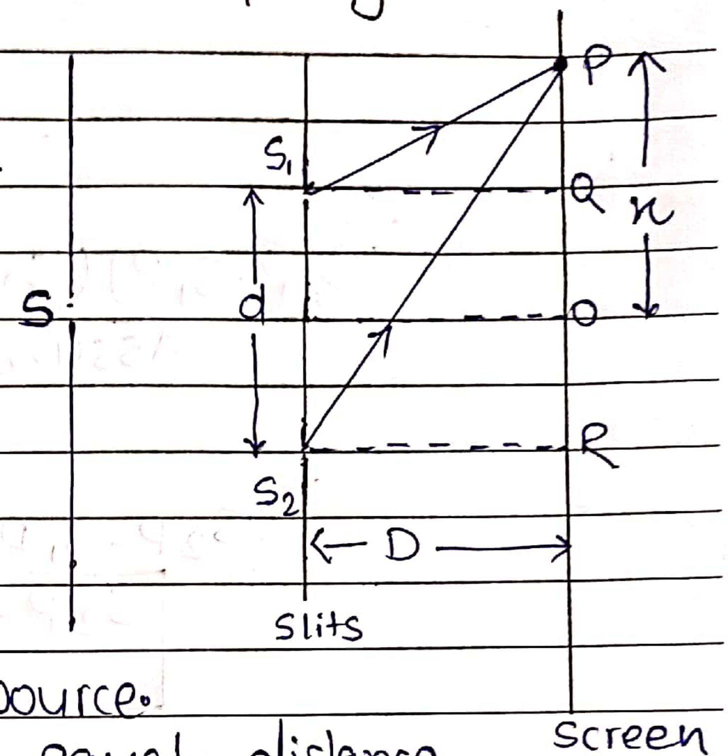
Q1. Explain the young's double slit experiment and find fringe width in young's double slit experiment.

Soln:



Consider a narrow monochromatic light source S and two parallel slits S_1 and S_2 are separated by 'd'. Both slits are at Equal distance from source S. let D be the distance b/w screen and coherent source.

Let point 'O' is at equal distance from S_1 and S_2 therefore the path difference between the two rays reaching at point 'O' is zero. Thus, that point has maximum intensity.

consider point 'P' at the distance x from O and wave reaches at point 'P' from S_1 and S_2.

Here,

$$PQ = x - \frac{d}{2} \quad \text{or} \quad PR = x + \frac{d}{2}$$

$$S_1P^2 = PQ^2 + D^2 \quad \text{or} \quad S_2P^2 = (PR)^2 + D^2$$

$$= \left(x - \frac{d}{2}\right)^2 + D^2 \qquad = \left(x + \frac{d}{2}\right)^2 + D^2$$

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$$S_2P^2 - S_1P^2 = \left(x+\frac{d}{2}\right)^2 + \cancel{D^2} - \left(x-\frac{d}{2}\right)^2 - \cancel{D^2}$$

$$= \cancel{x^2} + \cancel{\frac{d^2}{4}} + xd - \cancel{x^2} - \cancel{\frac{d^2}{4}} + xd$$

$$= 2xd$$

$\therefore$

$$(S_2P - S_1P)(S_2P + S_1P) = 2xd$$

Assume

$$S_2P \approx S_1P \approx D$$

$\therefore$

$$(S_2P - S_1P)(\cancel{2}D) = \cancel{2}xd$$

$$\boxed{S_2P - S_1P = \frac{xd}{D}}$$ (path difference)

The distance b/w any two consicutive dark ~~or~~ and bright fringe is known as fringe width and denoted by '$\bar{X}$'.

condition for bright fringe:

For bright fringes the path difference is an integral multiple of wave length λ.

$$\therefore \frac{xd}{D} = n\lambda$$

$$\boxed{x_n = \frac{n\lambda D}{d}}$$ where $n = 0, 1, 2 ....$

$$x_1 = \frac{\lambda D}{d}, \quad x_2 = \frac{2\lambda D}{d}, \quad x_3 = \frac{3\lambda D}{d}$$

So, $x_3 - x_2 = x_2 - x_1 = \frac{\lambda D}{d}$

$\therefore$ $\boxed{\text{fringe width } (\bar{x}) = \frac{\lambda D}{d}}$ —— (A)

for Dark fringes:
for dark fringes the path difference is an odd integral multiple of half wave length

$\therefore$ $\cancel{\lambda D}\ \frac{\lambda d}{D} = (2n+1)\frac{\lambda}{2}$

$\therefore$ $\boxed{x_n = (2n+1)\frac{\lambda D}{2d}}$ where $n = 0, 1, 2 \ldots$

$x_0 = \frac{\lambda D}{2d}$, $x_1 = \frac{3\lambda D}{2d}$, $x_2 = \frac{5\lambda D}{2d}$

$\therefore$ $x_1 - x_0 = x_2 - x_1 = \frac{\lambda D}{d}$

$\therefore$ $\boxed{\text{fringe width } (\bar{x}) = \frac{\lambda D}{d}}$ —— (B)

$\therefore$ It is clear that fringes may be either dark or bright but the fringe width remain same.
Also, we conclude that

$\boxed{\bar{x} \propto \lambda}$, $\boxed{\bar{x} \propto D}$, $\boxed{\bar{x} \propto \frac{1}{d}}$

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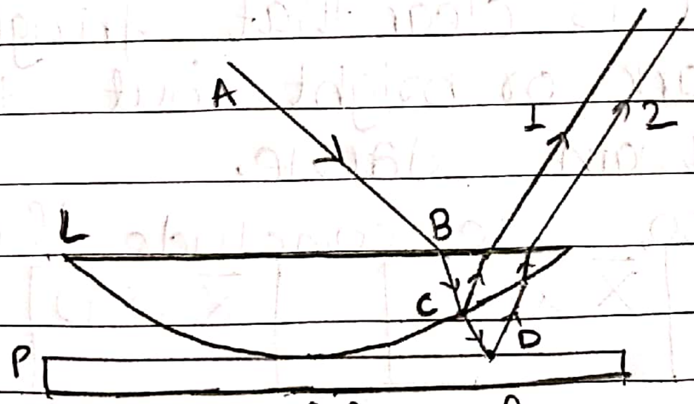
Q. Explain newton's ring Experiment and also find diameter of newton's ring in reflected system.

Solⁿ: When a plano- convex lens with its convex surface is place on a plane glass plate, an air film of gradually increasing thickness is formed between the two. The thickness of the film at the point of contact is zero. If the monochromatic light is allowed to fall normally, and the film is viewed in reflected light, alternate dark and bright concentric rings with their centre dark is formed between the lens and glass plate are seen.

