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Digital Image Processing

Image : An image may be defined as a 2D function, $f(x,y)$ where x and y are spatial coordinates and the amplitude of f at any pair of coordinates (x,y) is called the intensity at that point.

Digital Image : When x, y and the intensity values of f are all finite, discrete quantities, we call the image a digital image.

3-D Scene $\xrightarrow{\text{acquisition by physical camera}}$ $g(x,y)$ {analog signal} $\xrightarrow{\text{discretization}}$ {sampling + quantization} $g(i,j)$ $\xrightarrow{\text{Quantization}}$ $f(i,j)$ (Digital Image)

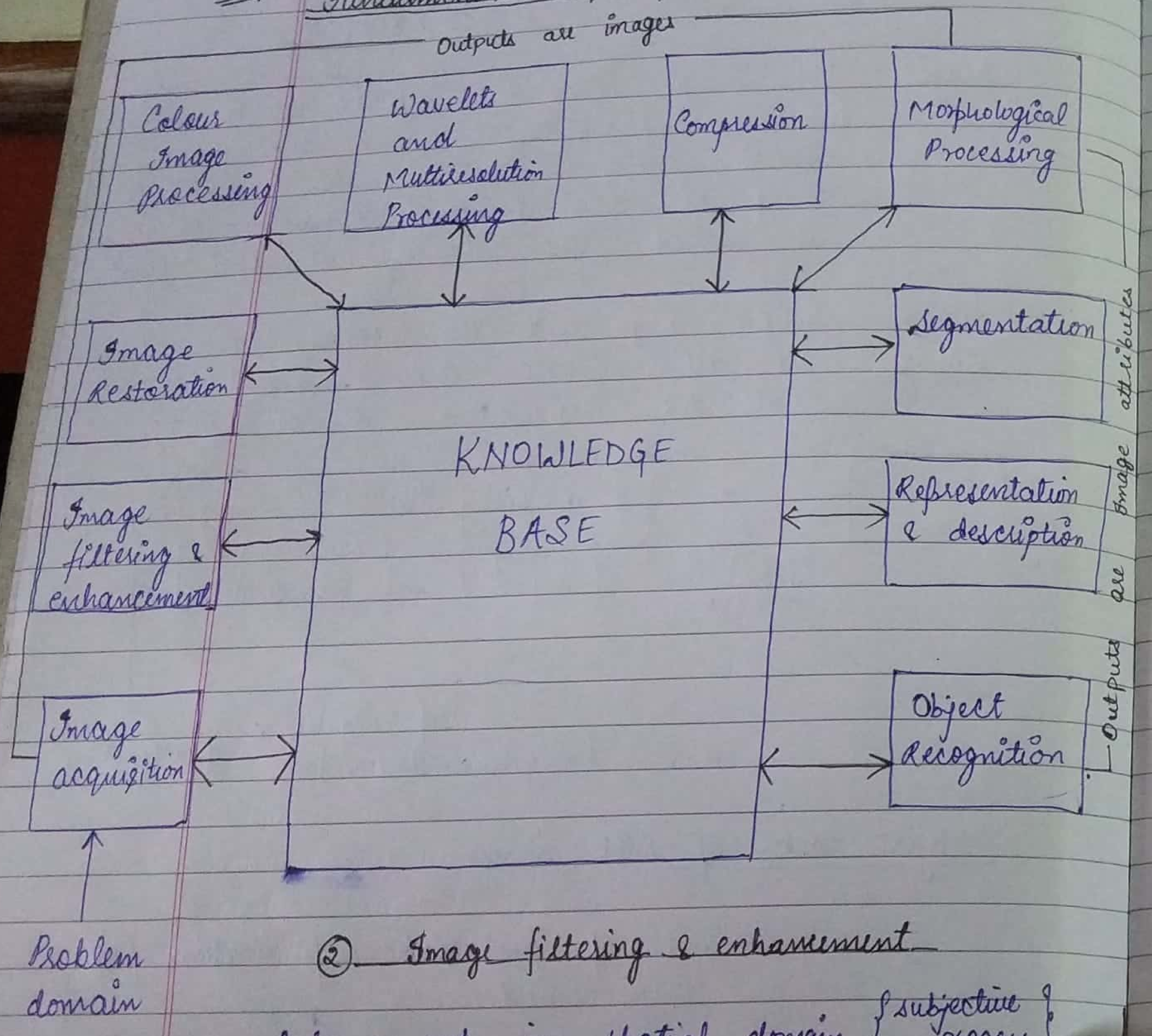
Major task of DIP

1. Improvement of pictorial information for human improvement.

2. Processing of image data for storage, transmission & representation for autonomous machine perception.

Analog image $\rightarrow$ Sampling $\rightarrow$ Quantization $\rightarrow$ Digital Image

Imp. **Fundamental steps of DIP**



② Image filtering & enhancement

- Enhancement in spatial domain {subjective process}
- Enhancement in frequency domain

→ Point

- done so that result is more suitable than original for specific application.

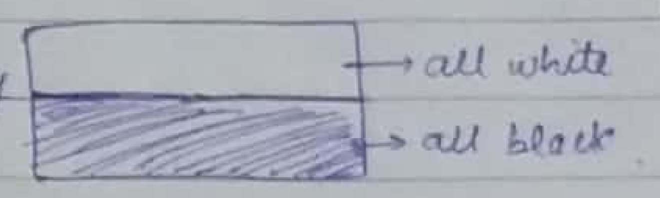
- Point processing → enhancing single pixel
- Neighbourhood processing → enhancing single pixel along with neighbourhood pixel as well as.

$\boxed{S = 255 - r}$ {getting image negatives}

↓ Output image ↓ Input image

✲ FILTERING

high frequency content



Background – low frequency content
Object – high frequency content

① Low pass filter : remove ~~low~~ high frequency content

② High pass filter : remove low frequency content.

③ Image Restoration

- ★ It is an objective process.
- ★ Generally happens when capturing moving objects.

We try to remove noise and convert blurred images into sharp images.

④ Colour ... Processing

- done as color is ... descriptor
- humans underst...

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⑥ Wavelet & multiresolution processing

Improving & Resolving images in multiple directions.

⑦ Morphological processing

Dealing with shape, size & ~~image~~ structure of image, then we call its morphological processing.

⇒ eg. Finger recognition, Medical diagnosis, Pattern recognition.

⑧ Segmentation

To describe features of large image, we segment it & then describe features.

- Based on discontinuities
  - ↓ identification of line, point and edges which are generally discontinous points where intensity changes suddenly.
- Based on similarities
  - ↓ Region based
  - → Threshold based

⑨ Representation & description

Represent image & describe it using its small features based on segmented image.

⑩ Object Recognition

- Assigning label to objects inside images.
- Goal is to recognise image with respect to Color, Texture & Shape.

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Image formation Model

✡ For image formation we need – Illumination
– Reflectance.

$0 < i(x,y) < \infty$ and $0 < r(x,y) < 1$

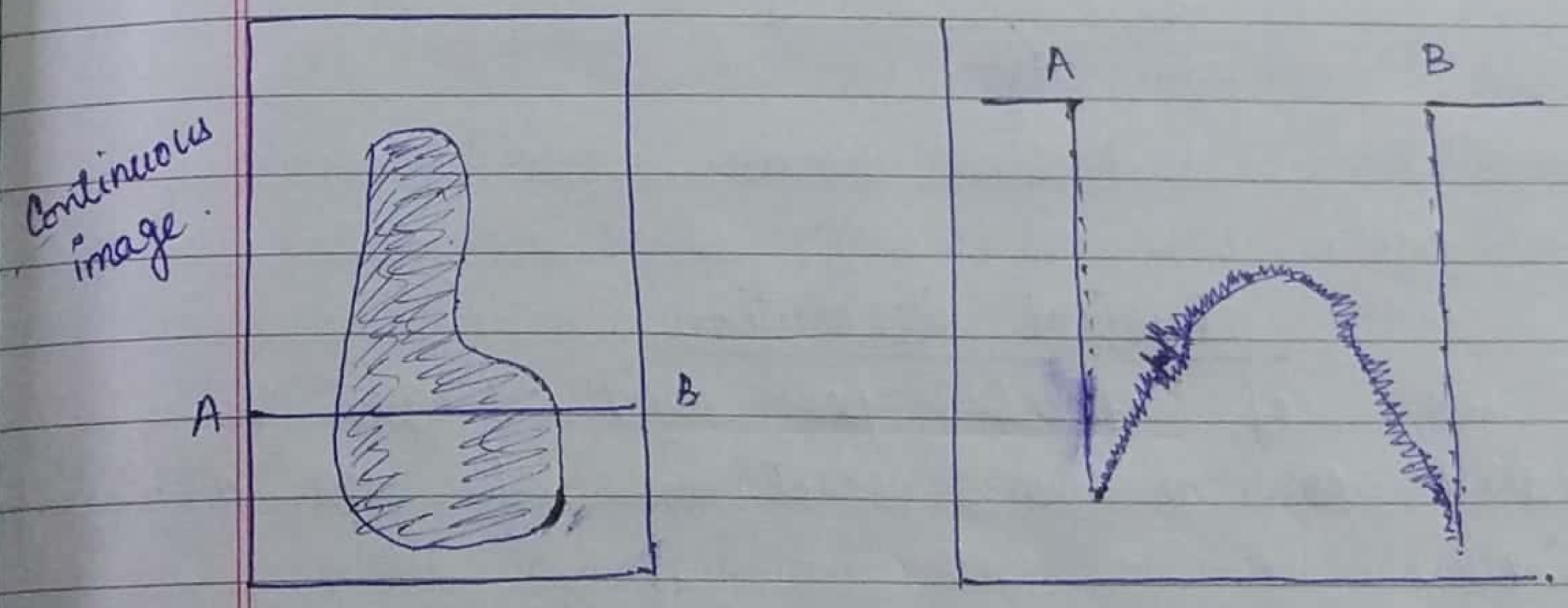
↑ total absorption ↖ total reflectance

V. Imp.

Sampling and Quantization

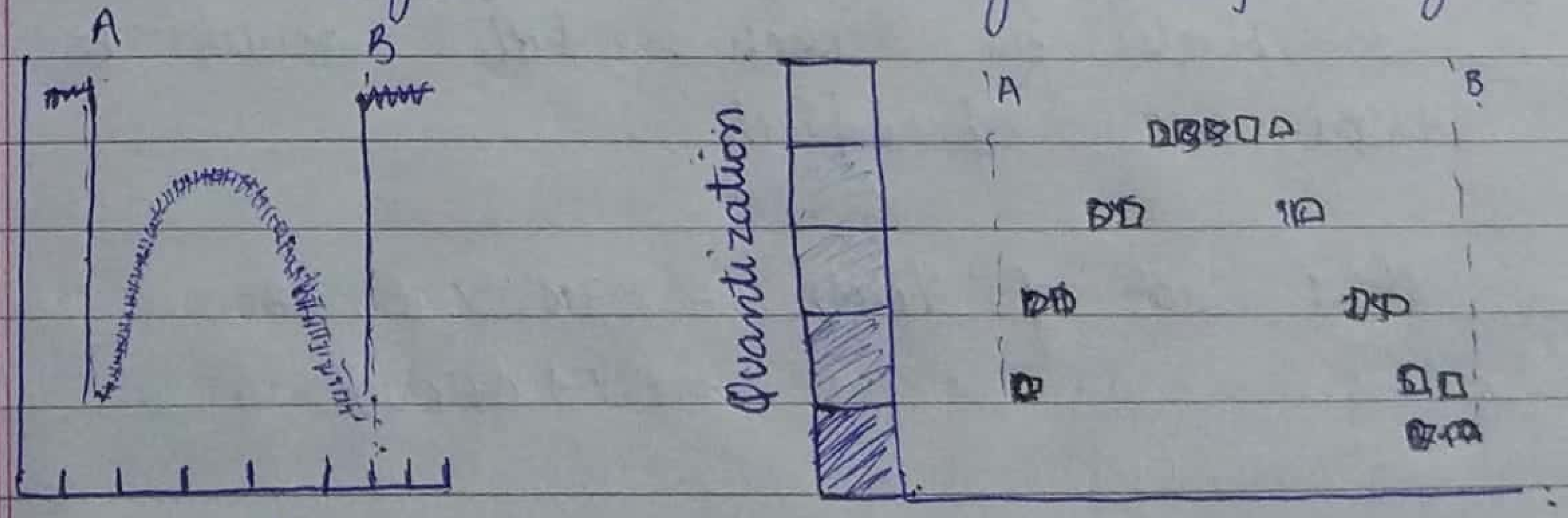
Sampling is related to coordinate values (Nyquist frequency)

Quantization is related to intensity values.



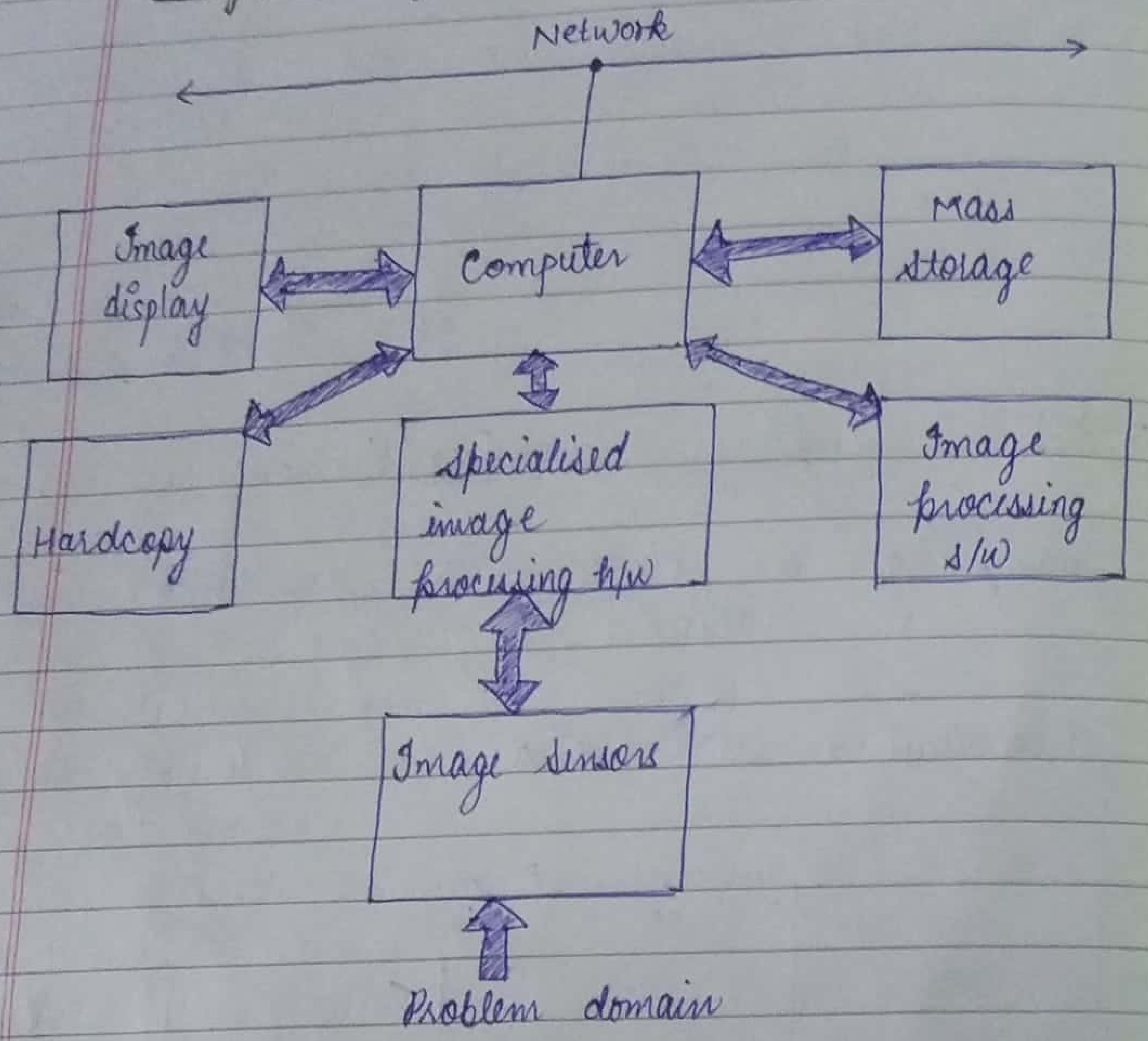
A scan line from A to B in continuous image

Fig. Scan line diagram of Image.



Sampling & Quantization Digitization

Components of Image Processing Systems



Physical Resolution

\## The no. of pixels per unit length is referred to as a resolution of the displaying device.

Q- In 3x2 inch image, and resolution of 300 pixels per inch, what would be the total no. of pixels.

Total no of pixels $= 3 \times 2 \times 300 \times 300$
$= 540000$

Image size - 1024×1024 × No of bits [8 for greyscale, 24 for colored]
width × height

Aspect Ratio - $\frac{\text{Ratio of width}}{\text{Ratio of height}} = \frac{w}{h}$

Q- We we want to resize a 1024 X 768 image to 1 i.e 600 pixel wide with the same aspect ratio as the original image, what should be the height of resized image.

$$\frac{1024}{768} = \frac{600}{h}$$

$$h = \frac{600 \times 768}{1024} = 450$$

Q- A common measure of transmission for digital data is the baud rate, defined as the no. of bits transmitted per second. Transmission is accomplished in packets, consisting of a start bit, a byte of information and a stop bit.

(a) How many minutes would it take to transmit a 1024 x 1024 image with 256 greylevels, if we use a 56k baud modem?

(b) What would be the time required if we use a 750k baud transmission line?

(a) No. of bits $= 1024 \times 1024 \times 10$
56k bits → 1 sec.
$1024 \times 1024 \times 10$ bits → $\frac{1024 \times 1024 \times 10}{56 \times 1024} = 182.85$ sec.
$= 3.05$ minutes.

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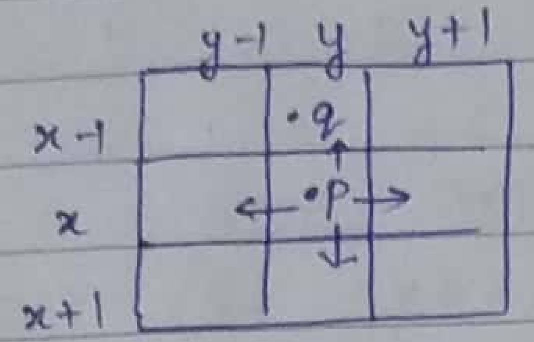
(b) Time = $\frac{1024 \times 1024 \times 10}{750 \times 1024}$ = 13.65 sec.

= 0.23 minutes.

V. Imp: **Basic Relationship b/w Pixels**

* p has 4 neighbourhood pixel.



* $P(x,y)$ = coordinate of p

$N_4(P) = (x, y-1), (x-1, y)$
$(x+1, y), (x, y+1)$ = 4 neighbourhood pixel.

* $N_D(p)$ = diagonal pixel of p

$N_8(p)$ = 8 neighbourhood pixel = $N_4(p) \cup N_D(p)$

Connectivity of the pixels / Adjacency of pixels

* Let V be the set of intensity values used to define connectivity in a binary image.

* If we are referring to the connectivity of pixel with value $V=\{1\}$, there are basically 3 types of connectivity -

(i) Four connectivity - 2 pixels p & q with values from V are four adjacent if q is in the set $N_4(p)$.

(ii) Eight connectivity - 2 pixels p & q with values from V are eight adjacent if q is in the set $N_8(p)$.

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(iii) M connectivity - Two pixels p & q with values from V are M adjacent if -

① q is in $N_4(p)$, or
② q is in $N_D(p)$, and $N_4(p) \cap N_4(q) = \phi$

Example

		p	q
	0	1	1
	0	1 p1	0
	0	0	1 q1

Given $V=\{1\}$ (Can be 0 or 1 for binary)

Find connectivity for $V=\{1\}$ i.e. pixel having values as 1.

ⓐ p and q are 4 connected, & 8 connected, & M connected also.

ⓑ p1 and q are 8 connected. (not M connected)

ⓒ p1 and q1 are 8 connected. (and M connected)

ⓓ Checking M connectivity of p1 & q.

- p1 & q = $N_4(p)$ ≠ not true
- q is in $N_D(p)$ = true and $N_4(p) \cap N_4(q) = p \neq \phi$ } condition false.

- Thus, p1 & q are not M connected

{ The intersecting pixel must be of same intensity then only we will consider the intersection }

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Path between the pixels

A path from pixel p with coordinates (x, y) to pixel q with coordinate (s, t) is a sequence of distinct pixels with coordinate $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$, where

$(x_0, y_0) = (x, y)$ and
$(x_n, y_n) = (s, t)$

Note: $\{(x_i, y_i)$ & (x_{i+1}, y_{i+1}) should be adjacent$\}$

Here n = length of the path
If $(x_0, y_0) = (x_n, y_n)$ = path is known as closed path.

Example : $V = \{1, 2\}$ – work only on these intensities.

	4	2	3	2 q.
	3	3	1	3
	2	3	2	2
p	2	1	2	3

(a) shortest 4 path
(b) shortest 8 path
(c) shortest m path)

If it is 4 adjacent – we have 4 path
If it is 8 adjacent – we have 8 path
If it is m adjacent – we have m path.

(a) 4 path

	4	2	3	2	q
	3	3	← x 1 x →	3	
	2	3	2 →	2	
p	2 →	1 →	2 →	3	

Thus shortest 4 path does not exist.

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(b) 8 path

	4	2	3	2	q
	3	3	1	3	
	2	x 3	2	2	
p	2 →	1	2	3	

Thus shortest path exists having path length = 4.

(c)

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2 →	1 →	2	3	

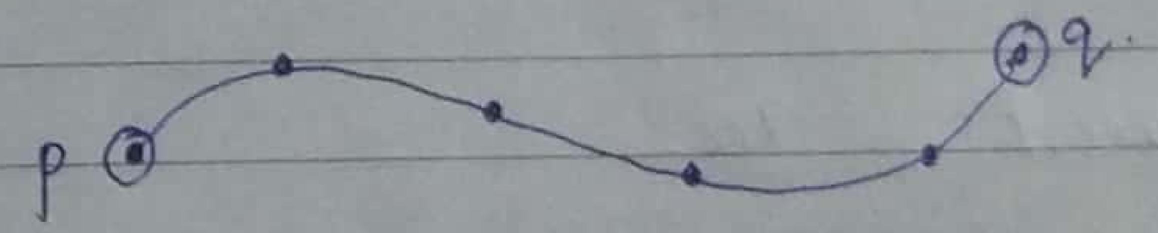
Thus shortest m connected path exists

Path length = 5.

* Let s represent a subset of pixels in an image.

* Two pixels p & q are said to be connected in s if there is a path b/w them consisting entirely of pixels in s.

* For any pixel p in s, the set of pixels that are connected to it is called a connected component of s.



* If it only has 1 connected component then set s is called a connected set.

Adjacent Regions

* Let R be a subset of images in an image. We call R a region of image if R is a connected set.

* Two regions are said to be adjacent regions if their union forms a connected set.

* The region that are not adjacent are said to be disjoint.

Example

Consider 2 images subset S_1 & S_2 shown in following figure:
For $V = \{1\}$ examine whether these 2 subsets are
- 4 adjacent
- 8 adjacent
- m adjacent.

	S_1				S_2				
0	0	0	0	0	0	0	1	1	0
1	0	0	1	0	0	1	0	0	1
1	0	0	1	0	1	1	0	0	0
0	0	1	1	1	0	0	0	0	0
0	0	1	1	1	0	0	1	1	1

4 adjacent = No

8 adjacent = Yes

m adjacent = Yes.

Boundary - The boundary of a region is the set of points that are adjacent to points in the complement of R.

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Talking about 4 adjacency -

1 & 0 not 4 adjacent

∴ 1 not member of border if talking about 4 adjacency in R.

