Unit - 4 Morphological Image Processing

Introduction, Logic operations involving Binary images, Dilation and Erosion, Opening and closing, Morphological Algorithms - Boundary Extraction, Region filling, Extraction of connected components, Convex hull, Thinning, Thickening

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The word morphology commonly denotes a branch of biology that deals with form and structure of animals & plants. In image processing, morphology is about region & shapes

It is a tool for extracting image components that are useful in the representation & description of region shapes such as boundaries, skeletons and convex hull.

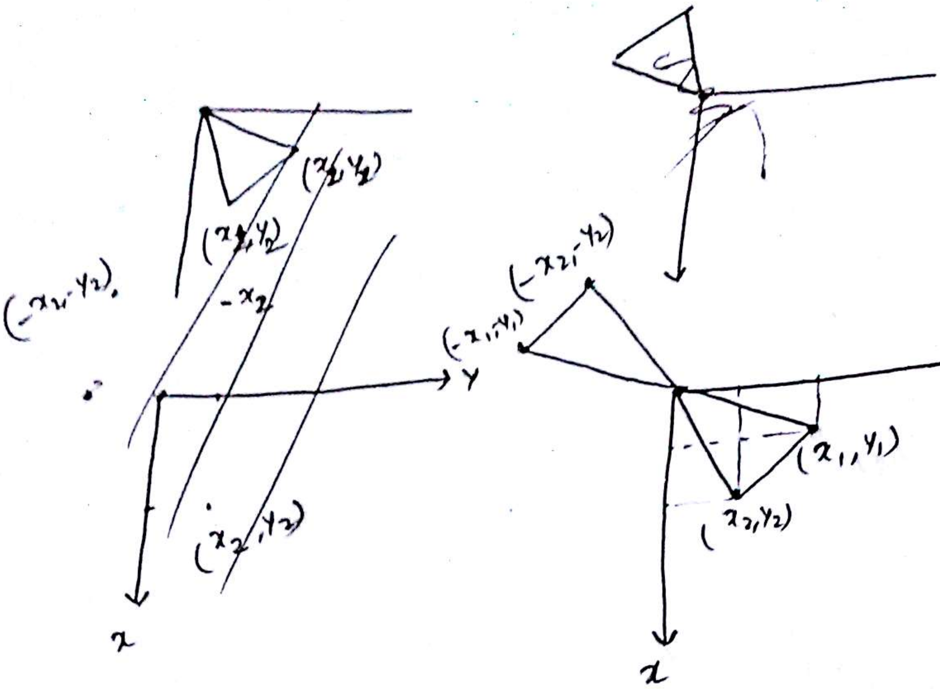
(for binary image)
$Z^2 \Rightarrow$ 2D Integer space, which is the set of all ordered pairs of elements (z_i, z_j) with z_i & z_j being integers from Z. Each elements are the (x,y) coordinates of a white pixel in the image

~~f(x)~~ Note! $f(x,y)$ is a digital image if (x,y) are integers from Z^2 and f is a function that assigns an intensity values to each distinct pair of coordinates (x,y)

Reflection :- The reflection of set B, denoted $\hat{B}$, is defined as

$$\hat{B} = \{ w / w = -b, \text{ for } b \in B \}$$

ie if B is the set of pixel (x,y) representing an object in an image, then $\hat{B}$ simply the set of pts in B whose (x,y) coordinates have been replaced by $(-x,-y)$.



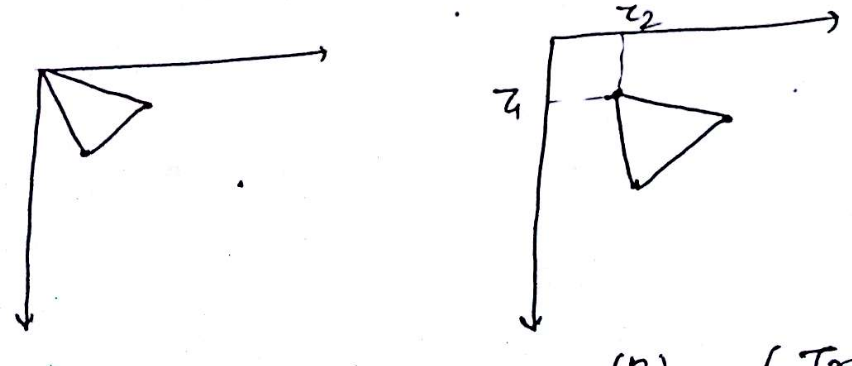
original image with its reflection

Translation :

The translation of a set B by point $z = (z_1, z_2)$, denoted $(B)_z$, is defined as

$$(B)_z = \{ c \mid c = b + z, \text{ for } b \in B \}$$

$(B)_z$ is the the set of points in B whose (x, y) coordinates have been replaced by $(x + z_1, y + z_2)$.