Formation of Newton's ring:
The formation of Newton's rings can be explained with the help of diagram drawn below. AB is a monochromatic ray of light which falls on the system. A part is reflected at C which goes out in the form of ray 1 without any phase reversal.



Wedge of film of Air

The ~~fo~~ other part is refracted along CD. At point D it is again reflected and goes out in the form ~~is~~ ~~refracted~~ of ray 2 with a phase reversal of π. The reflected ray 1 and 2 are in a position to produce interference fringes as they have been derived from the same ray AB and hence fulfill the condition of interference. The fringe are circular because the air film is symmetrical about the point of contact of the lens with the plane glass plate.

As the rings are observed in the reflacted ~~ray~~ light, the path diff. between them is

$$= 2\mu t \cos r + \lambda/2$$

for air film ($\mu = 1$) and $r = 0$

$$\therefore \boxed{\text{path diff.} = 2t + \lambda/2}$$

At the point of contact $t = 0$ and path diff $= \lambda/2$

This is the condition for minimum intensity hence, centre of Newton's ring is dark.

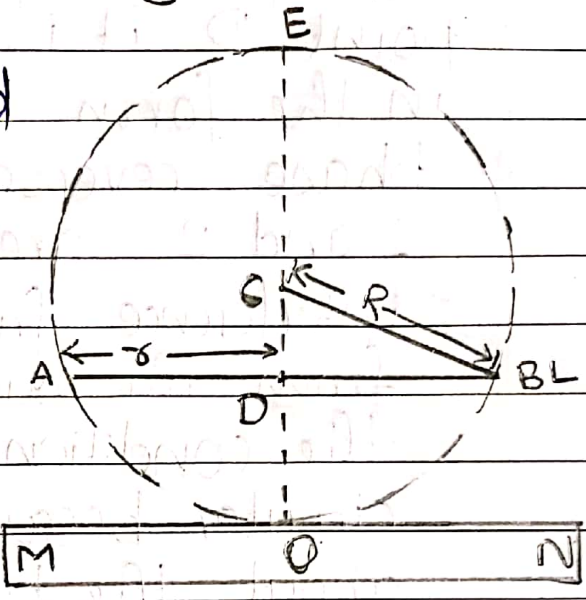
for n^{th} maximum

$$\boxed{2t = (2n-1)\frac{\lambda}{2}} \quad \text{— (1)}$$

for n^{th} minimum

$$\boxed{2t = n\lambda} \quad \text{— (2)}$$

Diameters of Dark and Bright rings:-



let a lens ADB is placed over a plane glass plate MN. let O is the point of contact and γ is the radius of curvature of the lens.
let r be the radius of Newton's ring and t is the thickness of an air film.
For the geometrical property of the circle

$$AD \times DB = OD \times DE$$
$$\gamma \times \gamma = t(2R - t)$$
$$\gamma^2 = 2Rt - t^2$$

t^2 can be neglected as thickness t is very small as compaired to radius of curvature R

so, $\gamma^2 = 2tR$

or $\boxed{2t = \frac{\gamma^2}{R}}$

put the value of 2t in bright ring in Eq. ①

$$2t = (2n-1)\lambda/2$$
$$\frac{\gamma^2}{R} = (2n-1)\lambda/2$$

$$\boxed{r_n = \sqrt{\frac{(2n-1)\lambda R}{2}}}$$

so, $D_n = 2r_n = \sqrt{2(2n-1)\lambda R}$

or

$$D_n = \sqrt{2\lambda R} \cdot \sqrt{2n-1}$$

$$\therefore \quad \boxed{D_n \propto \sqrt{2n-1}}$$

so, the diameter of n^{th} bright ring is proportional to the square root of odd natural number.

The diameter of first few bright rings are in ratio of

$$\sqrt{1} : \sqrt{3} : \sqrt{5} : \sqrt{7}$$

The separation b/w the rings decreases as the order of ring increases.

Now, put the value of 2t for dark ring in Eq (2)

$$2t = n\lambda$$

$$2\frac{r^2}{R} = n\lambda$$

$$r = \sqrt{n\lambda R}$$

so, Diameter $(D_n) = 2r_n = \sqrt{4n\lambda R}$

$$D_n = \sqrt{4\lambda R}\sqrt{n}$$

$$\boxed{D_n \propto \sqrt{n}}$$

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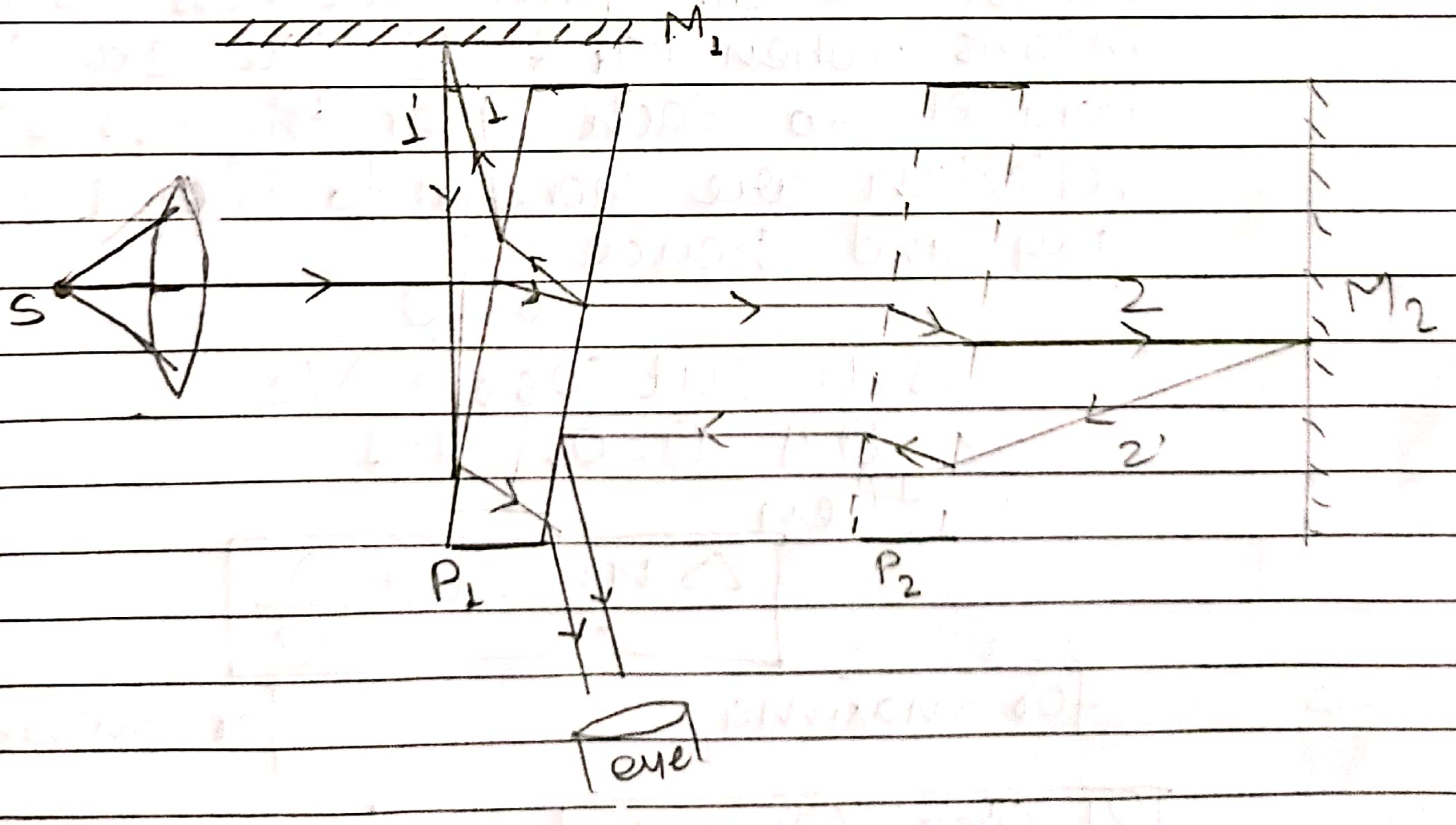
so, diameter of nth dark ring is proportional to square root of natural number. so, diameter of first few dark ring in ratio of

