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Stable Interaction of Autonomous Vehicle Platoons with Human-Driven Vehicles

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***Abstract*— A necessary prerequisite for the safe interaction of autonomous systems with a human-driven vehicle is for the overall closed-loop system (autonomous systems plus human-driven vehicle) to be stable. This paper studies the safe and stable interaction between a platoon of autonomous vehicles and a set of human-driven vehicles. Considering the longitudinal motion of the vehicles in the platoon, the problem is to ensure a safe emergency braking by the autonomous platoon considering the actions of human-driven vehicles, which may vary based on the driver type. We consider two types of platoon topologies, namely unidirectional and bidirectional. Safe emergency braking is characterized by a specific type of platoon stability, called head-to-tail stability (HTS). We present system-theoretic necessary and sufficient conditions for the combination of the autonomous platoon and human-driven vehicles to be HTS for two platoon control laws, namely the velocity tracking and the platoon formation. Modeling the input-output behavior of each vehicle via a transfer function, the HTS conditions restrict the human-driven vehicles' transfer functions to have H_∞ norms below certain thresholds. A safe interaction algorithm first identifies the transfer functions of the human-driven vehicles. Then, it tunes the platoon control gains such that the overall system meets HTS conditions. Theoretical results are validated with both experimental data with human subject studies and simulation studies.**

I. Introduction

A. Motivation

Intelligent transportation system is evolving through interconnected and autonomous vehicles that aim to reduced traffic congestion, environmental impact, and capital costs. To yield this, the safety of autonomous vehicles must be put foremost. One of the main aspects of autonomous vehicles' safety is their safe interactions with human-driven vehicles. Due to the large level of complexity in human behavior along with the sophisticated structure of urban traffic systems, providing a holistic view of the safe coexistence of autonomous and human-riven cars in future cities is challenging. This paper studies the interaction of human-driven vehicles with a platoon of autonomous vehicles in a longitudinal emergency braking scenario. We show that a safe interaction between vehicles, even for a simple traffic topology and vehicle 1-dimensional motions, demands subtle system-theoretic conditions. These conditions not only require knowledge about the platoon topology and control actions of the autonomous vehicles, but they also need to know the type of driving behaviors of the human-driven vehicles.

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B. Literature review

In early research on the interaction between autonomous and human-driven vehicles, autonomous cars where modeled as defensive objects with low abilities in interacting with human drivers. Although more simplistic vehicle maneuvers, such as obstacle avoidance, active steering and braking, and lane keeping, had been well studied for autonomous driving in the literature [1]–[3], considering the human actions required interactions between autonomous and human driven vehicles; a topic which has been studied in recent years [4], [5]. The ways that an autonomous car can leverage effects on a human-driven car in its decisions are discussed in [6], [7]. There, a high fidelity tactical horizon is used to predict the immediate interactions, assuming that the human can infer the autonomous car's planned trajectory and react accordingly. To address long-horizon interactions, dynamic game models were applied [8], [9]. The work [10] considers hierarchical dynamic games for an autonomous car interacting with a simulated human-driven car.

Extending the problem of interaction between autonomous and a human-driven vehicle to multiple vehicles has been studied recently [10], [11]. An important aspect of this problem is the way that autonomous vehicles interact with themselves and the procedure of information disseminated throughout the network. To do that, the first step is to specify the type of the traffic topology. One of the simplest, yet useful, vehicle networks is platooning, which is widely used in efficient freight transportation systems [12], [13]. A safe interaction between a platoon of autonomous vehicles and human driven vehicles has been a topic of research in recent years. Earlier works include maintaining tight vehicle gaps to discourage car cut-ins [14]. Other works propose opening up a wider vehicle gap when a car enters the platoon identified by sensors [15]. To provide less conservative results, a hierarchical dynamic game, played between the human driver and the nearest vehicle in the platoon, was proposed in [16], which leverages the effects of the human decisions.

C. Contributions

This paper provides a system-theoretic approach to safe interaction between an autonomous platoon an a set of human-driven vehicles. Our main contributions can be summarized as follows:

- For a platoon of autonomous vehicles with a velocity tracking control law and for bidirectional and uni-

directional communication topologies, necessa sufficient conditions for head-to-tail stability (H the platoon in the presence of a single humar vehicle are derived.
- The above-mentioned necessary and sufficient tions are derived for platoon formation dynami known as the cooperative adaptive cruise contr show that, unlike velocity tracking, the conditi HTS in platoon formation scale on the platoon
- The necessary and sufficient conditions for H extended to the case of multiple human driven v interacting with an autonomous platoon.
- Finally, we propose an (offline) system identi technique to find the type of the human driver input-output behaviour which is further used to assess the derived HTS conditions. Several simulations, based on the experimental data, presented to validate the theoretical results.

The paper is organized as follows. In Section II, we formulate the problem of an interaction between a platoon of autonomous vehicles and a set of human-driven vehicles. Section III discusses necessary and sufficient conditions for HTS for platoons with unidirectional and bidirectional communications and both velocity tracking and platoon formation dynamics. Section IV extends the results of Section III to multiple human-driven vehicles. Section VI belongs to the simulation results. Section VII concludes the paper.