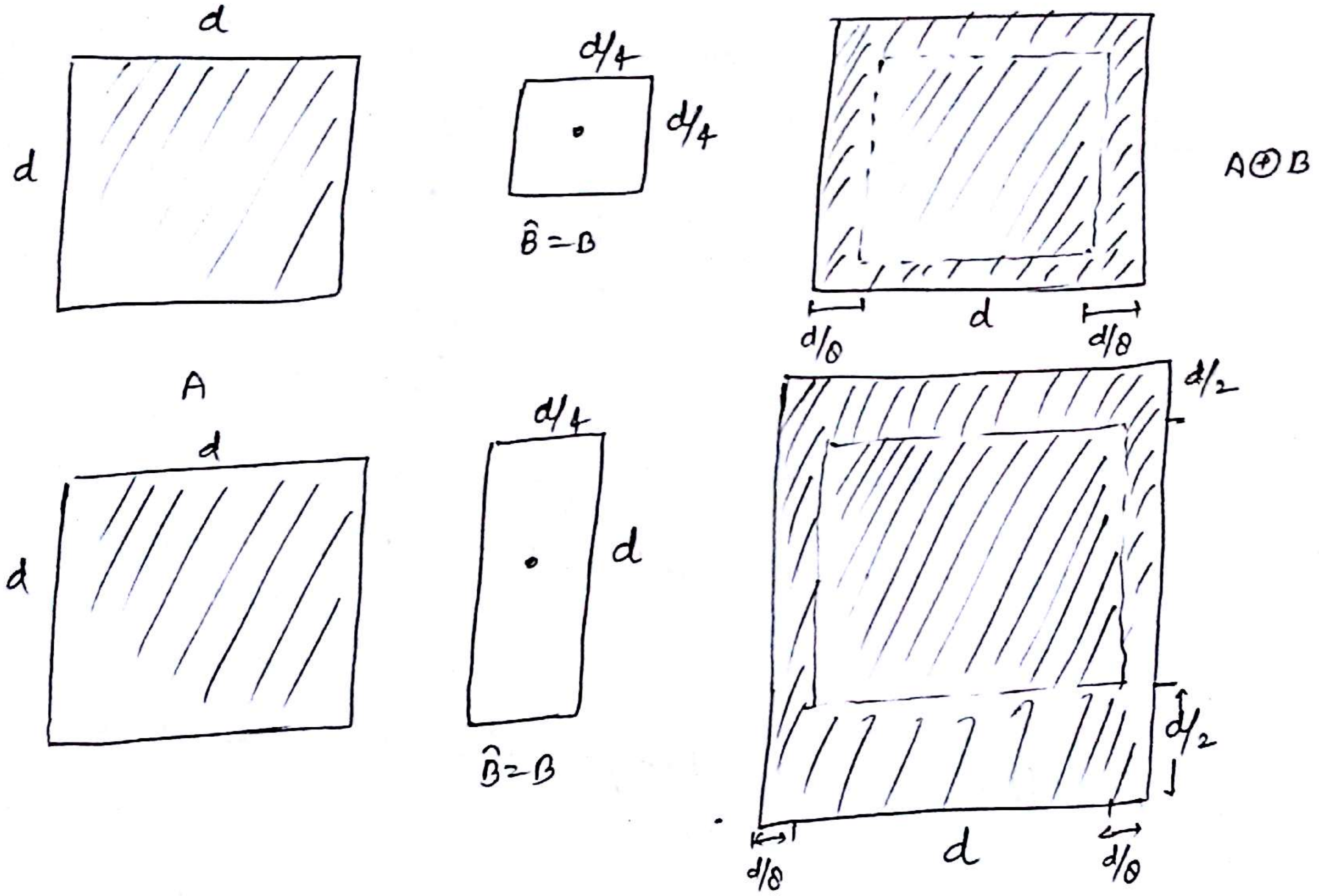


B

$(B)_z$ (Translation)

The dilation of A by B ~~then~~ is the set of all displacements z, such that $\hat{B}$ and A overlap by at least one element. Based on this the eqn can be represented as.

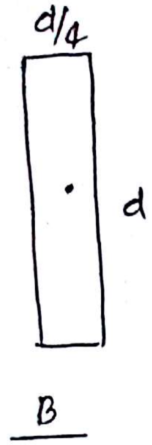
$$A \oplus B = \{ z / [(\hat{B})_z \cap A] \subseteq A \}$$

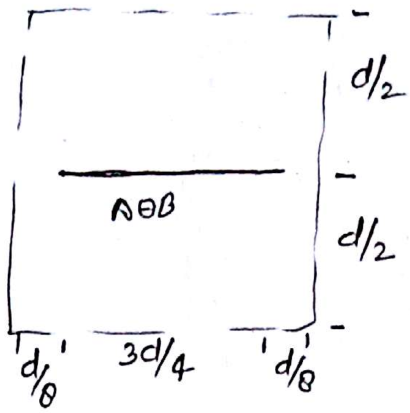


Gray scale dilation :

Let A(x,y) be a binary image and C(x,y) be the resulting image obtained after A(x,y) has been dilated with a m×n template B(i,j). B is called the structuring element where $0 \le i \le m-1$, $0 \le j \le n-1$

for dilation $\boxed{C(x,y) = \max\{ A(x-i, y-j) \times B(i,j) \}}$





Gray scale erosion :-

If $A(x,y)$ is the image & $B(i,j)$ is the structuring element then $C(x,y)$ which is the eroded image is given by

$$C(x,y) = \text{Minimum}\left\{ A(x-i,\, y-j) \times B(i,j) \right\}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A

$$B = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

B

NOTE : Erosion enlarges holes in the object, shrinks its boundary. It gets rid of irrelevant data by reducing its size.

