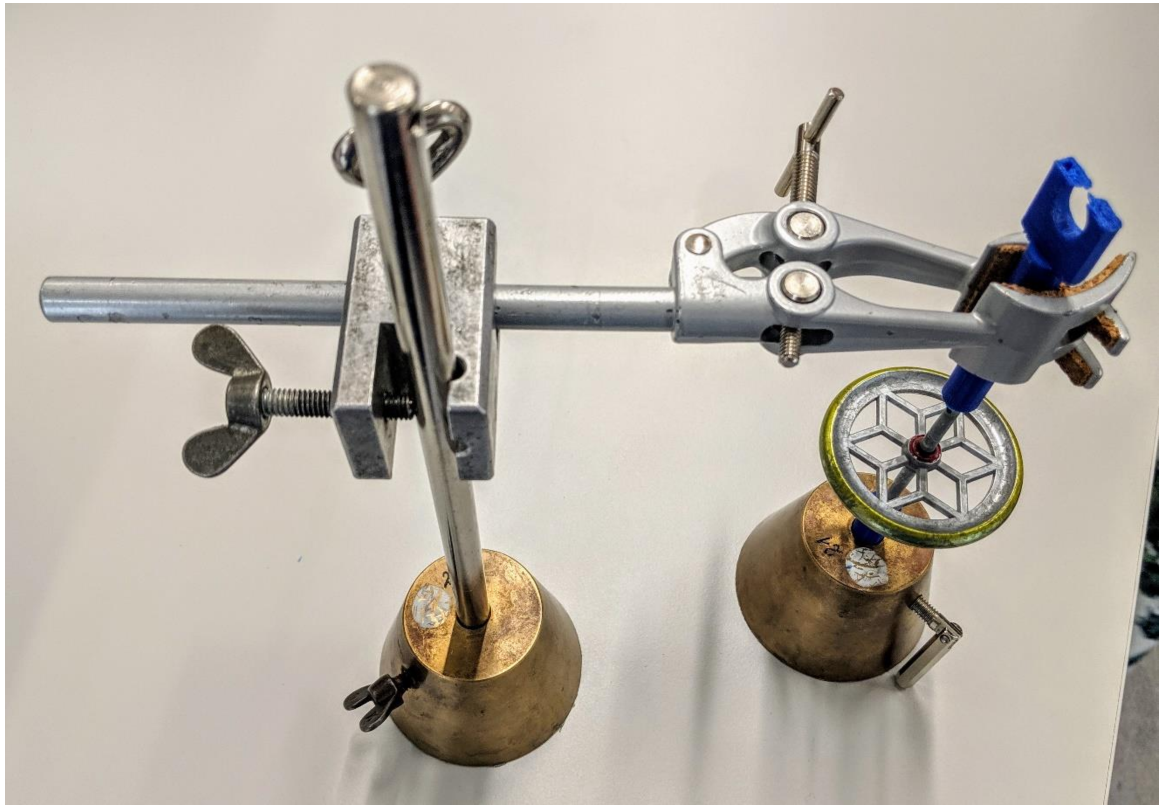
TEAM AUSTRIA

Rohan Walia presents problem No. 12
Gyroscope Teslameter

32nd IYPT 2019
WARSAW

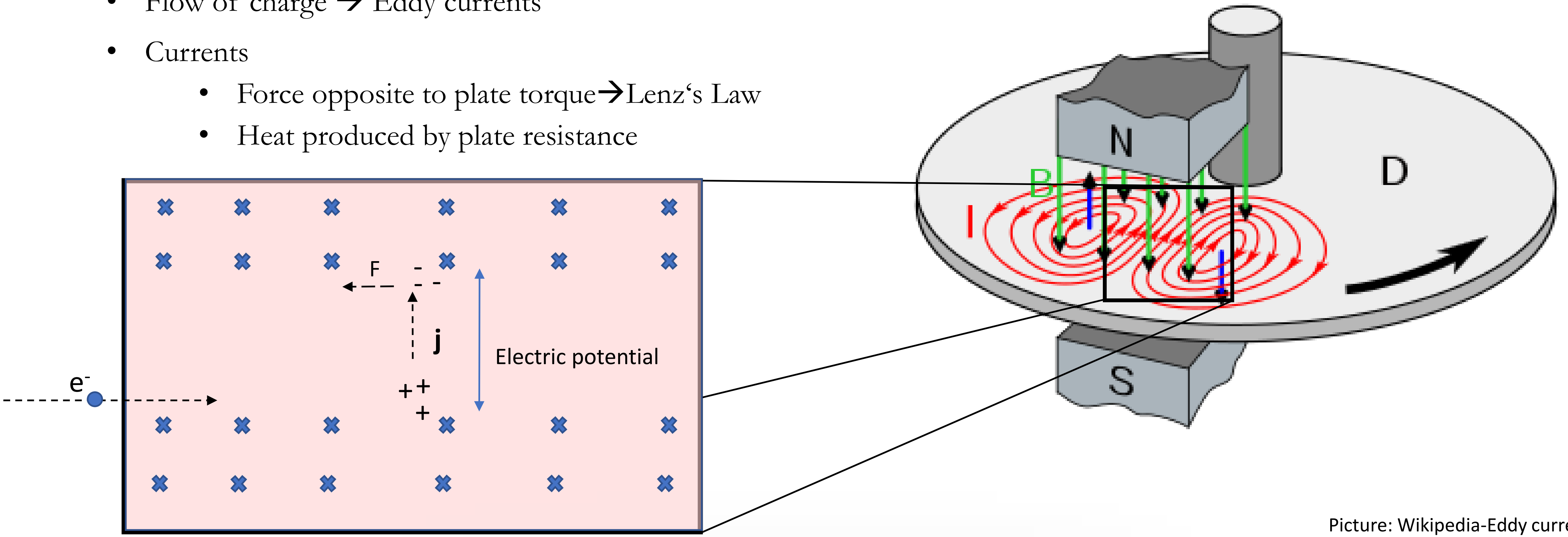
Problem statement

A spinning **gyroscope** made from a **conducting**, but **non-ferromagnetic** material slows down when placed in a magnetic field. Investigate how the **deceleration** depends on **relevant parameters**.



Basic concepts

- Lorenz Force acts on electrons
- Flow of charge → Eddy currents
- Currents
    - Force opposite to plate torque→Lenz‘s Law
