"UNIT-3" "IMAGE RESTORATION"

Image restoration is the process of removal or reduction of degradation in an image through linear or non linear filtering. Degradation are usually incurred during the acquisition of the image.

The aim of image restoration is to bring the image towards what it would have been if it had been recorded without degradation

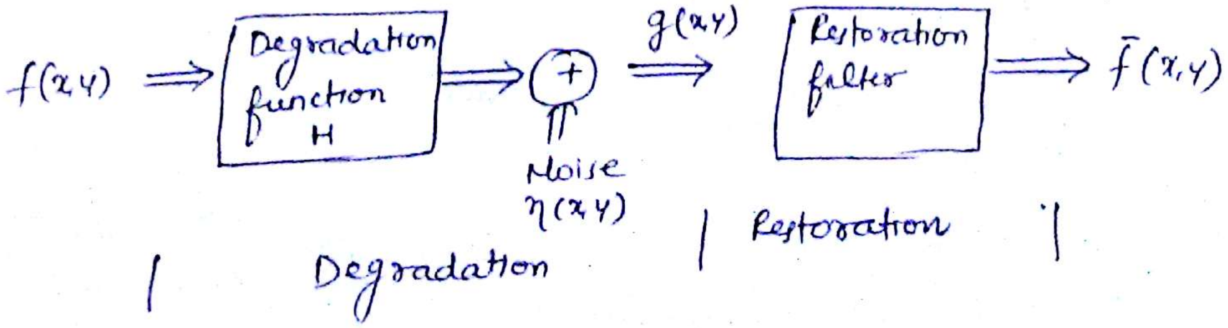
. Image restoration is used to improve the quality of an image but image restoration is an objective process while image enhancement is a subjective process

Restoration tries to reconstruct by using prior knowledge of the degradation phenomena.

Degradation can be due to

1) Image sensor noise
2) Blur due to miss focus
3) Blur due to motion
4) Noise from transmission channel

Linear model of Image degradation/ Restoration :-



$f(x,y)$ ⇒ Original image before degradation / I/p image

$g(x,y)$ ⇒ Degraded image

H ⇒ Degradation function

$\eta(x,y)$ ⇒ Noise

⇒ Given $g(x,y)$, some knowledge about H & some knowledge about $\eta(x,y)$. The objective of restoration is to obtain an estimate $\bar{f}(x,y)$ of the original image & the estimate to be as close as possible to the original image.

⇒ In general, the more we know about H & η, the closer $\bar{f}(x,y)$ will be to $f(x,y)$.

if H is a linear, position invariant process, then the degraded image is given in the spatial domain by -

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$h(x,y)$ - spatial representation of the degradation function

$*$ ⇒ Convolution

in frequency domain

$$\boxed{G(u,v) = H(u,v)\, F(u,v) + N(u,v)}$$

⇒ Convolution is analogous to multiplication in frequency domain.

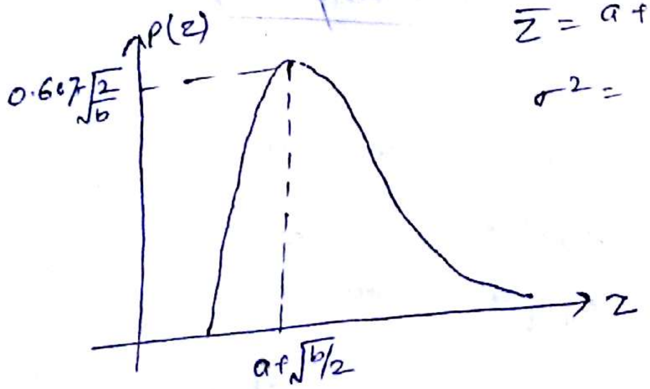
⇒ The term in capital letters are the Fourier transforms of the corresponding term.