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

A

1	1	1

B

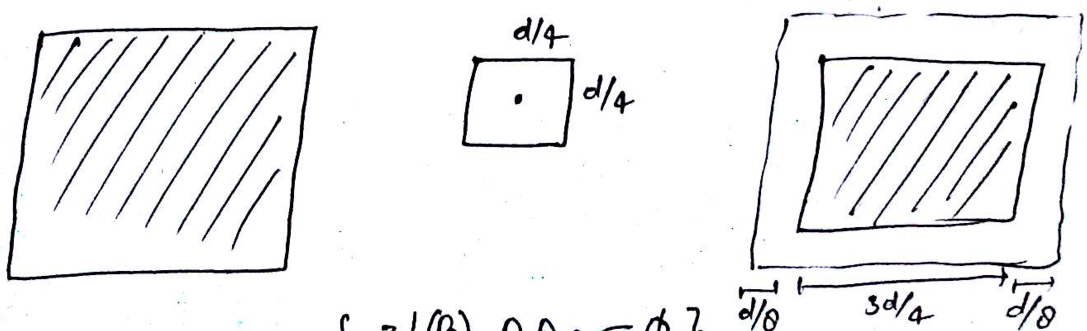
1	1	0	0	0
1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	0	1	1

Erosion :-

It is used for shrinking of element A by using element B. Erosion for set A & B in Z^2, is defined by the following eqn

$$A \ominus B = \{ z \mid (B)_z \subseteq A \}$$

It indicates that the erosion of A by B is the set of all points z such that B, translated by z, is ~~combined~~ contained in A.



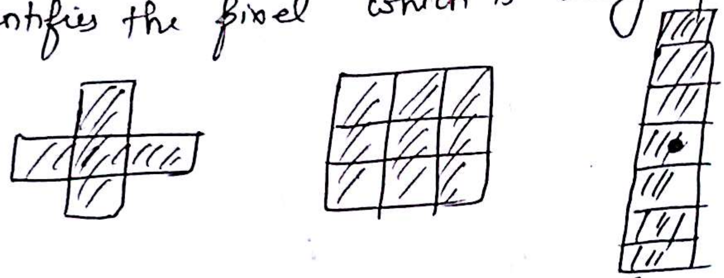
$$A \ominus B = \{ z \mid (B)_z \cap A_c = \phi \}$$

Que

Structuring element:

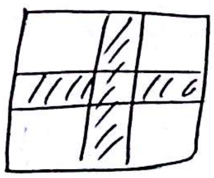
small sets or subimages used to probe an image under study for properties of interest. It consist of a matrix of 0 & 1 and the size is much smaller than the image.

The centre pixel of structuring element is called origin and it identifies the pixel which is being processed.



Examples of structing element

here the value of shaded portion is 1. & unshaded portion is 0.



structuring element converted to rectangular array.

Dilation & Erosion:

⇒ Dilation is used for expanding an element A by using structuring element B

⇒ with A & B as sets in Z^2, the dilation of A by B, denoted $A \oplus B$, is defined as.

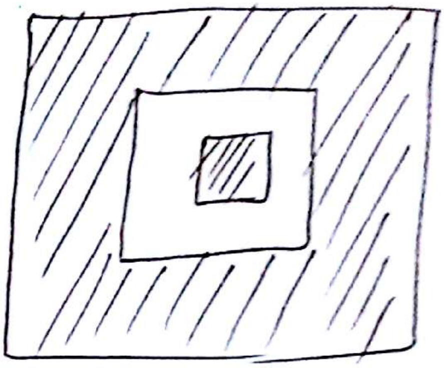
$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \phi \}$$

This eqn is based on reflecting B about its origin, and shifting this reflection by z.

Ques: Given 10×10 image; perform dilation using a structuring element

B =

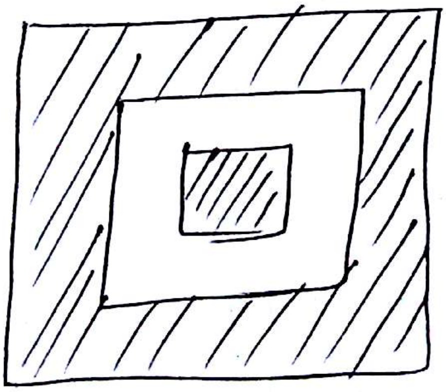
1	1	1
1	1	1
1	1	1

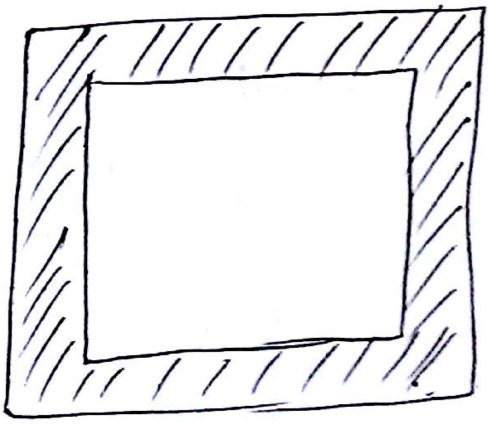
A = 

A =

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	0	0	1	1	0	0
0	0	1	1	0	0	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0





Dilation fills the holes & expand the boundaries

Duality :

Erosion & Dilation are dual of each other w.r.t set complementation & reflection

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

& $$(A \oplus B)^c = A^c \ominus \hat{B}$$

Note : Duality property is useful when the structuring element is symmetric w.r.t its origin so that $\hat{B} = B$.