    - Heat produced by plate resistance



Picture: Wikipedia-Eddy currents

Fundamentals of the theory

$\phi(x,y)$ and **j**(x,y) are assumed such that a **stationary state** is reached, **we assume that this happens instantaneously**

Force on charge vanishes in the **stationary state:**

(I) $$\boldsymbol{F_R} + q\left(-\nabla\phi + (\boldsymbol{v} \times \boldsymbol{B})\right) \approx \boldsymbol{0}$$

Ohms' law connects $\boldsymbol{F_R}$ and j

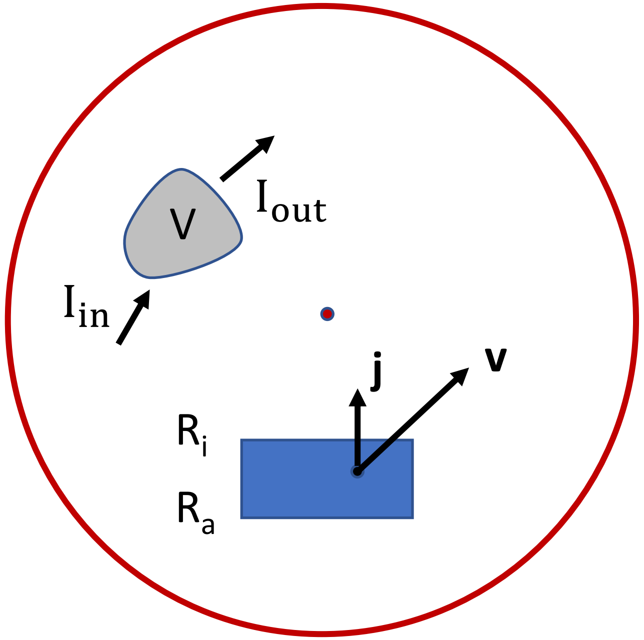
$$\boldsymbol{F_R} = -q\,\rho\,\mathbf{j}$$