Noise models :

1. Gaussian
2. salt & pepper (Impulse noise)
3. Rayleigh noise
4. Gamma noise (Erlang)
5. Exponential noise
6. Uniform noise
7. ~~Sinusoidal noise~~ 7. Periodic noise (Arises from electrical or electromechanical interference during image acquisition

Rayleigh noise :

$$p(z) = \begin{cases} \frac{2}{b}(z-a)\, e^{-(z-a)^2/b} & z \geq a \\ 0 & z < a \end{cases}$$



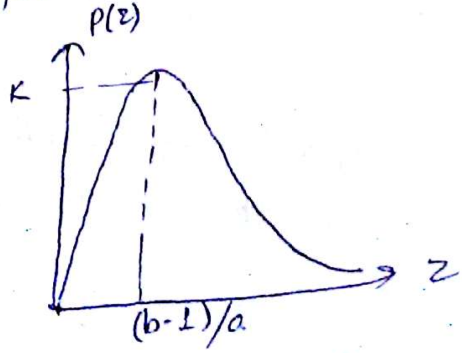
$\bar{z} = a + \sqrt{\pi b/4}$

$\sigma^2 = \frac{b(4-\pi)}{4}$

skewed to the right

Gamma Noise :

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$



here $a > 0$ & b is positive integer

mean $\bar{z} = b/a$

variance $\sigma^2 = b/a^2$

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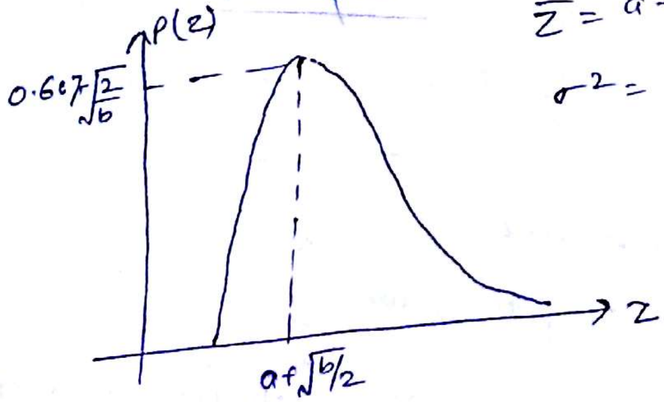

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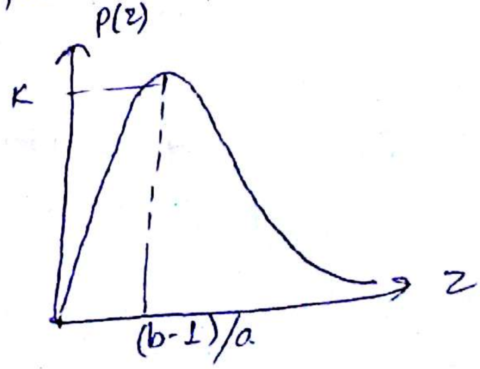
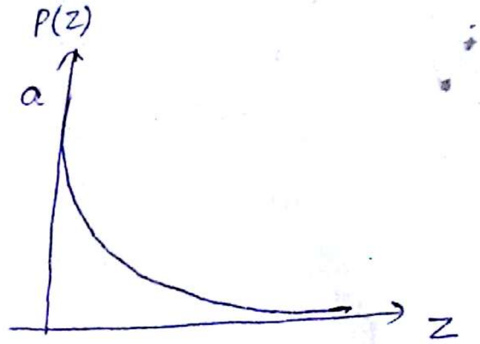