Then we can obtain the erosion of an image by B simply by dilating its background (A^c) with the same structuring element and complementing the result.

Opening & Closing :-

→ Opening of an image is basically erosion followed by Dilation, using the same structuring element

open (A, B) = D (E(A))

$$A \circ B = (A \ominus B) \oplus B$$

→ It smooths the contour of the image, breaks down narrow bridges and eliminate thin protrusion i.e it isolates objects which may be just touching one another

Application

(1) Analysis of wear particles in engine oil
(2) Ink particles in recycled paper
(3) Study of cells in cytology

Closing :-

⇒ Closing of an image is basically dilation followed by erosion, using the same structuring element

$$CLOSE(A, B) = E(D(A))$$

$$A \bullet B = (A \oplus B) \ominus B$$

⇒ Closing tends to fuse narrow breaks & elliminates small holes.

NOTE: ⇒ opening & closing are dual to each other.

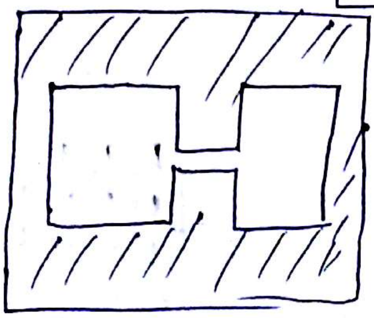
⇒ These operations can be applied few times, but has effect only once. $(A \bullet B)^c = (A^c \circ \hat{B})$

$(A \circ B)^c = (A^c \bullet \hat{B})$

Que Perform the opening operation on the image shown. Use the structuring element

1
1

& image is of size 10×10



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	1	1	1	0
0	1	1	1	0	0	1	1	1	0
0	1	1	1	1	1	1	1	1	0
0	1	1	1	0	0	1	1	1	0
0	1	1	1	0	0	1	1	1	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

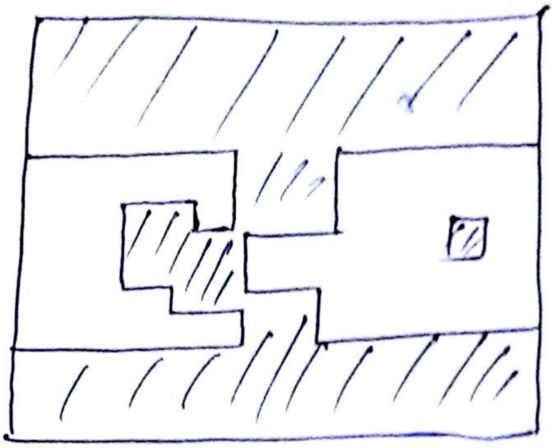
A

1
1

$A \ominus B \oplus B$ →

	1	1	1	0	0	1	1	1	
	1	1	1	0	0	1	1	0	
	1	1	1	0	0	1	1	1	
	1	1	1	0	0	1	1	1	
	1	1	1	0	0	1	1	1	
	1	1	1			1	1	1	

Que. Perform closing operation on the image shown below. The size of the image is 10×10. Use the same structuring element $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$



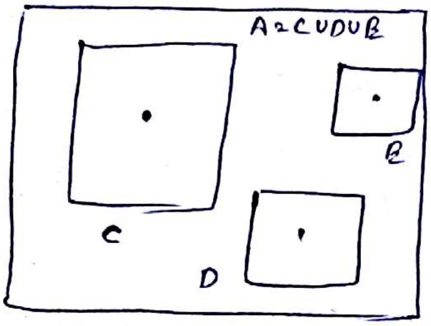
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
1	1	1	1	0	1	1	1	1	1
1	1	0	1	0	1	1	0	1	1
1	1	1	0	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$A \cdot B = (A \oplus B) \ominus B$

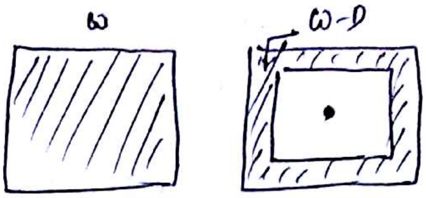
1	1	1	1	0	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1

Hit or miss Transformation :-

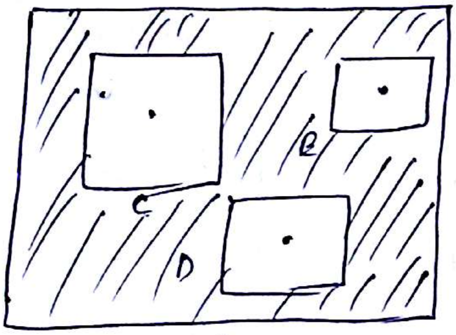
It is a basic tool for shape detection.