$$\sqrt{1} : \sqrt{2} : \sqrt{3}$$

Q.3. Explain Michelson's interferrometer and also Explain the shape of the fringes.

Soln: Michelson's made a interferrometer which is instrument based on the phenomenon of interference. This is the instrument to obtain interference fringes of various shapes which has a no. of application like:

1) To determine the wavelength of monochromatic light.
2) To find refractive index of thin slab.
3) To find the thickness of thin glass slab.
4) To find diff. b/w two wavelengths.



Ray 2' reflected from top surface of denser medium so, by stoke's law it suffers a phase change of π and hence there is additional path difference $\lambda/2$ is inclination of ray θ is the angle of air film and t is the thickness of air film.

Path difference:
When M_1 and M_2 are parallel or mirror M_1 and M_2 are perpendicular to each other where M_2' is the virtual image of mirror M_2.
When the air film thickness is constant means when M_1 & M_2' are exactly parallel to each other then ~~reflected~~ reflection are normal to the incident ray and hence

$$r = 0$$

$$\Delta x = 2\mu t \cos r + \lambda/2$$

if $\theta = 0$, $\mu = 1$
then

$$\boxed{\Delta x = 2t + \frac{\lambda}{2}}$$

for maxima

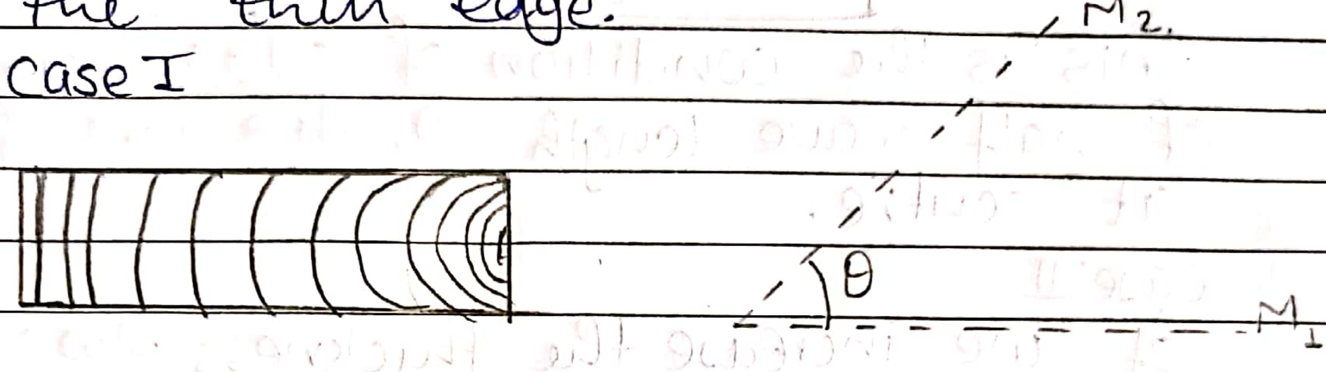
$$\boxed{2t \cos r = (2n-1)\frac{\lambda}{2}}$$

for minima

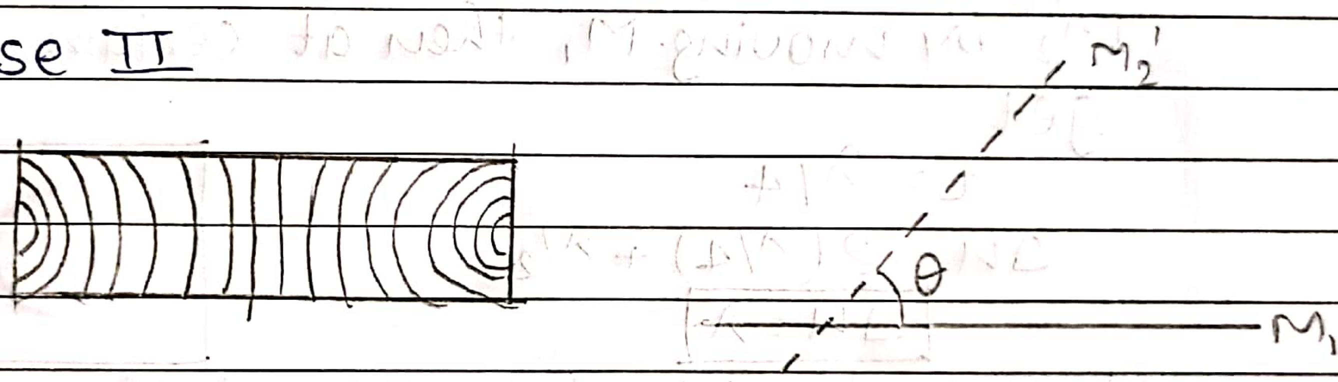
$$\boxed{2t \cos r = n\lambda}$$

1. Localised fringes:
fringes are curved, hyperbolic, straight fringes.
fringes formed with the convexity towards the thin edge.