Exponential noise :-

$$P(z) = \begin{cases} a e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

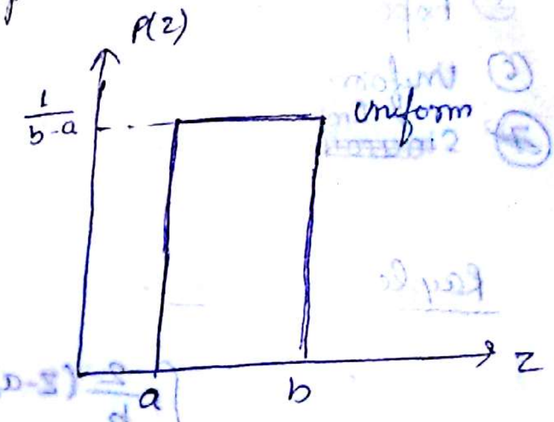


where $a > 0$ $\qquad \bar{z} = 1/a \qquad \sigma^2 = 1/a^2$

when $b = 1$ in Gamma it is just like exponential noise

Uniform noise :

$$P(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$



$\bar{z} = \frac{a+b}{2}$, $\sigma^2 = \frac{(b-a)^2}{12}$

Restoration in the presence of Noise only - Spatial filtering

when degradation is noise only then

$$g(x,y) = f(x,y) + \eta(x,y) \quad \text{--- (1)}$$

& $G(u,v) = F(u,v) + N(u,v)$ --- (2)

in frequency domain

To remove the noise we use some spatial filter -

(1) Mean filter:

I) Arithmetic mean filter :-

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$$

S_{xy} = set of coordinates of rectangular subimage window of size $(m \times n)$.

It computes the avg value of the corrupted image $g(x,y)$ in the area defined by S_{xy}. The value of the restored image $\hat{f}$ at point (x,y) is simply the arithmetic mean.

This operation can be implemented using a spatial filter of size $m \times n$ in which all coefficient have the value $\frac{1}{mn}$.

It smooths local variations in an image, and noise is reduced as a result of blurring.

2 (II) Geometric mean filter :-

$$\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{1/mn}$$

Each restored pixel is given by the product of the pixels in the subimage window, raised to the power $1/mn$.

Drawback: Lose, less image detail in the process

(III) Harmonic mean filter:-

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$$

Drawback: It does well for salt noise but fails for pepper noise.

~~Contra~~

(IV) Contraharmonic mean filter :

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

if $Q > 0$, filter eliminates pepper noise
$Q < 0$, " " salt noise.
$Q = 0$ Arithmatic mean filter
$Q = -1$ Harmonic mean filter

(2) Order statistic filters :-
The response of the filter is based on ordering (ranking) the values of the pixels contained in image area encompassed by the filter

(a) (I) Median filter
Best known order statistic filter which replaces the value of pixel by the median.

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}} \{ g(s,t) \}$$

(II) Max & Min filter :

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\max} \{ g(s,t) \}$$ (100 percentile filter)

It is useful for finding the brightest point in an image. Reduces pepper noise.

$$f(x,y) = \underset{(s,t)\in S_{xy}}{\min} \{ g(s,t) \}$$ (0th percentile filter)

It is useful for finding the darkest point in an image reduce salt noise

Mid point filter :-

Computes the mid point b/w max & min values.

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

It combines order statistics and averaging. It work best for randomly distributed noise like gaussian & uniform noise.

Alpha trimmed mean filter :-

Let we delete $d/2$ lowest and $d/2$ highest intensity values of $g(s,t)$ in the neighborhood S_{xy}. Let $g_r(s,t)$ represent the remaining $mn-d$ pixels.

A filter formed by these remaining pixels is called an alpha trimmed mean filter

$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

$0 \leq d \leq mn-1$

if $d=0$, it's arithmetic mean

$d=mn-1$, it becomes a median filter

It removes salt & pepper noise as well as gaussian noise.