30/1/19 (UNIT-2)

Image enhancement in spatial domain

- Point processing
- Neighbourhood processing

$g(x,y) = T\, f(x,y)$

T ↓ Transformation operator = 1×1 Operator

Point processing -

$S = T(r) \rightarrow$ I/P gray level

- Linear processing
- Non-linear processing

Linear processing

- Identity transformation
- Image negative
- Contrast stretching
- Thresholding
- Gray level slicing
- Bit plane slicing

① Identity Transformation

s = 255	r = 0
r = 0	s = 255

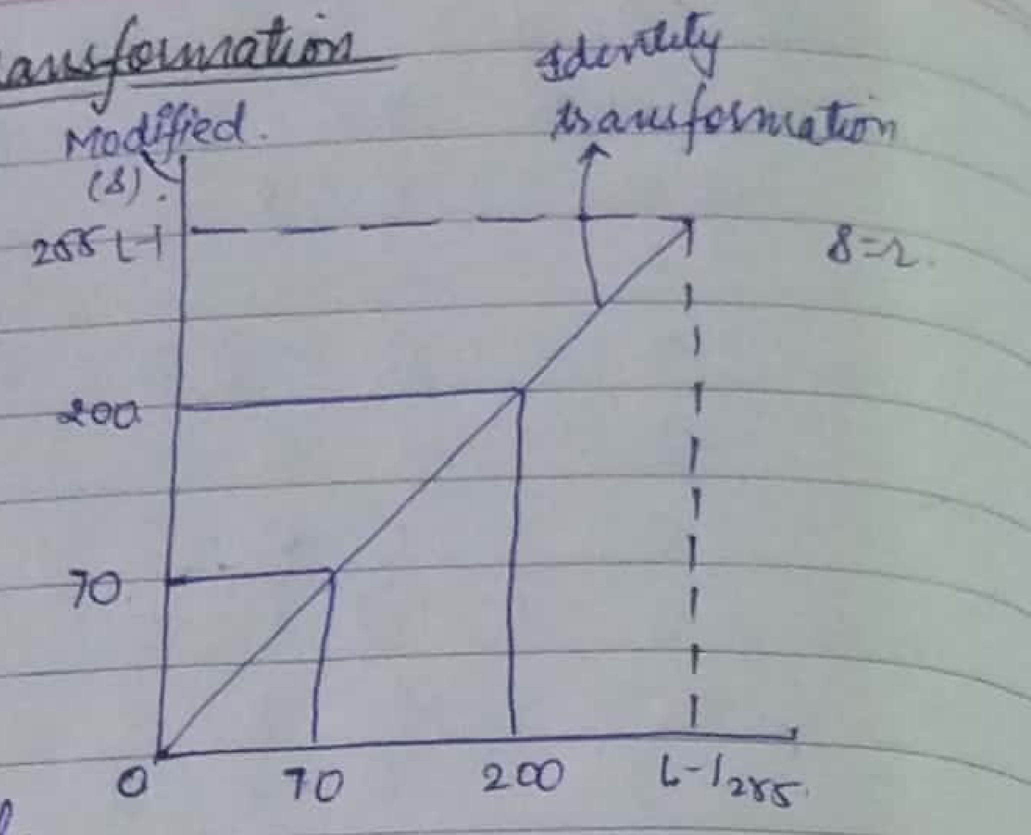


Image Negative :-

eg: $S = (256-1) - r$

$\boxed{S = 255 - r}$

$$\boxed{S = (L-1) - r}$$

S = output gray level

r = input gray level

L = max. no. of gray level

eg: Obtain digital -ve of the following 8 bit per pixel image (8 BPP image)

8BPP ⇒ $L = 2^8 = 256$

$S = L - 1 - r$

$S = 255 - 121$

$= 134$

$S = 255 - 139$

$= 116$

121	205	217	151
139	127	157	125
252	117	236	142
201	106	119	250

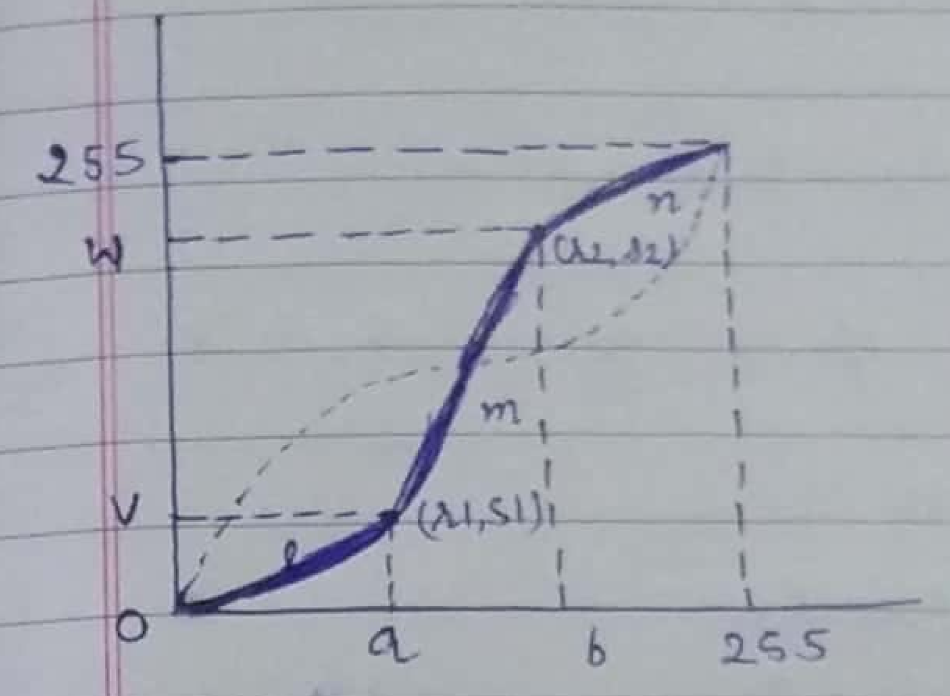
↓

134	50	38	104
116	128	98	130
3	138	125	113
54	149	136	5

→ Output gray level (S)

② Contrast stretching

It is used to increase the contrast of the image by making dark portion darker and bright portion lighter.



$$S = \begin{cases} lr & 0 \le r \le a \\ m(r-a)+v & a < r < b \\ n(r-b)+w & b < r < L-1 \end{cases}$$

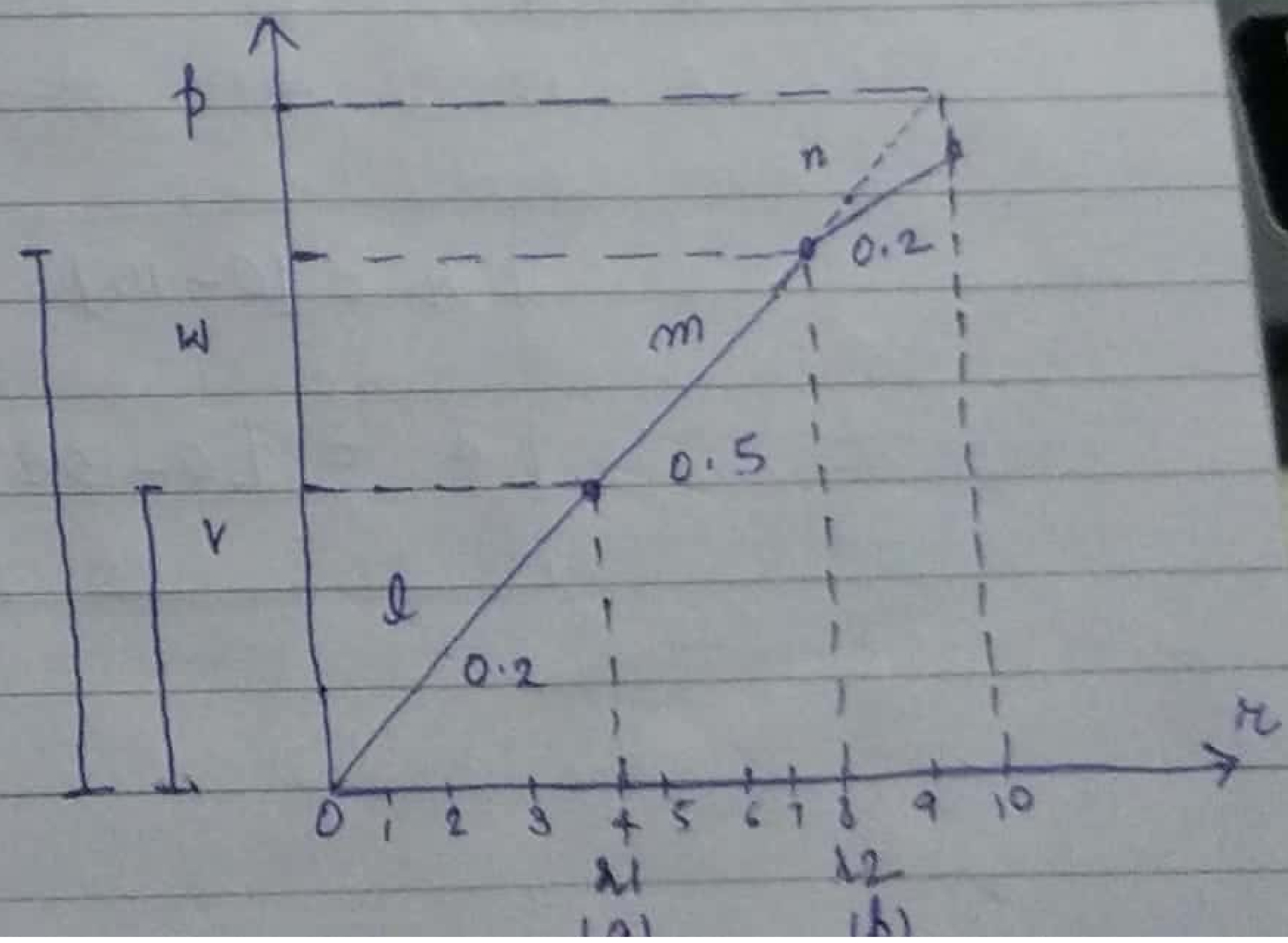
where l, m, n are the slopes, l and m are less than 1 and n is greater than 1.

Imp Q. What would happen to the dynamic range of an image if all the slopes in the contrast stretch algorithm (l, m, n) are less than 1? [Let the initial dynamic range of the original image be [0 - 10] $r_1 = 4$ and $r_2 = 8$, $l = 0.2$, $m = 0.5$, $n = 0.2$.]

① r [0-4]

$s = lr$

r = 0	s = 0
r = 1	s = 0.2
r = 2	s = 0.4
r = 3	s = 0.6
r = 4	s = 0.8



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② $r = [5-8]$

$s = m(r - r_1) + V$

$s = 0.5(r - 4) + V$

$d = \frac{V}{4} \Rightarrow V = 4 \times 0.2 = 0.8$

$\therefore s = 0.5(r - 4) + 0.8$

Now using values of r as 5, 6, 7, 8

$r = 5 \Rightarrow s = 1.3$

$r = 6 \Rightarrow s = 1.8$

$r = 7 \Rightarrow s = 2.3$

$r = 8 \Rightarrow s = 2.8$

③ $r = [9-10]$

$s = n[r - r_2] + W$

$s = 0.2[r - 8] + W$

$0.5W = \frac{W-V}{4} \Rightarrow W = 2.8$

$s = 0.2[r - 8] + 2.8$

$r = 9 \Rightarrow s = 3$

$r = 10 \Rightarrow s = 3.2$

$$\boxed{\begin{array}{l} r = [0-10] \\ s = [0-3.2] \end{array}}$$

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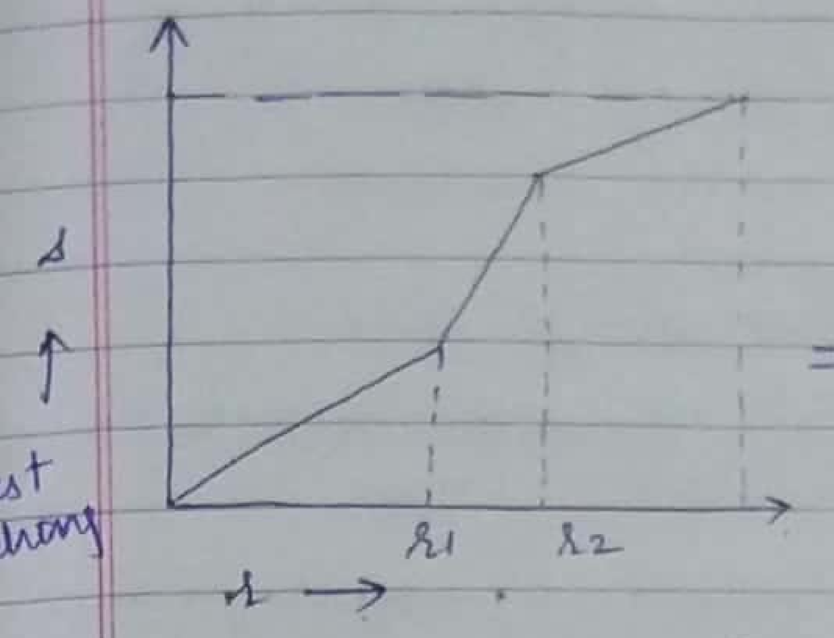
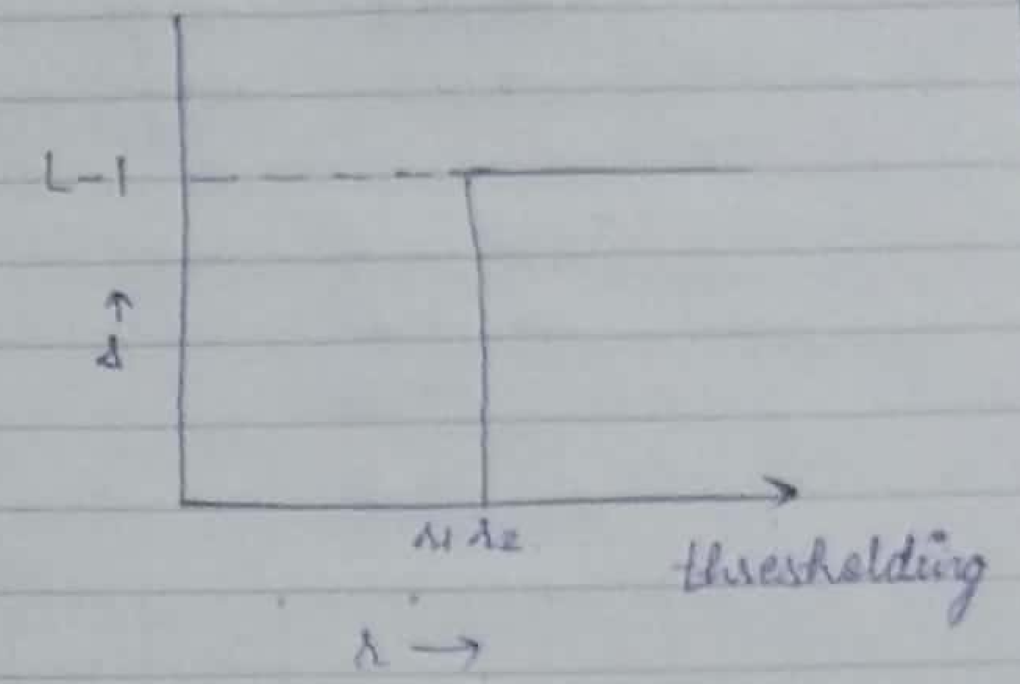
③ Thresholding – Extreme level of contrast stretching



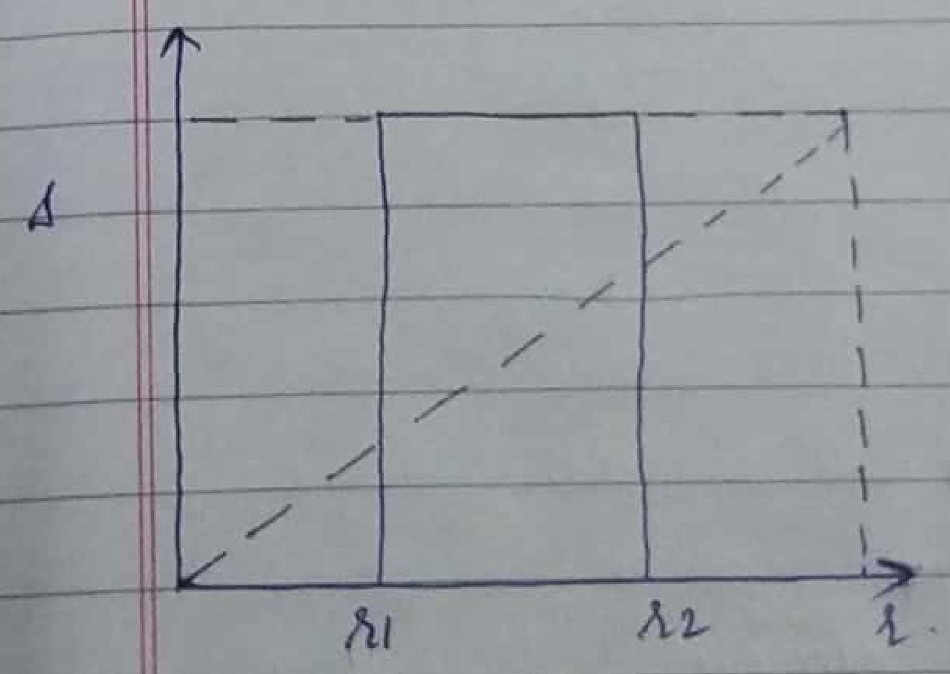


$s = 0 \quad r < r_1$

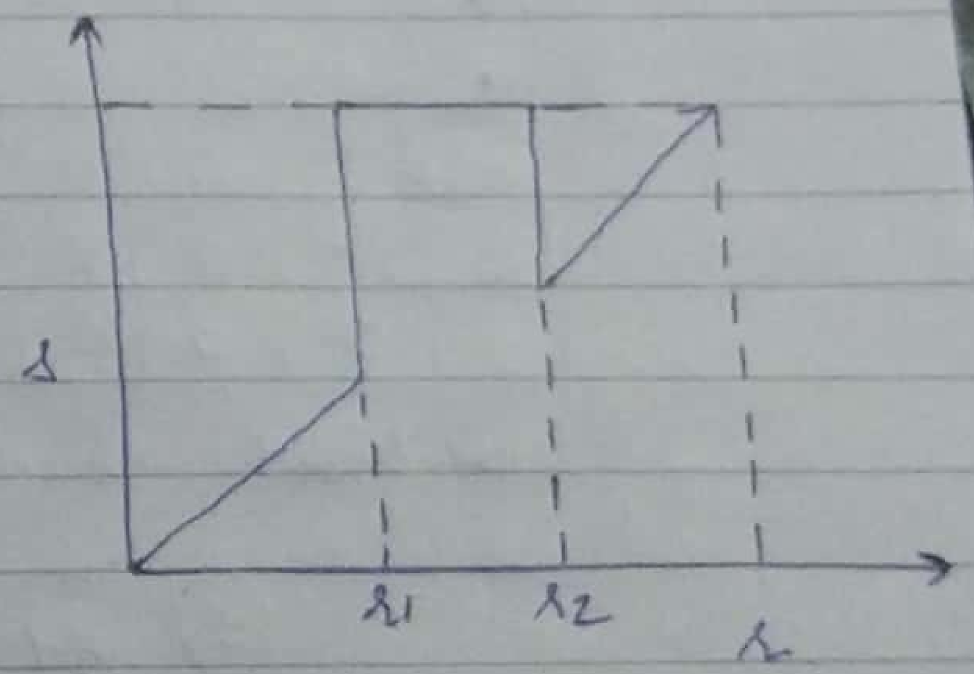
$s = L-1 \quad r \geq r_1$

$= 256 - 1 = 255$

④ Gray level slicing



Slicing without background



Slicing with background

I/P gray level

$S = L-1 \quad r_1 \leq r \leq r_2$

$S = 0 \quad$ otherwise

O/P gray level

$S = L-1 \quad r_1 \leq r \leq r_2$

$r \quad$ otherwise

Q- Perform intensity level slicing on 3BPP image. Let $r_1 = 3$ and $r_2 = 5$, draw the modified image using background and without background transformation.

$3BPP = 2^3 = L$

$S = L-1$

$= 8-1$

$= 7$

○ → Change to 7 in with background.

2	1	2	2	1
2	3	4	5	2
6	2	7	6	2
2	6	6	5	1
0	3	2	2	1

2	1	2	2	1
2	7	7	7	2
6	2	7	6	2
2	6	6	7	1
0	7	2	2	1

With background.

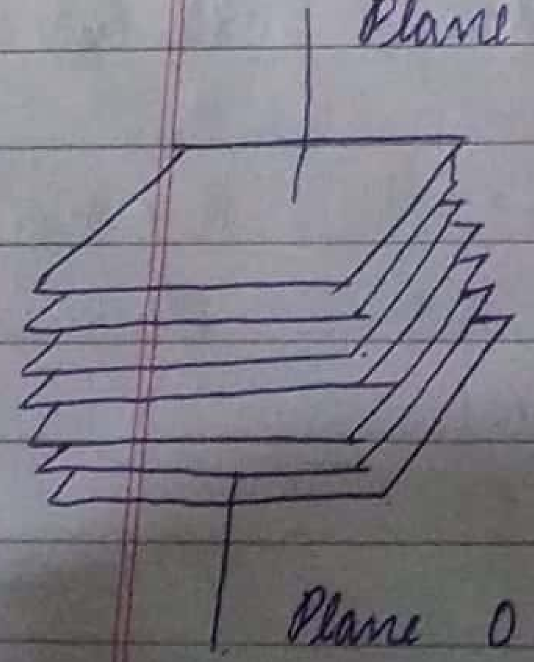
0	0	0	0	0
0	7	7	7	0
0	0	0	0	0
0	0	0	7	0
0	0	0	0	0

Without background.

(5) Bit plane slicing

- → Compression
- → Steganography

Plane 7 (MSB)



Plane 0 (LSB)

* In this we find out the contribution made by each bit to the final image.

* In bit plane slicing we see the importance of each bit in the final image.

* An 8-bit image may be considered as being composed of 8 1-bit plane with plane 1 contains the lower order bit of all pixels & plane 8 contains all higher order bits.

Q- Given a 3X3 image, plot its bit plane.

001	010	000
100	011	010
111	101	010

0	0	0
1	0	0
1	1	0

LSB

0	1	0
0	1	1
1	0	1

1	0	0
0	1	0
1	1	0

MSB

Q- Image shown below has 8 different gray levels. Plot this image using only 4 gray levels.

Gray level reduction transformation

0	1	1	1	1	4
1	1	2	3	2	2
1	1	2	2	3	3
1	2	4	6	2	3
1	2	4	2	4	4
1	2	3	7	2	5

$$S(X,Y) = \left\lfloor \frac{G \cdot r(x,y)}{K} \right\rfloor \frac{K}{G}$$

G = modified no. of gray levels

K = original no. of gray levels.

$$s(x,y) = \left\lfloor \frac{4\,r(x,y)}{8} \right\rfloor \frac{8}{4}$$

$r(x,y)$ = Input gray levels.

$S(x,y)$ = Output gray levels.

$r = 0, 1$

0	0	0	0	0	4
0	0	2	2	2	2
0	0	2	2	2	2
0	2	4	6	2	2
0	2	4	2	4	4
0	2	2	6	4	4

Modified image.

$S = \left[\frac{4}{8} \times 0 \right] \frac{8}{4}$

$S = 0 \quad \begin{cases} r = 0 \\ r = 1 \end{cases}$

$S = 2 \quad \begin{cases} r = 2 \\ r = 3 \end{cases}$

$S = 4 \quad \begin{cases} r = 4 \\ r = 5 \end{cases}$

$S = 6 \quad \begin{cases} r = 6 \\ r = 7 \end{cases}$

Point processing
- Linear
- Non linear.

⑥ Log Transformation

* This transformation maps a narrow range of low intensity values in the input into a wider range of the output levels. The opposite is true for higher values of input gray level.

* Log function compresses the dynamic range of images with large variation in pixel values.

* This technique of compressing the dynamic range is known as dynamic range compression.

* Log operation is an excellent compression function & hence the dynamic range compression is achieved by using log operator.

$\boxed{S = c \log (1+r)}$ c = +ve constant.

⑦ Power law transformation – different γ value for different devices.

$\gamma \uparrow$ bright
$\gamma \downarrow$ dark

CRT = 2.8 to 2.9

$\boxed{S = c r^{\gamma}}$

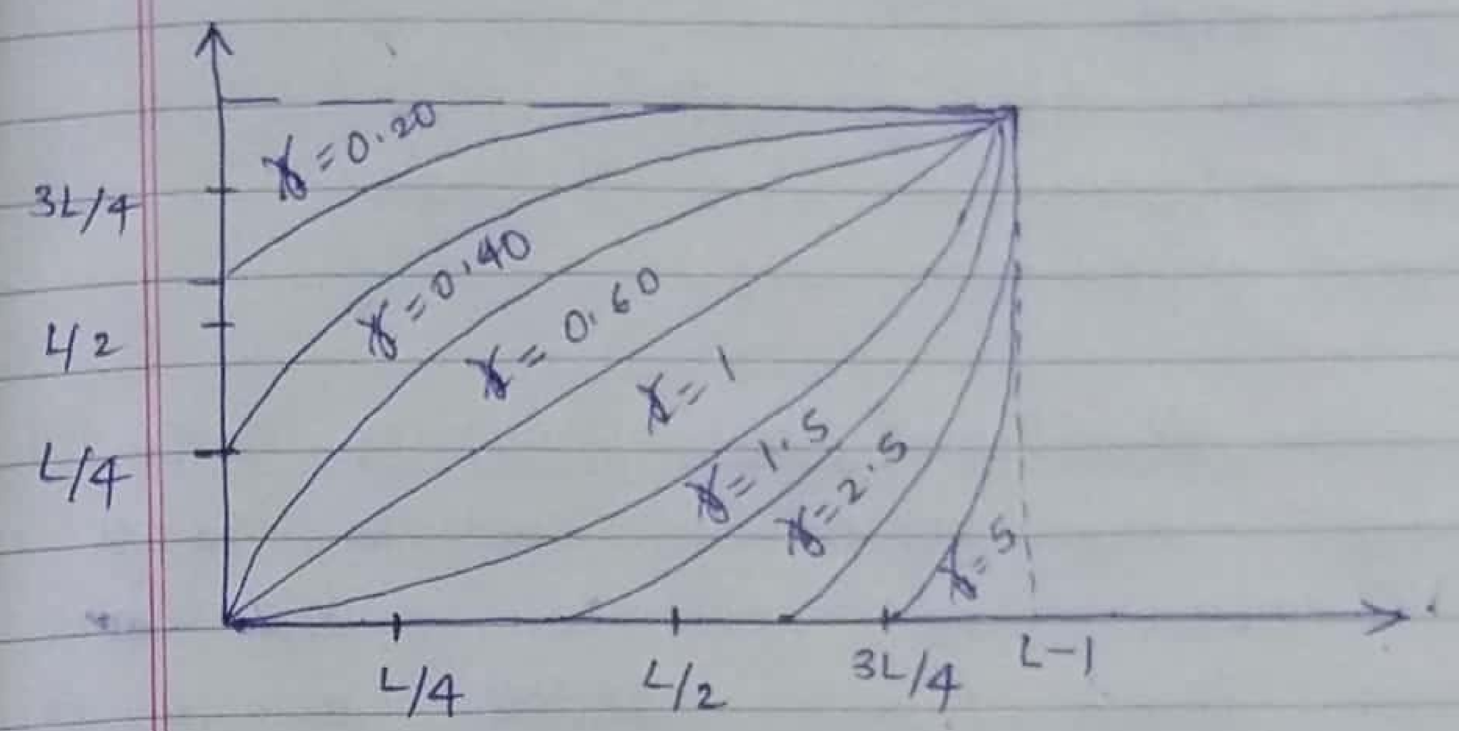


Image enhancement in spatial domain

- point processing
  - linear
  - non linear.
- neighbourhood processing
  - convolution
  - correlation
  - filtering
    - smoothing filters
      - to remove blurry (due to motion error)
      - noise removal.
    - ~~sloping~~ sharpening filter.

- Noise occurs when – image acquisition is done
  – image transmission is done.

Types of noise (Based on shape)

1. Gaussian noise {smoothing filter}
2. Rayleigh noise
3. Salt & pepper noise {smoothing filter}.
4. Gamma noise.
5. Uniform noise.

NOISE (high frequency content)

\# In image, any degradation in an image signal caused by external disturbance while an image is captured or being sent from one place to another place via satellite, wireless or network cable.

\# Based on the shape of noise, they are classified into the above 5 types of noise.

1) Gaussian noise :- It is caused by random fluctuation in the signal.