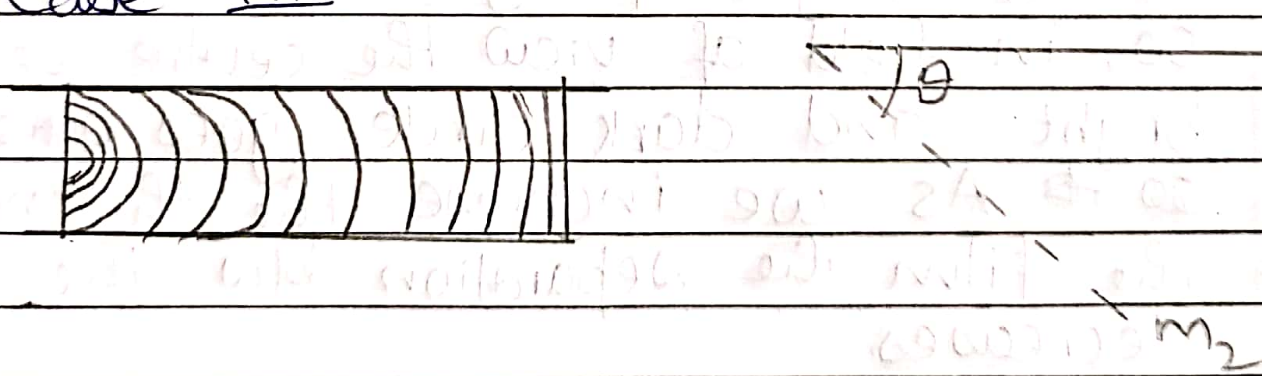
case I



Case II



Case III



2. Circular fringe (Haidinger fringe):
If M_1 and M_2' are parallel
then $\theta = 0$, $r = 0$, $\mu = 1$

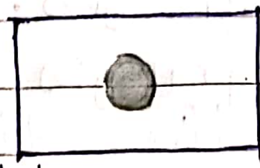
$$\boxed{\Delta x = 2t + \frac{\lambda}{2}}$$

Case I

When M_1 and M_2' coincide

$$t = 0$$

$$\boxed{\Delta x = \frac{\lambda}{2}}$$



This is the condition of odd integral multiple of half wave length so, ~~dra~~ dark fringe at centre.

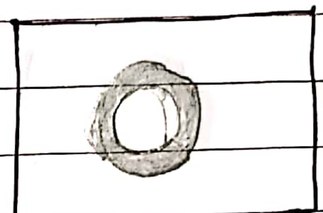
Case II

If we increase the thickness b/w M_1 and M_2' by moving M_1 then at certain point we get

$$t = \lambda/4$$

$$\Delta x = 2(\lambda/4) + \lambda/2$$

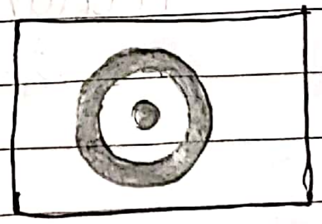
$$\boxed{\Delta x = \lambda}$$



Hence, bright fringes at the the centre so, in field of view the centre becomes bright and dark circle goes outside. ~~SO·~~ As we increase the thickness of the film the separation b/w the fringe decreases.

$$\Delta x = 2\left(\frac{\lambda}{2}\right) + \lambda/2$$

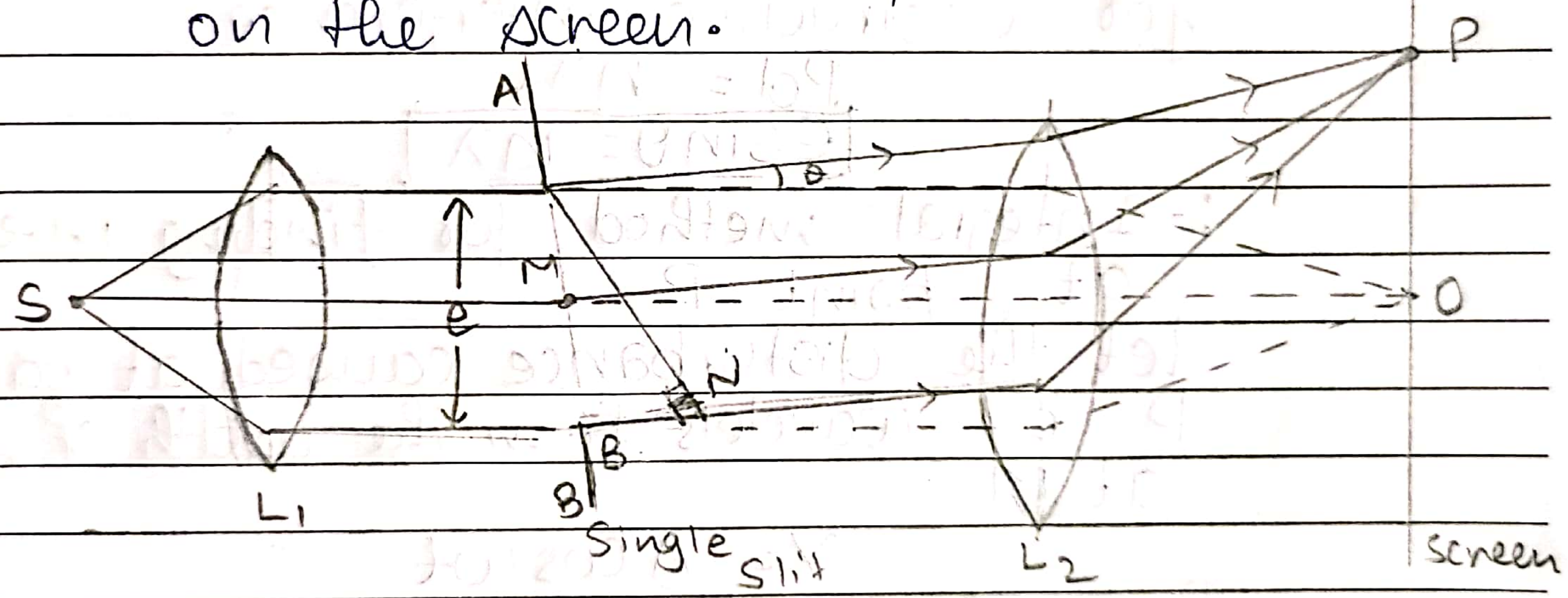
$$\boxed{\Delta x = \frac{3\lambda}{2}}$$



$$\boxed{\lambda = \frac{2t}{N}}$$

Q4. Explain fraunhofer diffraction due to single slit.