Que Given below is a 3X3 image. What would the value of the centre pixel change to when this image is passed through a

1) Arithmetic mean filter
2) Geometric mean filter
3) Harmonic mean filter
4) Max filter
5) Min filter

1	7	5
6	2	3
1	4	2

(I) $\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$

$= \frac{1}{3 \times 3} [1+7+5+6+2+3+1+4+2]$

$= \frac{1}{9} \times 31 \Rightarrow 3.44 \approx 3$

(II) $\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{1/mn}$

$= (1 \times 7 \times 5 \times 6 \times 2 \times 3 \times 1 \times 4 \times 2)^{1/9}$

$\Rightarrow 2.70 \approx 3$

(III) $\hat{f}(x,y) = \frac{m \times n}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}} = \frac{9}{\frac{1}{1} + \frac{1}{7} + \frac{1}{5} + \frac{1}{6} + \cdots \frac{1}{2}}$

$2.19 \approx 2$

(IV) $\hat{f}(x,y) = \max_{(s,t) \in S_{xy}} \{g(s,t)\} = 7$

(V) $\hat{f}(x,y) = \min_{(s,t) \in S_{xy}} \{g(s,t)\} = 1$

Periodic noise reduction by Frequency Domain filtering:

Periodic noise appears as concentrated bursts of energy in the Fourier Transform, at locations corresponding to the frequencies of periodic noise interference.

The 3 types of selective filters to isolate the noise are -

1) Band reject
2) Band pass
3) Notch filters

Periodic noise: Pure sine wave

Appear as a pair of impulse (conjugate) in frequency domain

$$\begin{cases} f(x,y) = A \sin(u_0 x + v_0 y) \\ F(u,v) = -j\frac{A}{2}\left[\delta\left(u - \frac{u_0}{2\pi}, v - \frac{v_0}{2\pi}\right) - \delta\left(u + \frac{u_0}{2\pi}, v + \frac{v_0}{2\pi}\right)\right] \end{cases}$$

① Band reject filters:

It is used in noise removal where the general location of noise components in frequency domain is approximately known

Ideal Band reject filter

$$H(u,v) = \begin{cases} 1 & D(u,v) < D_0 - W/2 \\ 0 & D_0 - W/2 \le D(u,v) \le D_0 + W/2 \\ 1 & D(u,v) > D_0 + W/2 \end{cases}$$

W is the width of Band

Butterworth Band reject

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)\,W}{D^2(u,v) - D_0^2}\right]^{2n}}$$

Gaussian Band reject

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0^2}{D(u,v)\,W}\right]^2}$$

$$D(u,v) = \left[\left(u - M/2\right)^2 + \left(v - N/2\right)^2\right]^{1/2}$$

Band Pass filter :

It performs opposite operation of a band reject filter

$$HPB \quad \boxed{H_{BP}(U,V) = 1 - H_{BR}(U,V)}$$

Notch filter :-

A notch filter rejects or passes frequencies in predefined neighborhoods about a centre frequency. Zero phase shift filters must be symmetric about the origin, so a notch with centre at (u_0, v_0) must have a corresponding notch at location $(-u_0, -v_0)$.

Ideal notch reject filter

$$H(U,V) = \begin{cases} 0 & D_1(U,V) \leq D_0 \text{ or } D_2(U,V) \leq D_0 \\ 1, & \text{otherwise} \end{cases}$$

$$D_1(U,V) = \left[(U - M/2 - u_0)^2 + (V - N/2 - v_0)^2 \right]^{1/2}$$

$$D_2(U,V) = \left[(U - M/2 + u_0)^2 + (V - N/2 + v_0)^2 \right]^{1/2}$$

~~Butterworth notch reject filters~~

$$\cancel{H(U,V) = 1 - e^{-1/2\left[\frac{D_1(U,V)\, D_2(U,V)}{D_0^2}\right]}}$$

Notch reject filters are constructed as product of high pass filters whose centre has been translated to the centre of notches.

$$H_{NR}(U,V) = \prod_{K=1}^{Q} H_K(U,V)\, H_{-K}(U,V)$$

$H_K(U,V)$ = H.P filters whose centres are at (u_0, v_0)

$H_{-K}(U,V)$ = " " $(-u_0, -v_0)$

Q - Number of notch pair

Notch pass filter

$$\boxed{H_{NP}(U,V) = 1 - H_{NR}(U,V)}$$

Optimum Notch filtering :

When several interference components are present, then we use optimum notch filtering because when we implement previous filter they remove too much image information.