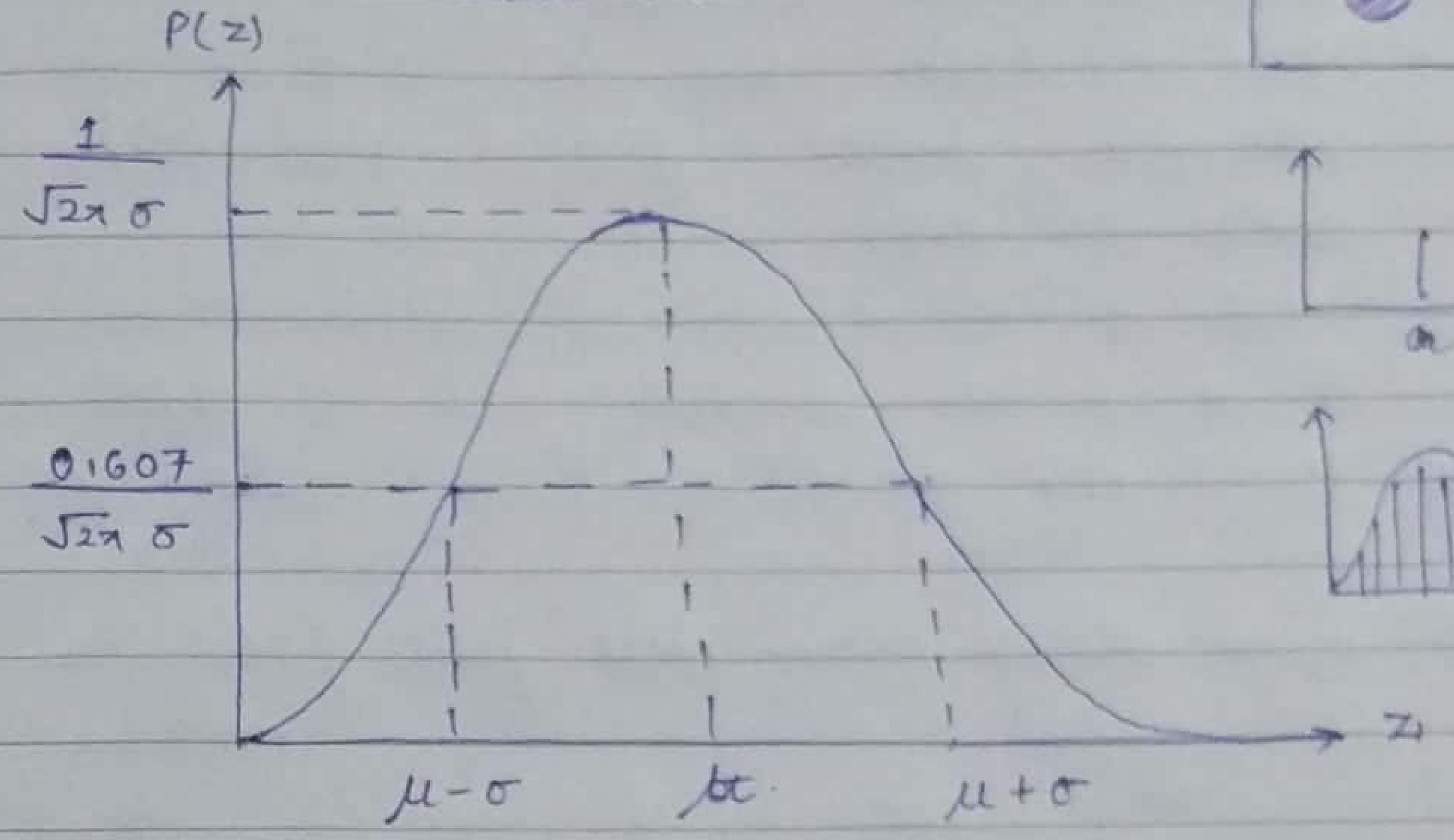
- It is modelled by random value added to an image.

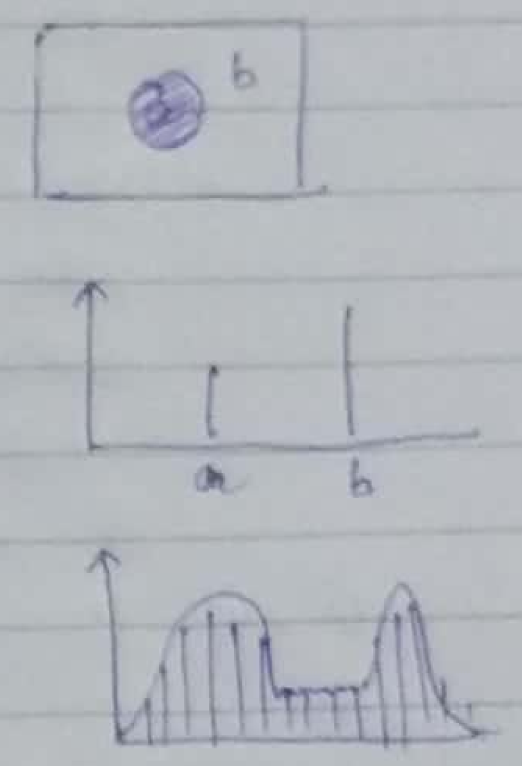
- The probability density function (PDF) of this noise is -

$$P(z) = \frac{e^{-(z-\mu)^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma}$$

→ Max value at μ ... off

where z = grey level
μ = mean of any value of z.
σ = standard deviation
σ^2 = Variance.

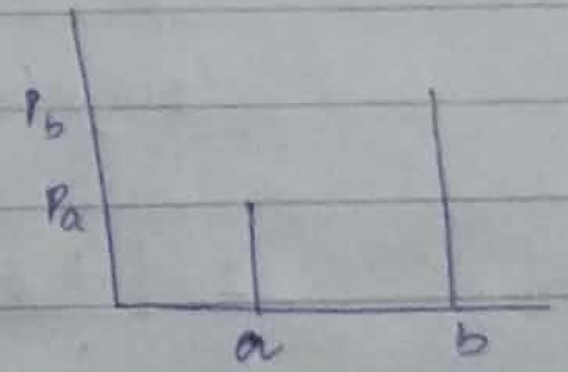




- Gaussian noise arises in an image due to factors such as electronic circuit noise, sensor noise caused by poor illumination & high temperature.

2) Salt & Pepper noise :- It is also known as impulse noise or bipolar noise.

$$P(z) = \begin{cases} P_a & \text{if } z = a \\ P_b & \text{if } z = b \\ 0 & \text{otherwise} \end{cases}$$



if P_a or $P_b = 0$, then it is unipolar noise.

- It arises in an image due to memory cell failure & by synchronisation error in image digitisation or transmission.

Smoothing spatial filter

Smoothing filters are used for blurring & noise reduction.

Noise reduction can be achieved by blurring with a linear filter and also by non-linear filter.

It is further divided into 2 parts —

(Gaussian) ① smoothing linear filter (low pass averaging filter)

(Salt & Pepper) ② Smoothing non-linear filter (low pass filter median filter) or (order statistics filter)

- When we use low pass filter we have +ve or -ve elements in mask, but in high pass filter, we can have both +ve & -ve elements in mask together.

- Low frequency content → same frequency content → eg: image background.

Low pass mask

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

Weighted mask

$$\frac{1}{16}\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix}$$

The o/p of smoothing linear spatial filter is simply the average of the pixels contained in the neighbourhood of the filtered mask.

An m×n mask would have normalizing constant equal to $\frac{1}{mn}$.

In low pass median filter, a mask of 3×3 which is empty is used.

When it is placed on image, we sort values either in increasing or decreasing order.

~~The fifth value will be replaced by the~~
The median value will be replaced by 5th value.

Empty mask

- Order statistics filter — It is non-linear filter whose response is based on ordering the pixels contained in the image area by the filter and then replacing the value of centre pixel with the value determined by ordering result. (median value).

Median filters are particularly effective in the presence of salt & pepper noise.

Sharpening spatial filter (high pass filter)

- We use derivative approach in this filter.

- The second derivative is called as Laplacian filter.

- The objective is to highlight the transitions in intensity.

- Sharpening can be achieved by spatial differentiation.

- The strength of the response of a derivative operator is proportional to degree of intensity which discontinuities of the image at the point at which the operator is applied, i.e. it can be performed by derivative approach.

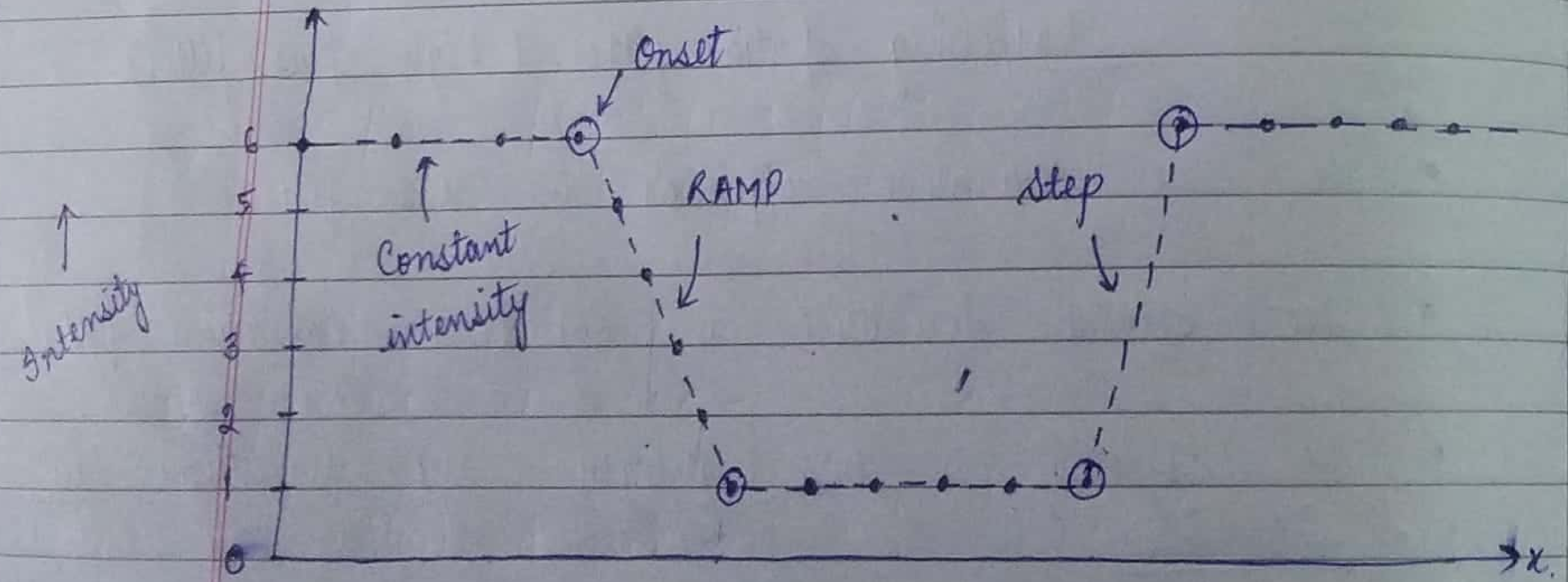
First derivative & IInd derivative of 1-D ~~multi~~function

Ist derivative of f(x).

$$\boxed{\frac{df}{dx} = f(x+1) - f(x)}$$

2nd derivative of f(x).

$$\boxed{\frac{d^2 f}{dx^2} = f(x+1) + f(x-1) - 2f(x)}$$



Scan line

6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6.

Ist derivative of scan line

0	0	0	-1	-1	-1	-1	-1	0	0	0	0	0	5	0	0	0	0

Property
- Constant intensity region → Ist derivative is zero.
- Step edge or ramp edge → Onset point → Non-negative Zero.
- First derivative of ramp edge → Non-zero.

IInd derivative of scan line

-	0	0	-1	0	0	0	0	1	0	0	0	0	5	-5	0	0	0

Property
- Onset and end point must be non-zero.
- Ramp edge with constant slope → ~~non~~ zero.
- Constant intensity region → IInd derivative is zero.

Note: Visualization is more clear & thus we prefer ~~Ist~~ IInd derivative over Ist derivative.

14/Feb/19

Ist & 2nd derivative of 2-D function

Ist derivative –

$$\frac{df}{dx} = f(x+1, y) - f(x,y)$$

$$\frac{df}{dy} = f(x, y+1) - f(x,y)$$

Because derivative of any order is ~~a~~ are linear operations, Laplacian is a linear operator

Isotropic filter → Response is independent of the direction of discontinuities in image to which it is applied.

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V.V. Imp.

2nd derivative – (Laplacian transformation)

$$\boxed{\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}}$$ (isotropic nature)

→ aim is to identify mask.
→ mask is called laplacian mask.
→ mask has isotropic nature, i.e. use mask on image to convolve it and then rotate it (the resultant image).
or
→ Rotate mask and then convolve it over image

Rotation Invariant

• Both the results will be same, thus it is transformation variant & called as isotropic filter/mask.

Representing laplacian operator in discrete form

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

$$\Rightarrow \boxed{\nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)}$$

1) Generate a mask –

Standard mask →

	y-1	y	y+1
x-1	0	1	0
x	1	-4	1
x+1	0	1	0

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Standard laplacian mask –

+1	1	1
1	-8	1
~~0~~ 1	1	1

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

(sum of all elements is zero)

→ After masking / using the laplacian filter, the image will get sharpen but the background will get clear. {as matrix sum is 0 & background has same intensity pixel}

→ To retain the background we use –

$$\boxed{g(x,y) = f(x,y) + c\,\nabla^2 f(x,y)}$$ ← Adding laplacian image to original image to retain background

c = -1 } for center value = -ve.
c = 1 } for center pixel value = +ve

Imp.

Unsharp masking and highboost filtering

Original image → $\boxed{\text{Low pass filter}}$ → Low pass original image.

$$\boxed{\text{Unsharp masking} = \text{Original image} - \text{Low passed image.}}$$

$$\boxed{g_{mask} = f(x,y) - \bar{f}(x,y)}$$ Sharp or ~~smooth~~ image.

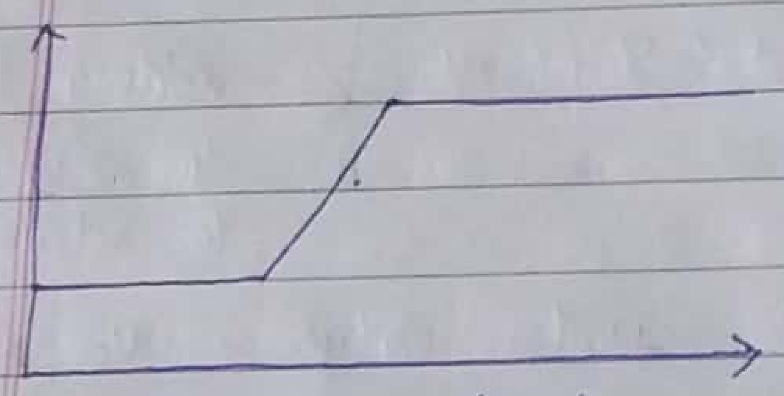
To retain background -

$$\boxed{g(x,y) = f(x,y) + k * g_{mask}(x,y)}$$

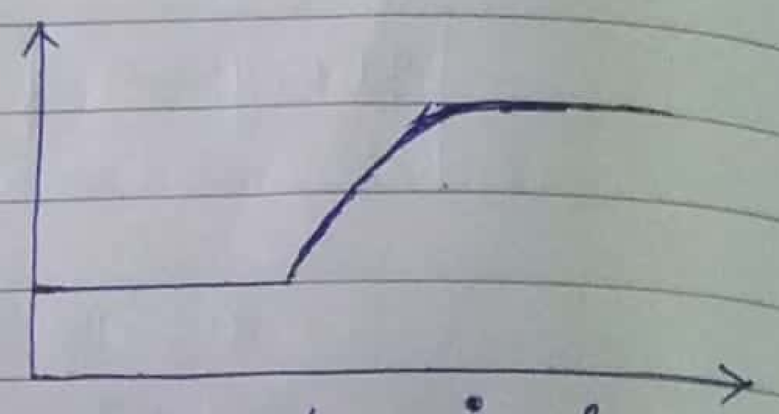
Generally $K \geq 0$.

When $K=1 \Rightarrow$ unsharp masking.
$K>1 \Rightarrow$ highboost filtering.

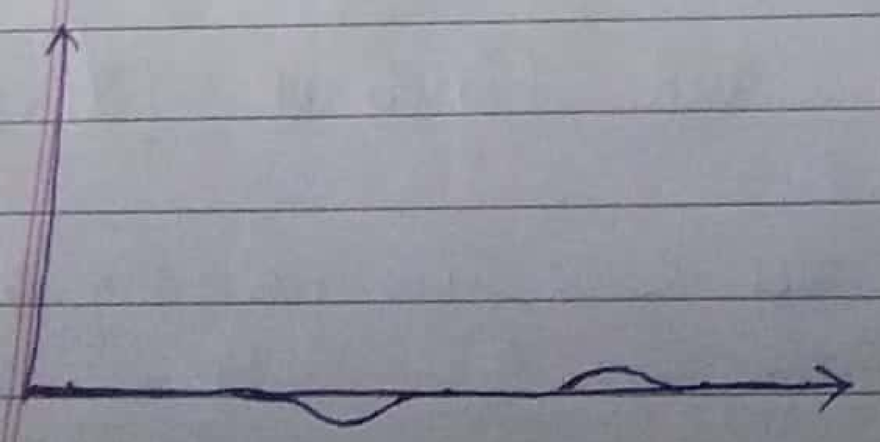
<u>Highboost filtering</u>: Sharpening the already sharped image with a factor of K, is called highboost filtering.



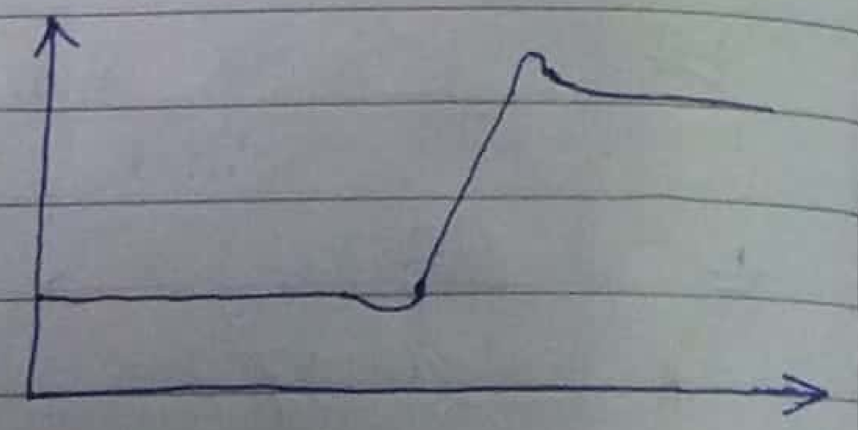
Original signal



Blurred signal



Unsharp mask



Sharpen signal.

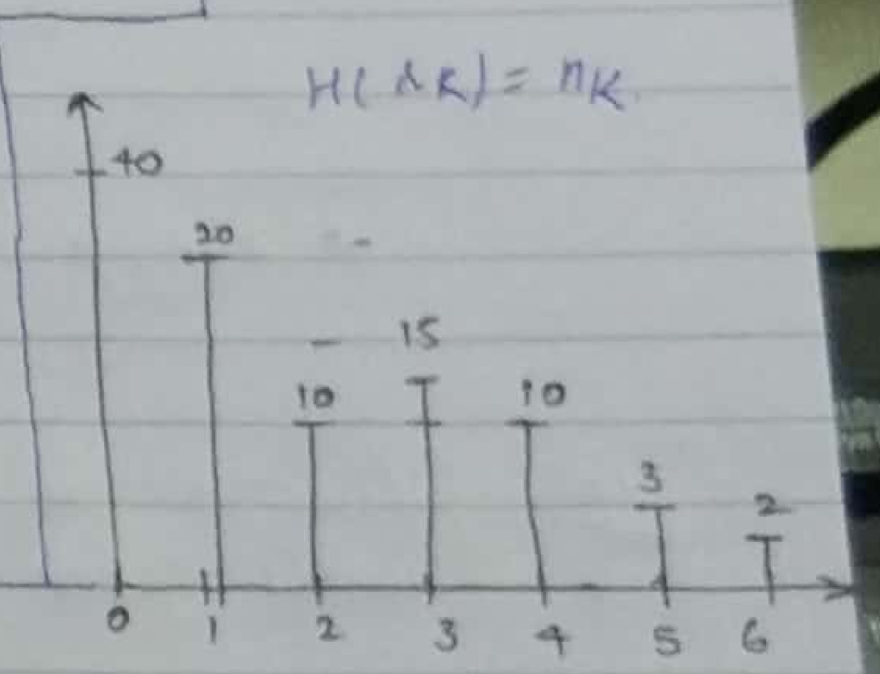
15/Feb/19

<u>Image enhancement based on histogram.</u>

→ Relative frequency available on a particular graylevel

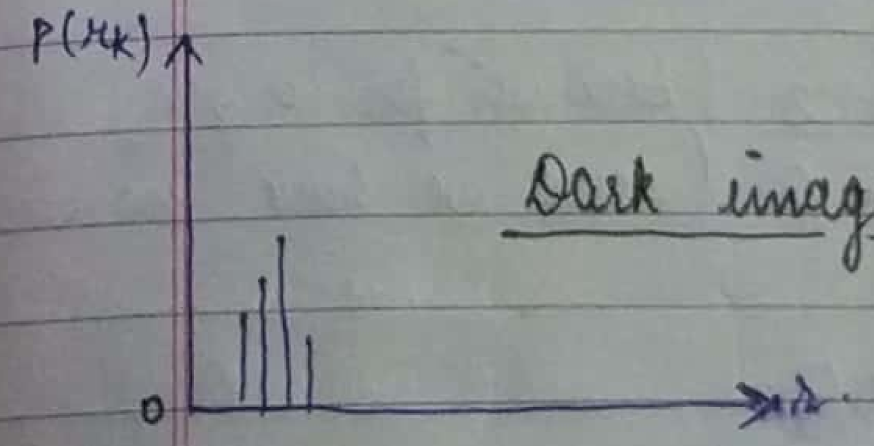
→ Normalised histogram
General histogram.

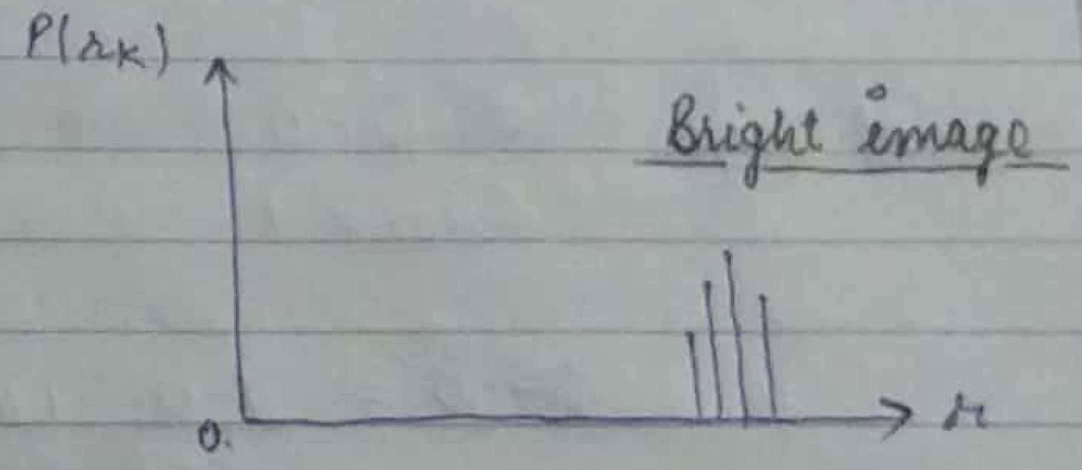
$P(r_k)$	Graylevel (r_k)	No. of pixels (n_k)
0.40	0	40
0.20	1	20
0.10	2	10
0.15	3	15
0.10	4	10
0.03	5	3
0.02	6	2
	7	$n = 100$

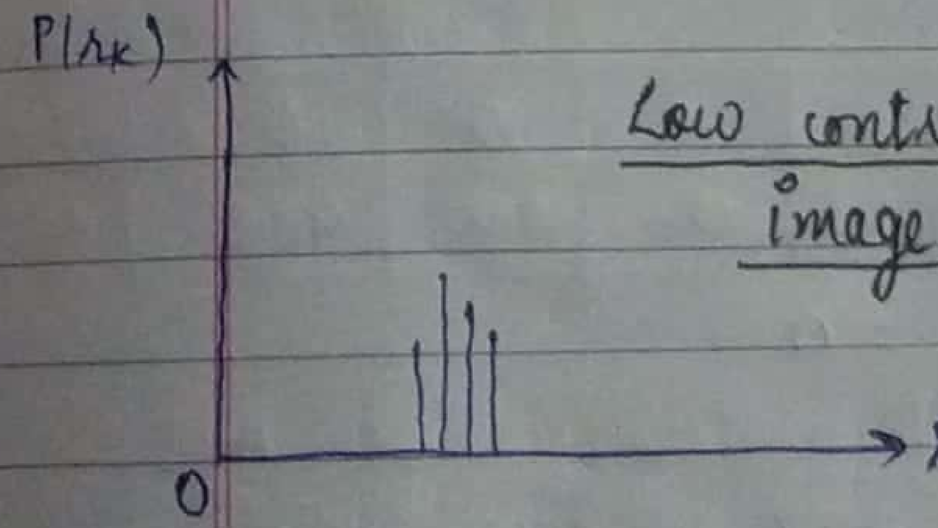


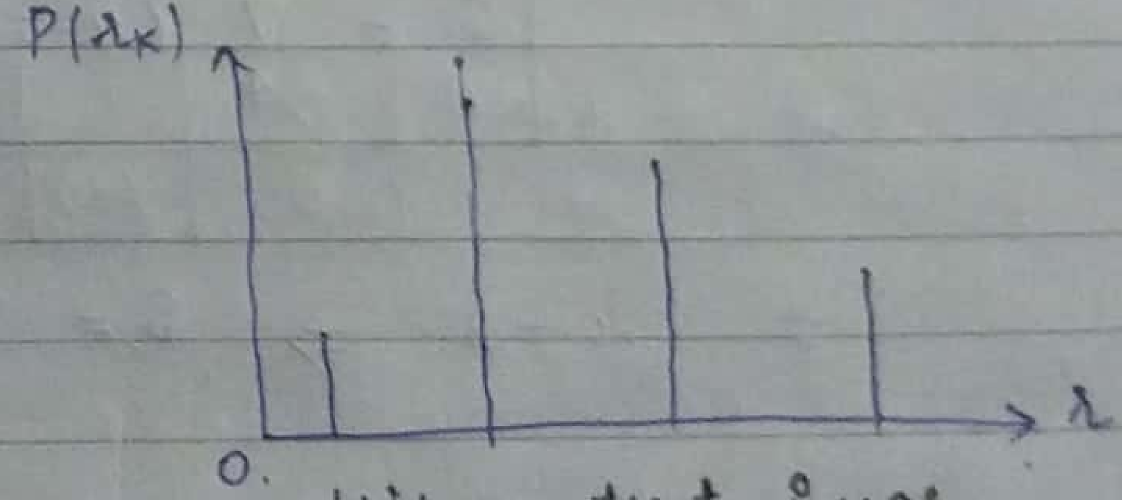
histogram

For a normalised histogram -
$$\boxed{P(r_k) = n_k / n}$$









High contrast image

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V. Imp.