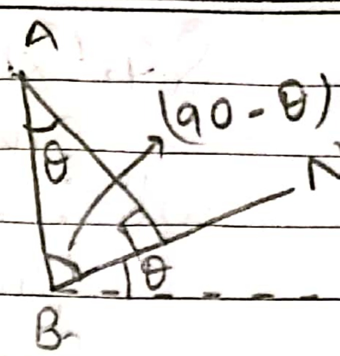
Soln: let a parallel beam of monochromatic light of wavelength λ incident normally on a slit AB whose width is 'e', a slit is rectangular ~~apparatus~~ apperture whose length is more than its width. The diffracted light is focused by a convex lens. and diffraction pattern is obtained on the screen.



According to Hygen's principle a plane wave front incident on slit AB and each point of AB generate out secondary wavelets. in all direction. The rate of proceding in the same direction as the incident ray focused at point O while other ray which are diffracted by an angle θ are focused at point P.

In the right ΔANB

$$\sin\theta = \frac{BN}{AB} = \frac{e\sin\theta}{AB}$$



Path diff $(BN) = e\sin\theta$

for constructive interference (Bright fringes)

$$Pd = (2n+1)\lambda/2$$

$$\boxed{e\sin\theta = (2n+1)\frac{\lambda}{2}}$$

for destructive interference

$$Pd = n\lambda$$

$$\boxed{e\sin\theta = n\lambda}$$

:- Integral method for finding intensity at point P

let the disturbance caused at point P by wavelets from the width of slit at M

$$Y = A\cos\omega t$$

Then the wavelets from a ~~ω~~ unit width dx at C

when it reaches at point p, it has the amplitude Adx & phase $\left(\omega t + \frac{2\pi}{\lambda} x\sin\theta\right)$

let this small disturbance be dy

$$\therefore\ dy = A\,dx\left(\cos\omega t + \frac{2\pi}{\lambda} x\sin\theta\right)$$

The total disturbance at point of observation at an ang. θ is

$$Y = \int_{-e/2}^{e/2} dY$$

$$Y = \int_{-e/2}^{e/2} A\cos\left(\omega t + \frac{2\pi u}{\lambda}\sin\theta\right) du$$

$$Y = A\int_{-e/2}^{e/2} \left[\cos\omega t \cos\left(\frac{2\pi u \sin\theta}{\lambda}\right) - \sin\omega t \sin\left(\frac{2\pi u \sin\theta}{\lambda}\right)\right] du$$

$$Y = A\cos\omega t \int_{-e/2}^{e/2} \left(\cos\frac{2\pi u \sin\theta}{\lambda}\right) du \quad (1)$$

$$- A\sin\omega t \int_{-e/2}^{e/2} \sin\left(\frac{2\pi u \sin\theta}{\lambda}\right) du \quad (11)$$

Eq (11)

$$= -A\sin\omega t \left[-\cos\left(\frac{2\pi u \sin\theta}{\lambda}\right) \Big/ \frac{2\pi \sin\theta}{\lambda}\right]_{e/2}^{e/2}$$

$$= +A\sin\omega t \left[\frac{\cos(\pi e\sin\theta/\lambda)}{2\pi\sin\theta/\lambda} - \frac{\cos(-\pi e \sin\theta/\lambda)}{2\pi\sin\theta/\lambda}\right]$$

as $\cos(-\theta) = \cos\theta$

$\therefore \quad = 0$

Eq (1)

$$Y = 2A\cos\omega t \int_{0}^{e/2} \cos\left(\frac{2\pi u \sin\theta}{\lambda}\right) du$$

$$= 2A\cos\omega t \left[\frac{\sin(2\pi u\sin\theta/\lambda)}{2\pi\sin\theta/\lambda}\right]_{0}^{e/2}$$

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$$Y = \cancel{2}A\cos\omega t\left[\frac{\sin(\pi e\sin\theta/\lambda)}{\cancel{2}\pi\sin\theta/\lambda}\right] - [\sin\theta]$$

$$Y = A_e\cos\omega t\left[\frac{\sin(\pi e\sin\theta/\lambda)}{(\pi e\sin\theta/\lambda)}\right]$$

where $A_e = A_0$ is Amplitude at $\theta = 0$.

put $\alpha = \dfrac{\pi e\sin\theta}{\lambda}$

$\therefore$ Resultant amplitude is

$$R = A_0\frac{\sin\alpha}{\alpha}$$

$$I = R^2$$

$$\therefore\ I = A_0^2\frac{\sin^2\alpha}{\alpha^2}$$

or $$\boxed{I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2}$$

Condition for maxima or minima

for maxima or minima, the derivative of I with respect to α must be zero

i.e., $\dfrac{dI}{d\alpha} = 0$

$$\frac{d}{d\alpha}\left(I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\right) = 0$$

or $I_0 \times 2\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\alpha\cos\alpha - \sin\alpha\cdot 1}{\alpha^2}\right) = 0$

Now, either $\frac{\sin\alpha}{\alpha} = 0$

or $\alpha\cos\alpha - \sin\alpha = 0$

$\therefore \boxed{\alpha = \tan\alpha}$

But it is clear that if $\frac{\sin\alpha}{\alpha} = 0$, the resultant intensity $I = 0$. Hence. $\sin\alpha/\alpha = 0$ represent the condition of minima and $\tan\alpha$ represent the condition of maxima.

For minimum

where $\frac{\sin\alpha}{\alpha} = 0$

$\alpha = \pm\pi, 2\pi, 3\pi \ldots, m\pi$

or

$\frac{\cancel{\pi} e\sin\theta}{\lambda} = \pm m\cancel{\pi}$

$\therefore \boxed{e\sin\theta = \pm m\lambda}$

where

$m = 1, 2, 3 \ldots$ and $I_{min} = 0$

for maximum

$\tan\alpha = \alpha$

to solve Eq. let $y = \tan\alpha$ and $y = \alpha$, plot the graph b/w them.