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Ex: Starlike components in fourier transformation spectrum indicate more than one sinusoidal pattern.

$$f(x,y) = g(x,y) - w(x,y)\,\eta(x,y)$$

$w(x,y)$ = weighting or modulation function

$w(x,y)$ has been selected in such a way that variance of $\hat{f}(x,y)$ is minimized over a specified neighborhood of every point (x,y).

Butterworth notch reject filter of order n containing 3 notch pairs

$$H_{NR}(u,v) = \prod_{k=1}^{3} \left[\frac{1}{1+[D_{0k}/D_{1}(u,v)]^{2n}}\right] \left[\frac{1}{1+[D_{0k}/D_{2}(u,v)]^{2n}}\right]$$

Inverse filtering :-

The simplest approach to restoration is direct inverse filtering, where we compute an estimate, $\hat{F}(u,v)$ of the transform of the original image.

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} \quad \text{—— (1)}$$

Division is an array operation

we know that

$$G(u,v) = F(u,v)\,H(u,v) + N(u,v) \quad \text{—— (2)}$$

substituting eqn (2) in eqn (1)

$$\hat{F}(u,v) = \frac{F(u,v)\,H(u,v) + N(u,v)}{H(u,v)}$$

$$\boxed{\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}}$$

even if we know the degradation function
That means, we can not recover the undegraded image exactly because $H(u,v)$ is not known.

If the degradation function has zero or very small values then the estimation $\hat{F}(u,v)$ is done by $N(u,v)/H(u,v)$

Minimum Mean square Error (Wiener) Filtering :-

The inverse filtering approach makes no explicit provision for handling noise.

In this, we discuss an approach that incorporates both the degradation function and statistical characteristics of noise into the restoration process

Here images and noise are considered as random variables and the objective is to find an estimate $\hat{f}$ of the uncorrupted image f such that the mean square error b/w them is minimized. This error measure is given by :-

$$e^2 = E\{(f-\hat{f})^2\}$$

where $E\{\cdot\}$ is the expected value of the argument
also known as the mean value of the argument
It is assumed that the noise and the image are uncorrelated. that one or the other has zero mean; and that the intensity levels in the estimate are a linear function of the levels in the degraded image.

Based on these conditions, the minimum of the error function is given in the frequency domain by -

$Cov[X,Y] = E[XY] - E[X]E[Y]$

Uncorrelated : Two random variable x & y are uncorrelated if their covariance $E[xy] - E[x]E[y] = 0$. If two variables are uncorrelated there is no linear relationship b/w them

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)\, S_f(u,v)}{S_f(u,v)\, |H(u,v)|^2 + S_\eta(u,v)} \right] G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

$$= \underbrace{\left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right]}_{\text{Minimum mean square error filter}} G(u,v)$$

Here we use the fact that the product of a complex quantity with its conjugate is equal to the magnitude of the complex quantity squared.

This result is known as weiner filter

$H(u,v) =$ Degradation function

$H^*(u,v) =$ Complex conjugate of $H(u,v)$

$|H(u,v)|^2 = H^*(u,v)\, H(u,v)$

$S_\eta(u,v) = |N(u,v)|^2 =$ Power spectrum of the noise

$S_f(u,v) = |F(u,v)|^2$ Power spectrum of the undegraded image

The restored image in the spatial domain is given by the inverse fourier transform of the frequency domain estimate $\hat{F}(u,v)$.

If Noise is zero, then noise power spectrum vanishes and weiner filter reduces to inverse filter.

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Signal to noise Ratio :-

$$SNR = \frac{\sum_{U=0}^{M-1} \sum_{V=0}^{N-1} |F(U,V)|^2}{\sum_{U=0}^{M-1} \sum_{V=0}^{N-1} |N(U,V)|^2}$$

It gives a measure of the level of information bearing signal power (i.e. of the original, undegraded image) to the level of noise power

Images with low noise tend to have a High SNR and, conversely the same image with a higher level of noise has lower SNR.