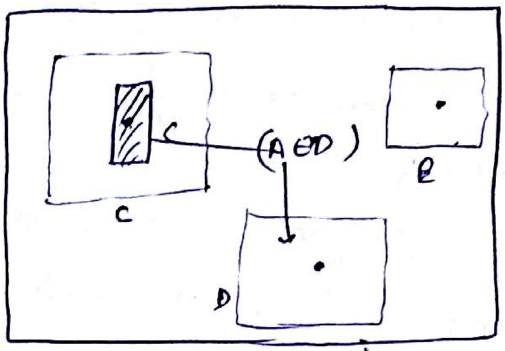
(a)



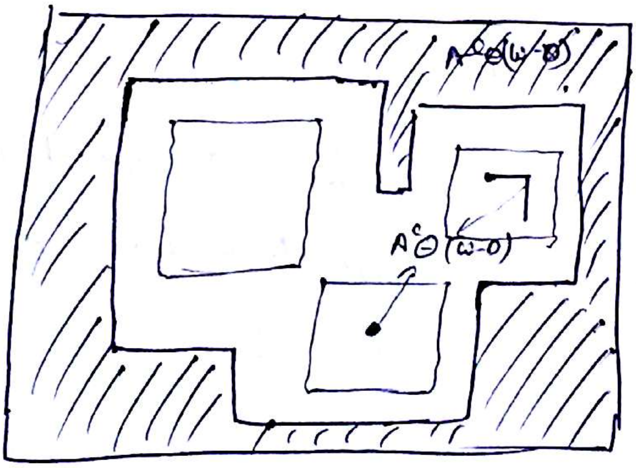
(b)



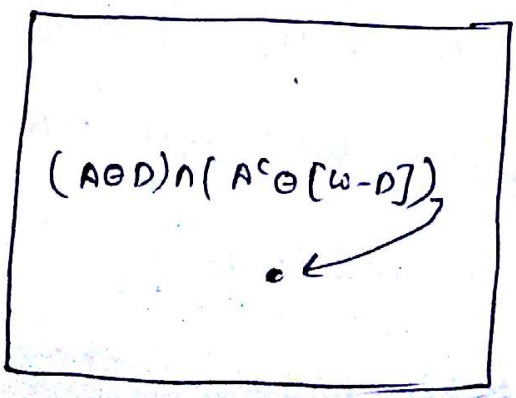
(c)



(d)



(e)



Let the origin of each shape be located at its centre of gravity

If we want to find the location of a shape, say D, at large image, say A:

Let D be enclosed by a small window say W.

The local background of D with respect to W is defined as the set difference (W-D).

Apply erosion operation of A by D, will get us the set of location of the origin of D, such that D is completely contained in A.

It may be also viewed geometrically as the set of all locations of the origin of D at which D found a match (hit) in A.

Apply erosion operation on the complement of A by the local background set (W-D)

The set of locations for which D exactly fits inside A is the intersection of these two last operators above

This intersection is the location of D.

Formally

If B denotes the set composed of D and its background

$B = (B_1, B_2)$ $B_1 = D$ & $B_2 = (W-D)$

The match or (set of matches) of B in A, denoted $A \circledast B$ is

$$\boxed{A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)}$$

It contains all the origin points at which, simultaneously, B_1 found a match ("hit") in A & B_2 find a match in A^c.

⇒ The reason for using these kind of structuring element $B = (B_1, B_2)$ is based on assumed definition that two or more objects are distinct only if they are disjoint (disconnected) sets.

⇒ In some cases, we may interested in detecting certain patterns (combination) of 1's & 0's and not for detecting individual objects. In this case a background is not required and the hit or miss transform reduces to simple erosion.

⇒ This pattern detection is used in some of the algorithm for identifying characters within a text

Basic morphological algorithms:

1. Boundary extraction
2. Region filling
3. Convex hull
4. Thinning
5. Thickening
6. Skeletons

Ques Given a 7×7 image, use the hit or miss transform to find the top edge of the 5×5 square. Use the two structuring elements shown below

$B_1 =$

0	0	0
0	1	1
0	1	0

$B_2 =$

0	1	0
0	0	0
0	0	0

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

$A =$

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0

$A \ominus B_1$

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$A^c =$

1	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
1	1	1	1	1	1	1

0	0	0	0	0	0	0
1	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	0	0	0	0	1
						1
1	0	0	0	0	0	1
1	0	0	0	0	0	
1	0	0	0	0	0	1

$A^c \ominus B_2$

$B_2 =$

0	1	0
0	0	0
0	0	0

$(A \ominus B_1) \cap (A^c \ominus B_2)$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

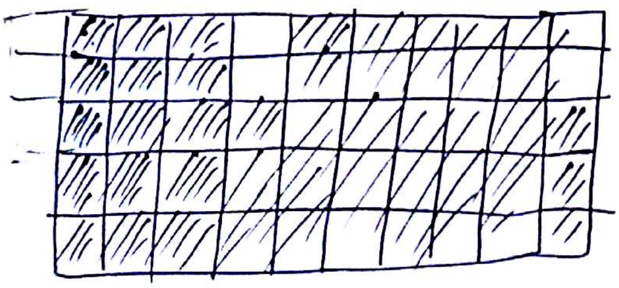
Boundary Extraction :.