II. Problem Formulation

Consider a platoon of n of autonomous vehicles, denoted by P_n, with a leading vehicle v_0. The leading vehicle is responsible for planning the platoon motion, including optimal velocity for fuel consumption, emergency braking, and lateral maneuvers [12], [17]. The position and velocity of vehicle v_i is denoted by $p_i(t)$ and $u_i(t)$, respectively. The communication topology of the platoon is either unidirectional (also known as predecessor following) or bidirectional, as shown in Fig. 1. The evolution of the state of vehicle v_i, denoted by $x_i(t) = [p_i(t)\; u_i(t)]^T$, depends on the communication topology, i.e., $x_i(t)$ evolves as

$$\begin{aligned} \dot{x}_i(t) &= f\left(x_{i-1}(t), x_{i+1}(t)\right), && \text{Bidirectional} \\ \dot{x}_i(t) &= f\left(x_{i-1}(t)\right), && \text{Unidirectional} \end{aligned} \tag{1}$$

where the updating rule $f(\cdot)$ can be of different types which will be discussed later.[1] When platoon P_n reaches an obstacle, the leader performs an emergency braking. The braking propagates through the platoon string. There are several measure to quantify the stability of the platoon which is subjected to an abrupt acceleration or braking. String stability is one of the widely used stability conditions for vehicle platoons. Among variations of string stability introduced in the literature (see [18] and references therein) we use the the following definition.

[1]From now, for the simplicity of the notations, we drop the time argument from the states.

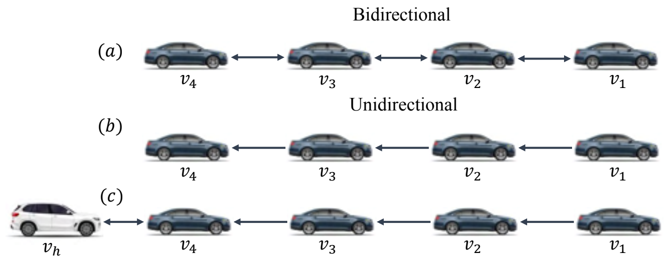

Fig. 1: (a) A platoon with a bidirectional communication, (b) A platoon with a unidirectional communication, (c) A platoon with a human-driven vehicle behind.

Definition 1 (The head to tail stability (HTS)): For platoon P_n, the system is HTS if the transfer function from the output of the leading vehicle v_1 to the output of the tail vehicle v_n, denoted by $G_{v_1 \to v_n}$, satisfies

$$\|G_{v_1 \to v_n}\|_{\mathcal{H}_\infty} \leq 1, \tag{2}$$

where $||G||_{\mathcal{H}_\infty} \triangleq \sup_{\omega \in \mathbb{R}} \lambda_{\max}^{\frac{1}{2}}(G^* G)$ is the $\mathcal{H}_\infty$ norm of transfer function G. □

The transfer function G mentioned above depends on vehicle's local dynamics which will be discussed later. If an autonomous platoon is HTS and the inter-vehicular distance is large enough, then the vehicles in the string can perform a safe emergency braking. However, this is only true when all vehicles in P_n follow the prescribed dynamics (1). In this paper, we also consider a set of human-driven vehicles, denoted by $\mathcal{H} = \{h_1, h_2, ..., h_H\}$, merging with platoon P_n. The dynamics of human driven vehicles are unknown, hence, the HTS is not necessarily guaranteed for the overall platoon, i.e., $P_n \cup \mathcal{H}$. The objective is to perform a safe emergency braking in the presence of the set of human-driven vehicles $\mathcal{H}$.

Problem 1: Consider a platoon of n vehicles P_n and a set of human driven vehicles $\mathcal{H}$ which are merged with P_n.[2] The objective for the platoon of autonomous and human-driven vehicles, $P_n \cup \mathcal{H}$, is to be HTS.

III. String Stability with a human-Driven Vehicle

We first consider a single human-driven vehicle, h, approaching the last vehicle in P_n, as shown in Fig. 1 (c). The information flow between the last autonomous vehicle and h is bidirectional. A safe braking of the overall platoon $P_n \cup h$ requires that P_n is HTS itself. Hence, we make the following assumption. Any string of m autonomous vehicles, $m \leq n$, is HTS.

We investigate two classes of vehicle local dynamics, i.e., function $f(\cdot)$ in (1). In each case, we discuss HTS conditions for both unidirectional and bidirectional platoon topologies.

A. Velocity Tracking

In the velocity tracking scenario, the objective for each vehicle is to track the reference velocity determined by the

[2]The way that the vehicles merge with the platoon requires to analyze the vehicles' lateral motions which is out of the scope of this paper.

leader. This can be considered as a cruise control performed in a cooperative manner. We study the velocity tracking for two platoon topologies below.