Histogram stretching | Histogram equalisation | Histogram specification

(1) Histogram stretching :

$$\boxed{s = T(r) = \left(\frac{s_{max} - s_{min}}{r_{max} - r_{min}}\right)(r - r_{min}) + s_{min}}$$

→ change in size but not shape.

eg: Perform histogram stretching so that the new image has a dynamic range [0-7].

Gray level (r_k)	0	1	2	3	4	5	6	7
No. of pixels (n_k)	0	0	50	60	50	20	10	0

Solution) $s_{min} = 0$ $\quad r_{min} = 2$ (because for 0 gray level we have no pixels)

$s_{max} = 7$ $\quad r_{max} = 6$

$s - 0 = \frac{7-0}{6-2}(r-2) + 0$

$$\boxed{s = \frac{7}{4}(r-2)}$$

$r = 2 \Rightarrow s = \frac{7}{4} \times 0 = 0$

$r = 3 \Rightarrow s = \frac{7}{4} \times 1 = \frac{7}{4} = 1.75 \approx 2$

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$r = 4 \Rightarrow s = \frac{7}{4}(4-2) = \frac{7}{2} = 3.5 \approx 4$

$r = 5 \Rightarrow s = \frac{7}{4}(5-2) = \frac{21}{4} = 5.2 \approx 5$

$r = 6 \Rightarrow s = \frac{7}{4}(6-2) = 7$

Input gray level (r)	Pixels (N_k)	Output gray level (s)
2	50	0
3	60	2
4	50	4
5	20	5
6	10.	7.

Modified gray level

Gray level	0	1	2	3	4	5	6	7
No. of pixels	50	0	60	0	50	20	0	10

Histogram

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Q- Perform histogram stretching so that the dynamic range is [0-7].

Gray level :	0	1	2	3	4	5	6	7
no. of pixels :	100	90	85	70	0	0	0	0

Solution) $S_{min} = 0$ $r_{min} = 0$
$S_{max} = 7$ $r_{max} = 3$

$$S = \frac{7-0}{3-0}(r-0) + 0$$

$$\boxed{S = \frac{7r}{3}}$$

r	S	r_K
0	0	100
1	2	90
2	5	85
3	7	70

Modified :

Grey level	0	1	2	3	4	5	6	7
no. of pixels	100	0	90	0	0	85	0	70

ANS

gray level	0	1	2	3	4	5	6	7
n_K	0	790	~~1023~~	1023	0	850	985	448

ANS

V. Imp

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② Histogram equalisation – ★ Aim is to find out the flat histogram i.e. for each gray level we try to find out the same no. of pixels.

★ It can be done in → continuous domain (probability density function)
→ discrete domain

★ It helps in improving / achieving the maximum quality of image.

★ Repeating histogram equalisation on already equalized image the result is same as image has already achieved its maximum quality.

- In discrete domain

Equalise the histogram given as follows

Gray level	0	1	2	3	4	5	6	7
no of pixels	790	1023	850	656	329	245	122	81

Solution) $P(r_K) = \frac{n_K}{n}$ $\sum_0^r P_r(r_K)$ $\boxed{L = 8}$

Gray level	n_K	P.D.F.	C.D.F (S_K)	$(L-1) \times S_K$	Round off
0	790	0.19	0.19	1.33	1 = 790
1	1023	0.25	0.44	3.08	3 = 1023
2	850	0.21	0.65	4.55	5 = 850
3	656	0.16	0.81	5.67	6 } 656+329
4	329	0.08	0.89	6.23	6 } = 985
5	245	0.06	0.95	6.65	7 } 245+122+81
6	122	0.03	0.98	6.86	7 } = 448
7	81	0.02	1.00	7.	7 }
	4096				

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- <u>In continuous domain</u>

\# In linear stretching the shape remains same but there are many application where we need a flat histogram.

\# A perfect image is one in which we have equal no. of pixels in all its gray level.

\# Hence, our objective is not only to spread the dynamic range but also to have equal pixels in all the gray levels.

\# This technique is known as histogram equalisation.

<u>Assumptions</u>: $S = T(r)$ $\quad 0 \le r \le L-1$

$0 \le S \le L-1 \quad 0 \le r \le L-1$

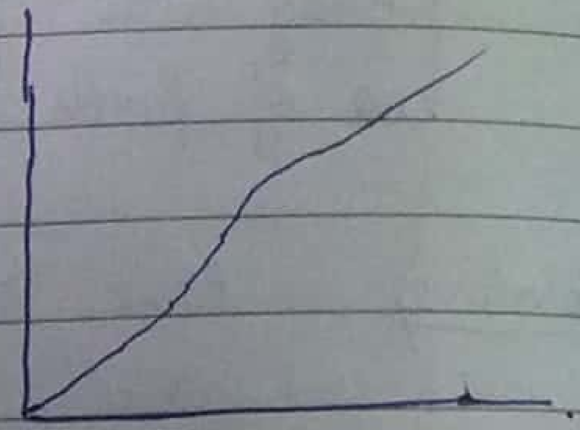
\# <u>Monotonically increasing function</u>

O/P intensity values will never be less than i/p values.

$r_1 \geq r_2$

$T(r_1) \geq T(r_2)$

$S_1 \geq S_2$.



single valued & monotonically increasing

\# Since the transformation is single valued & monotically increasing,

→ the inverse transformation exist

i.e. $\boxed{r = T^{-1}(s).}$ $\quad 0 \le s \le 1$

\# The grey level for continuous variable can be defined by the probability density function - $P_r(r)$ and $P_s(s)$.

\# From probability theory, we know that if $P_r(s)$ and T_r are known and if $T^{-1}(s)$ satisfies first condition (T_s is single valued & monotically increasing) then the PD of transformed gray level is -

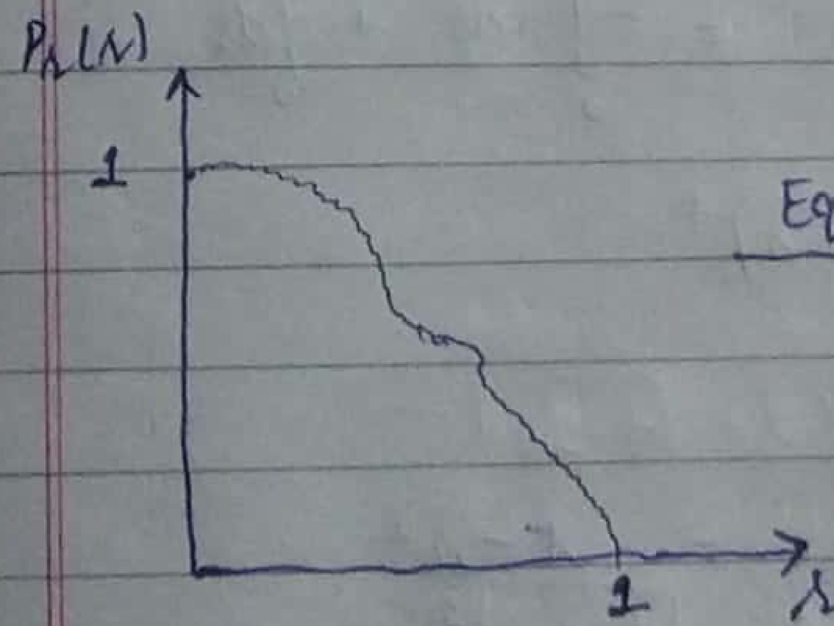
$$\boxed{P_s(s) = P_r(r) \frac{dr}{ds} \Big|_{r = T^{-1}(s)}}$$

$$S = \int_0^r P_r(r)\, dr.$$

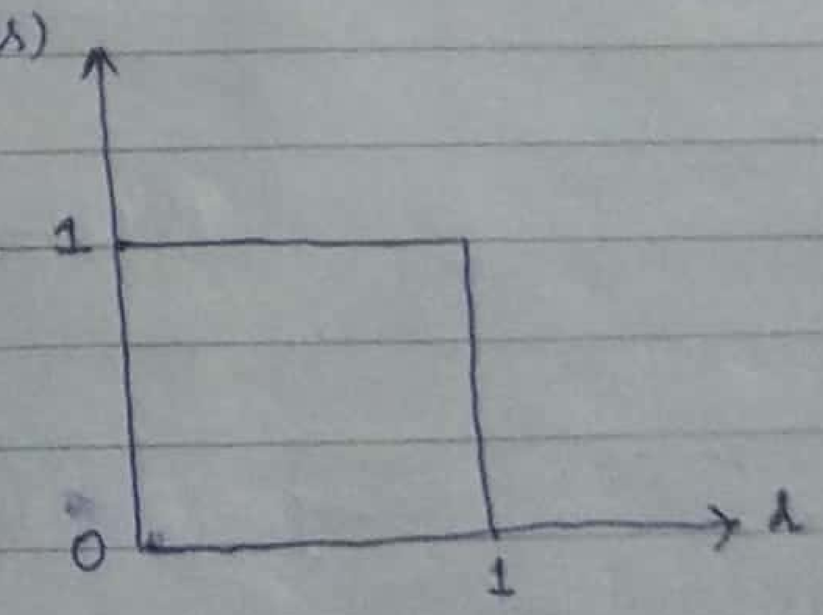
$$\frac{ds}{dr} = P_r(r) \qquad 0 \le r \le 1$$

$$\Rightarrow P_s(s) = P_r(r) \frac{1}{P_r(r)}$$

$$\boxed{P_s(s) = 1} \qquad 0 \le s \le 1$$

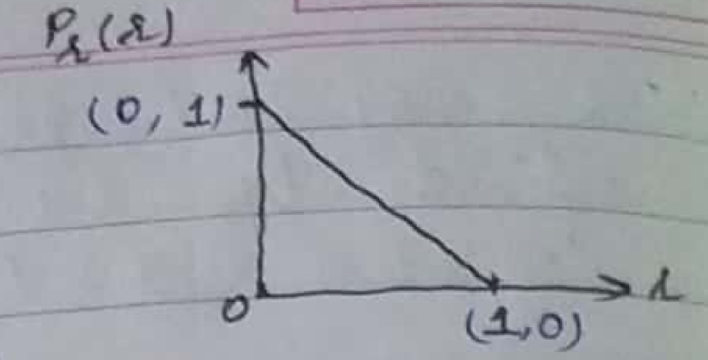


Equalization



Imp

Q. Given the histogram. Equalise it.



Solution) ① find probability density function

Using straight line equation -

$$(y-y_1) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-0 = \frac{1-0}{0-1}(x-1)$$

$$y = -1(x-1)$$

$$y = -x+1$$

$$\boxed{P_r(r) = -r+1}$$

$$P_r(r) = \begin{cases} -r+1 & 0\le r\le 1 \\ 0 & \text{otherwise}\end{cases}$$

$S = T(r) = \int_0^r P_r(r)\,dr$

$$= \int_0^r (-r+1)\,dr = -\int_0^r r\,dr + \int_0^r dr$$

$$= -\left[\frac{r^2}{2}\right]_0^r + [r]_0^r$$

$$S = -\left[\frac{r^2}{2}\right] + r = r - \frac{r^2}{2}$$

$$2S = 2r - r^2$$

$$2S-1 = 2r - r^2 - 1$$

$$1-2S = r^2 - 2r + 1$$

$$1-2S = (r-1)^2$$

$$\pm\sqrt{1-2S} = r-1$$

$$\boxed{1\pm\sqrt{1-2S} = r}$$ ⇒ {We skip the value $1+\sqrt{1-2S}$ as it will go out of our range i.e. $0\le r\le 1$}

⇒ $\boxed{r = 1-\sqrt{1-2S}}$

$P_s(s) = P_r(r)\frac{dr}{ds}$

$$= (-r+1)\frac{d}{ds}(1-\sqrt{1-2s})$$

$$= (-r+1)\left[\frac{d(1)}{ds} - \frac{d\sqrt{1-2s}}{ds}\right]$$

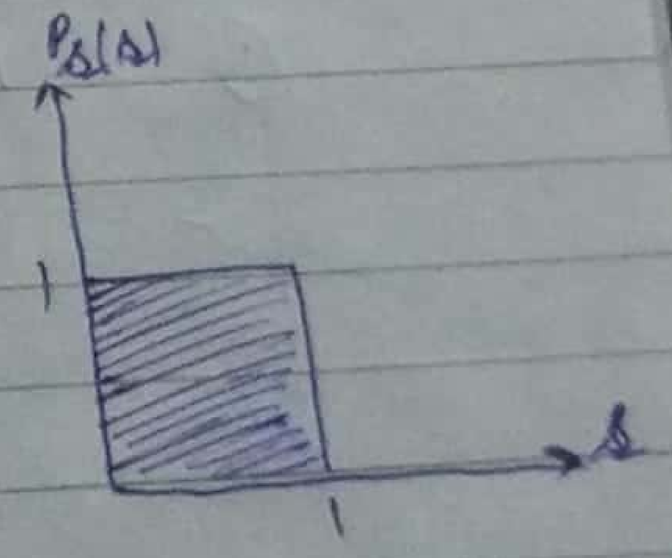
$$= (-r+1)\left(0 - \frac{1}{2}(1-2s)^{-1/2}\times(-2)\right)$$

$$= (-r+1)(1-2s)^{-1/2}$$

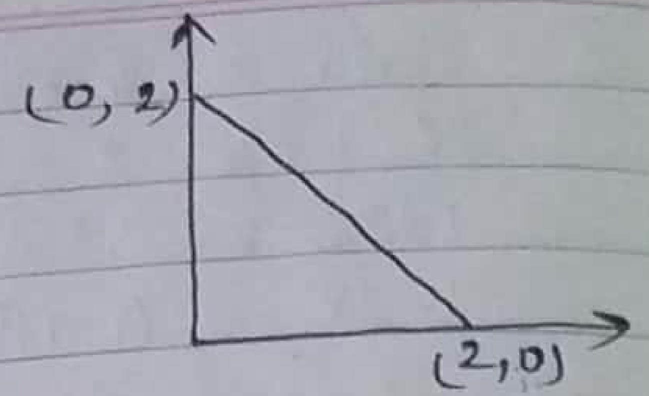
$$= (\cancel{1}+\sqrt{1-2s}-\cancel{1})(1-2s)^{-1/2}$$

$$= \sqrt{1-2s}\times\frac{1}{\sqrt{1-2s}}$$

$$\boxed{P_s(s) = 1}$$



Q- Given a histogram equalise it.



Solution) (1) Finding $P_r(r)$

$$y - 0 = \frac{2-0}{0-2}(x-2)$$

$$y = -x + 2$$

$$\boxed{P_r(r) = -r + 2}$$

$$P_r(r) = \begin{cases} -r+2 & 0 \le r \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

(2) $$S = T(r) = \int_0^r P_r(r)\, dr = \int_0^r (-r+2)\, dr$$

$$S = \frac{-r^2}{2} + 2r$$

$$2S = -r^2 + 4r$$

$$2S - 4 = -r^2 + 4r - 4$$

$$4 - 2S = (r-2)^2$$

$$\boxed{r = 2 \pm \sqrt{4-2S}} \Rightarrow \boxed{r = 2 - \sqrt{4-2S}}$$

(3) $$P_s(s) = P_r(r)\frac{dr}{ds}$$

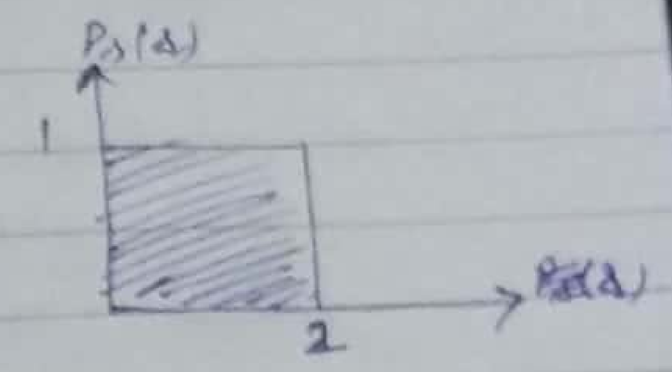
$$= (-r+2)\frac{d}{ds}\left(2 - \sqrt{4-2s}\right)$$

$$= \sqrt{4-2s}\,\frac{d}{ds}\left(2 - \sqrt{4-2s}\right)$$

$$= \sqrt{4-2s}\left(0 - \frac{1}{2}(4-2s)^{-1/2}(-2)\right)$$

$$= \sqrt{4-2s} \times \frac{1}{\sqrt{4-2s}}$$

$$\boxed{P_s(s) = 1}$$



L-1 = 3

Q-

Graylevel	0	1	2	3
n_k	70	20	7	3

Equalise the histogram twice

Graylevel	n_k	PDF	CDF(s_k)	(L-1) s_k	Roundoff
0	70	0.7	0.7	2.1	2 → 70
1	20	0.2	0.9	2.7	3 → 30
2	7	0.07	0.97	2.91	3 → 7
3	3	0.03	1.00	3	3 → 3
	100				

$$\left.\begin{matrix} 3 \to 30 \\ 3 \to 7 \\ 3 \to 3 \end{matrix}\right\} 20+7+3 = 30$$

L-1 = 3

Graylevel	n_k	PDF	CDF	(L-1) s_k	Roundoff
0	70	0.7	0.7	2.1 = 2.1	2 → 70
1	30	0.3	1	1×3 = 3	3 → 30
	100				

20/Feb/19

③ Histogram Specification - Known as histogram matching also, because 2 histograms are given and we match one to another.

\# Sometimes, it is useful to be able to specify the shape of the histogram that we wish ~~that~~ the processed image to have.

\# The method used to generate a processed image that has a specified histogram is called histogram matching or specification.

\# The reverse transformation function that we use in histogram specification is -

$$\boxed{S = T_2^{-1}[T_1(r)]}$$

\# Procedure of histogram specification can be computed as -

1) Equalise the graylevel of original image.

2) Specify the desired density function & obtain the transformation function.

3) Apply the inverse transformation function.

Q- Given histogram A and B. Modify histogram A as given by histogram B.

A)

Gray level	0	1	2	3	4	5	6	7
No. of pixels	790	1023	850	656	329	245	122	81

B)

Gray level	0	1	2	3	4	5	6	7
No. of pixels	0	0	0	614	819	1230	819	614

Solution) ① Equalise histogram A and B.
(A is already equalised in previous ques)

→ Equalising B. $L-1=7$

Gray level	n_K	P.D.F.	CDF (s_K)	$(L-1)s_K$	Round off
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	614	0.15	0.15	1.05	1 → 614
4	819	0.20	0.35	2.45	3 → 819
5	1230	0.30	0.65	4.55	5 → 1230
6	819	0.20	0.85	5.95	6 → 819
7	614	0.15	1.00	7.00	7 → 614
	4096				

0 (above the round-off column); $0+0+0=0$ (beside the round-off values of rows 0–2)

② Apply (histogram) inverse transformation on B.

Roundoff A	B Roundoff
1	→ 3
3	→ 4
5	→ 5
6	→ 6
7	→ 7

{no of pixels on 1 in histogram A will be shifted to no of pixels on 3 in histogram B}

{if we don't have value put 0 in B}

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Gray level	0	1	2	3	4	5	6	7
no. of pixels	0	0	0	790	1023	850	985	448

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Image Enhancement in frequency domain

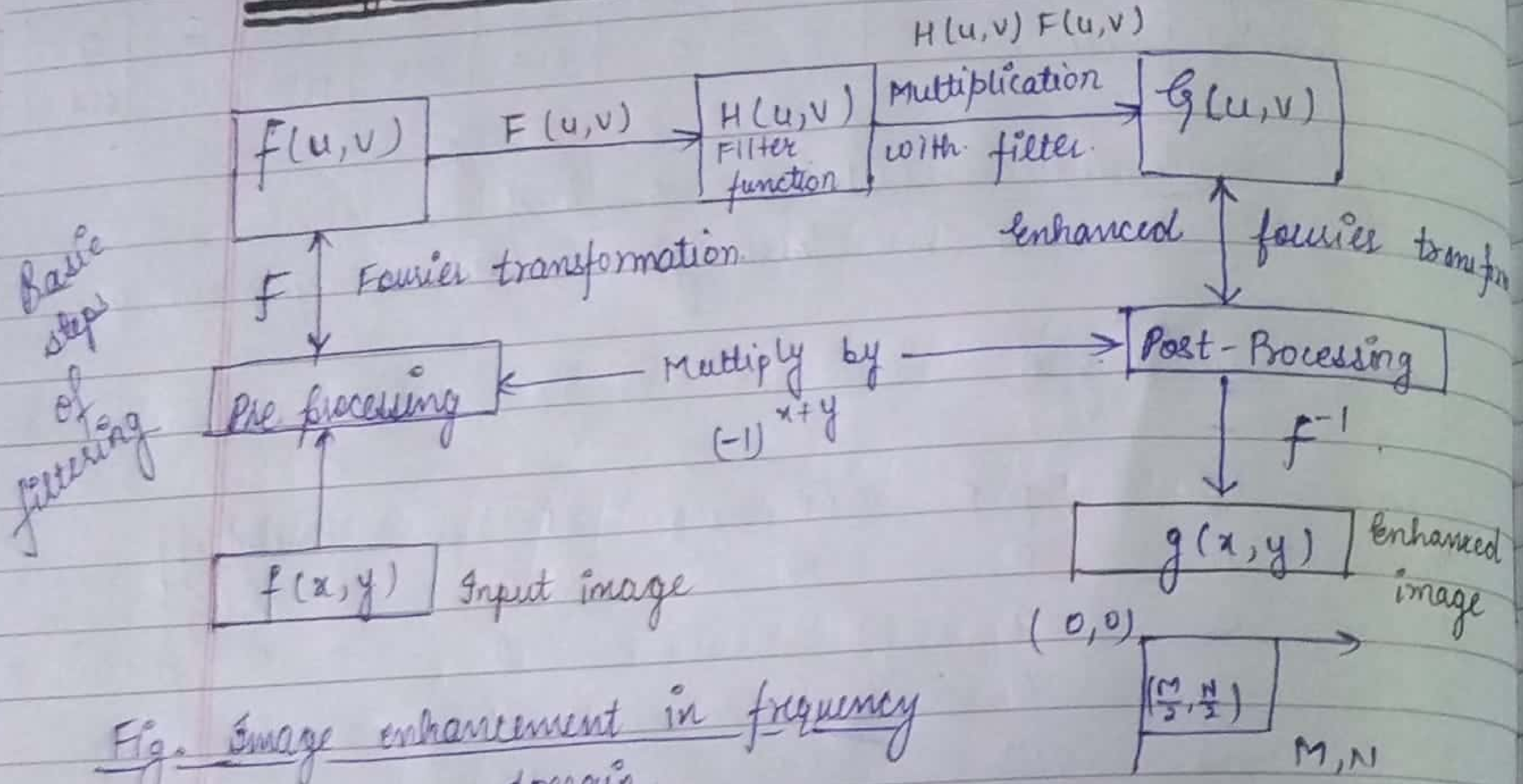

Fig. Image enhancement in frequency domain

1-D Fourier Transformation

$f(x) = f(u)$

$$F[f(x)] = f(u) = \int_{-\infty}^{\infty} f(x) \cdot e^{-j2\pi ux} \, dx$$

$$= \int_{-\infty}^{\infty} f(x) \left[\cos 2\pi ux - j \sin 2\pi ux\right]$$

$$\boxed{f(u) = |f(u)| e^{j\phi(u)}}$$

$\phi(u)$ = Phase average $= \tan^{-1}\left(\frac{I(u)}{R(u)}\right)$

Power Spectrum $P(u) = |f(u)|^2 = R^2(u) + I^2(u)$

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Inverse fourier $F^{-1}(f(u)) = \int_{-\infty}^{\infty} f(u) \, e^{j2\pi u} \, du$

2D Fourier Transformation

$f(x,y)$ = continuous

$F(f(x,y)) = f(u,v) = \int_{-\infty}^{\infty} f(x,y) \cdot e^{-j2\pi(ux+vy)} \, dx\,dy$

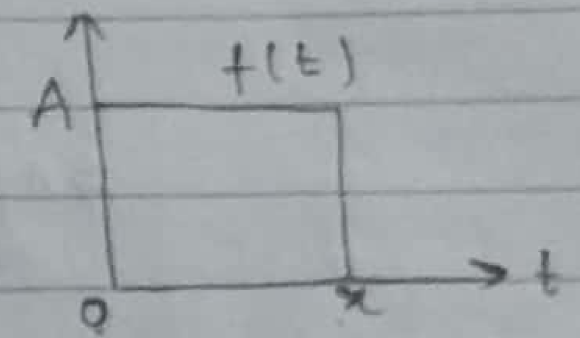
$F^{-1}(f(u,v)) = f(x,y) = \int_{-\infty}^{\infty} f(u,v) \, e^{-j2\pi(ux+vy)} \, du\,dv$

$e^{i\theta} = \cos\theta + i\sin\theta$

$= \cos 2\pi(ux+vy) - j\sin(ux+vy)2\pi$

$$\boxed{F(f(x,y)) = \int_{-\infty}^{\infty} f(x)\left[\cos 2\pi(ux+vy) - j\sin 2\pi(ux+vy)\right]}$$