If A is the image & B is the structuring element, then boundary extraction can be achieved using the formule Boundary (A) = $A - (A \ominus B)$

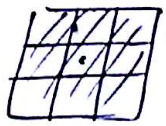
or

$\beta(A)$

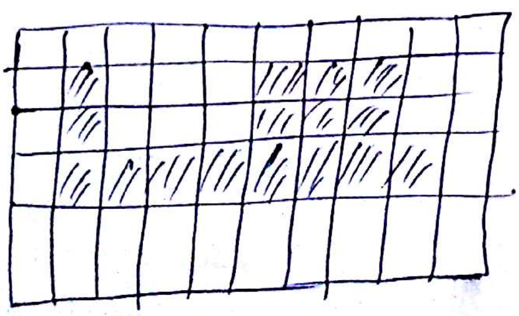
min {1, 1, 1, 1, 1, 1, 1, 0, 0, 0}



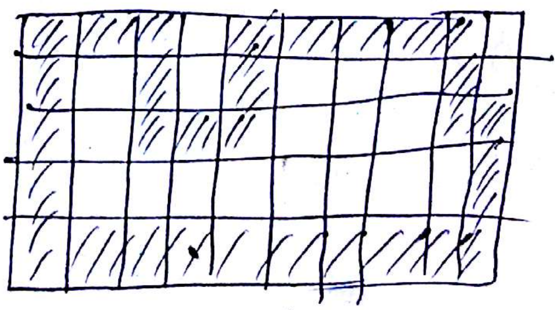
A



B



$A \ominus B$



$\beta(A)$

Region filling (or Hole filling)

A hole may be defined as a background region surrounded by a connected border of foreground pixels.

It uses set dilation, complementation, and intersection for filling holes in the image.

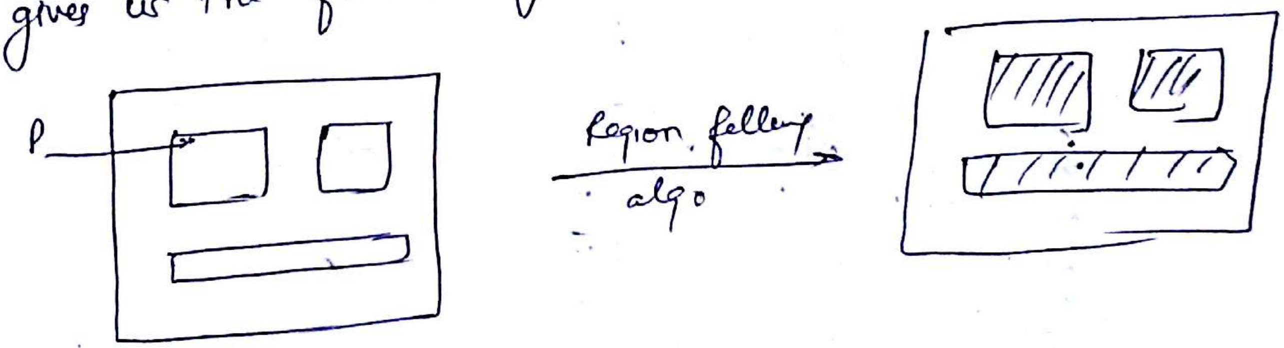
To start the procedure of region filling, we start with a pixel p and assign a value 1 to it

$$X_K = (X_{K-1} \oplus B) \cap A^C \quad K = 1, 2 .$$

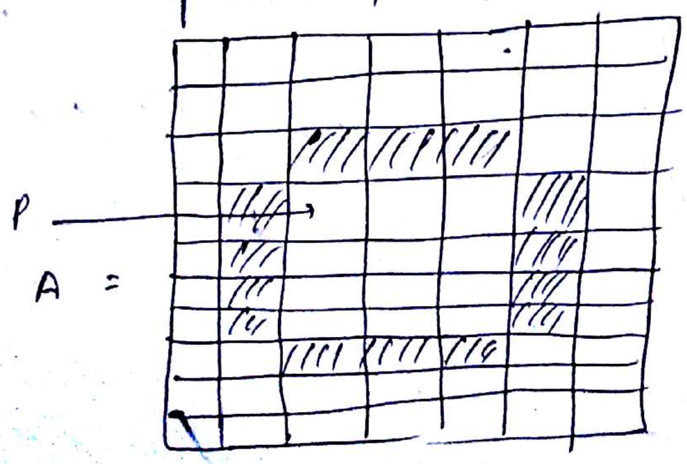
here $X_0 = P$ and B is structuring element

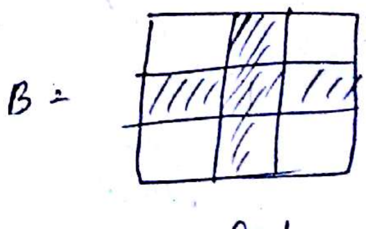
This algorithm terminates when $X_K = X_{K-1}$.

The union of the final X_K with the original image gives us the filled region.