1) Unidirectional Communication: For unidirectional communication, the following first order updating law represents the cruise control for each vehicle

$$\begin{aligned} \dot{u}_0 &= k(u_{\text{ref}} - u_0) + \zeta, \\ \dot{u}_i &= k(u_{i-1} - u_i), \qquad i = 1, ..., n-1. \end{aligned} \tag{3}$$

Here, u_{ref} is the reference velocity and ζ models the disturbance due to an emergency braking done by the leading vehicle v_0. k is the control gain chosen by vehicles in the platoon.[3] Since the objective for each vehicle is to eventually track the reference velocity u_{ref}, we introduce $e_i = u_i - u_{\text{ref}}$ for each vehicle v_i. The error dynamics become

$$\begin{aligned} \dot{e}_0 &= -ke_0 + \zeta, \\ \dot{e}_i &= k(e_{i-1} - e_i), \qquad i = 1, ..., n-1, \end{aligned} \tag{4}$$

and the transfer function from ζ to e_n can be simply calculated

$$G_{\zeta \to e_n} = \frac{k^{n-1}}{(s+k)^n}. \tag{5}$$

Now, suppose that a human driven vehicle h approaches the platoon. The transfer function from the n-th vehicle to h, $G_{e_n \to e_h}$, is called the human transfer function and is simply written as G_h. Thus, the overall transfer function is

$$G_{\zeta \to e_h} = G_h G_{\zeta \to e_n} = G_h \frac{k^{n-1}}{(s+k)^n}. \tag{6}$$

*Proposition 1 (**Velocity Tracking, Unidirectional**):* Consider platoon P_n with unidirectional topology and a human-driven vehicle h. For platoon $P_n \cup h$ to be HTS, it is necessary to have $k \geq G_h(0)$ and sufficient to have $k \geq \|G_h\|_{\mathcal{H}_\infty}$.

Proof: To show the necessary and sufficient conditions, we first need to show

$$\frac{1}{k} G_h(0) \leq \|G_{\zeta \to e_h}\|_{\mathcal{H}_\infty} \leq \frac{1}{k} \|G_h\|_{\mathcal{H}_\infty}, \tag{7}$$

The left inequality above comes from (6) and the fact that the $\mathcal{H}_\infty$ norm is not less than the DC gain. To show the right inequality, we write (4) in matrix form

$$\begin{aligned} \dot{\boldsymbol{e}} &= A_u \boldsymbol{e} + \mathbf{e}_1 \zeta \\ y &= \mathbf{e}_n^\top \boldsymbol{e} \end{aligned} \tag{8}$$

where $\mathbf{e}_i$ is the i-th vector of canonical basis. A_u is a grounded Laplacian matrix of a path graph [19], [20] and its elements are the control gains in (4). It can be verified that A_u is a Metzler matrix (a matrix in which all the off-diagonal components are nonnegative). Hence, (8) is a positive system and its $\mathcal{H}_\infty$ norm happens at zero frequency [21], i.e.,

$$\|G_{\zeta \to e_n}\|_{\mathcal{H}_\infty} = \frac{1}{k}.$$

[3]For the ease of the analysis, we assume uniform control gains. The results can be readily extended to non-uniform control gains.

Hence, by sub-multiplicative property, we have

$$\begin{aligned} \|G_{\zeta \to e_h}\|_{\mathcal{H}_\infty} &\leq \|G_{\zeta \to e_n}\|_{\mathcal{H}_\infty} \|G_h\|_{\mathcal{H}_\infty} \\ &= \frac{1}{k} \|G_h\|_{\mathcal{H}_\infty}. \end{aligned}$$

Necessary and sufficient conditions for HTS are readily obtained from (7). ■

Based on Proposition 1, the platoon can maintain string stability only by adjusting control gain k. The value of k must be at least equal to the DC gain of the human transfer function, $G_h(0)$. One can interpret the necessary condition for string stability mentioned in Proposition 1 in terms of the platoon safety. In particular, if vehicle h behaves unsafely near the zero frequency (i.e., in response to very slow time varying inputs), then it will be definitely unsafe in cases of harsher brakings.

2) Bidirectional Communication: For bidirectional communication, each vehicle receives information from its front and back vehicles. The updating rules of vehicles are

$$\begin{aligned} \dot{u}_0 &= k(u_{\text{ref}} - u_0) + \zeta, \\ \dot{u}_i &= k(u_{i-1} - u_i) + k(u_{i+1} - u_i), \quad i = 1, ..., n-2 \\ \dot{u}_{n-1} &= k(u_{n-2} - u_{n-1}). \end{aligned} \tag{9}$$

Similar as before, we define the state error $e_i = u_i - u_{\text{ref}}$ and the error dynamics can be written in the following vector form

$$\dot{\boldsymbol{e}} = A_b \boldsymbol{e} + \mathbf{e}_1 \zeta, \tag{10}$$

*Proposition 2 (**Velocity Tracking, Bidirectional**):* Consider platoon P_n with bidirectional topology and a human-driven vehicle h. For platoon $P_n \cup h$ to be HTS, it is necessary to have $k \geq G_h(0)$ and sufficient to have $k \geq \|G_h\|_{\mathcal{H}_\infty}$.

Proof: Similar to A_u, matrix A_b is Metzler and the system (10) is positive. Thus, the $\mathcal{H}_\infty$ norm is equal to the DC gain of the system. One can calculate the DC gain of (10) based on the properties of matrix A_b as follows [22]

$$\|G_{\zeta \to e_n}(0)\|_{\mathcal{H}_\infty} = \mathbf{e}_n^T A_b^{-1} \mathbf{e}_1 = \frac{1}{k}. \tag{11}$$

The rest of the proof is similar to Proposition 1. ■

Remark 1: According to Propositions 1 and 2, the $\mathcal{H}_\infty$ norms of head-to-tail transfer functions for both bidirectional and unidirectional communications are the same. Hence, the conditions for the platoon to be HTS are identical for both communication topologies. This observation becomes more interesting when we compare it with the case where $\mathcal{H}_\infty$ norm is calculated from the disturbances on all vehicles to the state of all vehicles, i.e., when the system is of the following form

$$\begin{aligned} \dot{\boldsymbol{e}} &= A\boldsymbol{e} + \mathbf{1}\zeta, \\ y &= \boldsymbol{e} \end{aligned} \tag{12}$$