No accumulation of currents in the **stationary state:**

$I_{in} = I_{out}$ (for an arbitrary Volume V)

This can also be formulated as

(II) $$\nabla \cdot \mathrm{j} = 0$$



ϕ[V]…electrical potential

j [A/m²]… current density

F_R[N]…force caused by resistance

v[m/s]… velocity of electron due to rotation of plate

B[T]…magnetic field strength

R_a[m]…outer radius of magnet

R_i [m]…inner radius of magnet

ρ [Ω m]… resistivity

I_{in} [A].. current into volume V

I_{out} [A].. current out of volume V

Area of constant magnetic field

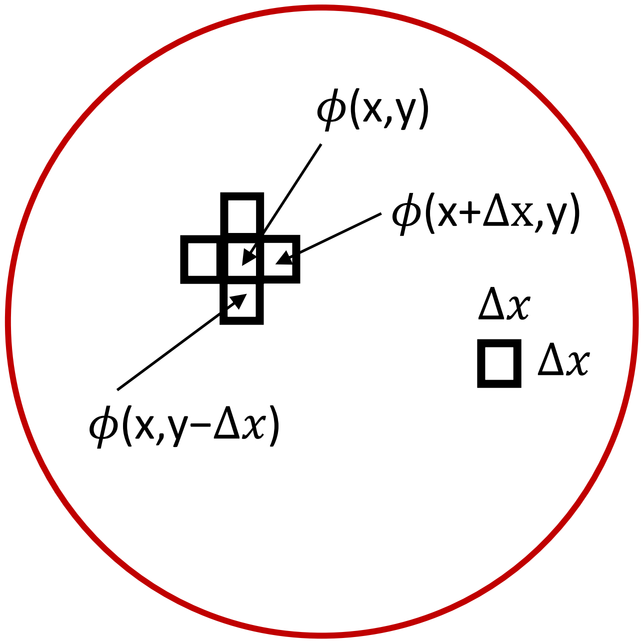
Numerical computation of $\phi(x, y)$

For ϕ(x,y) and **j**(x,y) we divide the plate into small squares of length Δx

In every square there is a value for ϕ and **j**

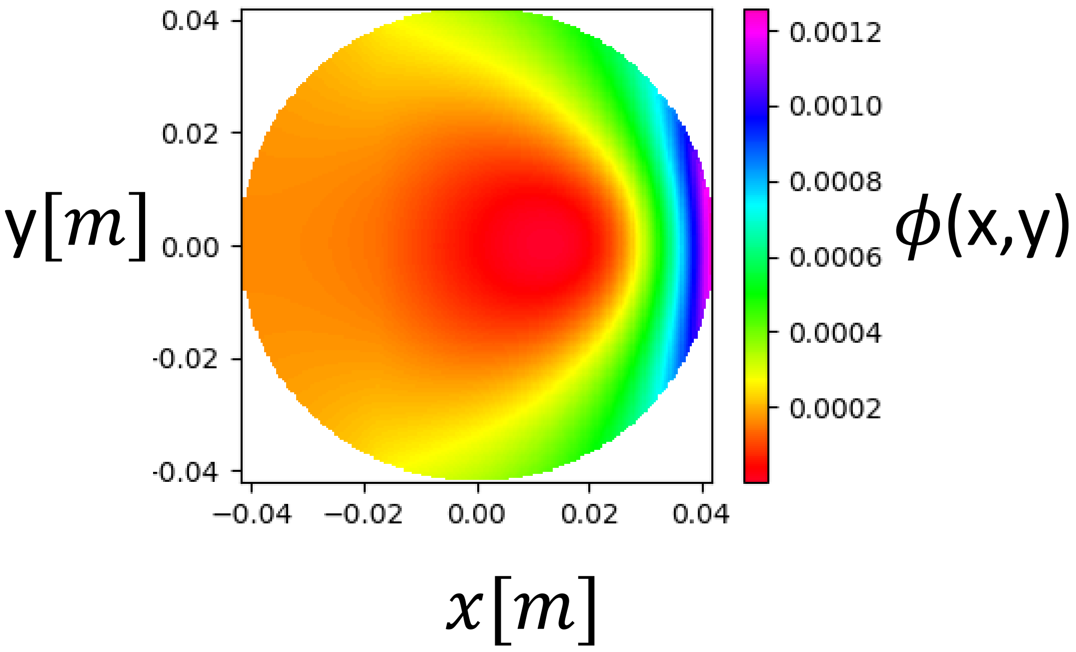
We can write eq. **(I)** for every square and use **(II)** to eliminate **j** in this equation, to obtain one equation for ϕ per square:

$$4\,\phi(x,y) = \phi(x+\Delta x,y) + \phi(x-\Delta x,y) + \phi(x,y+\Delta x) + \phi(x,y-\Delta x) - \int_{x}^{x+\Delta x} dx' B(x',y)\,\omega x' - \int_{x}^{x-\Delta x} dx' B(x',y)\,\omega x' - \int_{y}^{y+\Delta x} dy' B(x,y')\,\omega y' - \int_{y}^{y-\Delta x} dy' B(x,y')\,\omega y'$$



Numerical computation of $\phi(x, y)$

For resolution **200x200** squares
→a linear equation system with **40k** equations and **40k** unknown **ϕ(x,y)** values
→solved them numerically with the scipy libraries for Python

