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Colab Link-https://colab.research.google.com/drive/1SNnJ4rBEKSXJ0XqNMQBK7Uy

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Question 1: Hypothesis Test for Battery Lifetimes

Introduction:
A battery manufacturer asserts that the average lifetime of its batteries is 500 hours. This claim is tested using a hypothesis test, assuming a known standard deviation. A sample of 30 battery lifetimes is collected by a quality control team to verify this claim.

Data:

- Sample Size (n): 30
- Population Standard Deviation (σ): 100 hours
- Sample Lifetimes: `495, 520, 510, ..., 495`

Methodology:

- H_0: $\mu = 500$
- H_1: $\mu \neq 500$
- Test Type: Z-test
- Significance Level: $\alpha = 0.05$
- Formula: $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

Results:

- Sample Mean: 502.67
- Z-statistic: 0.146
- P-value: 0.883

Discussion:
The sample mean is very close to the claimed mean. The high p-value indicates a high probability of obtaining this result under H_0. Hence, there is no evidence against the claim.

Conclusion:
Fail to reject H_0. The data supports the manufacturer's claim.

Question 2: Water Usage Hypothesis Testing

Introduction:
A public health official claims the average daily household water usage is 350 gallons. A random sample of 20 homes was taken.

Data:
Sample Size: $n = 20$

(a) Known Variance (Z-Test)

- $\sigma^2 = 144 \Rightarrow \sigma = 12$
- Sample Mean: 353.80
- Z-statistic: 1.4162
- P-value: 0.1567

Discussion:
The sample mean is close to the claimed value. The p-value ¿ 0.05 suggests no statistically significant difference.

Conclusion:
Fail to reject H_0.

(b) Unknown Variance (T-Test)

- Sample Standard Deviation: 21.85
- T-statistic: 0.7778
- P-value: 0.4462

Discussion:
Even with unknown variance, the result is consistent with the Z-test. No strong evidence against the official's claim.

Conclusion:
Fail to reject H_0.

Question 3: Paired t-test for Diet Plan Effectiveness

Introduction:
To test the effectiveness of a new diet, weight data before and after one month was recorded for 10 individuals.

Data:

Participant	Before (kg)	After (kg)
1	85.2	82.5
2	78.5	75.8
3	92.3	90.1
4	80.0	77.2
5	88.7	85.4
6	76.4	74.5
7	90.5	87.6
8	84.1	81.3
9	79.0	76.8
10	86.2	83.0

Results:

- Mean Before: 84.09 kg
- Mean After: 81.42 kg
- Mean Difference: 2.67 kg
- T-statistic: 18.8745
- P-value: 0.0000

Discussion:
The large t-statistic and extremely small p-value provide strong evidence that the diet is effective.

Conclusion:
Reject H_0. The diet significantly affects body weight.

Question 4: Chi-Square Test for Variance in IV Fluids

Introduction:
The manufacturer claims the variance in IV fluid bottle volume is no more than 4 mL^2.

Data:
Sample Size: $n = 15$ Volumes: 502, 498, 505, ..., 504

Original Dataset

- Sample Variance: 11.67
- Chi-Square Statistic: 40.8333
- P-value: 0.0002

Discussion:
The p-value is very small, indicating the sample variance significantly exceeds the claimed maximum.

After Removing Outliers (<495 or >505)

- Sample Variance: 10.22
- Chi-Square Statistic: 33.2143
- P-value: 0.0016

Discussion:
Even after removing outliers, the variance remains significantly high.

Final Conclusion:
Reject H_0. The machine may violate the variance specification even after filtering.