Example Find the fourier transform of the signal shown below :-



$$F(f(x)) = \int_{-\infty}^{\infty} f(x) \cdot e^{-2\pi j ux} \, dx$$

$$= \int_{0}^{x} A \cdot e^{-j2\pi ut} \, dt$$

$$= \frac{A}{-j2\pi u}\left[e^{-j2\pi ut}\right]_0^x$$

$$= \frac{A}{-j2\pi u}\left[e^{-j2\pi ux} - 1\right]$$

$$= \frac{A}{j2\pi u} \times e^{-j\pi ux}\left[e^{j\pi ux} - e^{-j\pi ux}\right]$$

$$= \frac{A}{j2\pi u}\left[1 - e^{-j2\pi ux}\right] = \frac{A}{2j\pi u} 2j\sin \pi ux \cdot e^{-j\pi ux}$$

$$f(u) = \frac{A}{j2\pi ux} \cdot 2j \sin \pi ux \times e^{-j\pi ux} \times x = Ax \cdot \frac{2j \sin \pi ux}{\pi ux} \times e^{-j\pi ux}$$

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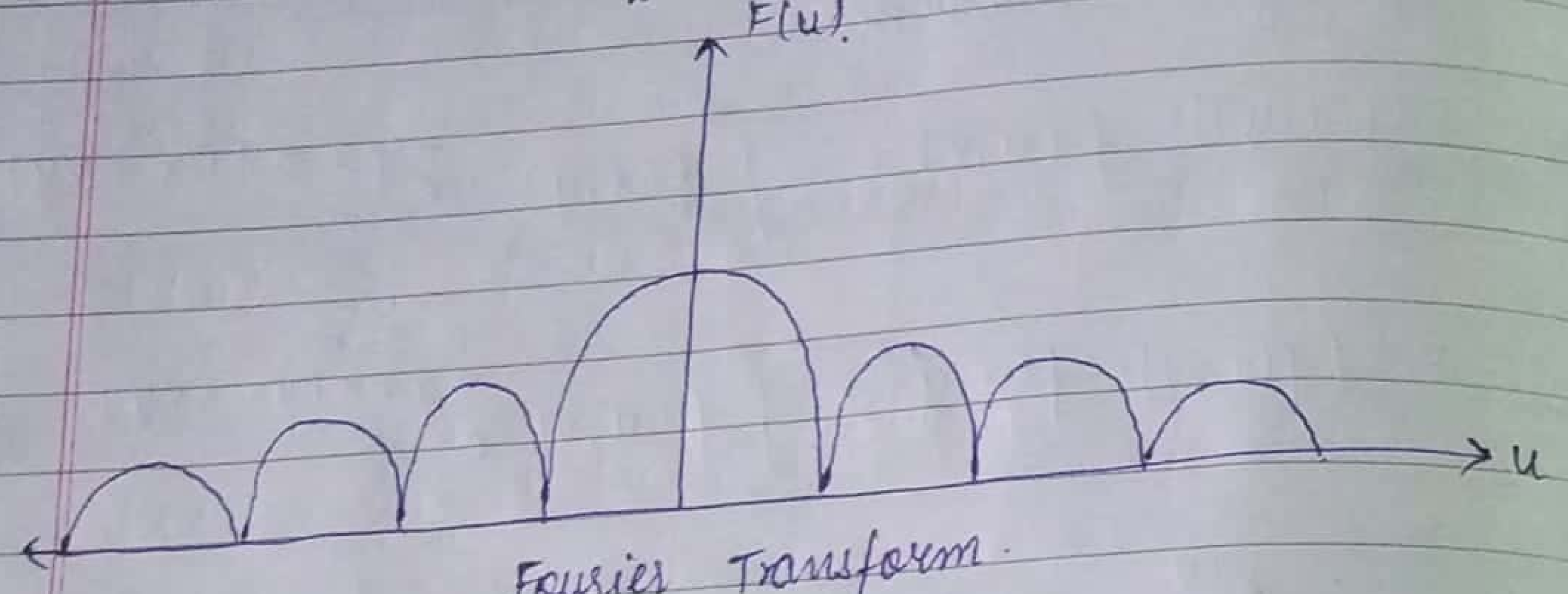
$$|f(u)| = Ax$$

$$|f(u)|_{u=0} = Ax$$

$$u = \frac{1}{x} = 0$$



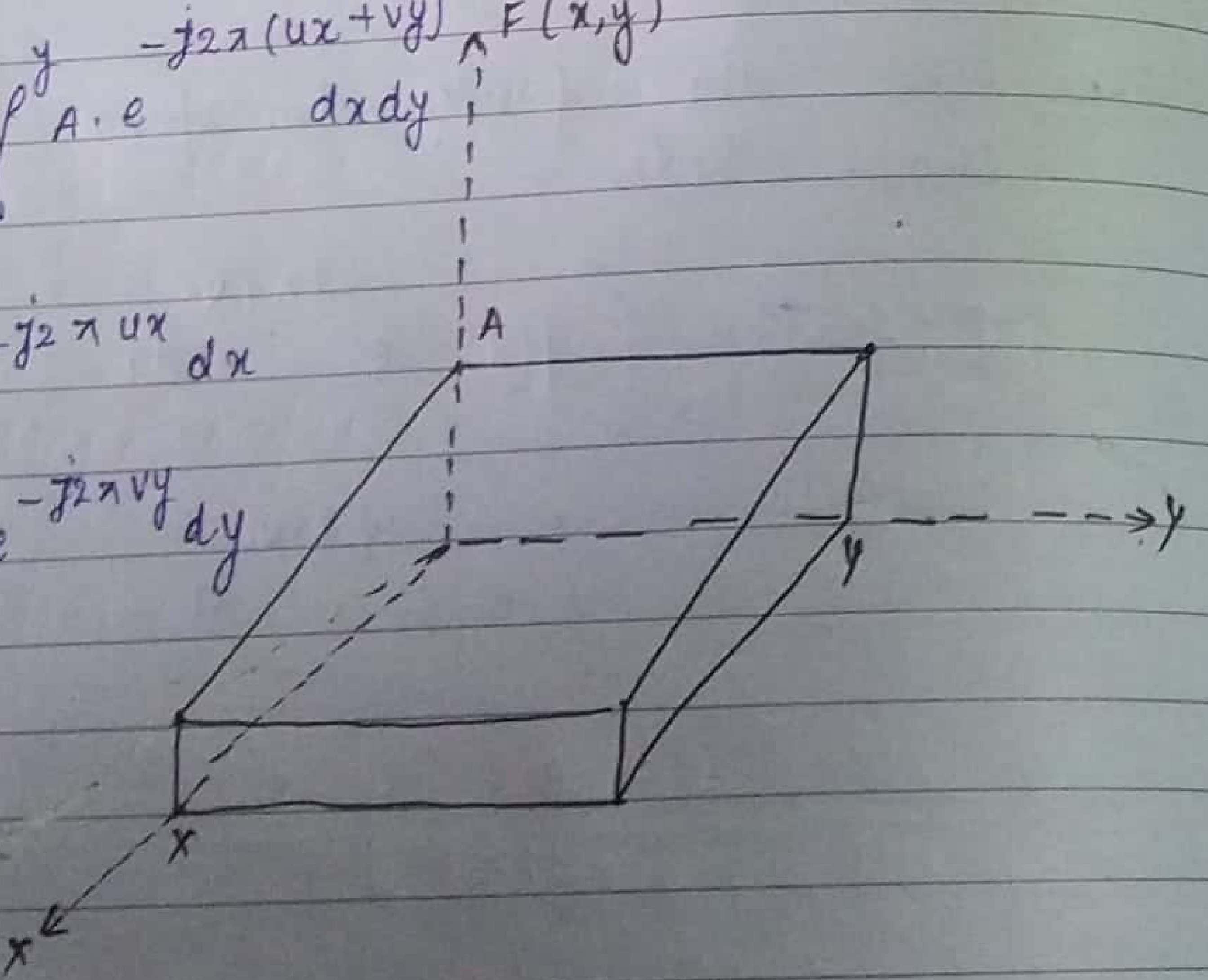
Fourier Transform

Example) Find the 2-D Fourier Transformation of following

$$F(u,v) \Rightarrow \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux-vy)}\, dx \cdot dy$$

$$\Rightarrow \int_0^x \int_0^y A \cdot e^{-j2\pi(ux+vy)}\, dx\, dy$$

$$\Rightarrow A\int_0^x e^{-j2\pi ux}\, dx \int_0^y e^{-j2\pi vy}\, dy$$



$$\Rightarrow \frac{A}{-j2\pi u}\left[e^{-j2\pi ux}\right]_0^x \left(-\frac{1}{j2\pi v}\left[e^{-j2\pi vy}\right]_0^y\right)$$

$$\Rightarrow \frac{A}{-j2\pi u}\left[e^{-j2\pi ux}-1\right]\left(-\frac{1}{j2\pi v}\left[e^{-j2\pi vy}-1\right]\right)$$

$$\Rightarrow \left[\frac{A}{j2\pi u}\left(1-e^{-j2\pi ux}\right)\right]\left[\frac{1}{j2\pi v}\left(1-e^{-j2\pi vy}\right)\right]$$

$$\Rightarrow \left[\frac{A}{j2\pi u}\left(e^{j2\pi ux}-e^{-j2\pi ux}\right)e^{-j2\pi ux}\right]\left[\frac{1}{j2\pi v}\left(e^{+j2\pi vy}-e^{-j2\pi vy}\right)e^{-j2\pi vy}\right]$$

$$\Rightarrow \frac{A}{j2\pi u}\left(2j \sin \pi ux\right) \cdot \frac{1}{j2\pi v}\, 2j \sin \pi vy \cdot e^{-j\pi ux}\, e^{-j\pi vy}$$

$$\Rightarrow A \cdot \left(\frac{\sin \pi ux}{\pi u}\right)\left(\frac{\sin \pi vy}{\pi v}\right)\left(e^{-j\pi ux}\right)\left(e^{-j\pi vy}\right)$$

$$F(u,v) \Rightarrow A \cdot xy \left(\underbrace{\frac{\sin \pi ux}{\pi ux}}_{1}\right)\left(\underbrace{\frac{\sin \pi vy}{\pi vy}}_{1}\right)\left(e^{-j\pi ux}\right)\left(e^{-j\pi vy}\right)$$

$$e^{-j\pi ux} = i \sin \pi ux + \cos \pi ux$$

$$|e^{-j\pi ux}| = \sqrt{\cos^2 \pi ux + \sin^2 \pi ux} = 1$$

$$\Rightarrow |Axy|\, |e^{-j\pi ux}|\, |e^{-j\pi vy}|$$

$$\boxed{|F(u,v)| \Rightarrow Axy}$$

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Discrete Fourier Transformation

→ If we have continuous image and we have sample of these image then it is called discrete values of continuous image.

$$F(u) = \sum_{x=0}^{M-1} f(x)\, e^{-j2\pi ux/M} \quad \text{for } u = 0,1,2 --- M-1$$

$$f(x) = \sum_{u=0}^{M-1} F(u)\, e^{+j2\pi ux/M} \quad \text{for } (x = 0,1,2 --- M-1)$$

Q- Find discrete fourier transformation for $f(x) = \{0,1,2,1\}$

Solution ⇒
$$F(u) = \sum_{x=0}^{M-1} f(x)\, e^{-j2\pi ux/M}.$$

$$= \sum_{x=0}^{3} f(x)\, e^{-j2\pi ux/4}$$

$$\boxed{F(u) = f(0)e^{0} + f(1)e^{-j\frac{2\pi u}{4}} + f(2)e^{-j\frac{\pi u 4}{4}} + f(3)\, e^{-j\frac{2\pi \times 3u}{4}}}$$

\# $F(0) = f(0) + f(1) + f(2) + f(3)$

$= 0 + 1 + 2 + 1$

$\boxed{F(0) = 4}$

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\# $F(1) = f(0) + f(1)\, e^{-j\frac{2\pi}{4 \times 2}} + f(2)\, e^{-j\pi u} + f(3)\, e^{-j\frac{6\pi u}{4}}$

$= 0 + \left(\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right) + 2\left(\cos\pi - j\sin\pi\right) + \left(\cos\frac{6\pi}{4} - j\sin\frac{6\pi}{4}\right)$

$= 0 + (0 - j) + 2(-1 - 0) + (\cos 270° - j\sin 270°)$

$= 0 - j - 2 + j$

$\boxed{F(1) = -2}$

\# $F(2) = f(0) + f(1)\, e^{-j\pi} + f(2)\, e^{-j2\pi} + f(3)\, e^{-j3\pi}$

$= 0 + (\cos\pi - j\sin\pi) + 2(\cos 2\pi - j\sin 2\pi)$
$e\,(\cos 3\pi - j\sin 3\pi)$

$= 0 + (-1 - 0) + 2(1 - 0) + (-0 - 1)$

$= -1 + 2 - 1$

$\boxed{F(2) = 0}$

\# $F(3) = f(0) + f(1)\, e^{-j\frac{6\pi}{4}} + f(2)^{-j3\pi} + f(3)\, e^{-j\frac{18\pi}{4}}$

$= 0 + \left(\cos\frac{6\pi}{4} - j\sin\frac{6\pi}{4}\right) + 2\left(\cos 3\pi - j\sin 3\pi\right)$
$+ \left(\cos\frac{18\pi}{4} - j\sin\frac{18\pi}{4}\right)$

$\boxed{F(3) = -2.}$

For shortcut ; Use twiddle factor

Twiddle factor $= e^{-j\frac{2\pi}{M}}$

$w_M = e^{-j\frac{2\pi}{M}}$ { It will be M×M matrix }

$w_4 = e^{-j\frac{2\pi}{4}}$

$$4\times4 \text{ matrix} = \begin{array}{c|c} & u \rightarrow \\ \hline x \downarrow & \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \end{array}$$

$$\boxed{w_M^{ux} = e^{-j\frac{2\pi ux}{M}}}$$

$$w_4^{ux} = e^{-j\frac{2\pi ux}{4}}$$

$$= \begin{array}{c|c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \hline \begin{array}{c} x=0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} w_4^0 & w_4^0 & w_4^0 & w_4^0 \\ w_4^0 & w_4^1 & w_4^2 & w_4^3 \\ w_4^0 & w_4^2 & w_4^4 & w_4^6 \\ w_4^0 & w_4^3 & w_4^6 & w_4^9 \end{bmatrix} \end{array}$$

$w_4^0 = e^0 = 1$

$w_4^1 = (\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}) = -j$

$w_4^2 = (\cos\pi - j\sin\pi) = -1$

$w_4^3 = (\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}) = j$

$$w_4^{ux} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\Rightarrow F(u) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$ ANS

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Q- Find the DFT of the image

0	1	2	1
1	2	3	2
2	3	4	3
1	2	3	2

Solution) ① Take each row and multiply by twiddle factor to get F(u).

$$F_{r1}(u) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

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$$F_{r2}(u) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

$$F_{r3}(u) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 12 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

$$F_{r4}(u) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

③ Now we take coloumn and multiply each by twiddle factor.

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$F_{c1}(u) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 12 \\ 8 \end{bmatrix}$

$$= \begin{bmatrix} 32 \\ -8 \\ 0 \\ -8 \end{bmatrix}$$

Intermediate result

$$\begin{bmatrix} 4 & -2 & 0 & -2 \\ 8 & -2 & 0 & -2 \\ 12 & -2 & 0 & -2 \\ 8 & -2 & 0 & -2 \end{bmatrix}$$

$F_{c2}(u) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} -2 \\ -2 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$F_{c3}(u) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$F_{c4}(u) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} -2 \\ -2 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Final Result

$$\begin{bmatrix} 32 & -8 & 0 & -8 \\ -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -8 & 0 & 0 & 0 \end{bmatrix}$$

ANS

Properties of DFT

1) Seberability Property

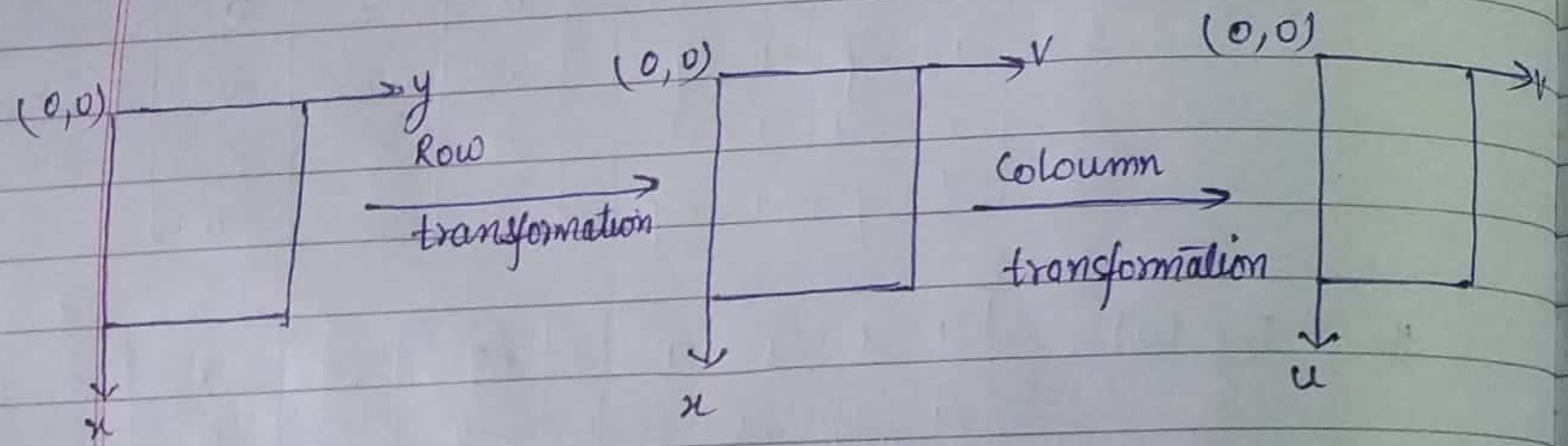
$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$F(u,v) = \sum_{x=0}^{M-1} e^{-j\frac{2\pi ux}{M}} \sum_{y=0}^{N-1} e^{-j\frac{2\pi vy}{N}} f(x,y)$$

If x is fixed

$$F(x,v) = \sum_{y=0}^{N-1} e^{-j\frac{2\pi vy}{N}} f(x,y)$$

$$F(u,v) = \sum_{x=0}^{M-1} F(x,v)\, e^{-j\frac{2\pi vx}{M}}$$



2) Distributivity & Scaling property

- Addition can be distributive but not multiplication

$$\mathcal{F}(af(x,y)) = aF(u,v)$$

$$\mathcal{F}(f(ax,by)) = \frac{1}{ab} F\left(\frac{u}{a},\frac{v}{b}\right)$$

3) Translation or Shifting property

$$f(x,y)\, e^{-j2\pi\left(\frac{u_0 x}{M},\frac{v_0 y}{N}\right)} \xrightarrow[\text{Fourier transform}]{} F(u-u_0, v-v_0)$$

Sub-property (If M & N is center)

$$f(x,y)(-1)^{x+y} \xrightarrow{\text{Fourier transform}} F\left(u-\frac{M}{2}, v-\frac{N}{2}\right)$$

Similarly,

$$f(x-x_0, y-y_0) \xrightarrow{\text{Fourier transform}} F(u,v)\, e^{-j2\pi\left(\frac{ux_0}{M}+\frac{vy_0}{N}\right)}$$

$$F(u-u_0, v-v_0) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{(u-u_0)x}{M}+\frac{(v-v_0)y}{N}\right)}$$

$$= \underbrace{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}}_{F(u,v) \text{ or } \mathcal{F}(f(x,y))} e^{j2\pi\left(\frac{u_0x}{M}+\frac{v_0y}{N}\right)}$$

$\mathcal{F} = F$

4) Rotation Property

Let $x = r\cos\theta$ $\quad u = \omega\cos\phi$

$y = r\sin\theta$ $\quad v = \omega\sin\phi$

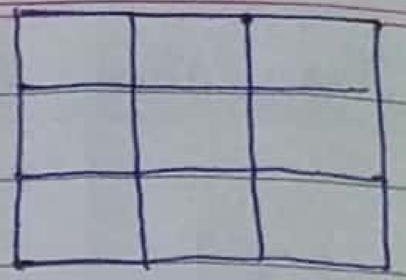
then $f(x,y) \Leftrightarrow f(r,\theta)$

$f(u,v) \Leftrightarrow F(\omega,\phi)$

then $\boxed{f(r,\theta+\theta_0) = F(\omega,\phi+\phi_0)}$

6) Average Value Property

$$\tilde{f}(x,y) = \frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)$$



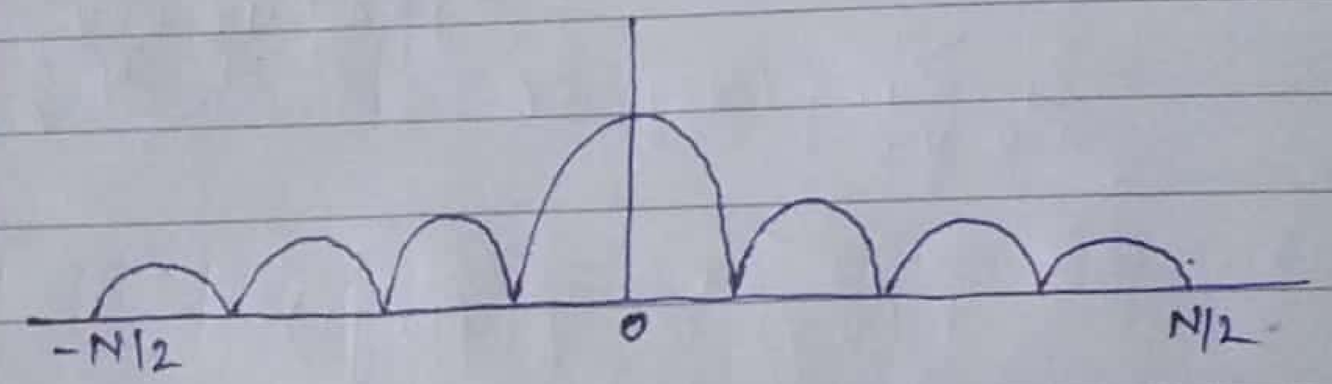
$$F(u,v) = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux+vy}{N}\right)}$$

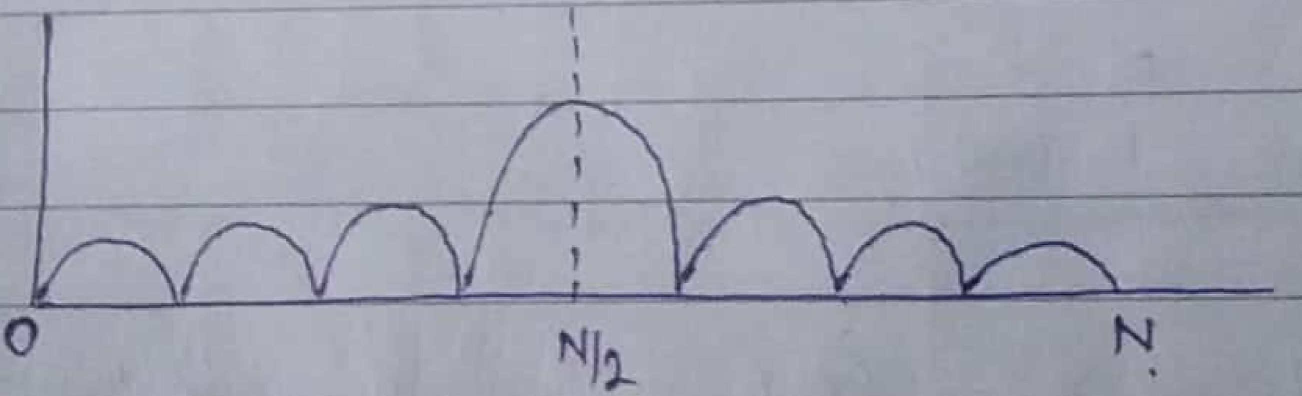
$$F(0,0) = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)$$

$$\boxed{\tilde{f}(x,y) = \frac{1}{N^2}\, F(0,0)}$$

7) Periodicity property

$$F(u,v) = F(u+N,v) = F(u,v+N) = F(u+N,v+N)$$





$$F(u+N,v+N) = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{(u+N)x}{N}+\frac{(v+N)y}{N}\right)}$$

$$= \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux+vy}{N}\right)}\, e^{-j2\pi(x+y)}$$

$$= \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux+vy}{N}\right)}\, 1^{(x+y)}$$

$$= \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux+vy}{N}\right)} = F(u,v)$$

V. imp.

8) Laplacian Property

$$\nabla^2 f(x,y) = \frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2}$$

$$\boxed{F\left(\nabla^2 f(x,y)\right) = -(2\pi)^2\,(u^2+v^2)\,F(u,v)}$$

Inverse DFT (2D)

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\, e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$\frac{\delta f(x,y)}{\delta x} = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\, e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)} \cdot \frac{j2\pi u}{M}$$

Taking $M=N$ & ignoring denominator terms

$$\frac{\delta f(x,y)}{\delta x} = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} F(u,v)\, e^{j2\pi\left(\frac{ux+vy}{N}\right)} \cdot j2\pi u$$

$$= F^{-1}\left\{(j2\pi u)\,F(u,v)\right\}$$

$$\boxed{F\left(\frac{\delta f(x,y)}{\delta x}\right) = [j2\pi u]\,F(u,v)}$$

Similarly

$$F\left(\frac{\delta^2 f(x,y)}{\delta x^2}\right) = (j2\pi u)^2 \, F(u,v) \quad \text{——— (1)}$$

Similarly

$$F\left(\frac{\delta^2 f(x,y)}{\delta y^2}\right) = (j2\pi v)^2 \, F(u,v) \quad \text{——— (2)}$$

Adding 1 & 2

$$F\left(\frac{\delta^2 f(x,y)}{\delta x^2} + \frac{\delta^2 f(x,y)}{\delta y^2}\right) = (j2\pi u)^2 F(u,v) + (j2\pi v)^2 F(u,v)$$

$$F(\nabla^2 f(x,y)) = (2\pi)^2 \, j^2 \, (u^2+v^2) \, F(u,v)$$

$$j = \sqrt{-1}$$

$$j^2 = -1$$

$$\boxed{F(\nabla^2 f(x,y)) = -(2\pi)^2 \, (u^2+v^2) \, F(u,v)}$$

hence proved

9) Convolution Property

$$f(x,y) \cdot g(x,y) = F(u,v) * G(u,v)$$

$$f(x,y) * g(x,y) = F(u,v) \times G(u,v)$$

Convolution → Multiplication (in spatial domain)

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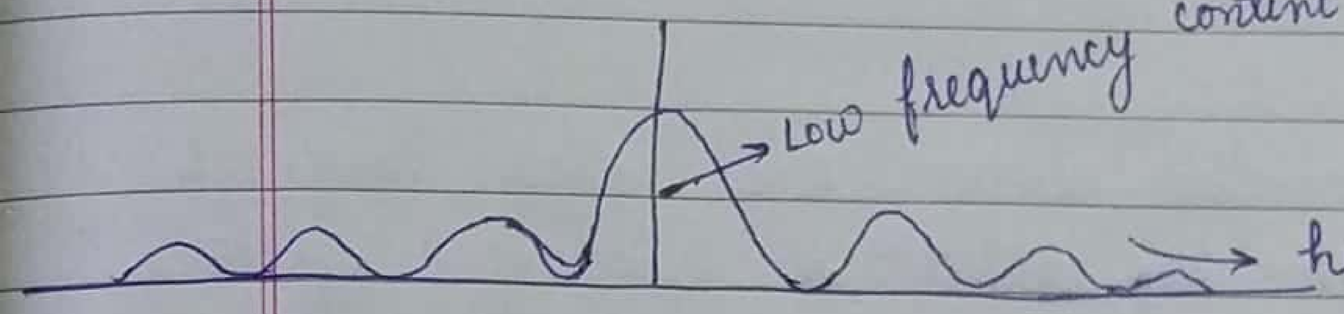
Low Pass Filter

- Ideal low pass filter (Ringing effect)
- Butterworth (ringing effect) (of higher order)
- Gaussian — remove drawback of previous two

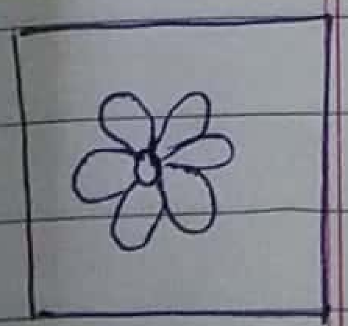
$$\boxed{G(u,v) = F(u,v) \times H(u,v)}$$ → Mask.