In that case, the $\mathcal{H}_\infty$ norm for bidirectional and unidirectional topologies are substantially different and we have [23]

$$\|G_{\zeta \to e_n}(0)\|_{\mathcal{H}_\infty,u} = \left(\|G_{\zeta \to e_n}(0)\|_{\mathcal{H}_\infty,b}\right)^{\frac{1}{2}}, \tag{13}$$

where indices b and u denote bidirectional and unidirectional topologies. □

B. Platoon Formation

In platoon formation scenario, in addition to tracking the leader velocity, each vehicle preserves a safe distance from its front vehicle. This is also known as cooperative adaptive cruise control [24]. Recall that the position and velocity of vehicle v_i are denoted by p_i and v_i, respectively. The desired safe distance between consecutive vehicles is Δ.[4]

1) Unidirectional Communication: Assuming $p_0(0) = 0$, the updating rule of the leader, v_0, is

$$\ddot{p}_0 = k_p(u_{\rm ref}.t - p_0) + k_u(u_{\rm ref} - \dot{p}_0) + \zeta, \tag{14}$$

For the rest of the vehicles, the updating rule is

$$\ddot{p}_i = k_p(p_{i-1} - p_i + \Delta) + k_u(\dot{p}_{i-1} - \dot{p}_i), \quad \forall i = 1, ..., n-1, \tag{15}$$

Here, k_p and k_u are position and velocity gains, respectively. In order to find the effect of the disturbance ζ to the last vehicle, we define auxiliary variable $e_i \triangleq p_i - p_i^*$ for each vehicle v_i, where $p_i^* = i.\Delta + u_{\rm ref}.t$ is the reference path trajectory for vehicle v_i. Hence, one can write the following error dynamics

$$\begin{aligned} &\ddot{e}_0 = -k_p e_0 - k_u \dot{e}_0 + \zeta, \\ &\ddot{e}_i = k_p(e_{i-1} - e_i) + k_u(\dot{e}_{i-1} - \dot{e}_i), \quad \forall i = 1, ..., n-1. \end{aligned} \tag{16}$$

By writing (16) in matrix form, the transfer function from ζ to the last vehicle becomes

$$G_{\zeta \to e_n} = \mathbf{e}_n^\top \left(s^2 I + k_u s A_u + k_p A_u\right)^{-1} \mathbf{e}_1. \tag{17}$$

Here, $A_u \in \{0,1\}^{n\times n}$ in which the nonzero entries follow the pattern of matrix A_u defined in Section III-A. Considering the human-driven vehicle h, the transfer function $G_{\zeta\to h}$ has the following form

$$G_{\zeta \to h} = G_h.\mathbf{e}_n^\top \left(s^2 I + k_u s A_u + k_p A_u\right)^{-1} \mathbf{e}_1, \tag{18}$$

where G_h is the same as defined in Section III-A.

*Proposition 3 (**Platoon Formation, Unidirectional**):* Consider platoon P_n with unidirectional communication and human-driven vehicle h. For platoon $P_n \cup h$ to be HTS, i.e., $\|G_{\zeta\to h}\|_{\mathcal{H}_\infty} \le 1$, it is necessary to have $k_p \ge G_h(0)$ and sufficient to have

$$\|G_h\|_{\mathcal{H}_\infty} \le \frac{1}{\beta}\alpha^{-n}, \tag{19}$$

where

$$\beta = \|\frac{1}{s^2 + k_u s}\|_{\mathcal{H}_\infty}, \quad \alpha = \|\frac{k_u s + k_p}{s^2 + k_u s + k_p}\|_{\mathcal{H}_\infty}. \tag{20}$$

Proof: Similar to the velocity tracking, we have the following bounds for $\|G_{\zeta\to e_h}\|_{\mathcal{H}_\infty}$

$$\begin{aligned} G_{\zeta\to e_n}(0)G_h(0) = \frac{1}{k_p}G_h(0) &\le \|G_{\zeta\to e_h}\|_{\mathcal{H}_\infty} \\ &\le \|G_{\zeta\to e_n}\|_{\mathcal{H}_\infty}\|G_h\|_{\mathcal{H}_\infty}. \end{aligned} \tag{21}$$

However, unlike the velocity tracking case, the platoon formation dynamics is not a positive system. Hence, the $\mathcal{H}_\infty$ norm of (18) does not happen in zero frequency and one must find the frequency for which the maximum system norm happens. One approach is to work with scaling results presented in [25]. In particular, for $n \gg 1$, the following approximation holds

$$\|G_{\zeta\to e_n}\|_{\mathcal{H}_\infty} = \beta\alpha^n, \tag{22}$$

where α and β are as mentioned in the statement of the proposition. Substituting (22) into (21) yields the result. ■

2) Bidirectional Communication: For bidirectional communication, the leader's dynamics is the same as before and the dynamics of the followers are

$$\begin{aligned} \ddot{p}_i = &\,k_p(p_{i-1} - p_i + \Delta) + k_p(p_{i+1} - p_i + \Delta) \\ &+ k_u(\dot{p}_{i-1} - \dot{p}_i) + k_u(\dot{p}_{i+1} - \dot{p}_i), \quad \forall i = 1, ..., n-1. \end{aligned} \tag{23}$$

By writing the error dynamics, similar to the unidirectional case, the transfer function from ζ to h becomes

$$G_{\zeta\to h} = G_h.\mathbf{e}_n^T \left(s^2 I + k_u s A_b + k_p A_b\right)^{-1} \mathbf{e}_1, \tag{24}$$

where $A_b \in \{0,1\}^{n\times n}$ in which the nonzero entries follow the pattern of matrix A_b defined in Section III-A.