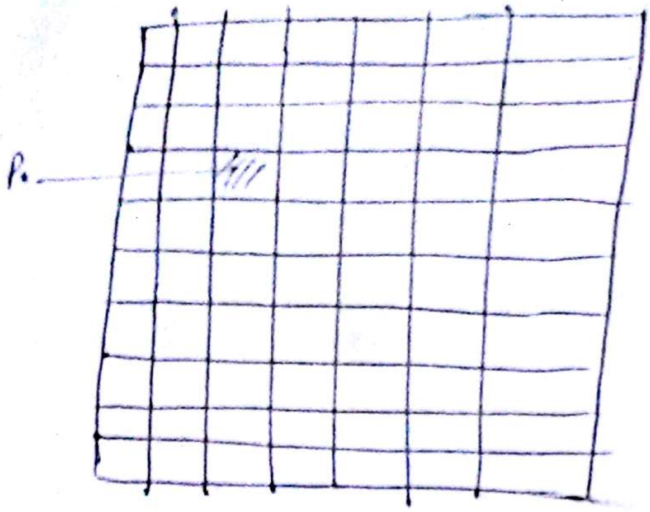


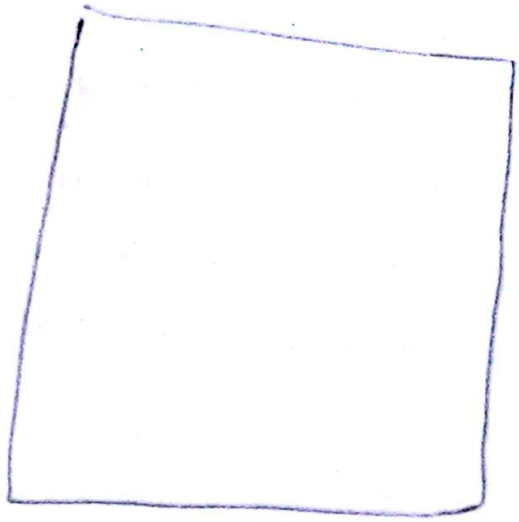
Que: Given the image below, use region filling to fill up the image. The structuring element is also given





shaded portion are considered as 1



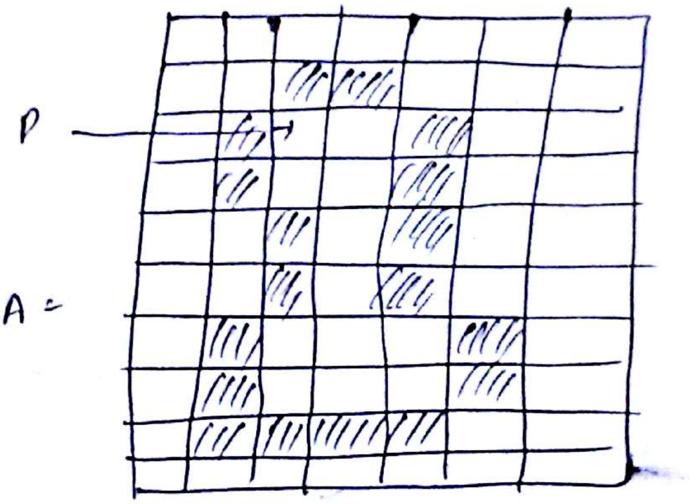
X_0

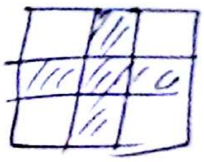
$X_1 = (X_0 \oplus B) \cap A^c$

$X_4 = X_5$

$X_4 \cup A$ is the final result

Que: Perform region filling of the given image

A =


B =


$X_7 = X_6$

$X_K = X_6$

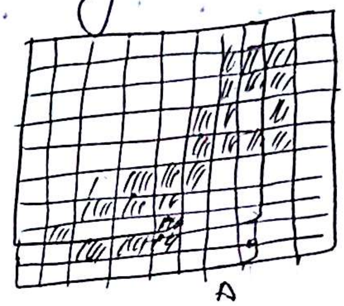
$A \cup X_K = A \cup X_6$

Extraction of connected components :

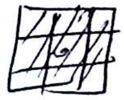
Here the of objective is to start with X_0 and find all the connected components.

$$X_K = (X_{K-1} \oplus B) \cap A \quad \text{for } K = 1, 2, 3 \ldots$$

The procedure terminates when $X_K = X_{K-1}$, with X_K containing all the connected components of the I/p image



A



B

Thinning & Thickening operation :-

⇒ The thinning of set A by a structuring element B, denoted $A \otimes B$, can be defined in terms of their hit or miss transform

$$A \otimes B = A - (A \circledast B)$$

$$= A \cap (A \circledast B)^c$$

⇒ A more useful expression for thinning A symmetrically is based on a sequence of structuring elements.

$$\{B\} = \{B^1, B^2, B^3, \ldots B^n\}$$

where B^i is rotated version of B^{i-1}. Using this the thinning is defined by a sequence of structuring elements as

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$$

⇒ The entire process is repeated until no further changes occur.

⇒ Thickening is the morphological dual of thinning.
Thickening is defined as

$$A \odot B = A \cup (A \circledast B)$$

⇒ In thinning a part of the boundary of the object is subtracted from the object and in thickening, a part of the boundary of the background is added to the object

⇒ As in thinning, thickening can be defined as a sequential operation.

$$A \odot \{B\} = ((\cdots((A \odot B^1) \odot B^2) \cdots) \odot B^n)$$

⇒ Thinning & thickening are dual operation.

$$\boxed{(A \odot B)^c = A^c \circledast B}$$

$B = (B_1, B_2)$

B_1 :

0	0	0
0	1	0
0	1	0

B_2 :

0	1	0
0	0	0
0	0	0

A

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0

$A \circledast B$ =

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$A - (A \circledast B)$ =

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0

Que. Given a 10×10 image, use the hit or miss transformation.

Use the following structuring elements.