Mean square error :-

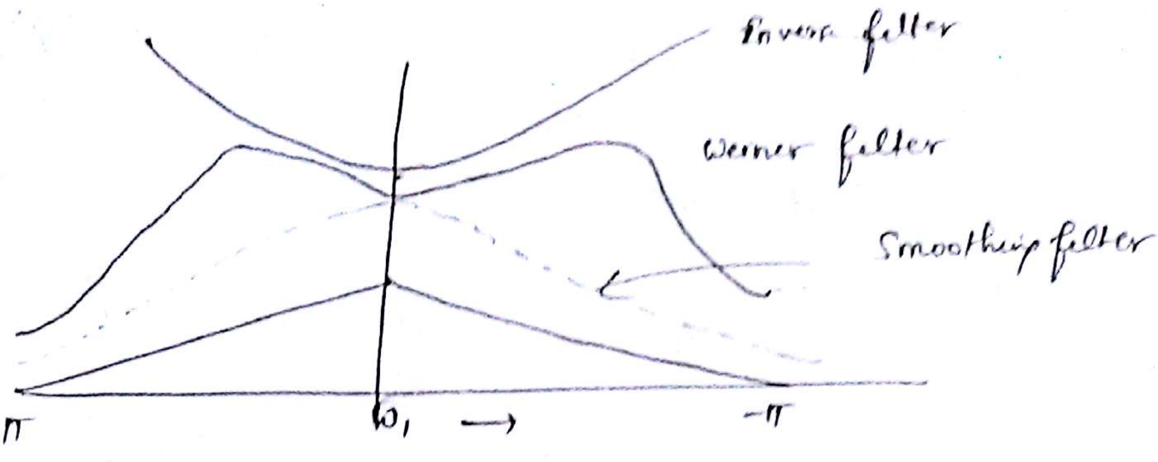
It can be approximated in terms of summation involving the original and restored images.

$$MSE = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x,y) - \hat{f}(x,y)]^2$$

If we consider the restored image to be "signal" and the original to be noise, the difference b/w image and the original in the spatial domain we can define signal to noise ratio as

$$SNR = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \hat{f}(x,y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x,y) - \hat{f}(x,y)]^2}$$

The closer f & $\hat{f}$, the larger this ratio will be.



Frequency response of weiner filter

~~Drawback of weiner filter :-~~

In the presence of noise only, the weiner filer acts as a low pass filter while in the presence of only ~~blur~~ degradation, the wener filter acts as a high pass filter

when both noise & degradation are present, the weiner filter achieves a compromise b/w low pass & highpass filter resulting in a band pass filter.

Drawback of weiner filter :-

① M.S.E is not very effective when images are restored for human eyes because M.S.E weights all errors equally regardless of their location in the image. The human eyes on the other hand is more tolerant to errors in dark areas and high transition areas. In minimising the m.s.e. the weiner filter tends to smooth the image more than what the human eye would prefer

(2) Standard weiner filter can not handle spatially variant blurring point spread functions like motion blur that involves curvature of ~~fellt~~ and rotation.

(3) It can not handle the common cause of non stationary signals and noise

(4) It performs poorly if both the signal & the noise are non stationary (Large constant region seperated by sharp transition)

~~There are 2 alternative technique to improve the weiner~~ filter

(1) P.S.E (Power spectrum equalization)

~~The following filter restore the power spectrum of the degraded image to its original amplitude~~

$$\hat{H}_R(U,V) = \left[\frac{S_f(U,V)}{|H(U,V)|^2 \, S_f(U,V) + S_N(U,V)} \right]^{1/2}$$

Geometric mean filter & Power spectrum equalization :-

$$\hat{F}(U,V) = \left[\frac{H^*(U,V)}{|H(U,V)|^2} \right]^{\alpha} \left[\frac{H^*(U,V)}{|H(U,V)|^2 + \beta \left[\frac{S_\eta(U,V)}{S_f(U,V)} \right]} \right]^{1-\alpha} G(U,V)$$