*Proposition 4 (**Platoon Formation, Bidirectional**):* Consider platoon P_n with bidirectional communication and human-driven vehicle h. For platoon $P_n \cup h$ to be HTS, i.e., $\|G_{\zeta\to h}\|_{\mathcal{H}_\infty} \le 1$, it is necessary to have $k_p \ge G_h(0)$ and sufficient to have

$$\|G_h\|_{\mathcal{H}_\infty} \le \frac{k_u\sqrt{k_p}\pi^2}{8n}. \tag{25}$$

Proof: The proof of the following proposition is similar to that of Proposition 3 with the only difference in scaling of the $\mathcal{H}_\infty$ norm for bidirectional platoon [25]. ■

Remark 2: It is worth noting that the conditions mentioned in Propositions 1 and 2 for velocity tracking scenario are independent of the platoon size while the conditions in Propositions 3 and 4 for platoon formation are highly sensitive to the platoon size. In particular, by increasing the platoon size, the sufficient conditions for the platoon to be HTS become stronger. □

Note that the first order model for velocity tracking discussed in the Section III-A can be viewed as an approximation of the second order model when $k_p \ll k_u$. This means that the controller relies more on the relative velocity feedback compared to the relative distance. It is physically valid when the sensors measure the relative velocity more accurately that the relative distance.

IV. Multiple human-driven vehicles

In this section, we study the case where there are H human-driven vehicles, denoted by $\mathcal{H} = \{h_1, h_2, ..., h_H\}$ located between the vehicles in platoon P_n. The objective is to determine conditions in which the overall platoon, i.e., $P_n \cup \mathcal{H}$, is HTS. The human-driven vehicles break the platoon

[4]For simplicity, here we assume that the safe distance is the same for all vehicle pairs.

Fig. 2: A platoon of autonomous vehicles with two human-driven vehicles.

vehicle in each sub-platoon (e.g., vehicle v_3 in Fig. 2) does not receive information from its predecessor autonomous vehicle (as it is blocked by a human-driven vehicle). Hence, this first vehicle must act as a leader for its corresponding sub-platoon. The transfer function of the i-th sub-platoon is denoted by $\tilde{G}_i$ and the transfer function of the i-th human-driven vehicle is denoted by G_i. The following two theorems extend the results of Propositions 1 and 4 to the case of multiple human-driven vehicles.

*Theorem 1 (**Velocity Tracking**):* Let P_n be a unidirectional or bidirectional platoon with velocity tracking dynamics (3) and $\mathcal{H}$ be a set of human-driven vehicles. Then, for $P_n \cup \mathcal{H}$ to be HTS, i.e., $\|G_{\zeta\to h_H}\|_{\mathcal{H}_\infty} \leq 1$, it is necessary to have

$$k^H \geq \prod_{i=1}^{H} G_i(0),$$

and sufficient to have

$$k^H \geq \prod_{i=1}^{H} \|G_i\|_{\mathcal{H}_\infty}.$$

□

Proof: Extending the approach we adopt in Section II to multiple human-driven vehicles, one can break the head-to-tail transfer function $G_{\zeta\to h_H}$ into H sub-platoons and write

$$G_{\zeta\to h_H} = G_{\zeta\to h_1} G_{h_1\to h_2} ... G_{h_{H-1}\to h_H}. \tag{26}$$

Recalling that $\tilde{G}_i$ and G_i are the transfer functions of the i-th autonomous sub-platoon and the i-th human-driven vehicle, we can write $G_{h_{i-1}\to h_i} = G_i\tilde{G}_i$. Thus, we rewrite (26) as

$$G_{\zeta\to h_H} = G_1\tilde{G}_1 G_2\tilde{G}_2 ... G_H\tilde{G}_H. \tag{27}$$

Hence, $G_{\zeta\to h_H}$ is bounded by

$$\underbrace{G_1(0)...G_H(0)}_{\text{human}}\underbrace{\tilde{G}_1(0)...\tilde{G}_H(0)}_{\text{autonomous}} \leq \|G_{\zeta\to h_H}\|_{\mathcal{H}_\infty} \leq \underbrace{\|G_1\|_{\mathcal{H}_\infty}...\|G_H\|_{\mathcal{H}_\infty}}_{\text{human}}\underbrace{\|\tilde{G}_1\|_{\mathcal{H}_\infty}...\|\tilde{G}_H\|_{\mathcal{H}_\infty}}_{\text{autonomous}} \tag{28}$$

Thus, the necessary and sufficient conditions follow the procedure in Proposition 1. ■

The proof of the following theorem follows the same procedure as Theorem 1.