B_1 =

x	1	x
0	1	1
0	0	x

B_2

x	1	x
1	1	0
x	0	0

B_3

x	0	0
1	1	0
x	1	x

B_4 =

0	0	x
0	1	1
x	1	x

1 = Foreground
0 = Background
x = ~~Ref~~ don't care

0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	0	0	0
0	1	1	1	1	1	0	0	0	0
0	1	1	1	1	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

we found R_1, R_2, R_3 & R_4

$R_{result} = R_1 \cup R_2 \cup R_3 \cup R_4$

Scanned by CamScanner

Given a 8×8 image, Perform sequential binary thinning

A =

0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0

B_1 =

0	0	0
x	1	x
1	1	1

B_2 =

x	0	0
1	1	0
1	1	x

B_3 =

1	x	0
1	1	0
1	x	0

B_4 =

1	1	x
1	1	0
x	0	0

B_5 =

1	1	1
x	1	x
0	0	0

B_6 =

x	1	1
0	1	1
0	0	x

B_7 =

0	x	1
0	1	1
0	x	1

B_8 =

0	0	x
0	1	1
x	1	1

Convex Hull:

⇒ A set A is said to be convex if the straight line segment joining any two points in A lies entirely within A.

⇒ The convex hull H of an arbitrary set S is the smallest convex set containing S.

⇒ The set difference H-S is called the convex deficiency of S.

⇒ It is useful for object description

Let B^i, $i = 1,2,3,4$ represent the four structuring elements

$$X_K^i = (X_{K-1} \circledast B^i) \cup A \qquad i = 1,2,3,4 \ldots \quad \& \ K = 1,2,3 \ldots$$

with $X_0^i = A$

when $(X_K^i = X_{K-1}^i)$ we let $\boxed{D^i = X_K^i}$

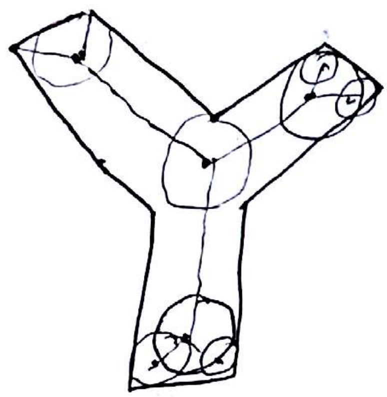
Then the convex hull of A is $\boxed{C(A) = \bigcup_{i=1}^{4} D^i}$

Skeletons :-

Let skeleton of set A is S(A) then

a) if z is a point of S(A) and $(D)_z$ is the largest disk centered in z and contained in A (One can not find a larger disk that fulfil this terms). that disk is called a "maximum disk".

b) The disk $(D)_z$ touches the boundary of A at two or more diffrent places



The skeleton of A is defined by terms of erosions & openings

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with $\boxed{S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B}$

where B = structuring element

$(A \ominus kB) = (..((A \ominus B) \ominus B) \ominus ...) \ominus B$

indicates k successive ~~...~~ erosions of A.

& K is the last iterative step before A erodes to an empty set $K = \max\{k \mid (A \ominus kB) \neq \phi\}$

In conclusion S(A) can be obtained as the union of skeleton subsets $S_k(A)$.

A can also be reconstructed from subset $S_K(A)$ by using the equation

$$A = \bigcup_{K=0}^{K} (S_K(A) \oplus KB)$$

It shows K succesive dilations of $S_K(A)$ that is.

$$(S_K(A) \oplus KB) = ((\cdots(S_K(A) \oplus B) \oplus B) \oplus \cdots) \oplus B$$

Pruning :-

Thinning & skeletonization can reduce the thickness of an object in an image to a one pixel wide skeleton representation

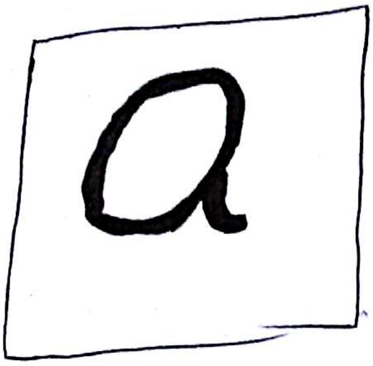
The problem with these operation is parasitic component that need to be "cleaned up" by postprocessing.

The process cleaning up these pixels is known as pruning

Morphological pruning ~~can be~~ is defined as-

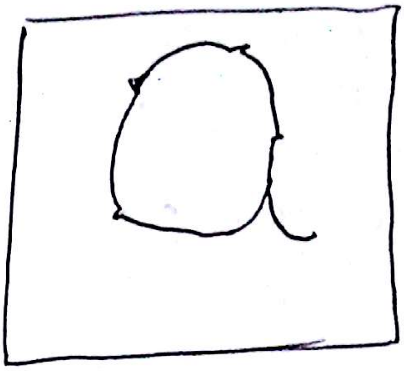
$$\boxed{A_{pruning} = A_{thinning} \otimes B_i}$$

Here B_i, $i = 1, 2, 3 \ldots 8$ are the 8 structuring element that were discussed in case of thinning & thickening operation



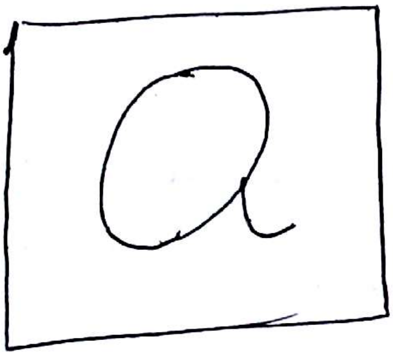
(a)

original image



(b)

Image after thinning operation



Pruned image