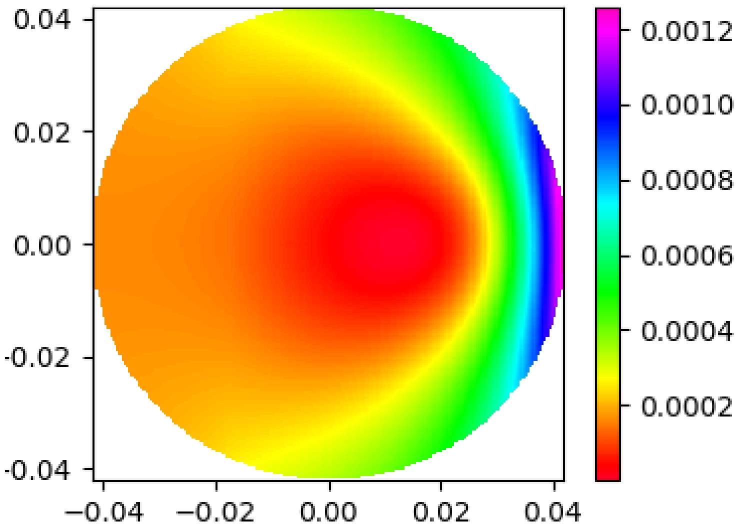


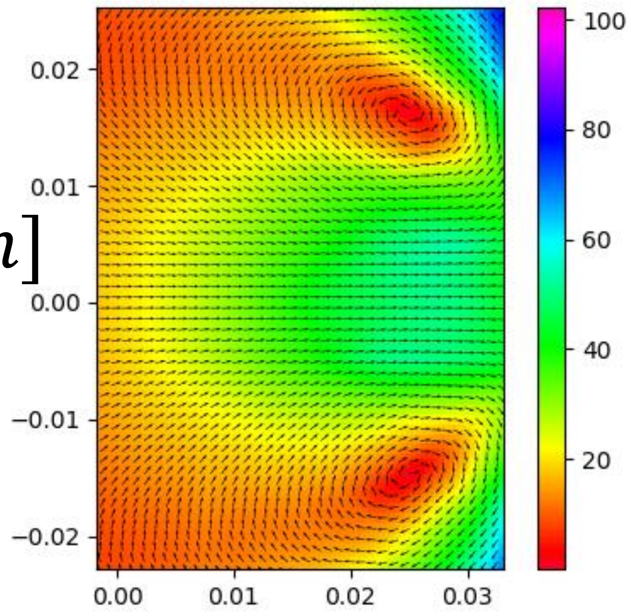
Numerical computation of $\mathbf{j}$(x,y)

$$\boldsymbol{I_x} = -\frac{\phi(\mathrm{x}-\Delta\mathrm{x},\mathrm{y})\cdot d}{\rho} - \int_{x}^{x-\Delta x} \left(\frac{dx' B(x',y)\,\omega x' \cdot d}{\rho}\right)$$

$$\boldsymbol{j}(x,y) = \frac{1}{d\cdot\Delta x}\cdot\begin{pmatrix}\boldsymbol{I_x}\\ \boldsymbol{I_y}\end{pmatrix}$$

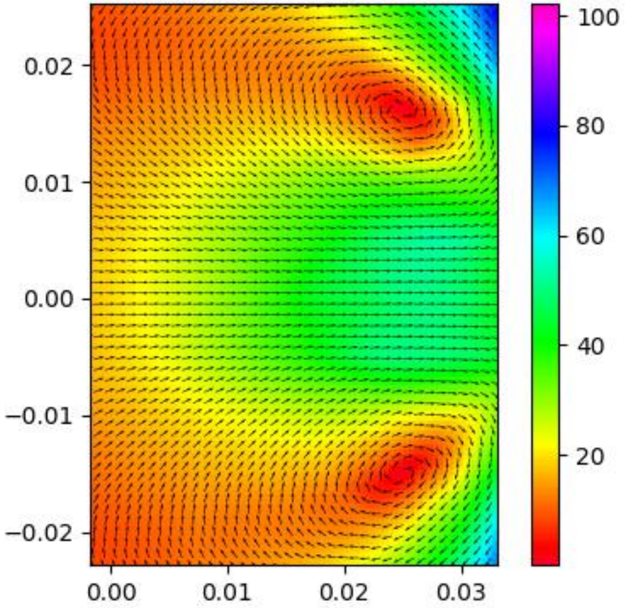
$$\boldsymbol{I_y} = -\frac{\phi(\mathrm{x},\mathrm{y}-\Delta\mathrm{x})\cdot d}{\rho} - \int_{y}^{y-\Delta x} \left(\frac{dy' B(x,y')\omega y' \cdot d}{\rho}\right)$$





Computing the deceleration power P_{dec}

$\mathbf{f}$(x,y) = Force per Area
due to interaction of $\boldsymbol{j}$ with $\mathbf{B}$(x,y)