*Theorem 2 (**Platoon Formation**):* Let P_n be a platoon with network formation dynamics and $\mathcal{H}$ be a set of human-driven vehicles. Then, for $P_n \cup \mathcal{H}$ to be HTS, i.e., $\|G_{\zeta\to h_H}\|_{\mathcal{H}_\infty} \leq 1$, for both unidirectional and bidirectional topologies, it is necessary to have

$$k_p^H \geq \prod_{i=1}^{H} G_i(0),$$

and sufficient to have

$$\begin{aligned} \prod_{i=1}^{H} \|G_i\|_{\mathcal{H}_\infty} &\leq \frac{1}{\beta}\prod_{i=1}^{H} \alpha^{m_i} && \text{Unidirectional} \\ \prod_{i=1}^{H} \|G_i\|_{\mathcal{H}_\infty} &\leq \frac{k_u\sqrt{k_p}\pi^2}{8}\prod_{i=1}^{H} \frac{1}{m_i}, && \text{Bidirectional} \end{aligned} \tag{29}$$

where m_i is the length of the i-th sub-platoon and α and β are the same as defined in proposition 3. □

V. Human behaviour model

In the previous sections, we discussed system-theoretic conditions to ensure string stability of an autonomous platoon P_n in the presence of a set of human-driven vehicles $\mathcal{H}$. Those conditions were in terms of the $\mathcal{H}_\infty$ norm of the transfer functions of the human-driven vehicles. Thus, one must have this quantity in hand to assess HTS of the system. With this in mind, the next step is to find a model for the human driver which, consequently, yields the transfer function G_h.

A. Transfer function model with reaction delay

Considering the basic limitations and characteristics of the human cognitive and neuromuscular system, the following transfer function has been proposed and widely used to model human response [26]

$$G_h(s) \approx K\frac{1+T_z s}{1+2\gamma T_w s + T_w^2 s^2}e^{-T_d s} = \frac{\dot{P}_h(s)}{\dot{P}_n(s)}. \tag{30}$$

Here $\dot{P}_h(s)$ and $\dot{P}_n(s)$ represent the Laplace transform of the the velocity of the human driven car ($\dot{p}_h(t)$) and a car in front with velocity $\dot{p}_n(t)$, respectively. The parameters in (30) determine the characteristics of a specific human driver. Here, T_d represents the inherent cognitive and muscular delay in human response. The other parameters in G_h characterise the responses of the human driver. Once the parameters of transfer function (30) are identified, we only need to calculate the $\mathcal{H}_\infty$ norm of G_h and check the HTS conditions mentioned in the previous sections.

B. Experimentally identifying the human behavior model

We carried out a human study with three drivers[5]. In a first-person driving game created in Unity for this purpose, the drivers use the brake and throttle of a Logitech G29 car controller to control a car (with autonomous steering) that is placed behind an autonomous platoon of 2 cars. The movement of all cars is restricted to a single lane.

[5]Limitations due to the Covid-19 pandemic restricted the participants to being a subset of the authors of this paper.

Fig. 3: In a platoon comprised of autonomous and human-c
the type of the human driving behaviour should be identifiec
the analysis of the platoon stability. Here, a *distracted* driver
an experiment to gather data for the driver model identific

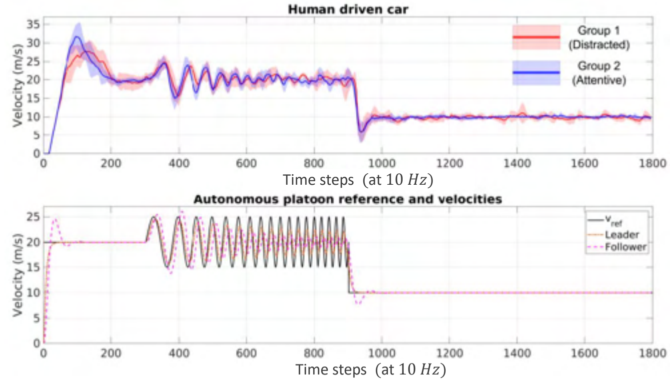

Fig. 4: Mean and standard deviation of car velocity for the two groups

1) Simulating different driver types: The pa
the experiment twice. The first time when they
with having to perform a cognitive task (m
algebra questions on their phone) in additic
the car. This methodology is common in hum
and simulates a group of distracted drivers. Fi
a distracted participant and the experimental
second time, the 3 participants drive the car
distraction. We refer to this group as the *atten*

The experiments involve each driver follo
tonomous platoon for 2 identical trials of 3 r
Two trials involve the lead autonomous vehicl
chirp-like reference velocity signal u_{ref}. Figu
shows the reference velocity for the autono
leader, the velocity of the leader, and the ve
autonomous follower in the platoon[7]. It also sh
resulting average velocity (and standard devi
human-driven vehicle behind it (for both the groups), across the trials. Based on this figure, the response of the attentive driver has a larger overshoot and faster time-to-peak, showing a more agile response compared to the distracted driver. This faster response stem from smaller response delay T_d and the damping factor γ, as will be shown in the next subsection. Finally, figure 5 shows the distances between vehicles for these trials.