(i) Ideal low pass filter :

$$H(u,v) = \begin{cases} 1, & \text{if } D(u,v) \le D_0 \\ 0, & \text{Otherwise} \end{cases}$$

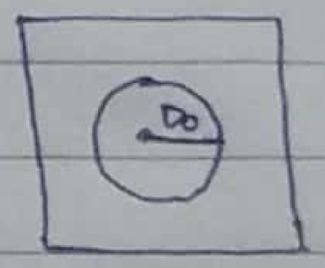


$$D(u,v) = \sqrt{\left(u-\frac{M}{2}\right)^2 + \left(v-\frac{N}{2}\right)^2}$$ = distance of (u,v) from centre $\left(\frac{M}{2}, \frac{N}{2}\right)$.

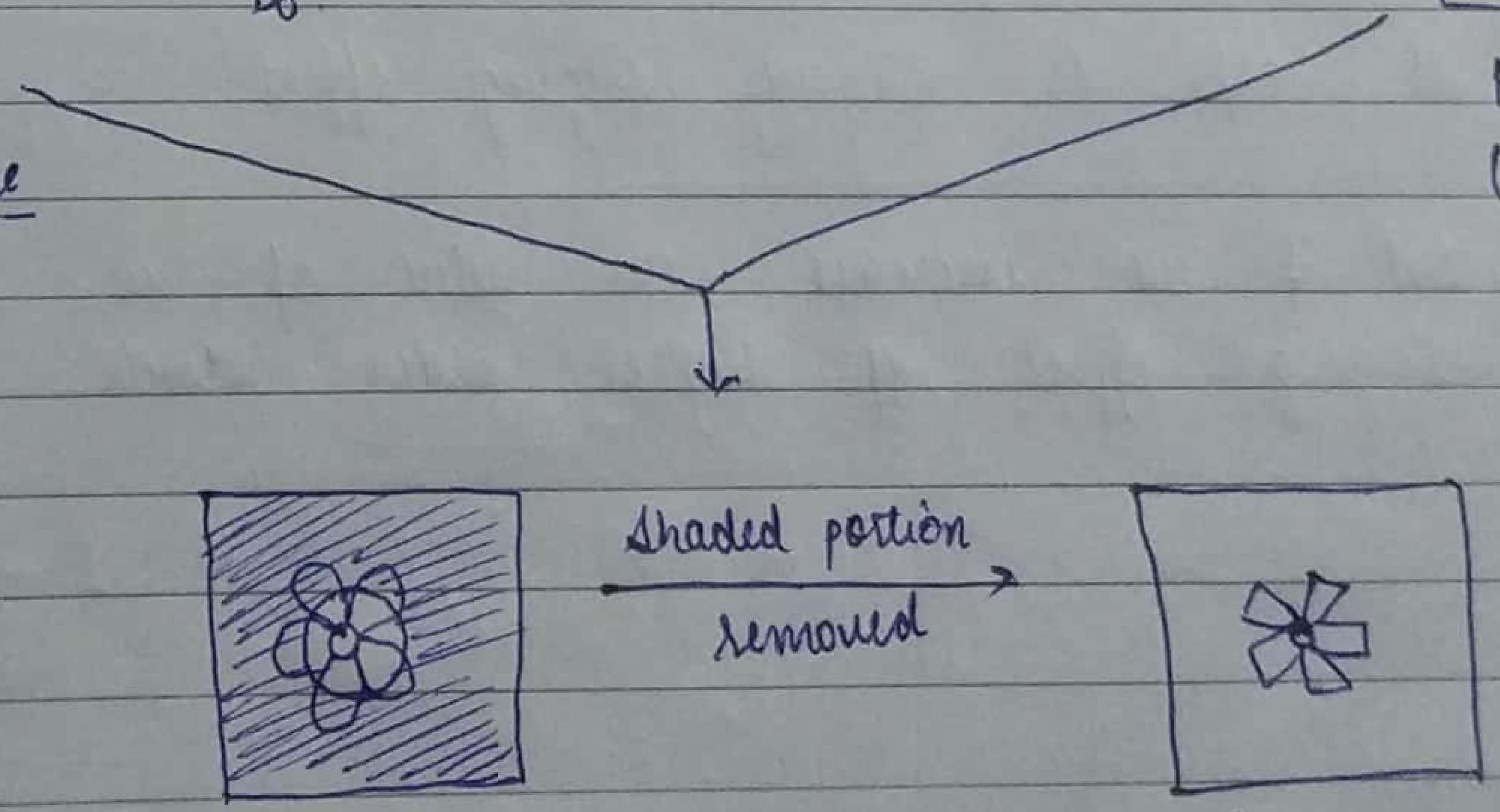


F(u,v)
Original image

D_0 = cutoff frequencies.
= circular crossection with radius D_0.



H(u,v)
(Mask or Filter)



Filtered image

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Drawback

→ In this a problem of Ringing Effect occurs.

→ Ringing Effect -

- In this when image is filtered, smooth part is cutoff & sharp part of image is left.

- Now, when this image undergoes fourier transformation, then the resultant image when seen as a spectrum appears as concentric circles.

- These concentric circles are called rings & this effect is called as ringing effect.

1/March/19

2) Butterworth low pass filter -

$$H(u,v) = \frac{1}{1+\left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

Order $= n$.

- It helps to remove ringing effect.

- But if we increase the value of n, then ringing effect of higher order starts occurring.

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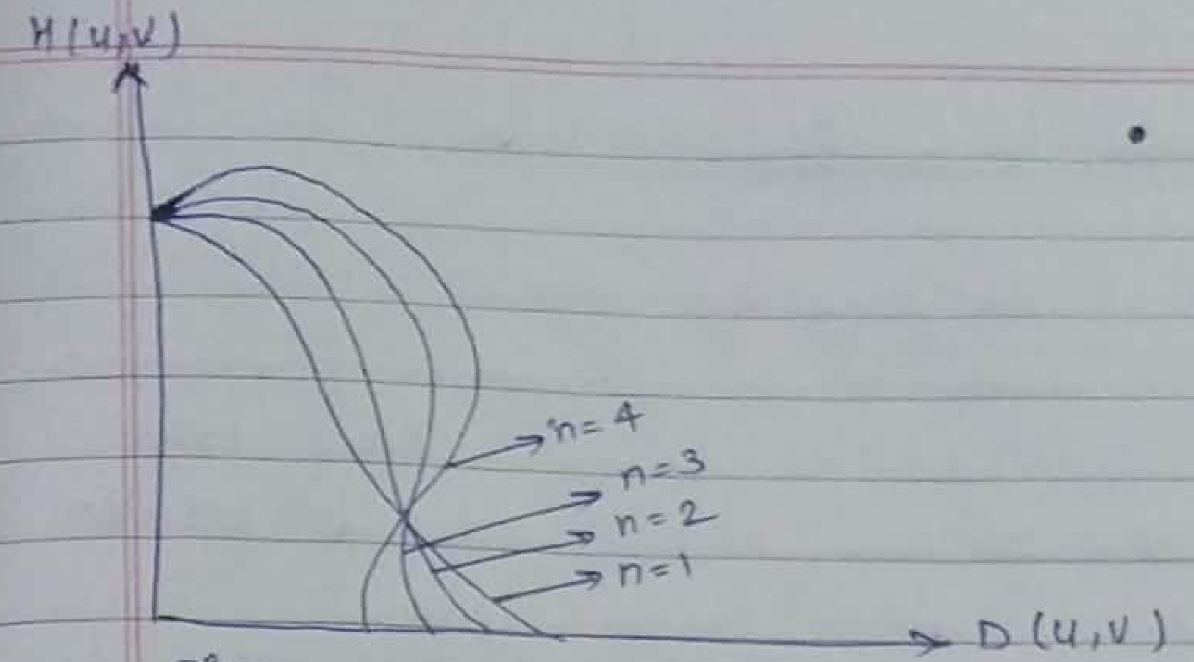
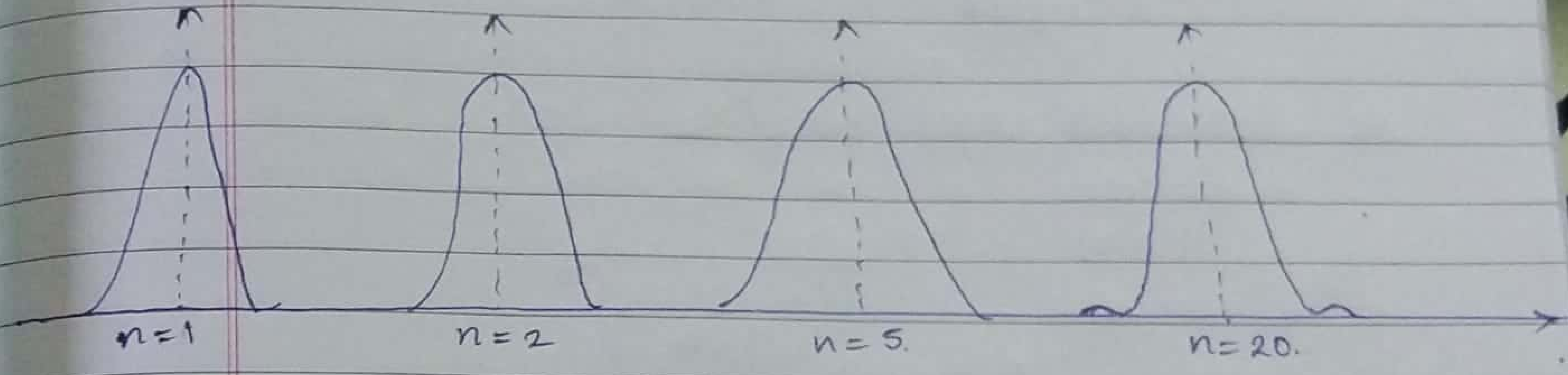


Filter radial cross section

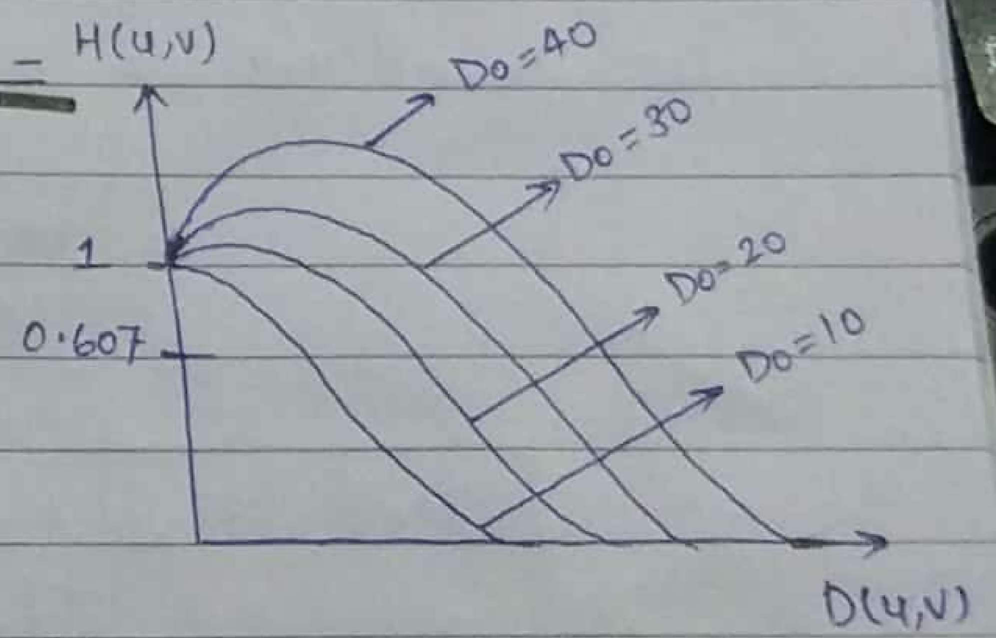
- It does not have a sharp discontinuity that gives a clear cutoff b/w passed and filtered frequencies.



No ringing effect —— Ringing effect

3) Gaussian low pass filter -

$$H(u,v) = e^{-\frac{D^2(u,v)}{2\sigma^2}}$$



σ = represents spread of gaussian curve.

\# If $\sigma = D_0$.

$$H(u,v) = e^{-\frac{D^2(u,v)}{2D_0^2}}$$

★ It has same shape in spatial & frequency domain & thus no ringing effect occurs here.

\# If $D(u,v) = D_0$

→ whole graph shifts to 0.607 as its highest peak.

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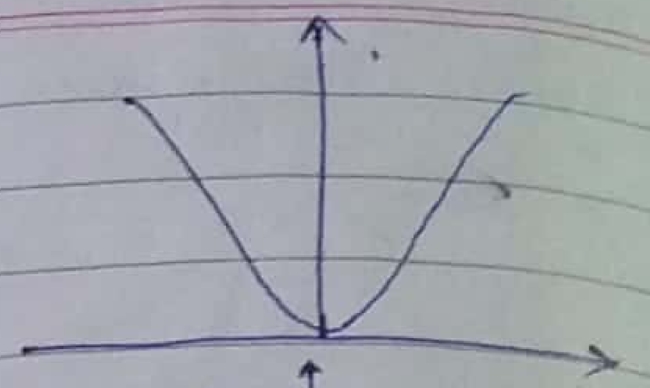
High Pass Filter

$$\boxed{H_{HP} = 1 - H_{LP}}$$



Centre value representation

(a) Ideal high pass filter

$$H = \begin{cases} 0 & D(u,v) \le D_0 \\ 1 & \text{otherwise.} \end{cases}$$

[Theory same as previous]

(b) Butterworth high pass filter

$$H(u,v) = 1 - \frac{1}{1 + \left(\frac{D(u,v)}{D_0}\right)^{2n}}$$

\# If $\left(\frac{D(u,v)}{D_0}\right)^{2n} = x$

$$H(u,v) = 1 - \frac{1}{1+x}$$

$$= \frac{1+x-1}{1+x} = \frac{x}{1+x} = \frac{1}{\frac{1+x}{x}} = \frac{1}{\frac{1}{x}+1}$$

$$\boxed{H(u,v) = \frac{1}{1 + \left[\frac{D_0}{D(u,v)}\right]^{2n}}}$$

(c) Gaussian high pass filter

$$H(u,v) = 1 - e^{-\frac{D^2(u,v)}{2\sigma^2}} = 1 - e^{-\frac{D^2(u,v)}{2D_0^2}}$$

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Unsharp Masking & high boost filtering

\# Unsharp masking = Original image + Sharpen image.

$$g_{mask}(x,y) = f(x,y) - f_{LP}(x,y).$$

\# $$\boxed{g_{final}(x,y) = f(x,y) + k * g_{mask}(x,y)}$$

\# If $k = 1$; unsharp masking
$k > 1$; high boost filtering.

$$f_{LP}(x,y) = H_{LP}(x,y) * f(x,y)$$

Low pass filter — Convolution — original image

→ For frequency domain, we implement fourier transformation on above :

$$g(x,y) = f(x,y) + k * [f(x,y) - f_{LP}(x,y)]$$

$$g(x,y) = f(x,y) + k * [f(x,y) - (H_{LP}(x,y) * f(x,y))]$$

\# Fourier transform on above (Convolution → Multiplication)

$$G(u,v) = F(u,v) + k \times [F(u,v) - \{H_{LP}(u,v) \times F(u,v)\}]$$

$$= F(u,v) + (k \times F(u,v))[1 - H_{LP}(u,v)]$$

$$= F(u,v)\left[1 + k \times [1 - H_{LP}(u,v)]\right]$$

$$\boxed{G(u,v) = F(u,v)\left[1 + k \times H_{HP}(u,v)\right]}$$

ANS

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v.v. Imp.

Homomorphic filter

used for image enhancement when an image is subject to multiplicative noise or interference.

→ Illumination – Low frequency content
Reflectance – high frequency content.

→ To decrease the illumination & increase reflectance is the work of homomorphic filters.

$$f(x,y) = i(x,y) \cdot r(x,y)$$

→ Direct fourier transformation is not possible as multiplication is not distributive in DFT.

→ So we take,

$$Z(x,y) = \ln[f(x,y)]$$
$$Z(x,y) = \ln(i(x,y) \cdot r(x,y))$$
$$Z(x,y) = \ln(i(x,y)) + \ln(r(x,y))$$

– Taking fourier transformation

$$Z(u,v) = F\{\ln(i(x,y))\} + F\{\ln(r(x,y))\}$$

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

– Let enhanced image in frequency domain be –

$$S(u,v) = H(u,v) \times Z(u,v)$$
$$= H(u,v) \times [F_i(u,v) + F_r(u,v)]$$

$$= H(u,v) \times F_i(u,v) + H(u,v) \times F_r(u,v)$$

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→ Implement inverse fourier transformation.

$$S(x,y) = F^{-1}\{H(u,v) \cdot F_i(u,v)\} + F^{-1}\{H(u,v) \cdot F_r(u,v)\}$$

$$\boxed{S(x,y) = i'(x,y) + r'(x,y)}$$

→ Actual enhanced image – (By removing ln)

$$g(x,y) = e^{S(x,y)}$$
$$= e^{[i'(x,y) + r'(x,y)]}$$
$$= e^{i'(x,y)} \cdot e^{r'(x,y)}$$

$$\boxed{g(x,y) = i_0(x,y) \cdot r_0(x,y)}$$

Seperation of illumination & reflactance component.

\# The key approach in homomorphic filtering is to seperate the illumination & reflectance component.

\# Between them $i(x,y)$ contributes to the low frequency and $r(x,y)$ is the high frequency component.

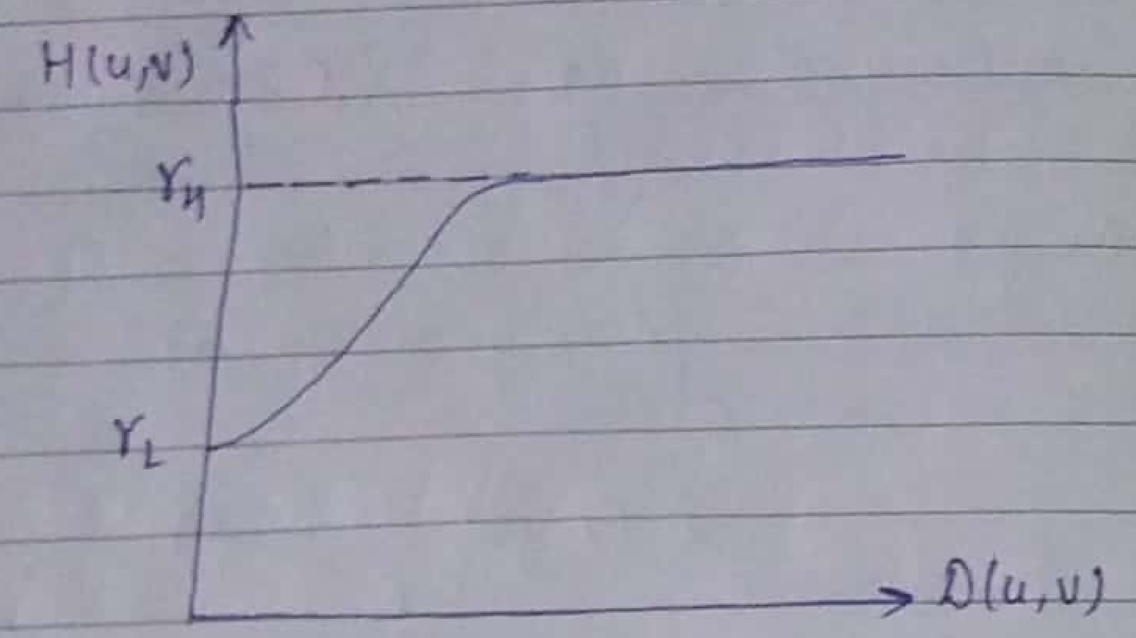
\# Homomorphic filters tends to decrease the contribution made by low frequencies (illumination) & of amplify the contribution made by high frequencies (reflectance).

$H(u,v)$ can be implemented by –

$$\boxed{H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-c\left[\frac{D^2(u,v)}{D_0^2}\right]}\right] + \gamma_L}$$

$\gamma_L < 1$ and $\gamma_H > 1$

#



γ_H and γ_L are the controlling factors to decrease $i(x,y)$ and increase $r(x,y)$.

γ_H & γ_L = constant

c = controlling factor that controls sharpness of the slope.

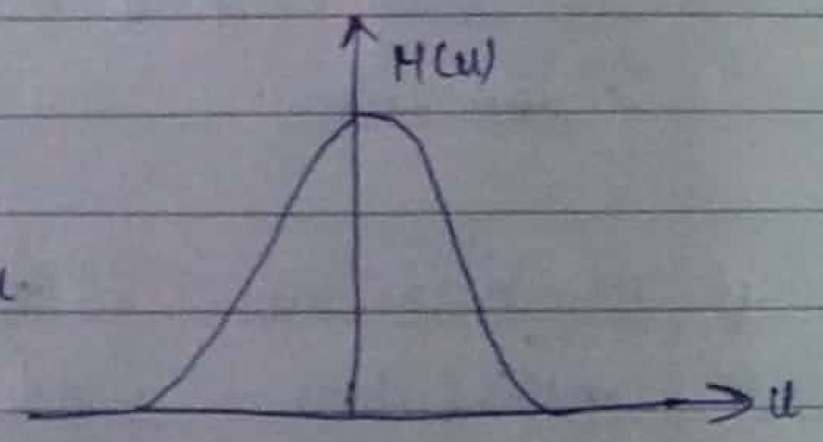
4/March/19

V.V. Imp

Correspondence b/w filters

1) Generating $h(x,y)$ from $H(u,v)$:

$$H(u) = A e^{-u^2/2\sigma^2}$$



Inverse fourier transform ⇒

$$h(x) = \int_{-\infty}^{\infty} H(u)\, e^{-j2\pi ux}\, du$$

$$= \int_{-\infty}^{\infty} A e^{-u^2/2\sigma^2} e^{-j2\pi ux}\, du$$

$$= \int_{-\infty}^{\infty} A e^{-\frac{1}{2\sigma^2}\left[u^2 - j4\pi ux\sigma^2\right]}\, du$$

$$\therefore\ e^{-(2\pi)^2 x^2\sigma^2/2} \cdot e^{(2\pi)^2 x^2\sigma^2/2} = 1$$

hence

$$A\int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}\left[u^2 - j4\pi ux\sigma^2 - (2\pi)^2 x^2\sigma^4\right]}\, e^{-2\pi^2x^2\sigma^2}\, du$$

#

$$h(x) = A\int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}\left[u - j2\pi x\sigma^2\right]^2}\, e^{-2\pi^2x^2\sigma^2}\, du$$

Let $u - j2\pi x\sigma^2 = s$.

$du = ds$.

$$= A\int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}s^2}\, e^{-2\pi^2x^2\sigma^2}\, ds.$$

$$= A e^{-2\pi^2x^2\sigma^2}\int_{-\infty}^{\infty} e^{-s^2/2\sigma^2}\, ds$$

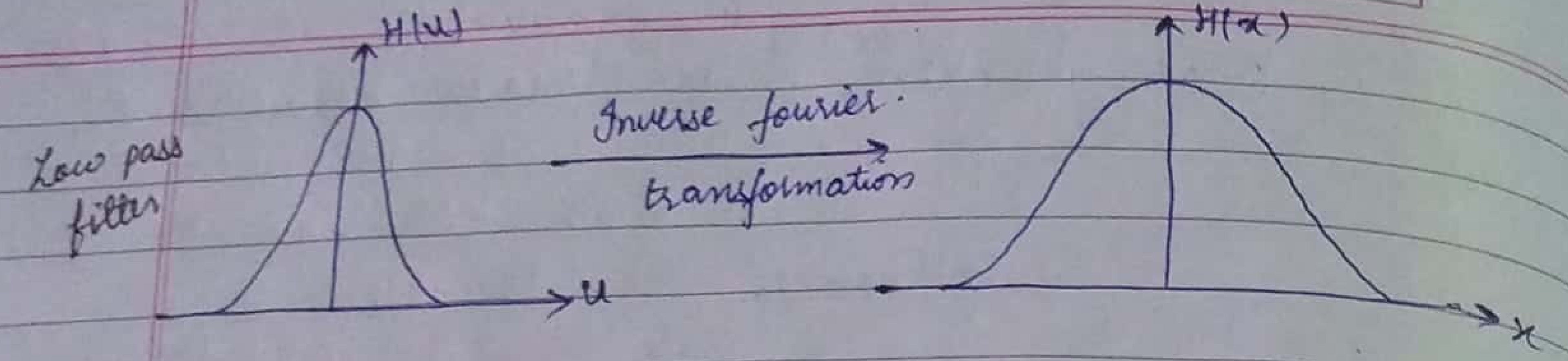
Multiply & divide by $\sqrt{2\pi}\sigma$.

$$= A\sqrt{2\pi}\sigma\, e^{-2\pi^2x^2\sigma^2}\underbrace{\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma} e^{-s^2/2\sigma^2}\, ds}$$

equal to 1 since integrating all probabilities gives us 1.