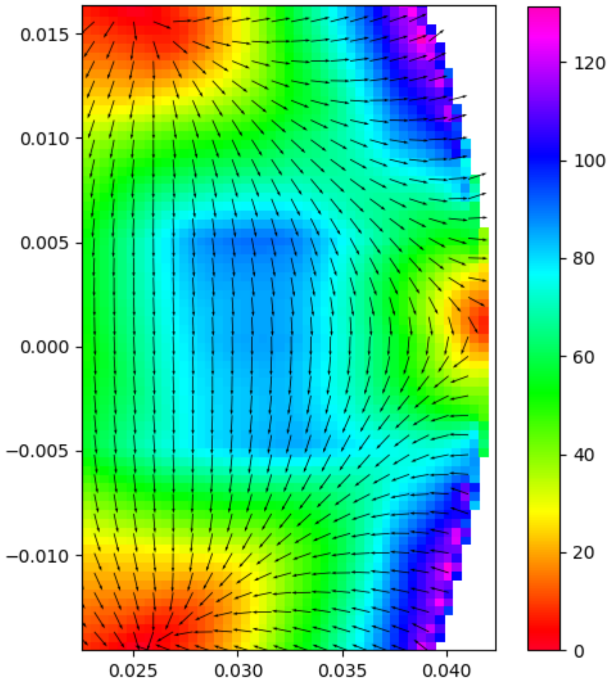


y[m]

x[m]

$\boldsymbol{j}$[A/cm^2]

$$\mathbf{f} = \mathrm{d} \cdot (\mathbf{j} \times \boldsymbol{B})$$



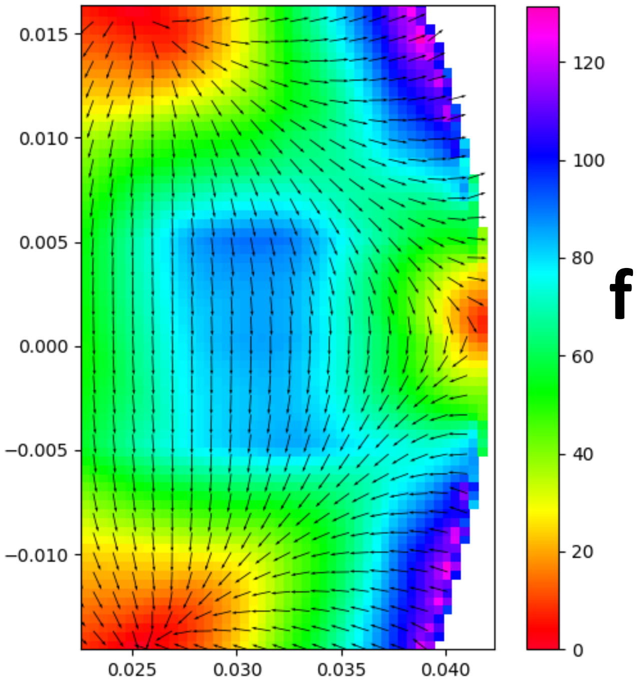
y[m]

x[m]

