the frequency of 1 MHz. The shielding should be such that the ratio of the incident electric field from outside to that of entering inside the enclosure is 10^6 . The conductivity of Aluminium is $3.7 \times 10^7\text{ S/m}$, taking permittivity and permeability that of the free space find the minimum thickness of the aluminium sheet required to make the enclosure (neglect the reflection). If the aluminium sheet is to be replaced with the iron sheet ($\mu=100\mu_0$, $\sigma=10^7\text{ S/m}$) what is the thickness required and hence conclude which material is better for such applications.
7. A communication is to be made from the surface of the ocean to the submarine submerged in side the sea at a frequency of 1 MHz (radio wave range). For sea water; $\epsilon=8\epsilon_0$, $\mu=\mu_0$ and $\sigma=4\text{S/m}$. The detector in the submarine can detect a minimum magnetic field intensity (H) of $1\text{ }\mu\text{A/m}$. Calculate the maximum depth (also termed as the range) up to which communication can be made with the submarine if the electromagnetic wave having magnetic field intensity of $10,000\text{A/m}$ is launched from the surface of the sea. What is going to be the depth for 100Hz signal assuming all the other parameters to be same. What is the wavelengths in the sea water at these two frequencies?
8. The permittivity of water in the visible frequency range (e.g. $\lambda \sim 500\text{nm}$) is $\sim 1.8\epsilon_0$ and that of at AC line frequency ($\sim 50\text{ Hz}$) is $\sim 80\epsilon_0$. Its conductivity (assuming independent of frequency) is $\sim 4 \times 10^{-6}\text{ S/m}$. Consider water as a nonmagnetic medium. Calculate the skin depth and speed of propagation at these frequencies. Hence conclude whether the water is conductor and or insulator at these frequencies.
9. Obtain the propagation equation for \mathbf{J} , current density, (similar to that of \mathbf{E} or \mathbf{B}) and then work out the solution for \mathbf{J} . Compare the expression for \mathbf{J} with \mathbf{E} (or \mathbf{B}).