2) Identified models for the different driving types: Using this data, we identified the following transfer functions for each group (with a fit of $\approx 80\%$ for each group):

$$G_h^{\text{distracted}}(s) = \frac{1 + 6.96s}{1 + 2(0.65)(4.76)s + (4.76)^2 s^2} e^{-0.512s} \tag{31a}$$

$$G_h^{\text{attentive}}(s) = \frac{1 + 5.41s}{1 + 2(0.54)(4.15)s + (4.15)^2 s^2} e^{-0.324s} \tag{31b}$$

For the distracted driving group, the identified delay in response $(0.512s)$ is higher than that for the attentive group $(0.324s)$. This is in line with trends observed in other human studies [26]. The damping factor for the distracted group $\gamma = 0.65$ is higher than for the attentive group $\gamma = 0.54$, which intuitively corresponds to a slightly more sluggish response

[6]See https://youtu.be/mfPzYDQrvV4 for video playback.

[7]The simulator for autonomous vehicles is deterministic, therefore the autonomous vehicles had identical behaviors for each trials

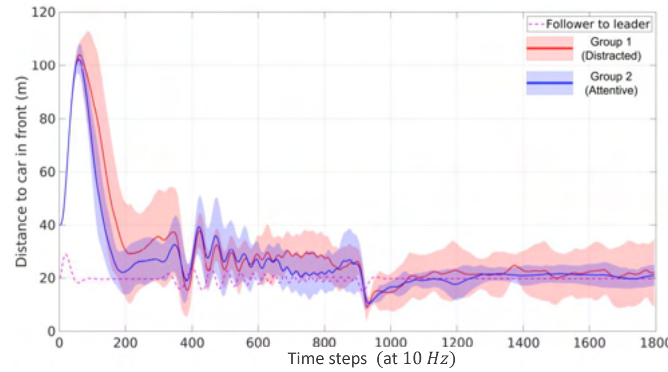

Fig. 5: Mean and standard deviation of distance of the autonomous follower from the human-driven vehicle for the two groups of participants (top). Also shown is the distance of the autonomous follower to the leader of the platoon.

from the distracted drivers. The models identified from this experimental data shows that under different conditions, drivers can have different driving responses. Further, from these identified transfer functions, we have the following DC gains and $\mathcal{H}_\infty$ norms

$$\begin{aligned} G_h^{\text{distracted}}(0) = 1, \quad & ||G_h^{\text{distracted}}||_{\mathcal{H}_\infty} = 1.397 \\ G_h^{\text{attentive}}(0) = 1, \quad & ||G_h^{\text{attentive}}||_{\mathcal{H}_\infty} = 1.538 \end{aligned} \tag{32}$$

The gap between the DC gains and $\mathcal{H}_\infty$ norms in (32) show the tightness of the necessary and sufficient conditions in Propositions 1 and 2.

VI. Simulation and Experimental Studies

Using the identified model of human behavior (from section V, we design the platoon gains to satisfy the conditions identified in sections III and IV. This section outlines a simulation case study that shows interaction of the autonomous platoon with a human driven vehicle. The simulation study was implemented in MATLAB R2021a on a computer running Ubuntu 20.04.

A. Simulation setup

Platoon configuration: We consider a simulation with a total of 4 autonomous vehicles interacting with 2 human-driven vehicles. The configuration for this simulation study involves a platoon of 2 autonomous vehicles being trailed by a human-driven vehicle, which is then followed by a similar configuration of 2 autonomous and 1 human-driven vehicle (see figure 2). As in section IV, this can be broken down into two platoons of two autonomous vehicles being trailed by one human driver vehicle. For the leader of the platoon in front, we set a reference velocity u_{ref} as shown in figure 4 (bottom). The acceleration disturbance ζ follows the same shape, but with a magnitude of 0.01 compared u_{ref} . For the leader of the second platoon, the reference velocity is the velocity of the human-driven vehicle in front.

Initialization: We initialize each autonomous vehicle to be $\Delta = 20m$ behind the vehicle in front, with the human driven vehicles at $\Delta = 20m$ behind the autonomous vehicle in front of them. All time 0, all vehicles have the same velocity of $5ms^{-1}$. We consider the platoon formation case, with unidirectional communication. Finally, the first human driven car in the formation is driven by a distracted driver, and the second is driven by an attentive driver. The transfer functions for simulating these vehicles are the ones identified from the human studies (31).

Assumptions: We first assume that the human-driver car is a connected vehicle, and can communicate the driver type (attentive or distracted) with the autonomous platoon. This information can be used by the platoon to change their control gains if required. The details of such a setup are beyond the scope of this work. We also assume that the human driver type does not change over the course of the simulation.

Platoon control gains: We set the platoon gains to be $k_p = 1.1$, $k_u = 3.5$. This satisfies the necessary and sufficient conditions for HTS identified in Theorem 2, irrespective of which of the two identified driver types the two drivers are. In general, based on the types of human drivers interacting with the platoon, different control gains can be used for the platoon.

B. Simulation results

Figure 6 shows the positions of the autonomous and human-driven vehicles for the simulation duration of $180s$. Figure 7 shows the distance between consecutive vehicles in the platoon. As expected from the controller design, the vehicles are head-to-tail stable and the oscillations from the lead autonomous vehicle are gradually attenuated in the following vehicles. The positions of the vehicles are also such that no collisions occurred between the vehicles[8].

C. Experiments with a human-driver

We also carried out experiments with three drivers (same as in section V, with each driver performing one trial at a time with and without a distracting cognitive task (for a total of 6 trials). Here, the drivers interact with a platoon of two autonomous vehicles in front of them. Figure 8 shows the reference velocity for the autonomous platoon, the velocities

[8]In general, HTS stability is a necessary but not sufficient condition for safety of the overall platoon.