$\mathbf{f}[\frac{\boldsymbol{N}}{\boldsymbol{m}^2}]$

Computing the deceleration power P_{dec}

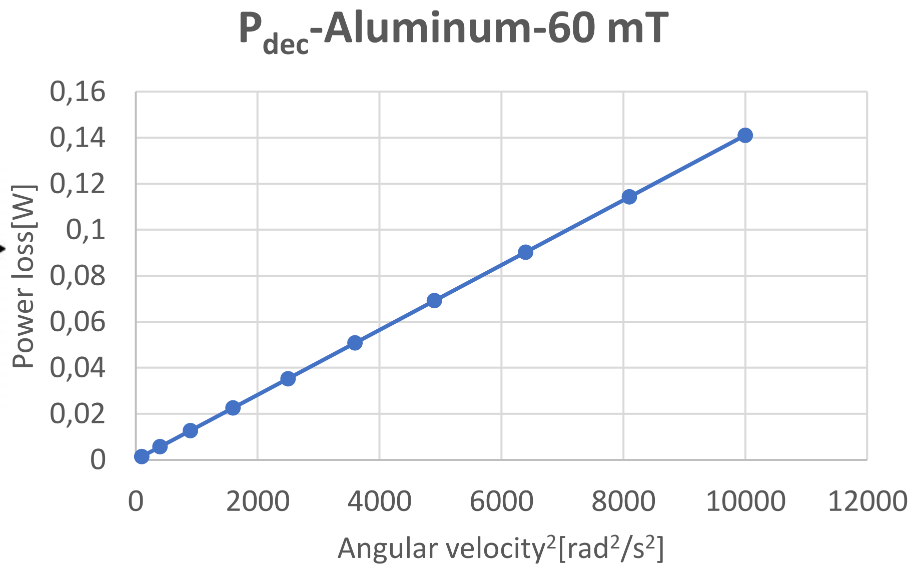
$y[m]$



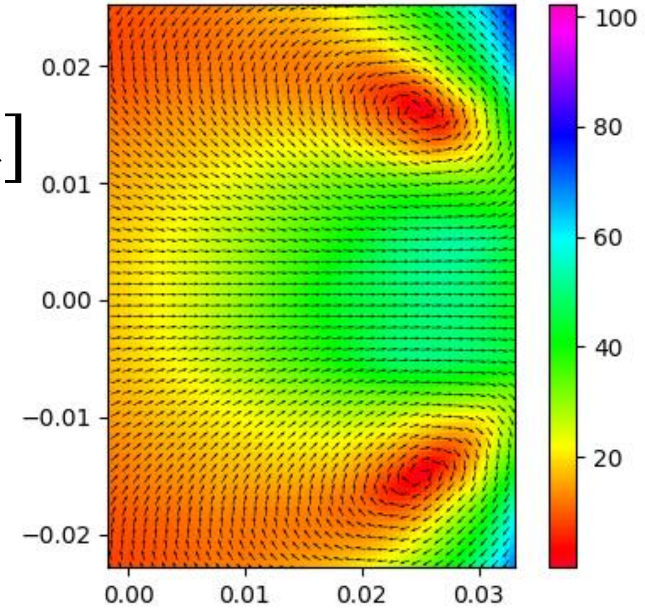
$x[m]$