α & β are positive real constant

when

$\alpha = 1$, ⇒ Inverse filter

$\alpha = 0$ ⇒ Parametric weiner filter, which reduced to standard weiner filter when $\beta = 1$

$\alpha = 1/2$ ⇒ Geometric mean filter

$\beta = 1$ & α decreases below $1/2$ = Performance tend more towards inverse filter

$\beta = 1$ & α increases above $1/2$ = It will behave more like weiner filter

$\alpha = 1/2$, $\beta = 1$ ⇒ Power spectrum equalization (~~It represents family~~ of we know that $\boxed{g = Hf + \eta}$

In weiner filter the power spectra of undegraded image & Noise must be known. It is a drawback of weiner filter.

The method discussed here is only require the knowledge of mean & variance of noise.

Frequency domain soln to optimization problem.

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right] G(u,v)$$

γ is a parameter that must be adjusted so that the constrain $\|g - H\hat{f}\|^2 = \|\eta\|^2$ is satisfied.

where $\|w\|^2 \triangleq w^T \cdot w = \sum_{k=1}^{n} w_k^2$

$P(u,v)$ is the fourier transform of the function

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

if $\gamma = 0$, it reduces to inverse filter

γ is manually chosen to provide the better visualization

Actually H is very sensitive to noise. To provide the optimal soln of restoration we measure the smoothness, such as the second derivative of an image. Thus to perform restoration it is desired to find a minimum of the criteria function

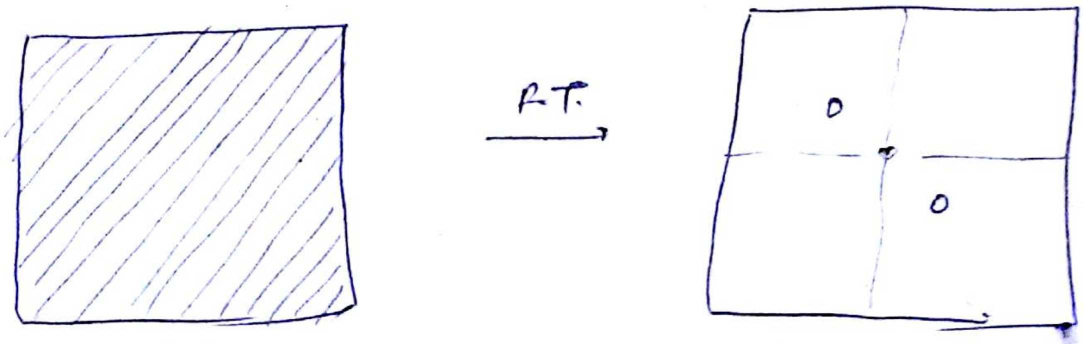
$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\nabla^2 f(x,y) \right]^2$$

subject to constraint $\|g - H\hat{f}\|^2 = \|\eta\|^2$

where $\|w\|^2 \triangleq w^T \cdot w = \sum_{k=1}^{n} w_k^2$ $\qquad w_k =$ kth component of w

Periodic Noise (Sinusoidal Noise)

It is a special kind of noise that can not be eliminated in the spatial domain. Periodic noise occured in the image due to electrical and electro mechanical interface during acquisition.



spatial domain corrupted by noise

Frequency response

Our job is to design a filter $H(u,v)$ such that it elleminates these spikes without disturbing the original spectrum.

$H(u,v)$ for periodic noise removal is not centred. Thus $H(u,v)$ will change depending on the position of spikes.

$$r(x,y) = A\left[\cos(u_0x+v_0y) + \sin(u_0x+v_0y)\right]$$

Here u_0 & v_0 determine the periodic frequencies w.r.t x & y.
The position of the spikes changes when u_0 & v_0 are changed