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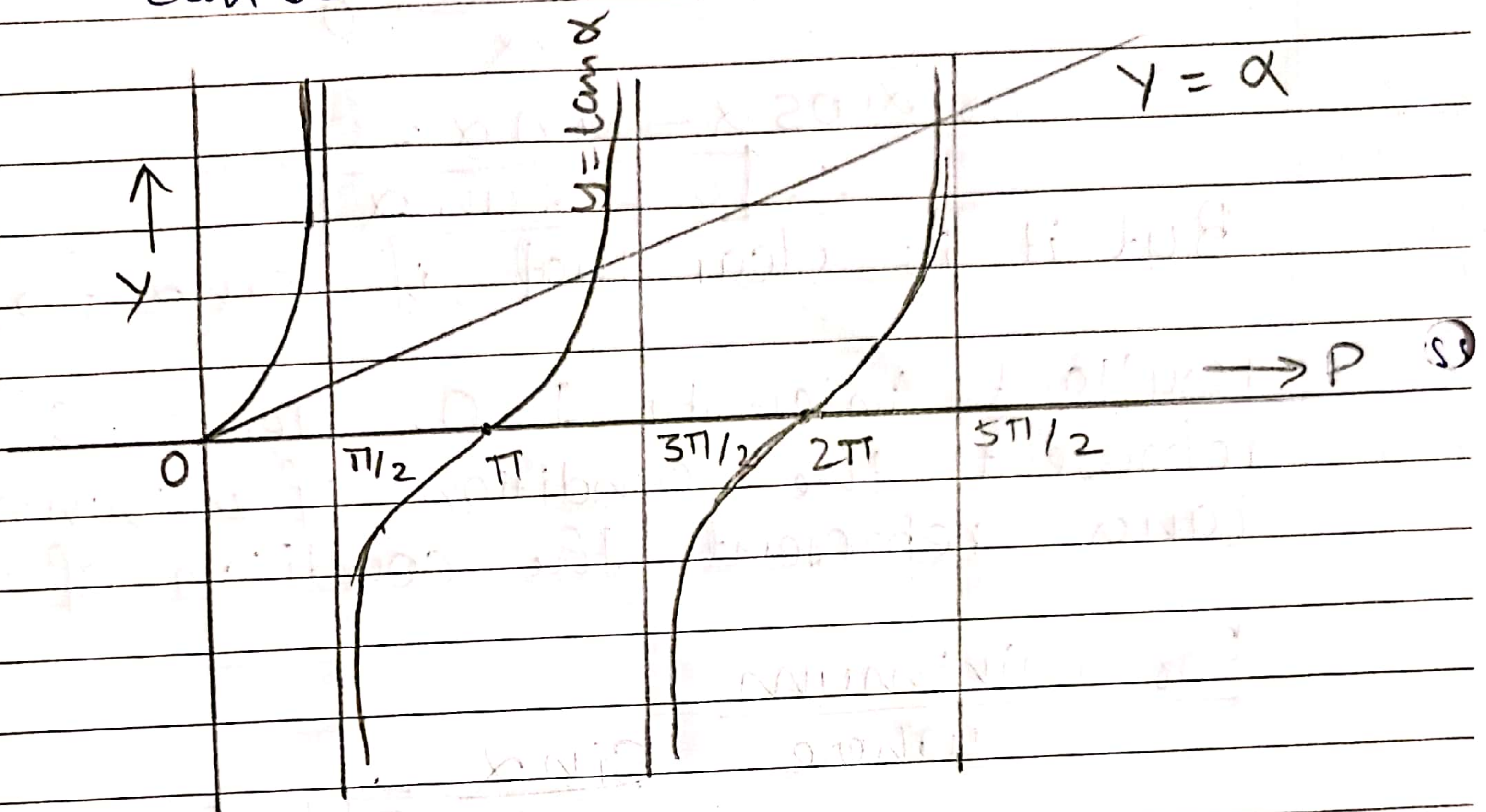
$Y = \alpha, \quad Y = \tan\alpha$

$\tan 0 = \pm n\pi = 0, \pm\pi, \pm 2\pi$

$\tan\infty = (2n+1)\pi/2 = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2$



from the graph

$\alpha = \pm(2n+1)\pi/2$

$\cancel{\frac{\pi e \sin\theta}{\lambda}} = \pm(2n+1)\frac{\pi}{2}$

$\sin\theta \approx \theta$

$$\boxed{e\theta_n = \pm(2n+1)\frac{\lambda}{2e}}$$

and

$\phi \quad \alpha = \frac{3\pi}{2e}, \frac{5\pi}{2e} \ldots$

Q.5. Explain fraunhofer diffraction due to n- parallel slits.

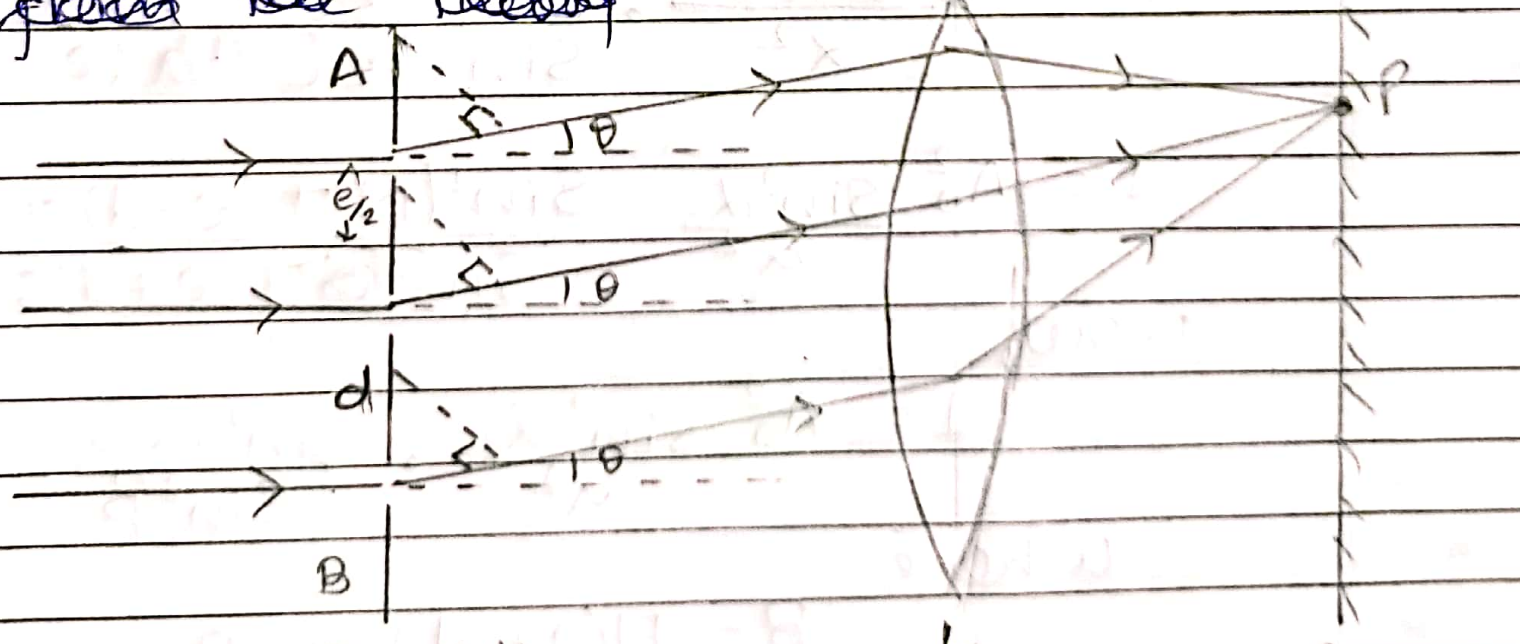
Sol^n: let a parallel beam of light (monochromatic) incident on grating and diffracted by an angle θ, from slits. then we have N waves each of amplitude $A_0 \frac{\sin\alpha}{\alpha}$ and its successive phase diff. :

