Gabor Functions for Interpolation on Hexagonal Lattice

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Abstract

An interpolation model using Gabor Filter is demonstrated on hexagonally sampled data, which outperform classical B-splines and MOMS. Our method has optimal approximation theoretic performances, for a good quality image. The computational cost is considerably low when compared to similar processing in the rectangular domain. In this paper the parameter sigma of 2/pi is found to satisfy most of the image interpolation requirements in terms of high Peak Signal-to-Noise Ratio (PSNR) , lower Mean Squared Error (MSE) and better image quality by adopting a windowing technique.

Keywords

Hexagonal lattices, Gabor filter, Interpolation, image processing.

I. Introduction

Image reconstruction through interpolation is routine task in image processing during all transformation that is made on an image. Such transformations include scaling, rotation, registration, and edge detection. Considerable interest on hexagonal sampling is shown recently due to the interest in human vision systems, some of the image acquisition systems using hexagonally laid pixels, imaging radars and nuclear medicine. They also possess better topological and geometrical properties, resulting in a more efficient signal representation in two dimensions [1,2-4].

Interpolation is the process of estimation of the underlying representative function of the data points on a lattice. Here we consider the nodes corresponding to

$$R = \sqrt{2/\sqrt{3}}\begin{bmatrix} 1 & 1/2 \\ 0 & \sqrt{3}/2 \end{bmatrix}$$

Images are generally constructed out of discrete sample points and if the fitted function is band limited, we can reconstruct the signal using interpolation by a Sinc-function, which is a low pass filter in the frequency domain. Ideally this interpolator should not create any artifacts on the resultant interpolation, but it introduces artifacts due to its truncation or by windowing for computational economics. This has prompted scientists to explore compactly supported interpolating functions φ(x) which are approximations to obtain a perfect reconstruction. It has also a filtering effect as some noise components get suppressed during reconstruction. Much work is available for interpolation in rectangular lattices using parameterized wavelets and piecewise polynomial functions, the most important being that is using B-Splines [5]. B-Spline methods enable approximation in a simple way using the orthogonal properties of B-Splines [6]. In hexagonal grid, three methods namely, box splines, hex-splines and generalized splines of maximal order and minimum support are proposed [7-8]. The first one exploits the six fold symmetry and the second, a twelve fold symmetry. However the third one is a generalization to get best results out of an interpolation with a trade off between the interpolated result and computational complexity. This is achieved from error minimization using the spectral components with an assumption that the signal energy is concentrated over the low-frequency region. We propose a method employing Gabor filters on the hexagonal lattice to get enhanced quality image reconstruction using Gabor kernels with some extra computational power.

II. Spline Interpolation on the Hexagonal Lattice

For a successful interpolation, the mapping function g(x) has to be approximately equal to the real function f(x). The spectrum of hexagonally sampled signal is a rotated by 90 degrees and scaled version of the signal itself. It means that the effect of sampling a function f(x) on the hexagonal lattice is to replicate its spectrum $\hat{f}(\omega)$ at the lattice sites $2\pi\hat{R}k$, where $\hat{R} = (R^{-1})^T$ .Accordingly, the Fourier transform of a discrete signal $s = (s[k])_{k\in z^2}$ sampled on the hexagonal lattice is

$$\hat{s}(\omega) = \Sigma_{k\in z^2} s[k]\exp(-j\omega^T Rk) \quad (1)$$

The three directional box-splines are piecewise polynomial functions that form two different multi-dimensional extensions of the 1-D B-splines, appropriate for interpolation on the hexagonal lattice. Hex-splines are another family of functions, proposed by D.Van De Ville, T. Blu,M.Unser,etc.,built by successive convolution: $\eta L = \eta L - 1 * \eta 1$, for every L > 1, where the first-order hex-spline $\eta 1$ is simply the indicator function of the Voronoi cell of the lattice Λ. The expressions of the hex-splines in the Fourier domain, as well as their properties, are given in [9].

III. New Hexagonal Spline Generators

A. Gabor Filters

Hubel and Wiesel [10] found simple cells in a cat's visual cortex, which was sensitive to frequency and orientation of an image perceived. Experiments revealed that a Gabor filter takes the form of a Gaussian modulated complex sinusoid in the spatial domain. There is no standard definition of a two dimensional Gabor function. We adopted the one used in B. S. Manjunath and W. Y. Ma [11].A two dimensional Gabor function g(x, y) and its Fourier transform G(u,v) is given as:

$$g(x,y) = \left(\frac{1}{2\pi\sigma_x\sigma_y}\right)\exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}\right)+2\pi jWx\right] \quad (2)$$

In the frequency domain,

$$G(u,v) = \exp\left\{-\frac{1}{2}\left[\frac{(u-W)^2}{\sigma_u^2}+\frac{v^2}{\sigma_v^2}\right]\right\} \quad (3)$$

Where

$$\sigma_u = \frac{1}{2\pi\sigma x} \quad and \quad \sigma_v = \frac{1}{2\pi\sigma y}$$

while σx and σy are the standard deviations of the elliptical Gaussian along x and y axes.