#

$$\boxed{h(x) = A\sqrt{2\pi}\sigma\, e^{-2\pi^2x^2\sigma^2}}$$

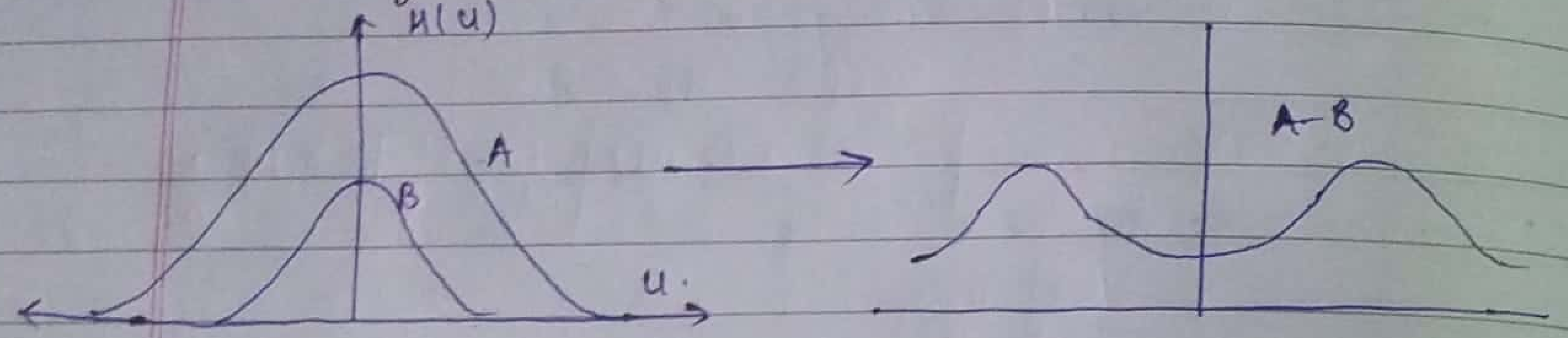


Narrower the H(u), broader the h(x).

<u>In 2D representation</u>

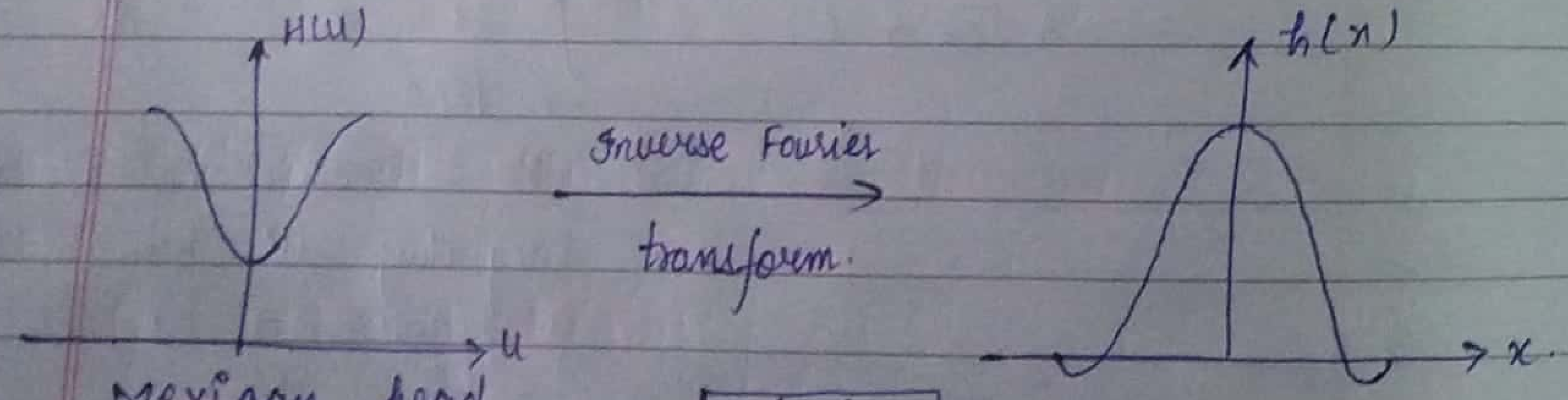
$$h(x,y) = A\sqrt{2\pi}\,\sigma\, e^{-2\pi^2(x^2+y^2)\sigma^2}$$

<u>For high pass filter</u>



$$H(u) = Ae^{-u^2/2\sigma^2} - Be^{-u^2/2\sigma^2}$$

$$\boxed{h(x) = A\sqrt{2\pi}\sigma_1 e^{-2\pi^2x^2\sigma_1^2} - B\sqrt{2\pi}\,\sigma_2 e^{-2\pi^2x^2\sigma_2^2}}$$



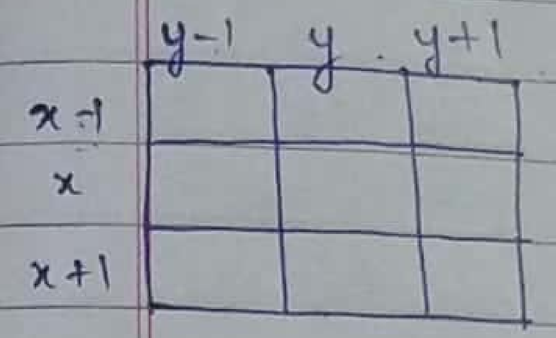
Mexican head shape filter

-1	-1	-1
-1	8	-1
-1	-1	-1

Mask filter

2) <u>Generating H(u,v) from h(x,y)</u>

$$f(x-x_0, y-y_0) = F(u,v)\, e^{-j2\pi\left(\frac{ux_0+vy_0}{N}\right)}$$



<u>h(x,y).</u>
<u>Image. (3×3)</u>

$\longrightarrow f(x-1, y-1)$

$\boxed{x_0 = 1 \;;\; y_0 = 1}$

$$= F(u,v)\, e^{-j2\pi\left(\frac{u+v}{N}\right)}$$

$f(x-1, y) = F(u,v)\, e^{\frac{-j2\pi u}{N}}$

$f(x-1, y+1) = F(u,v)\, e^{-j2\pi\left(\frac{u-v}{N}\right)}$

$f(x, y-1) = F(u,v)\, e^{\frac{-j2\pi v}{N}}$

$f(x, y) = F(u,v)\, e^{-j2\pi \times 0} = F(u,v)$

$f(x, y+1) = F(u,v)\, e^{\frac{j2\pi v}{N}}$

$f(x+1, y-1) = F(u,v)\, e^{-j2\pi\left(\frac{v-u}{N}\right)}$

$f(x+1, y) = F(u,v)\, e^{\frac{j2\pi u}{N}}$

$f(x+1, y+1) = F(u,v)\, e^{j2\pi\left(\frac{u+v}{N}\right)}$

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Imp.
Q- Given a spatial mask $h(x,y)$. Find the equivalent filter $H(u,v)$ in the frequency domain. Also comment on what type of filter $h(x,y)$ is?

	$y-1$	y	$y+1$
$x-1$	0	1/6	0
x	1/6	1/3	1/6
$x+1$	0	1/6	0

$h(x,y)$.
Low pass filter

Since all elements are +ve, it is a low pass filter.

Converting it into frequency domain

$$\frac{1}{6} f(x-1,y) = \frac{1}{6}\left[F(u,v)\, e^{-j\frac{2\pi u}{N}}\right]$$

$$\left\{\begin{array}{l} af(x,y) \longrightarrow aF(u,v) \\ f(ax,by) \longrightarrow \frac{1}{ab} F\left(\frac{u}{a},\frac{v}{b}\right) \end{array}\right\}$$

Scaling Property

$$\frac{1}{6} f(x,y-1) = \frac{1}{6}\left[F(u,v)\, e^{-j\frac{2\pi v}{N}}\right]$$

$$\frac{1}{6} f(x,y+1) = \frac{1}{6}\left[F(u,v)\, e^{j\frac{2\pi v}{N}}\right]$$

$$\frac{1}{3} f(x,y) = \frac{1}{3}\left[F(u,v)\right]$$

$$\frac{1}{6} f(x+1,y) = \frac{1}{6}\left[F(u,v)\, e^{+j\frac{2\pi u}{N}}\right]$$

Adding all the above values we get-

$$G(u,v) = \frac{1}{6} F(u,v)\left[e^{-j\frac{2\pi u}{N}} + e^{-j\frac{2\pi v}{N}} + e^{j\frac{2\pi v}{N}} + e^{j\frac{2\pi u}{N}}\right] + \frac{1}{3} F(u,v)$$

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$G(u,v) = F(u,v)\left\{\frac{1}{6}\left(e^{-j\frac{2\pi u}{N}} + e^{j\frac{2\pi u}{N}}\right) + \frac{1}{6}\left(e^{-j\frac{2\pi v}{N}} + e^{j\frac{2\pi v}{N}}\right)\right\} + \frac{1}{3} F(u,v)$

Note:

$$\Rightarrow e^{-j\frac{2\pi u}{N}} + e^{j\frac{2\pi u}{N}} = \cos\frac{2\pi u}{N} - j\sin\frac{2\pi u}{N} + \cos\frac{2\pi u}{N} + j\sin\frac{2\pi u}{N}$$

$$\Rightarrow e^{-j\frac{2\pi u}{N}} + e^{j\frac{2\pi u}{N}} = 2\cos\frac{2\pi u}{N}$$

$G(u,v) = \frac{1}{6} F(u,v)\left(2\cos\frac{2\pi u}{N} + 2\cos\frac{2\pi v}{N}\right) + \frac{1}{3} F(u,v)$

$$\boxed{G(u,v) = \frac{1}{3} F(u,v)\left[\cos\frac{2\pi u}{N} + \cos\frac{2\pi v}{N} + 1\right]}$$

We also know $G(u,v) = F(u,v) \times H(u,v)$

$$\boxed{H(u,v) = \frac{1}{3}\left[\cos\frac{2\pi u}{N} + \cos\frac{2\pi v}{N} + 1\right]}$$

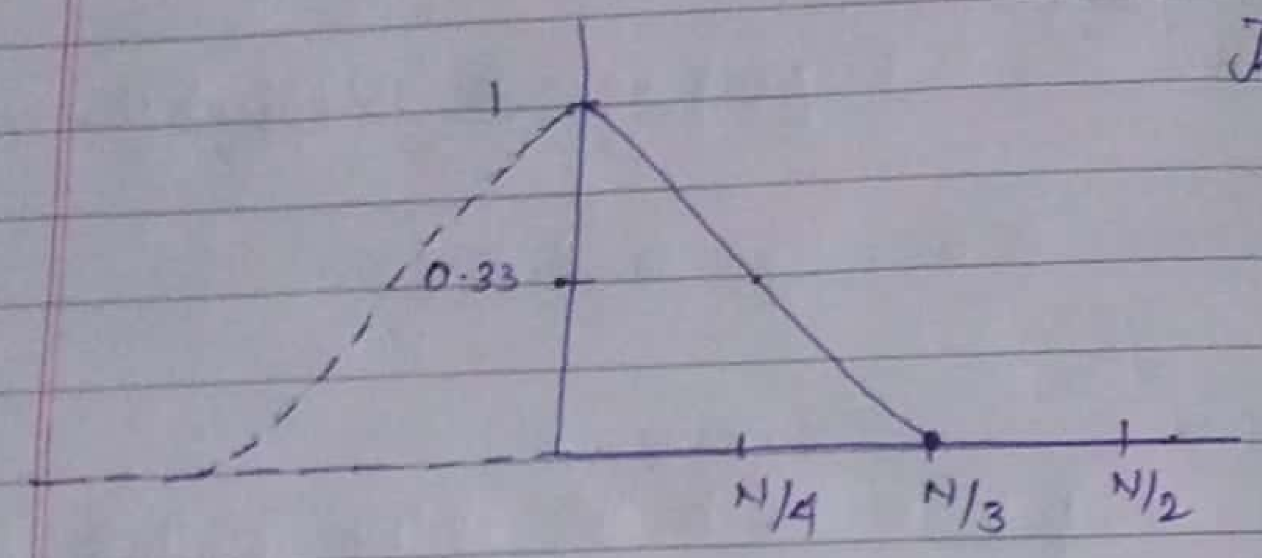
$H(u,v)\Big|_{u=0,\ v=0} = \frac{1}{3}\left[1+1+1\right] = \frac{3}{3} = 1.$

$H(u,v)\Big|_{u=v=\frac{N}{4}} = \frac{1}{3}\left[0+0+1\right] = \frac{1}{3} = 0.33$

Scanned by CamScanner

$H(u,v)\Big|_{u=v=\frac{N}{3}} = \frac{1}{3}\left[\frac{-1}{2} - \frac{1}{2} + 1\right] = 0$

Plotting the graph we will see which type filter it is -



This shows, it is a Low pass filter.

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Q- Given $h(x,y)$

	y-1	y	y+1
x-1	1	-3	1
x	-3	9	-3
x+1	1	-3	1

Find frequency domain filter $H(u,v)$ & also comment on the type of filter.

Solution) It is a high pass filter (both +ve & -ve values)

Converting into frequency domain

$$f(x-1, y-1) = F(u,v)\, e^{-j2\pi\left(\frac{u+v}{N}\right)}$$

$$-3 f(x-1, y) = -3\left[F(u,v)\, e^{-j\frac{2\pi u}{N}}\right]$$

$$f(x-1, y+1) = F(u,v)\, e^{-j2\pi\left(\frac{u-v}{N}\right)}$$

$$-3 f(x, y-1) = -3\left[F(u,v)\, e^{-j\frac{2\pi v}{N}}\right]$$

$$9 f(x,y) = 9\left[F(u,v)\, e^{-j2\pi v \times 0}\right] = 9 F(u,v)$$

$$-3 f(x, y+1) = -3\left[F(u,v)\, e^{j\frac{2\pi v}{N}}\right]$$

$$f(x+1, y-1) = F(u,v)\, e^{-j2\pi\left(\frac{v-u}{N}\right)}$$

$$-3 f(x+1, y) = -3\left[F(u,v)\, e^{j\frac{2\pi u}{N}}\right]$$

$$f(x+1, y+1) = F(u,v)\, e^{j2\pi\left(\frac{u+v}{N}\right)}$$

Adding all the terms we get-

$$G(u,v) = F(u,v)\Big[e^{-j2\pi\left(\frac{u+v}{N}\right)} - 3e^{-j\frac{2\pi u}{N}} + e^{-j2\pi\left(\frac{u-v}{N}\right)} - 3e^{-j\frac{2\pi v}{N}} + 9 + e^{-j2\pi\left(\frac{v-u}{N}\right)} - 3e^{j\frac{2\pi v}{N}} + e^{j2\pi\left(\frac{u+v}{N}\right)} - 3e^{j\frac{2\pi u}{N}}\Big]$$

$$= F(u,v)\Big[\left(e^{j2\pi\left(\frac{u+v}{N}\right)} + e^{-j2\pi\left(\frac{u+v}{N}\right)}\right) + \left(e^{-j2\pi\left(\frac{u-v}{N}\right)} + e^{+j2\pi\left(\frac{u-v}{N}\right)}\right) + -3\left(e^{j\frac{2\pi u}{N}} + e^{-j\frac{2\pi u}{N}}\right) - 3\left(e^{j\frac{2\pi v}{N}} + e^{-j\frac{2\pi v}{N}}\right) + 9\Big]$$

$$= F(u,v)\left[2\cos 2\pi\left(\frac{u+v}{N}\right) + 2\cos 2\pi\left(\frac{u-v}{N}\right) - 6\cos\frac{2\pi u}{N} - 6\cos\frac{2\pi v}{N} + 9\right]$$

\# So we get – $\boxed{G(u,v) = F(u,v) \times H(u,v)}$

$$H(u,v) = 2\cos 2\pi\left(\frac{u+v}{N}\right) + 2\cos 2\pi\left(\frac{u-v}{N}\right) - 6\cos\frac{2\pi u}{N} - 6\cos\frac{2\pi v}{N} + 9.$$

\# Putting values to draw a graph

$$H(u,v)\Big|_{\substack{u=0\\v=0}} = 2 + 2 - 6 - 6 + 9 = 13 - 12 = 1$$

$$H(u,v)\Big|_{u=v=\frac{N}{4}} = 0 + 2 - 0 - 0 + 9 = 11$$

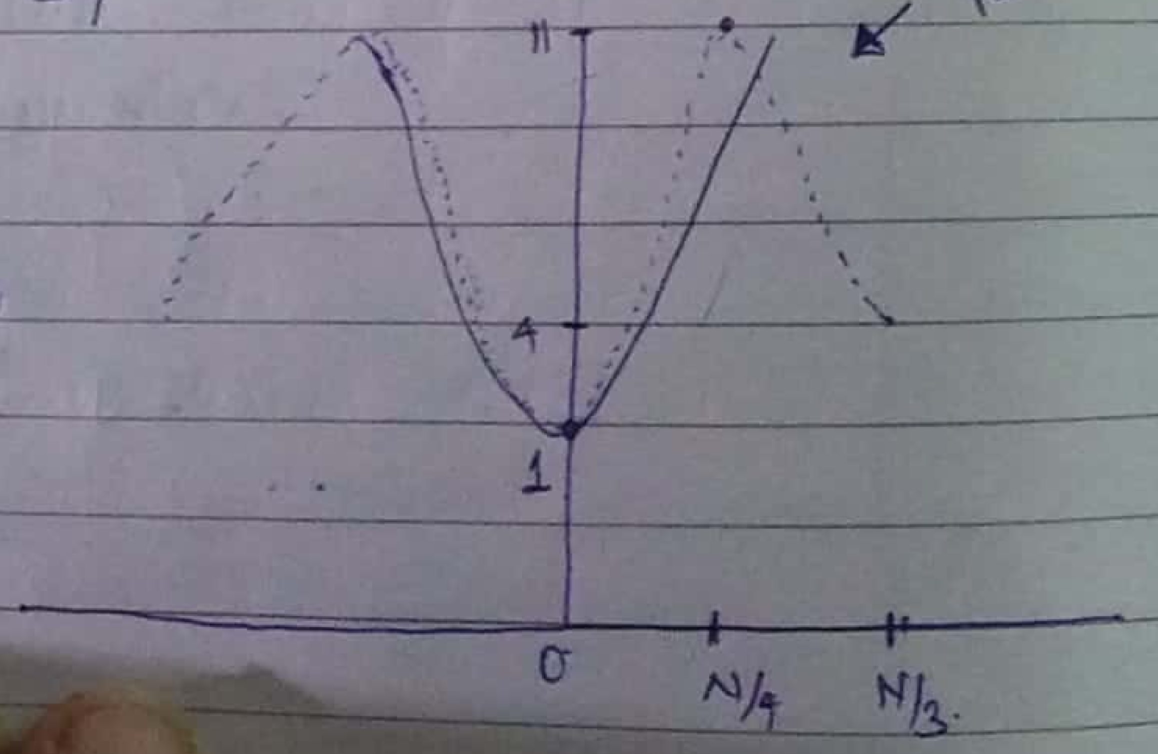
$$H(u,v)\Big|_{u=v=\frac{N}{3}} = -2\cos\frac{\pi}{3} + 2 - 6\cos\frac{\pi}{3} - 6\cos\frac{\pi}{3} + 9$$

$$= -2\times\frac{1}{2} + 2 - 6\times\frac{1}{2} - 6\times\frac{1}{2} + 9$$

$$= -1 + 2 - 3 - 3 + 9$$

$$= 11 - 7$$

$$= 4$$



\# Thus, it is a high pass filter.

Ans

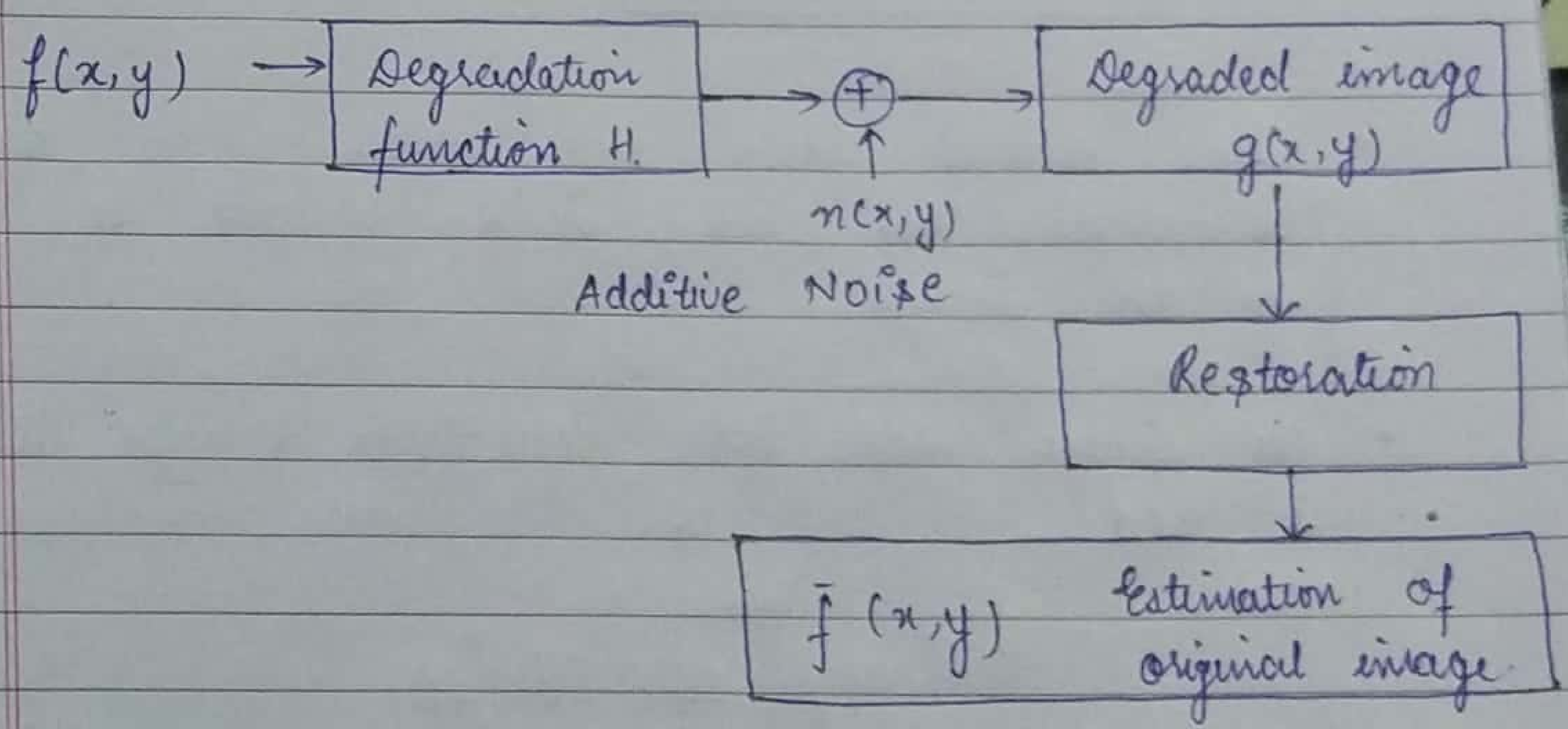
UNIT – 3

6/March/19

Image Restoration

→ Not a technique but approximation to get back the original image.

→ It is an objective process i.e. predefined mathematical formula is used and everyone gets the same kind of image using this process.

→ 

→ $g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$

where $f(x,y)$ = original image
$g(x,y)$ = Degraded image
$h(x,y)$ = Degradation function
$*$ = Convolution.
$\eta(x,y)$ = Noise.

→ In frequency domain

$$\boxed{G(u,v) = F(u,v) \times H(u,v) + N(u,v)}$$

① Inverse filter

$$G(u,v) = F(u,v) \times H(u,v) + N(u,v)$$

$$F(u,v) = \frac{G(u,v)}{H(u,v)} - \frac{N(u,v)}{H(u,v)}$$

For inverse filter we say $N(u,v) = 0$. So –

$$\boxed{F(u,v) = \frac{G(u,v)}{H(u,v)}}$$

- Thus inverse filter only works when degradation is there but no noise should be present in the image.
- For image with both degradation & noise Wiener filter is used.