$$\boxed{\Delta\phi = \delta = \frac{2\pi}{\lambda}(e+d)\sin\theta}$$

The resultant disturbance is given by

$$Y = A'[\cos\omega t + \cos(\omega t+\delta) + \cos(\omega t+2\delta) + \ldots + \cos(\omega t+N\delta)]$$

~~from the theory~~



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$$\sin\theta = \frac{P.d}{e+d}$$

$$\boxed{Pd = (e+d)\sin\theta}$$

from the theory of N harmonic vibration the resultant amplitude to all the wave is

$$R = \frac{A_\theta \sin(N\delta/2)}{\sin(\delta/2)}$$

as $I = R^2$

$$I = A_\theta^2 \frac{\sin(N\delta/2)}{\sin\delta/2}$$

$$A_\theta = A_0 \frac{\sin\alpha}{\alpha} \qquad \text{as } \theta = 0$$

$$I = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2(N\delta/2)}{\sin^2(\delta/2)}$$

$$I = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N/2\,[2\pi/\lambda\,(e+d)\sin\theta]}{\sin^2 1/2\,[2\pi/\lambda\,(e+d)\sin\theta]}$$

$$I = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2(N\pi(e+d)\sin\theta]/\lambda}{\sin^2(\pi(e+d)\sin\theta)\lambda}$$

Now,

$$I = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta} \quad \text{--- (1)}$$

Where

$$\beta = \frac{\pi(e+d)\sin\theta}{\lambda}$$

Hence, the intensity distribution is the product of two terms. the first term is $\frac{A_0^2 \sin^2\alpha}{\alpha^2}$ represents.

The diffraction pattern due to single slit and the second term $\frac{\sin^2 N\beta}{\sin^2\beta}$ represents the interference pattern due to N Slits.

from Eq. ①

Put $N = 1$

$$I_0 = \frac{A_0^2 \sin^2\alpha}{\alpha^2} \quad \left[\text{single slit diffraction}\right]$$

when $N = 2$

$$I_0 = \frac{A_0^2 \sin^2\alpha}{\alpha^2} \frac{\sin^2 2\beta}{\sin^2\beta}$$

$$I_0 = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \cdot \frac{2^2 \cancel{\sin^2\beta} \cos^2\beta}{\cancel{\sin^2\beta}}$$

$$I_0 = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \cdot 4\cos^2\beta$$

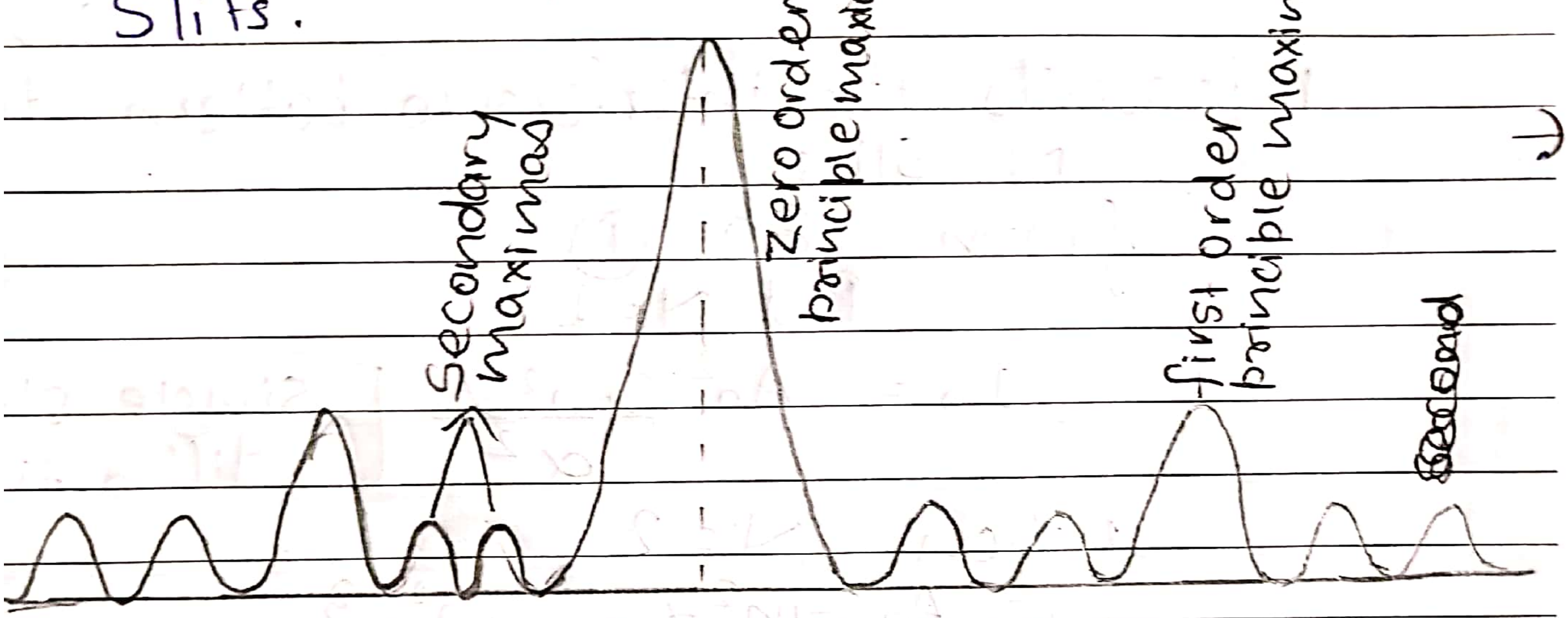
$$\boxed{I_0 = 4A_0^2 \frac{\sin^2\alpha \cos^2\beta}{\alpha^2}}$$

Intensity distribution with double slit.