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B. Importance of windowing

Apodization windows are often used in signal processing to reduce the ill effects of finite processing windows. The windowed signal Xw(k) is thus represented as the product of the signal and of the weighted window : Xw(k) = x(k).w(k)
where x(k) is the signal to be analysed and w(k) the weighting or temporal window of null value outside the observation interval. We need a window which does not sharply cut off the signal at its edges, but rather one which smooths the signal gradually down to zero at its edges. One candidate for this is selected as the Hanning window [12]. This is defined by

$$h_k = 0.54 - 0.46\cos\left(\frac{2\pi k}{N-1}\right), \qquad 0 \le k \le N \tag{4}$$

Effect of the Gabor filter with Hanning window on the texture image with hexagonal sampling is discussed in the section IV.

C. Gabor as a discrete filter for Hexagonal lattice

To transform continuous domain entities to the discrete domain, one always needs to be sure that they are represented with sufficient accuracy in order to reapply the continuous domain results. Real signals are usually strictly amplitude limited and quantization is not an issue. In applications, a proper construction depends more on the sampling theorem. By obeying the sampling theorem, an accurate and aliasing free construction of Gabor filters can be made in the frequency domain. A new strategy was proposed in [13] to design recursive implementations of the Gabor filters. The approach was based on the Gaussian filter as concatenation of two recursive filters (Forward recursion and backward recursion - Fig.2).

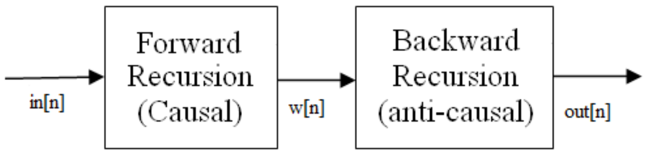


Fig.1 : Gabor as discrete filter

Forward Filtering for Gaussian is

$$H_+(z) = \frac{1}{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}$$

Backward Filtering for Gaussian is

$$H_-(z) = \frac{B}{b_0 + b_1 z^{1} + b_2 z^{2} + b_3 z^{3}} \tag{5}$$

With the replacement of z with $e^{-j\Omega_0} z$ which represents a rotation of angle Ω_0 around the point z = (0,0) in the complex z-plane, then the Gaussian filter pair becomes Gabor filter pair B_+ (z) and B_-(z).
Where

$$B_+(z,\Omega_0) = \frac{1}{\sum_{k=0}^{3} (b_k e^{jk\Omega_0} z^{-k})} = \frac{1}{P_+(z)}$$

$$B_-(z,\Omega_0) = \frac{B}{\sum_{k=0}^{3} (b_k e^{-jk\Omega_0} z^{k})} = \frac{1}{P_-(z)} \tag{6}$$

Where $P_+(z)$ and $P_-(z)$ are polynomials in z.
The Gabor filter transfer function G (z) is thus given by

$$G(z) = B_+(z,\Omega_0) B_-(z,\Omega_0) = \frac{1}{P_+(z) P_-(z)} \tag{7}$$

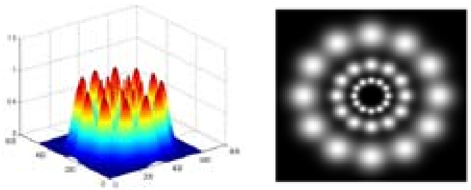
The coefficient of each term zk in $P_-(z)$ is just the complex conjugate of the term in z-k in $P_+(z)$.

We then consider more elaborate decompositions such as a steerable filter and a Gabor decomposition, which provide more texture classes, i.e.,0 degree,60 degree and 120 degree. We show that the proposed methodology works effectively with any complete/over complete directional decomposition.

IV. Computational results

A. Gabor Filter Bank

The filter bank was carried out with six orientations as 0°, 30°, 60°, 90°, 120° and 150° and three radial frequency values: F = (0.3536, 0.1768 and 0.0884). This leaves us with a total of 18 filters that cover the frequency domain. Different standard deviation values of the Gaussian curve were tested, those being the three values used in the study σg = (2.91, 5.82 and 11.64) [Equation (2)].