Types of Noise

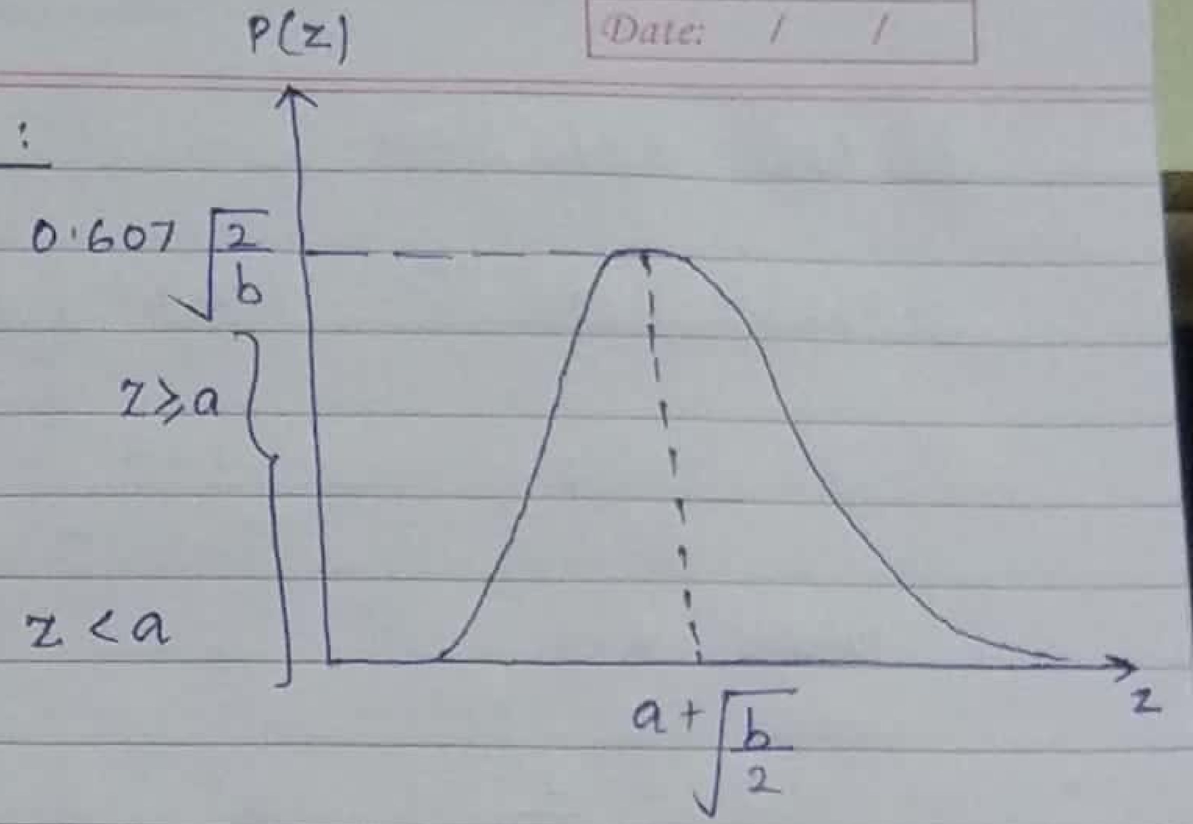
Source of noise

① Gaussian Noise : Poor illumination
② Salt & Pepper Noise (Impulse Noise) : ~~Rav~~ Faulty switch
③ Rayleigh Noise : Range imaging
④ Gamma Noise : Laser imaging
⑤ Exponential Noise : Laser imaging
⑥ Uniform Noise : Simulations
⑦ Periodic Noise.

Gaussian & Salt and Pepper is in UNIT-1.

④ Rayleigh Noise :

$$P(z) = \begin{cases} \frac{2}{b}(z-a)\, e^{-\frac{(z-a)^2}{b}} & z \geq a \\ 0 & z < a \end{cases}$$



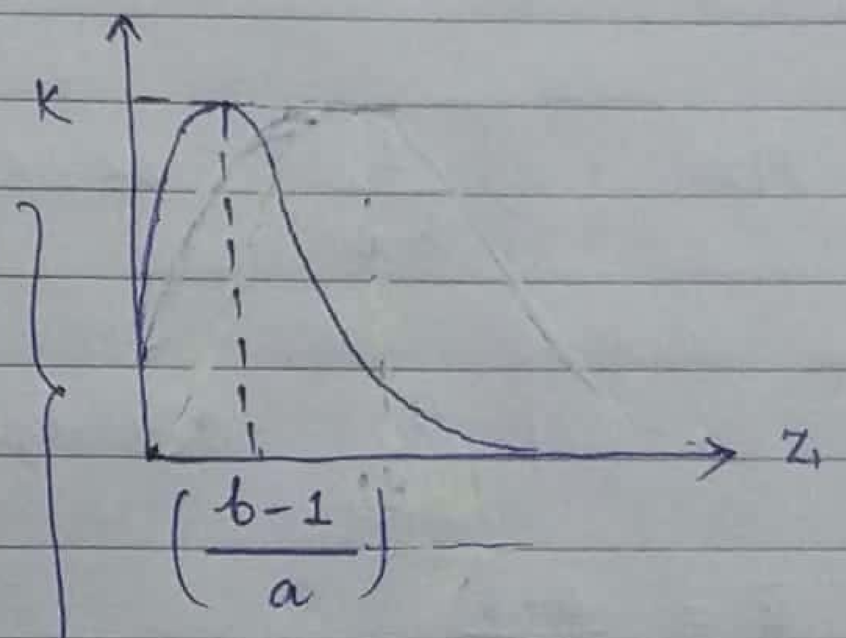
$$\bar{z} = a + \sqrt{\frac{\pi b}{4}}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

- Whenever we implement watermarking in an image then Rayleigh noise may occur.

⑤ Gamma Noise :

$$P(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & z < 0 \end{cases}$$



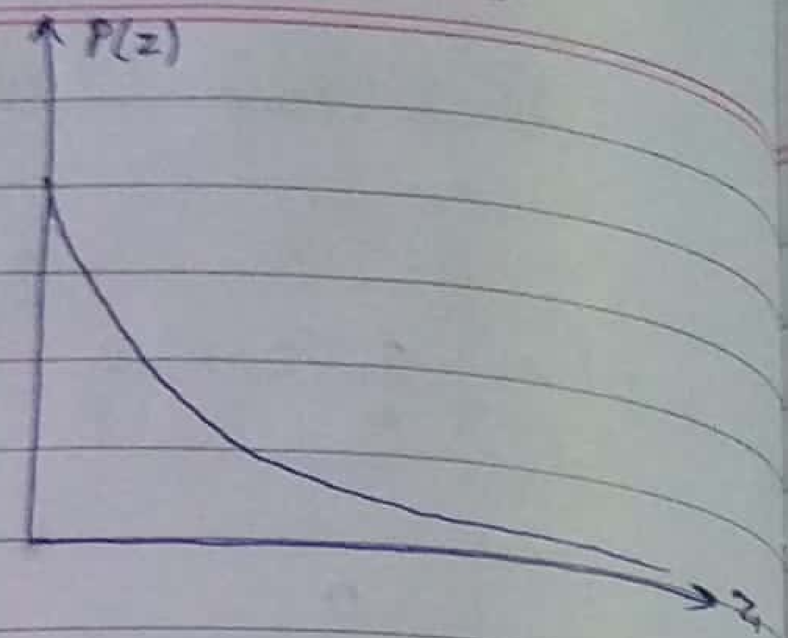
here $a > 0$ & b is +ve constant

$$\bar{z} = b/a$$

$$\sigma^2 = b/a^2$$

$$K = \frac{a(b-1)^{b-1}}{(b-1)!} e^{-(b-1)}$$

③ Exponential Noise :



$$P(z) = \begin{cases} a e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

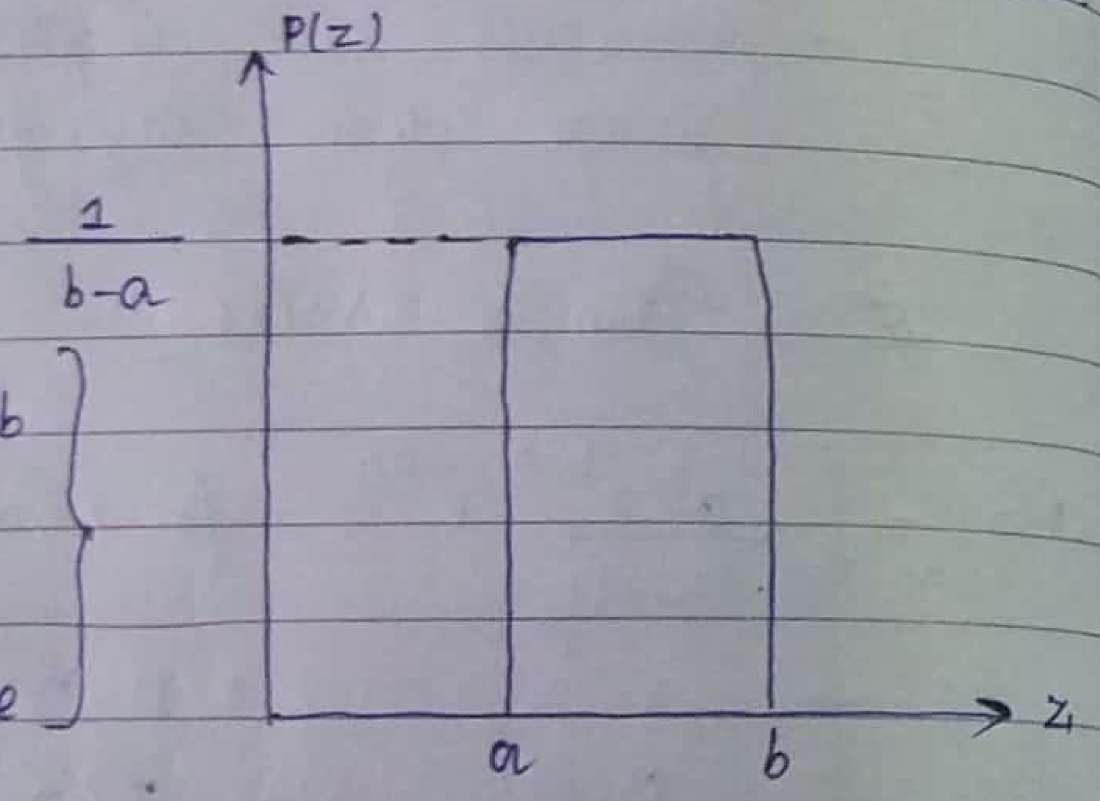
where $a > 0$

$$\bar{z} = \frac{1}{a}$$

$$\sigma^2 = \frac{1}{a^2}$$

When $b = 1$; Gamma noise becomes exponential noise.

④ Uniform Noise :



$$P(z) = \begin{cases} \frac{1}{b-a} & a \leq z \leq b \\ 0 & \text{Otherwise} \end{cases}$$

$$\bar{z} = \frac{a+b}{2}$$

$$\sigma^2 = \frac{(b-a)^2}{12}$$

★ It is quite useful as the basis for numerous random no. generators that are used in simulations.

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⑤ Periodic Noise : If we have a image, we can find fourier spectrum and can show whether periodic noise is there or not.

It occurs due to electromechanical issue & happens at the time of image acquisition.

Restoration in the presence of noise only

We know that -

$$G(u,v) = F(u,v) \times H(u,v) + N(u,v)$$

When only noise is present & no degradation then

$$G(u,v) = F(u,v) + N(u,v) \quad \text{(In frequency domain)}$$

$$g(x,y) = f(x,y) + n(x,y) \quad \text{(In spatial domain)}$$

Type of filters

① Mean filter
② Order statistics filter
③ Mid point filter
④ Alpha trimmed filter.

Mean filter → AM
Mean filter → GM
Mean filter → HM → Contra-harmonic mean filter

Order statistics filter → Median filter.
Order statistics filter → Min-Max filter

① Mean filter :

(I) Arithmetic Mean filter

$$\bar{f}(x,y) = \frac{1}{MN} \sum_{(s,t) \in S_{xy}} g(s,t)$$

where S_{xy} = set of coordinates of rectangular subimage window of size (M×N).

(II) Geometric Mean filter

$$\bar{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{1/MN}$$

Drawback - we lose some details of the image

\# Weighted filter $= \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

\# By default filter $= \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ → standard filter to be used if not given.

(III) Harmonic Mean filter

$$\bar{f}(x,y) = \frac{MN}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$$

Drawback: It can easily remove salt noise but fails in removing pepper noise.

(IV) Contra-Harmonic Mean filter

$$\bar{f}(x,y) = \frac{\sum_{(s,t) \in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s,t)^{Q}}$$

where Q = Order of the filter

Conditions :
- If $Q > 0$; eliminate pepper noise
- If $Q < 0$; eliminate salt noise
- If $Q = 0$; AM filter
- If $Q = 1$; HM filter.

Drawback: Remove both salt & pepper noise but seperately.

② Order - Statistics filter :

(I) Median filter

$$\bar{f}(x,y) = \underset{(s,t) \in S_{xy}}{\text{median}} \{g(s,t)\}$$

(II) Max and Min filter

$$\bar{f}(x,y) = \max_{(s,t) \in S_{xy}} \{g(s,t)\}$$

(also called 100 percentile filter)

* It is basically use to remove the pepper noise.

* Min filter is use to remove the salt noise

* It is also called as 0^{th} percentile filter.

$$\bar{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

③ Mid Point filter :

$$\bar{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min_{(s,t)\in S_{xy}} \{g(s,t)\}\right]$$

- It is used for randomly distributed noise like gaussian noise.

④ Alpha Trimmed Mean filter :

$$\bar{f}(x,y) = \frac{1}{MN-d} \sum_{(s,t)\in S_{xy}} g(s,t)$$

where : $0 \le d \le MN-1$

eg: If we have intensity value 10, 20, 30, 40, 50, 60, 70, 80

$d = 2$. (10 → Remove, 80 → Remove)

$$\bar{f}(x,y) = \frac{1}{8-2} = \frac{1}{6} \sum \left[20+30+40+50+60+70\right] =$$

* For alpha trimmed mean filter - we remove both low & high intensity pixel.

* For low intensity alpha trimmed filter - we only remove low intensity pixel.

* For high intensity alpha trimmed filter - we only remove so high intensity pixel.

Q- Given below is a 3x3 image. What will be the value of centre value pixel change when the image is passed through-

(a) AM filter
(b) GM filter
(c) HM filter
(d) 0^{th} percentile filter
(e) 100 percentile filter

1	7	5
6	2	3
1	4	2

(a) AM filter

$$\bar{f}(x,y) = \frac{1}{9}\left[1\times1 + 7\times1 + 5\times1 + 6\times1 + 2\times1 + 3\times1 + 1\times1 + 4\times1 + 2\times1\right] \quad g(x,y)$$

$$= \frac{1}{9}\left[1+7+5+6+2+3+1+4+2\right]$$

$$= \frac{1}{9}\left[31\right] = 3.44 \cong 3$$

(b) GM filter

$$\bar{f}(x,y) = \left[1\times7\times5\times6\times2\times3\times1\times4\times2\right]^{1/3\times3}$$

$$= (10080)^{1/9} = 2.78 \cong 3$$

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(c) HM filter

$$\hat{f}(x,y) = \frac{9}{\frac{1}{1}+\frac{1}{7}+\frac{1}{5}+\frac{1}{6}+\frac{1}{2}+\frac{1}{3}+\frac{1}{1}+\frac{1}{4}+\frac{1}{2}}$$

$$= \frac{9}{4.0928} = 2.198 \approx 2.$$

(d) 0^{th} percentile filter

$\hat{f}(x,y) = \min\{1,7,5,6,2,3,1,4,2\}$
$= 1$

(e) 100^{th} percentile filter

$\hat{f}(x,y) = \max\{1,7,5,6,2,3,1,4,2\}$
$= 7$

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v.v.imp

Wiener Filter / Mean square error filter

\# when image has both degradation and noise we use this filter.

$$\boxed{e^2 = E\{f(x,y) - \hat{f}(x,y)\}}$$

where $E\{\cdot\}$ = expected value / Mean value.
e^2 = Mean square error.

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$f(x,y)$ = Original image.

$\hat{f}(x,y)$ = Estimation of original image.

$f(x,y) - \hat{f}(x,y)$ = should be minimum to get better restored value.

\# Goal: Minimising mean square error.

\# Here, images and noise are considered as random variables & the objective is to find out an estimate i.e. $\hat{f}(x,y)$ of the original image $f(x,y)$ such that the mean square error b/w them is minimized.

\# This error can be represented as by the equation of e^2.

\# In Wiener filter, we have some assumptions-

1. The noise and image are uncorrelated.
2. One or other has zero mean.
3. The intensity levels in the estimate are a linear function of the levels in the degraded image.

$$g(x,y) \xrightarrow[\text{linear transformation}]{\text{filter}} \hat{f}(x,y)$$

Linear → Values

Non-linear → position

* Based on these assumptions, the minimum of the error function is given in frequency domain as -

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)\, S_f(u,v)}{S_f(u,v)\, |H(u,v)|^2 + S_n(u,v)} \right] \cdot G(u,v)$$

where

$H(u,v)$ = degradation function

$H^*(u,v)$ = complex conjugate degradation function

$S_n(u,v)$ = power density of noise $\Rightarrow |N(u,v)|^2$

$S_f(u,v)$ = Undegraded image power density $\Rightarrow |F(u,v)|^2$

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \frac{S_n(u,v)}{S_f(u,v)}} \right] G\cdot(u,v)$$

$$= \frac{1}{H(u,v)} \left[\frac{H(u,v)\cdot H^*(u,v)}{|H(u,v)|^2 + \frac{S_n(u,v)}{S_f(u,v)}} \right] \cdot G(u,v)$$

$$= \frac{1}{H(u,v)} \left[\frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] \cdot G(u,v)$$

$$\|a+b\|^2 = \|a\|^2 + |b|^2 + a^*b + ab^*$$

$\left\{ \begin{array}{l} \text{If } a = x+iy \\ a^* = x-iy \end{array}\right.$, similarly $\left. \begin{array}{l} b = x'+iy' \\ b^* = x'-iy' \end{array} \right\}$

where

$$|H(u,v)|^2 = H(u,v) \times H^*(u,v)$$

Since $S_n(u,v)$ & $S_f(u,v)$ is not known, we use hit and trial method.

If there is no noise - Weiner filter works as Inverse filter.

<u>Drawback</u> - If the object is moving as well as rotating then it is not applicable.

- Not applicable for human eyes directly.

① Mean square error is not very efficient when images are restored for human eyes, because m.s.e. weight all errors equally regardless of their location in the image.

② Standard Weiner filter can not handle variant blurring point spread functions like curvature of field and motion blur that involves rotation.

Signal to Noise Ratio (SNR)

$$\# \quad SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} |F(u,v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} |N(u,v)|^2}$$

or

$$\# \quad SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} \hat{f}(x,y)^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} \left(f(x,y) - \hat{f}(x,y)\right)^2}$$

Geometric Mean filter & Power spectrum equalisation

$$\# \quad \hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2}\right]^{\alpha} \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \beta\left[\frac{S_n(u,v)}{S_f(u,v)}\right]}\right]^{1-\alpha} \cdot G(u,v)$$

where α & β are +ve real constants.

\# When $\alpha = 1$; act like Inverse filter.

When $\alpha = 0$; act like parametric Weiner filter which reduce to standard Weiner filter when $\beta = 1$.

When $\alpha = \frac{1}{2}$; act like Geometric Mean Filter.

When $\beta = 1$, α = decreases below $\frac{1}{2}$; then performance tends more towards the Inverse filter.

When $\beta = 1$, α = increases above $\frac{1}{2}$; it will behave like Weiner filter.

When $\alpha = 1/2$, $\beta = 1$; It is known as Power Spectrum Equalisation (PSE).

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Adaptive filter

- Adaptive local noise reduction (linear)
- Adaptive median filter (Non-Linear)

☆ Depends on pixel, pixel location & image characteristic

☆ Stastical characteristic refer to Mean & Variance.

☆ Variance refers to sharpness of image.

☆ That is why it is different from other filters

Adaptive filter behaviour changes based on the stastical characteristic of the image.

Inside the filter region, defined by $M \times M$ window S_{xy}.

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Two types of adaptive filters are
① Adaptive local noise (filter reduction)
② Adaptive Median filter

1) Adaptive local noise reduction filter
The response of the filter at any point (x, y) on which the region is centered is based on 4 quantities.

(a) ~~fxy~~ $g(x,y)$ → degraded image / Noisy image.

The values of noisy image at (x,y).

(b) σ_n^2 → variance of noise.

Sharpness of ~~original~~ noise image that corrupt the image.

(c) m_L → local mean of pixels in S_{xy}.

(d) σ_L^2 → local variance of pixels in S_{xy}.

We want the behavior of filter as follows -

1) If $\sigma_n^2 = 0$; Means we do not have any kind of noise in our image, thus filter should return the value of $g(x,y)$.

2) If $\sigma_L^2 > \sigma_n^2$; The filter should return the value close to $g(x,y)$.

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3) If $\sigma_L^2 = \sigma_n^2$; The filter should return the arithmetic mean value of the pixel S_{xy}.

$$\boxed{\hat{f}(x,y) = g(x,y) - \frac{\sigma_n^2}{\sigma_L^2}\left[g(x,y) - m_L\right]}$$

2) Adaptive Median filter.

Algorithm :

14	16	15
15	29	14
19	22	16

Original image.

Level A :

$A_1 = Z_{med} - Z_{min}$
$A_2 = Z_{med} - Z_{max}$

if $A_1 > 0$ AND $A_2 < 0$
  GoTo Level B
else
  Increase the window size

if Window size $< S_{max}$
  repeat Level A
else
  Output Z_{xy}.

Level B :

$B_1 = Z_{xy} - Z_{min}$
$B_2 = Z_{xy} - Z_{max}$

if $B_1 > 0$ AND $B_2 < 0$
  Output is Z_{xy}
else
  Output is Z_{med}.

It is similar to low pass median filter with the difference that we check the median value before replacing the centre value.

# If median value has noise & centre value has no noise → do not change.
else
we change the value.

eg:

14	16	15
15	29	14
19	22	16

14 14 15 15 16
16 19 22 29

* $Z_{min} = 14$
  $Z_{max} = 29$
  $Z_{med} = 16$

* $A_1 = 16 - 14 = 2$
  $A_2 = 16 - 29 = -13$

* $A1 > 0$ & $A2 < 0$ → Goto Level B.

* $Z_{xy} = 29$
  $B_1 = 29 - 14 = 15$
  $B_2 = 29 - 29 = 0$

* $B1 > 0$ & But $B2 \not< 0$
else
Output is Z_{med}.

Thus $\boxed{O/P = 16}$

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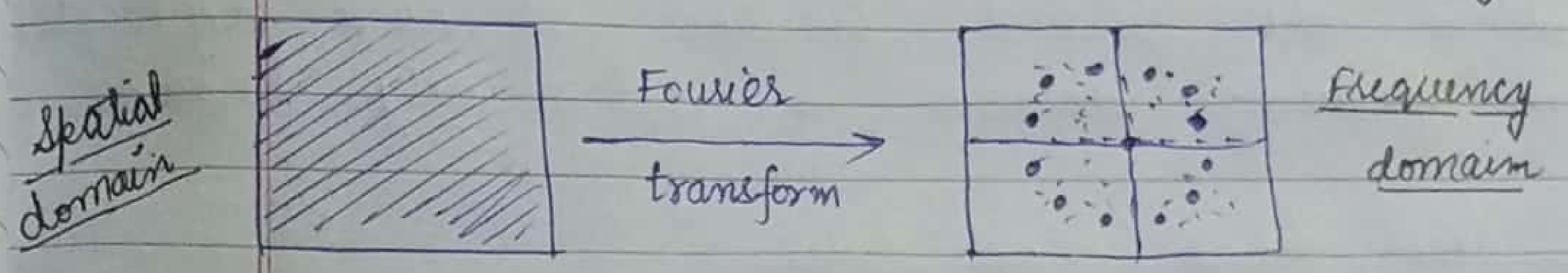
Periodic Noise Reduction in frequency domain



$$\boxed{\sin(2\pi K_0 x) = \frac{1}{2} j \left[\delta(K+K_0) - \delta(K-K_0) \right]}$$

Selective filter is used to remove periodic noise in frequency domain, because fourier transform spectrum of image is symmetric and we can easily select the region using selective filter.

Selective filters.
- Band pass
  - Ideal
  - Butterworth
  - Gaussian
- Band Reject
  - Ideal
  - Butterworth
  - Gaussian
- Notch filter
  - Ideal
  - Butterworth
  - Gaussian

If we have frequency range = 150- 200 Hz.

If we want this frequency range only then we use band pass filter.

But, if we want to reject this particular frequency range, then we use band reject filter.

Now if the frequency range is short = 150-152 Hz, in that case we use Notch filter.

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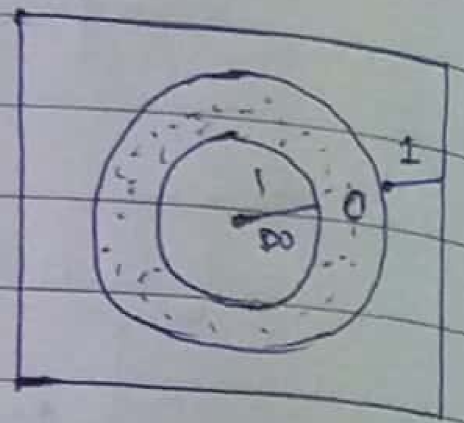
1) Band ~~pass~~ reject filter: It is used in noise removal where the general location of noise component in frequency domain is approximately known.

(i) Ideal band reject filter

$$H(u,v) = \begin{cases} 1 & D(u,v) < D_0 - w/2 \\ 0 & D_0 - w/2 \le D(u,v) \le D_0 + w/2 \\ 1 & D(u,v) > D_0 + w/2 \end{cases}$$

where w = width of frequency band

$$D(u,v) = \left[\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N}{2}\right)^2\right]^{1/2}$$



(ii) Butterworth band reject filter

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)\, w}{D^2(u,v) - D_0^2}\right]^{2n}}$$

(iii) Gaussian band reject filter

$$H(u,v) = 1 - e^{-\left[\frac{D^2(u,v) - D_0^2}{D(u,v)\, w}\right]^2}$$

2) Band pass filter:

$$\boxed{H_{BP} = 1 - H_{BR}}$$

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3) Notch filter: Used in small frequency range.

(i) Ideal notch filter:

$$H(u,v) = \begin{cases} 0 & D_1(u,v) \le D_0 \text{ or } D_2(u,v) \le D_0 \\ 1 & \text{Otherwise.} \end{cases}$$

$$D_1(u,v) = \left[\left(u - \frac{M}{2} - u_0\right)^2 + \left(v - \frac{N}{2} - v_0\right)^2\right]^{1/2}$$

$$D_2(u,v) = \left[\left(u - \frac{M}{2} + u_0\right)^2 + \left(v - \frac{N}{2} + v_0\right)^2\right]^{1/2}$$

(ii) Butterworth notch filter:

$$H(u,v) = \frac{1}{1 + \left\{\frac{D_0^2}{D_1(u,v)\, D_2(u,v)}\right\}^{2n}}$$

(iii) ~~Butterworth~~ Gaussian notch filter:

$$H(u,v) = 1 - e^{-\left\{\frac{D_1(u,v)\, D_2(u,v)}{D_0^2}\right\}}$$

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