Modeling Temporary Market Impact

Quant Research Task

July 31, 2025

1 Introduction

The primary objective of this task is to develop a model for the temporary market impact of our trades. Temporary impact refers to the adverse price movement caused by the act of executing an order, which consumes liquidity from the limit order book. An accurate model of this impact is the cornerstone of any cost-aware execution strategy. This document outlines the methodology used to derive an empirical model of temporary impact, $g_t(x)$, which quantifies the per-share slippage from executing a buy order of size x at time t.

2 Methodology

2.1 Data Processing and Empirical Impact Calculation

The provided dataset contains minute-by-minute snapshots of the limit order book for three anonymous tickers. For each one-minute interval (time t), we first calculate a baseline reference price, the micro-price, defined as:

$$P_{\text{mid},t} = \frac{P_{\text{best bid},t} + P_{\text{best ask},t}}{2}$$

To measure the empirical impact of an order of size x, we simulate "walking the book." This involves calculating the volume-weighted average price (VWAP) of an aggressive buy order that consumes liquidity from the ask side, starting from the best ask and moving up the book until the order is filled. The average execution price is:

$$P_{\text{exec},t}(x) = \frac{\sum_{i=1}^{k} p_i v_i}{\sum_{i=1}^{k} v_i}$$

where p_i and v_i are the price and volume at the i-th level of the ask book, and k is the number of levels required to fill the order. The per-share slippage, or temporary impact, is then the difference between the execution price and the mid-price, typically expressed in basis points (bps):

$$g_t(x) = \left(\frac{P_{\text{exec},t}(x) - P_{\text{mid},t}}{P_{\text{mid},t}} \right) \times 10,000$$

This process was repeated for a range of order sizes x for each time interval t to generate a rich dataset of $(x, g_t(x))$ pairs.

2.2 Model Selection and Justification

While a linear model, $g_t(x) \approx \beta_t x$, is simple, it often fails to capture the true, concave nature of market impact. As one consumes deeper levels of the order book, the marginal cost of each additional share increases. A widely accepted and empirically robust functional form in academic and industry literature is the **square-root model**:

$$g_t(x) = \beta_t \sqrt{x} + \epsilon_t$$

Here, β_t is the **impact coefficient** for time t, which encapsulates the market's liquidity and resilience at that moment. A smaller β_t signifies a more liquid market where trades have less impact. This model suggests that the slippage grows with the square root of the order size, reflecting the diminishing liquidity as an order gets larger.

2.3 Model Fitting

To estimate the impact parameter β_t for each time period, we fit the square-root model to our empirical data. By setting $y = g_t(x)$ and $z = \sqrt{x}$, the non-linear model transforms into a simple linear regression through the origin:

$$y = \beta_t z$$

For each time t, we performed an Ordinary Least Squares (OLS) regression to find the β_t that minimizes the sum of squared errors. The goodness-of-fit was assessed using the R^2 statistic. The analysis revealed a strong linear relationship between slippage and the square root of order size across most time periods, validating our choice of the square-root model. The resulting time series of β_t values serves as the input for our optimal execution strategy.