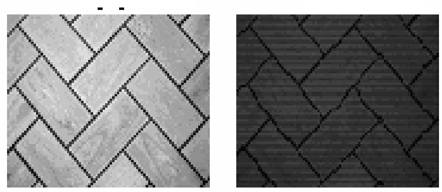


(a) (b)

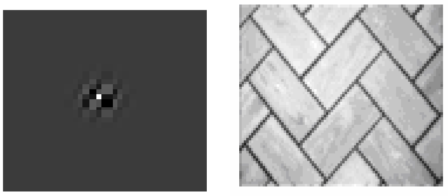
Fig.2 : (a) Gabor kernel and (b) Gabor filter bank with σg =11.64; F=0.0884

B. Gabor filter bank for hexagonal sampled Image

Next the filter bank was carried out with three orientations 0^0, 60^0 and 120^0. The filter bank was applied to the hexagonal sampled image shown in Fig.3(b) which is of poor quality obtained for the input image shown in Fig.3(a). We used half pixel shift method for the simulation of hexagonal sampled image as reported in our earlier paper [14-15].
The image resulting from the filtering process is shown in Fig.3(d) with clarity in the image. The results show that that the proposed methodology works effectively with any complete directional decomposition.



(a) (b)

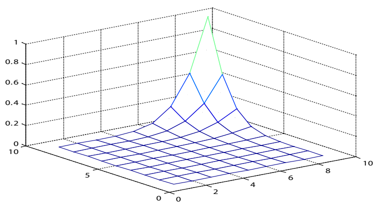


(c) (d)

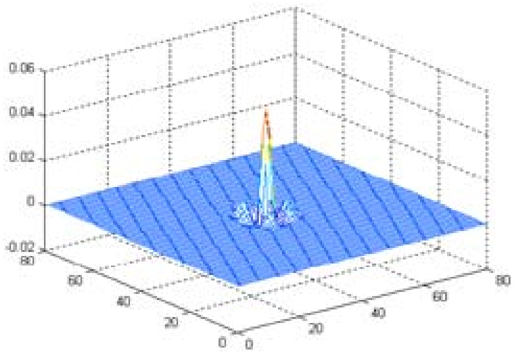
Fig. 3 : (a) Original image (b) Hexagonal sampled image (c) Filter bank with 3 orientations θ = 0^0 , 60^0 and 120^0 (d) Gabor filtered image with the effect of filter bank

C. Gabor as discrete filter

In order to analyse the Gabor filter as discrete filter, we used the implementation steps as mentioned in the section III C. First we have designed 1D filter and using it, 2D filter coefficients were calculated. For hexagonal grid we have chosen three orientations as θ = 00 , 600 and 1200 because of three axis symmetry.Using the obtained coefficients, the Gabor kernel is plotted as shown in Fig. 4.



(a)



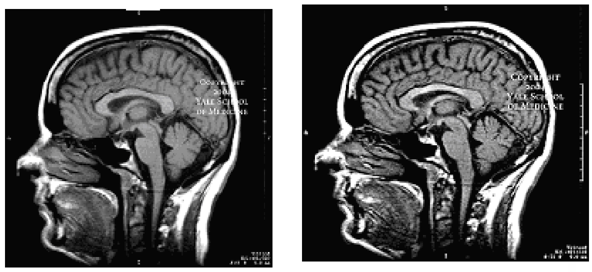
(b)

Fig.4 : (a) Gabor kernel with the obtained coefficients for σ = 2 θ = 60 degree (b) Gabor kernel with hanning window

Similarly, we have obtained coefficients for 0 degrees and 120 degrees and plotted the kernel which is same as Fig.4(a). From the implementation of the Gabor as discrete filter, we observed that the coefficient of each term zk in $P_{-}(z)$ is just the complex conjugate of the term in z-k in $P_{+}(z)$ (Equation 7).

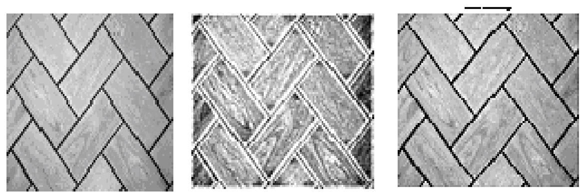
D. Gabor kernel with windowing technique

Fig.4(b) shows the Gabor kernel with Hanning window discussed in section III-B. Fig.5 shows the response of image obtained by convolving the kernel of Fig. 4 (b). The results show that the obtained image quality is better, smoothed and the orientations/edges are more clear (soft tissues are clearly visible) for the hexagonal sampled image which is shown in Fig. 5(b).