for N slits

$$\boxed{I = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}}$$

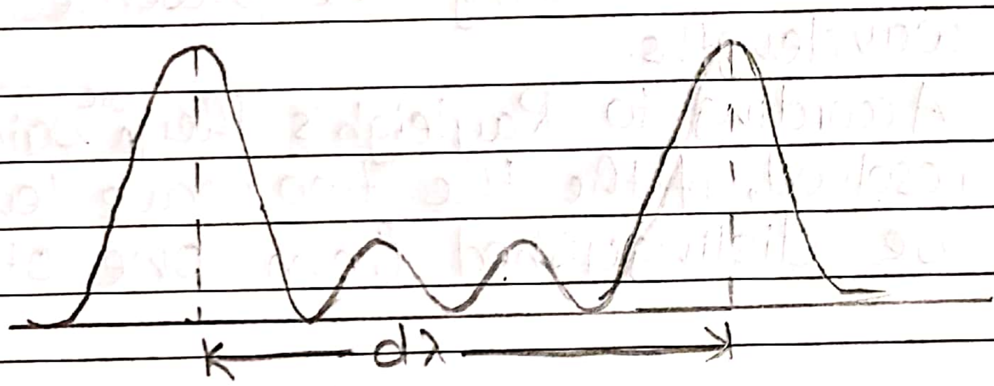
It is the generalized Expression for the intensity distribution due to N Slits.



Q.6. What is Rayleigh criteria?

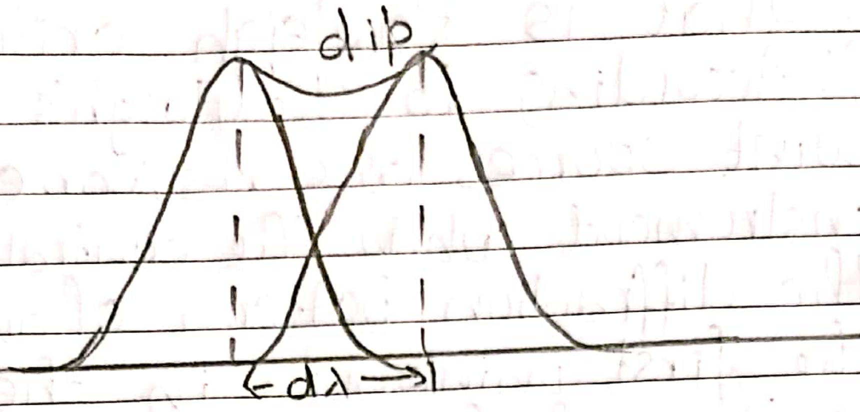
Sol^n: According to Rayleigh's criterion, two point sources are resoluable by an optical instrument when the central maximum in the diffraction pattern of one falls over the first minimum in the diffraction pattern of the other and vice versa.

Now, consider the resolution of two wavelength λ_1 and λ_2 by a grating.



It shows intensity curves of the diffraction patterns of two wavelengths. The difference in wavelength is such that the principle maxima are separately visible. There is a distint point of zero intensity in between the two. Hence the wavelengths are well resolved.

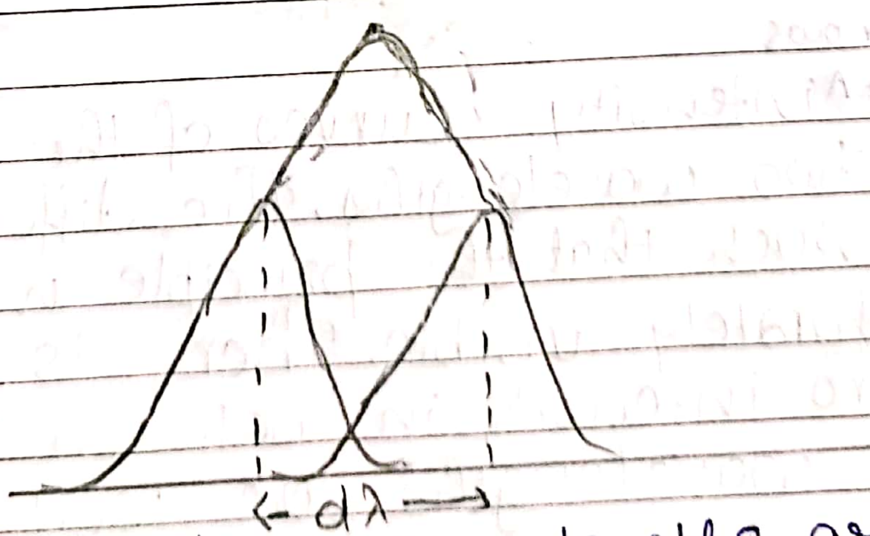
Now, consider the case when the difference in wavelength is smaller and such that the central maxima of wavelengths coincides with the first minimum of the other.



NOW,

A noticeable decrease in intensity b/w the two central maxima indicating the two central maxima indicating the presence of two diff wavelengths.

According to Rayleigh's they are said to Just resolved, if the the two wave lengths can be distinguished from one other.



Hence, the two wave lengths are not resolved.

Thus, two spectral line can be resolved only upto a certain limit expressed by Rayleigh criteria.

Q.7. Explain resolving power of diffraction grating.

Soln: When two objects are very close together they may appear as one object and it may be difficult for the naked eye to see them as separate. Similarly, if there are two point sources very close together, the two diffraction patterns produced by each of them may overlap and ~~here~~ hence it may be difficult to distinguish them as separate. To see the two objects or two spectral lines which are very close together, optical instruments like telescopes, microscopes, prism, grating etc. are employed. The capacity of an optical instrument to show two close objects separately is called resoultion and ability of an optical instrument to resolve the image of two close point objects is called its "resolving power."

A diffraction grating is an arrangement equivalent to n parallel slits of equal width and separated from one another by equal obey space.

$$\text{Resolving power}_{\text{grating}} = \frac{\lambda}{d\lambda}$$

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$$\cancel{n+\lambda+\frac{\lambda}{N} = n(\lambda+d\lambda)}$$

$$\cancel{\frac{\lambda}{N} = nd\lambda}$$

$$\cancel{\frac{\lambda}{d\lambda} = nN}$$

$$(e+d)\sin\theta_n = n\lambda \quad -①$$

$$(e+d)\sin(\theta_n+d\theta_n) = n\lambda+\frac{\lambda}{N} \quad -②$$

$$(e+d)\sin(\theta_n+d\theta_n) = n^N(\lambda+d\lambda) \quad -③$$

$$n+\lambda+\frac{\lambda}{N} = n(\lambda+d\lambda)$$

$$\frac{\lambda}{N} = nd\lambda$$

$$\boxed{\frac{\lambda}{d\lambda} = nN}$$