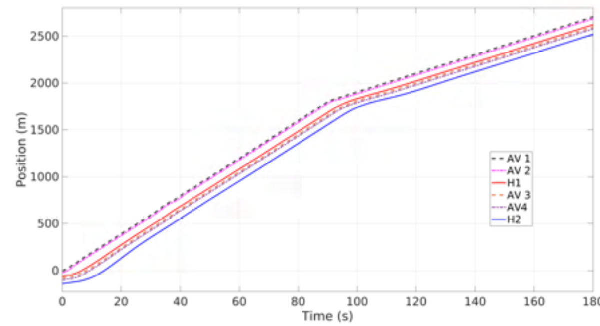

Fig. 6: Simulated positions of the four autonomous vehicles (AV) and two Human-driven vehicles (H). Note, H_1 is a distracted driver, while H_2 is an attentive driver.

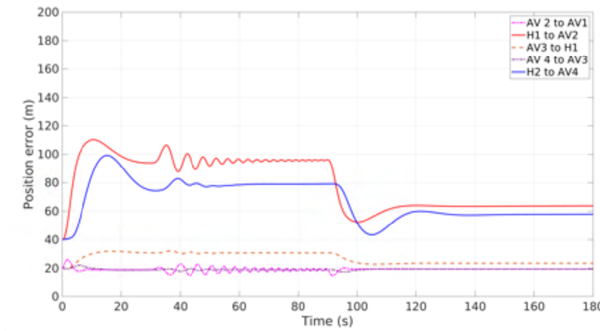

Fig. 7: Distances between vehicles in the simulation study with 4 autonomous and 2 human-driven vehicles.

of the two autonomous vehicles in the platoon, as well as the velocities of the human driven vehicle (mean and standard deviation for both groups). Figure 9 shows the resulting distance between cars in the platoon.

Results and conclusion: Similar to the results in the simulation, the overall platoon (autonomous and human-driven vehicles) are string stable, i.e., changes in the autonomous leaders velocity do not result in increasing tracking error (distance between vehicles) of the following vehicles.

VII. Discussion

Summary. This paper investigated system-theoretic conditions for head-to-tail stability of an autonomous platoon in the presence of human-driven vehicles. For platoons with both unidirectional and bidirectional topologies, and for both velocity tracking and network formation control laws, necessary and sufficient HTS conditions for the combination

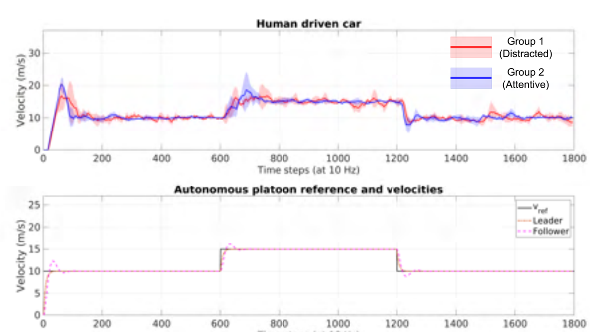

Fig. 8: Mean and standard deviation of car velocity for the two groups of participants (top). The participants are tracking the velocity of the autonomous vehicle in front of them, the velocity profile of which is shown in lower figure.

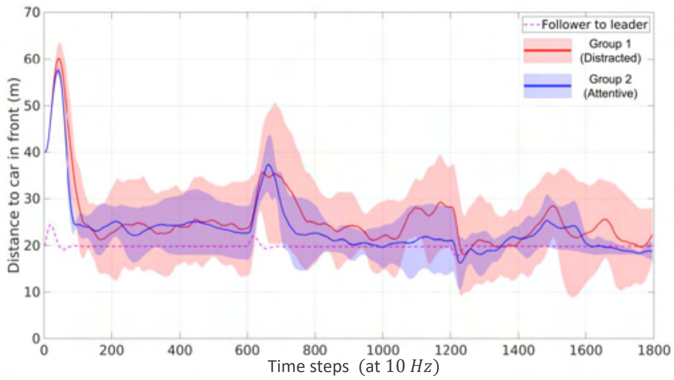

Fig. 9: Distance of the human-driven car to the follower in the autonomous platoon (mean and standard deviation, for both groups) in the human-subject experiment. As is to be expected, the distracted drivers get closer to the autonomous car in front.

of the platoon and human-driven vehicles are derived. An offline identification of the human transfer function was then proposed which was used to assess the HTS conditions. Simulations and experimental studies show the validity of our approach.

Limitations and Future Work. This paper is limited in many ways. First, we only considered the longitudinal motion of the vehicles. An avenue for further research is to investigate more diverse vehicle motions, e.g., including lateral motion and platoon merging. In this paper, we used a deterministic second order model for the human driver, and focus only on stability of the vehicles (which does not necessarily imply safety). Future work will incorporate probabilistic models of human behavior, and thus focus on chance-constrained notions of safety, in addition to stability. Finally, in this work, we make the assumption that the human driver type is known to the autonomous platoon a priori. Online methods to identify the human driver type [7], as well as individual tendencies, could be a valuable addition to this work.

Conclusion. This work takes a classical control-based look at the important problem of autonomous vehicle platoons interacting with human-driven vehicles. The initial results show the promise of this approach, and open the problem to solutions that bring together classical control with contemporary learning-based approaches.

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