(a) (b)

Fig. 5 : (a) Original image (b) Effect of Gabor kernel with Hanning window on hexagonal sampled image



(a) (b) (c)

Fig. 6 : (a) Original image (b) Gabor filtered image without window (c) Gabor filtered image with hanning window for sigma = 2/pi (using kernel of Fig.4 (a))

From the results, it is found that with the effect of window, image is free from spurious shading and the texture features of the image is very clear compared with the original image of Fig.3 (a) and the Gabor filtered image without window as shown in Fig.6 (b). Performance measures in terms of Peak Signal-to-Noise ratio (PSNR) and Mean Squared Error were analysed and listed in Table.1,2 and 3 which shows marginal improvement when operating the Gabor filter on the image with windowing technique, with an improved visual quality.

Table 1: Performance measures of Gabor filter with and without windowing technique for the hexagonal sampled images

	Gabor filter response With out window		Gabor filter response With hanning window	
	PSNR	MSE	PSNR	MSE
Wood	52.89	0.3340	57.5	0.108
Lena	54.75	0.2177	54.99	0.205
barbara	54.98	0.2063	59.7	0.06
Peppers	57.76	0.1087	58.01	0.1028
mri	65.29	0.0197	65.50	0.0183

Table 2 : Gabor Filter response with hanning window

	Image Type : wood.jpg					
Sigma	0.1	0.5	0.637	1	1.5	2
PSNR	24.5	54.3	57.7	57.06	54.3	52.8
MSE	227	0.24	0.108	0.127	0.239	0.334

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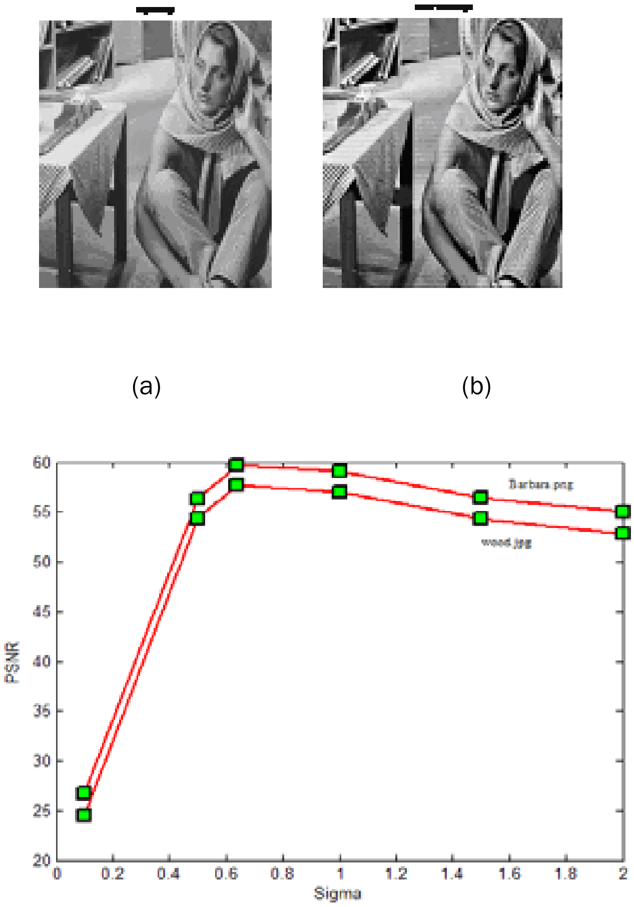
Table 3 :

	Image Type : Barbara.png					
Sigma	0.1	0.5	0.637	1	1.5	2
PSNR	26.6	56.3	59.7	59.07	56.4	54.9
MSE	139	0.14	0.06	0.08	0.14	0.20



(a) (b)

(C)

Fig.7 : (a) Barbara image (b) Gabor filtered image for the hexagonal sampled Barbara image at sigma =2/pi = 0.637 (c) Plot of sigma vs PSNR for the wood image shown in Fig.6(a) and Barbara image shown in Fig. 7(a).

V. CONCLUSIONS

We have proposed an implementation of Gabor filters on the hexagonal lattice for resampling and interpolation. In an earlier paper [14, 15] we have reported the benefits of hex Gabor filters for the purpose of filtering, edge detection and registration. In this paper the parameter sigma of 2/pi is found to satisfy most of the image interpolation requirements as per the results obtained in previous section. We also prove that at this sigma, the area of the interpolating kernel viewed in the time and frequency domain remains the same.

Interpolation methods using hexsplines and other interpolating functions have a smoothing effect on the image, whereas for Gabor, three orientation windowed filter show that the edges are better preserved, while keeping the low frequency information in tact. It gives best results for the textured images. Therefore this method is ideal for generation of medical atlas from MRI images and for high definition TV.

The image quality obtained with Gabor filter has superior visual quality over their spline counterparts over the hexagonal lattice, leading to high-quality visualization of images (see Fig.6,7). Notice that the hazy edges are modified to very sharp boundries.

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