Linear Algebra (as relevant to ML)

Vectors

Scalar: A single value

Vector: A list of values of fixed length d

Examples:

$$a = \begin{bmatrix} 0.2 \\ 0.0 \\ 1.0 \\ 2.0 \end{bmatrix} \quad x = \begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \\ 1.0 \\ 0.0 \end{bmatrix} \quad w = \begin{bmatrix} -1.0 \\ 2.2 \\ -0.1 \\ 0.0 \\ 1.0 \end{bmatrix}$$

$d = 4$ $\quad$ $d = 5$ $\quad$ $d = 5$

$x \in \mathbb{R}^d$

$w \in \mathbb{R}^d$

Vectors

c $\quad$ a $\quad$ b

$$\begin{bmatrix} 1.1 \\ 3.2 \\ 4.9 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.2 \\ -0.1 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

- **Simple operations**

$$c = a + b \quad \text{where } c_i = a_i + b_i$$

$$c = \alpha \cdot b \quad \text{where } c_i = \alpha b_i$$

$\alpha \in R$ d

$b \in R$

$$c = \sin a \quad \text{where } c_i = \sin a_i$$

- **Length (L2 Norm)**

$$\|a\|_2 = \left[\sum_{i=1}^{\cancel{m}\, d} a_i^2\right]^{\frac{1}{2}}$$

$$L_r = \left(\sum_{i=1}^{d} |a_i|^r\right)^{1/r}$$

$$\|a\|_2 \geq 0 \text{ for all } a$$

$$\|a + b\| \leq \|a\| + \|b\|$$

$$\|\alpha \cdot b\| = |\alpha| \cdot \|b\|$$

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Vectors

- **Dot product of two vectors**

$$a^{T} b = \sum_i a_i b_i$$

$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_d \end{bmatrix}$ $a^T = [a_1\ a_2 \cdots a_d]$

$$\mathrm{w}^{\mathrm{T}}\ \mathrm{x} = \mathbf{w}\ \mathbf{x} = \mathbf{w.x} = <\mathbf{w}, \mathbf{x}> = \sum_{\{j=1\}}^{d} w_j\ x_j$$

$\langle w, x \rangle$

Linear combination of x

The dot-product of two vectors yields a scalar

Notations: $\mathbf{w} \in R^d, \boldsymbol{x} \in R^d$

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Matrices

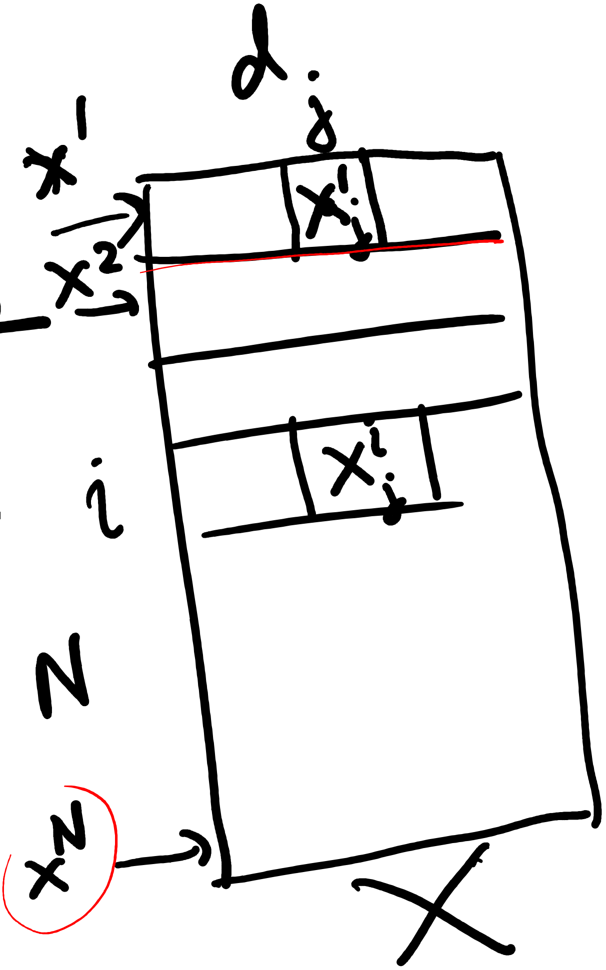
Two-dimensional Arrays.

Example:

$$A = \begin{array}{c l} \begin{bmatrix} 0.1 & 0.3 & 0 \\ 0.2 & 3 & 10 \end{bmatrix} & \begin{array}{l} \leftarrow \text{row 1} \\ \leftarrow \text{row 2} \end{array} \end{array}$$

A common matrix in ML: Training data (X).

$$x^i_j = X_{ij}$$



Matrices

- **Simple operations**

$$C = A + B \qquad \text{where } C_{ij} = A_{ij} + B_{ij}$$

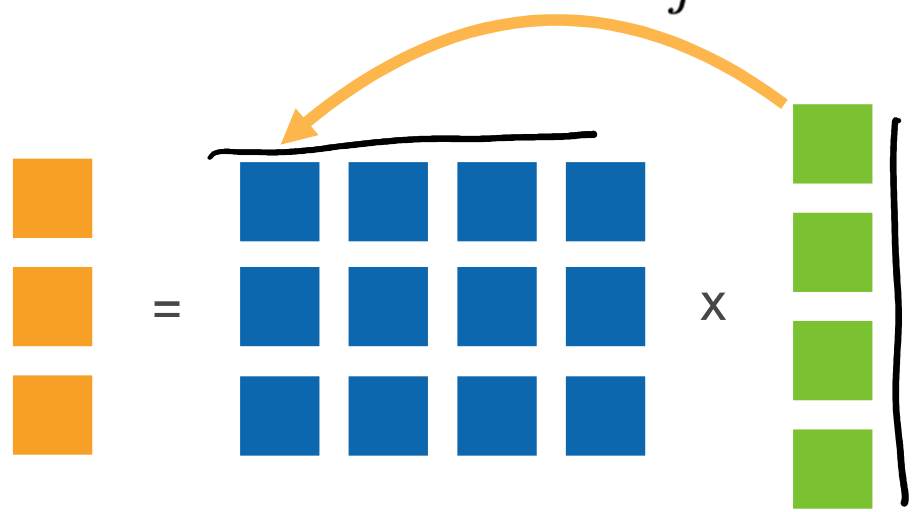
$$C = \alpha \cdot B \qquad \text{where } C_{ij} = \alpha B_{ij}$$

$$C = \sin A \qquad \text{where } C_{ij} = \sin A_{ij}$$

Matrices

- **Multiplications (matrix vector)**

$$c = Ab \text{ where } c_i = \sum_j A_{ij} b_j$$



Matrices

- **Multiplications (matrix matrix)**

$$C = AB \text{ where } C_{ik} = \sum_j A_{ij} B_{jk}$$