$\mathbf{f}[\frac{\boldsymbol{N}}{\boldsymbol{m}^2}]$

$$P_{dec} = \int dA\, \mathbf{f} \cdot \underbrace{\boldsymbol{\omega} \times \boldsymbol{r}}_{\mathbf{v}}$$



Power converted into heat P_{Ohm}

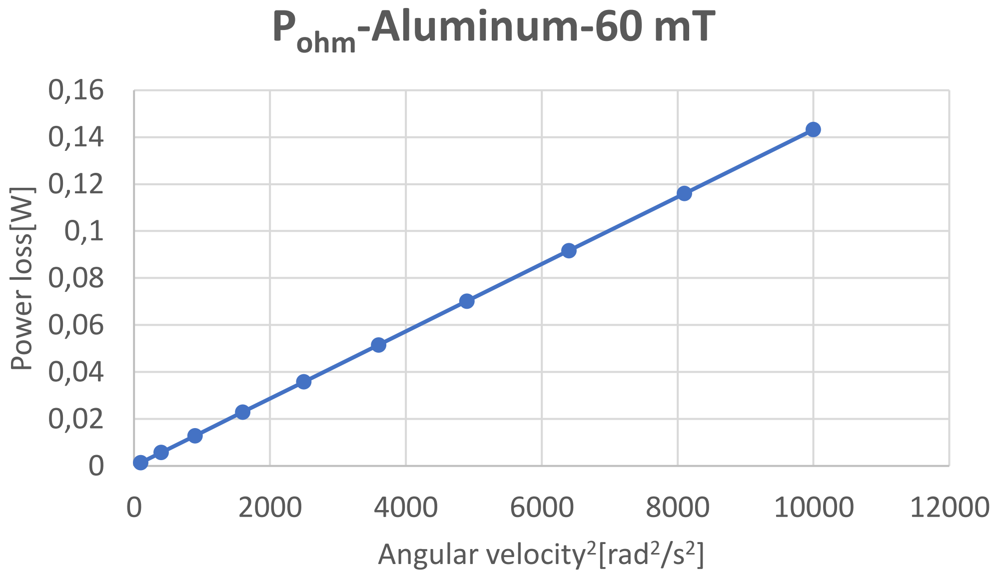


y[m]

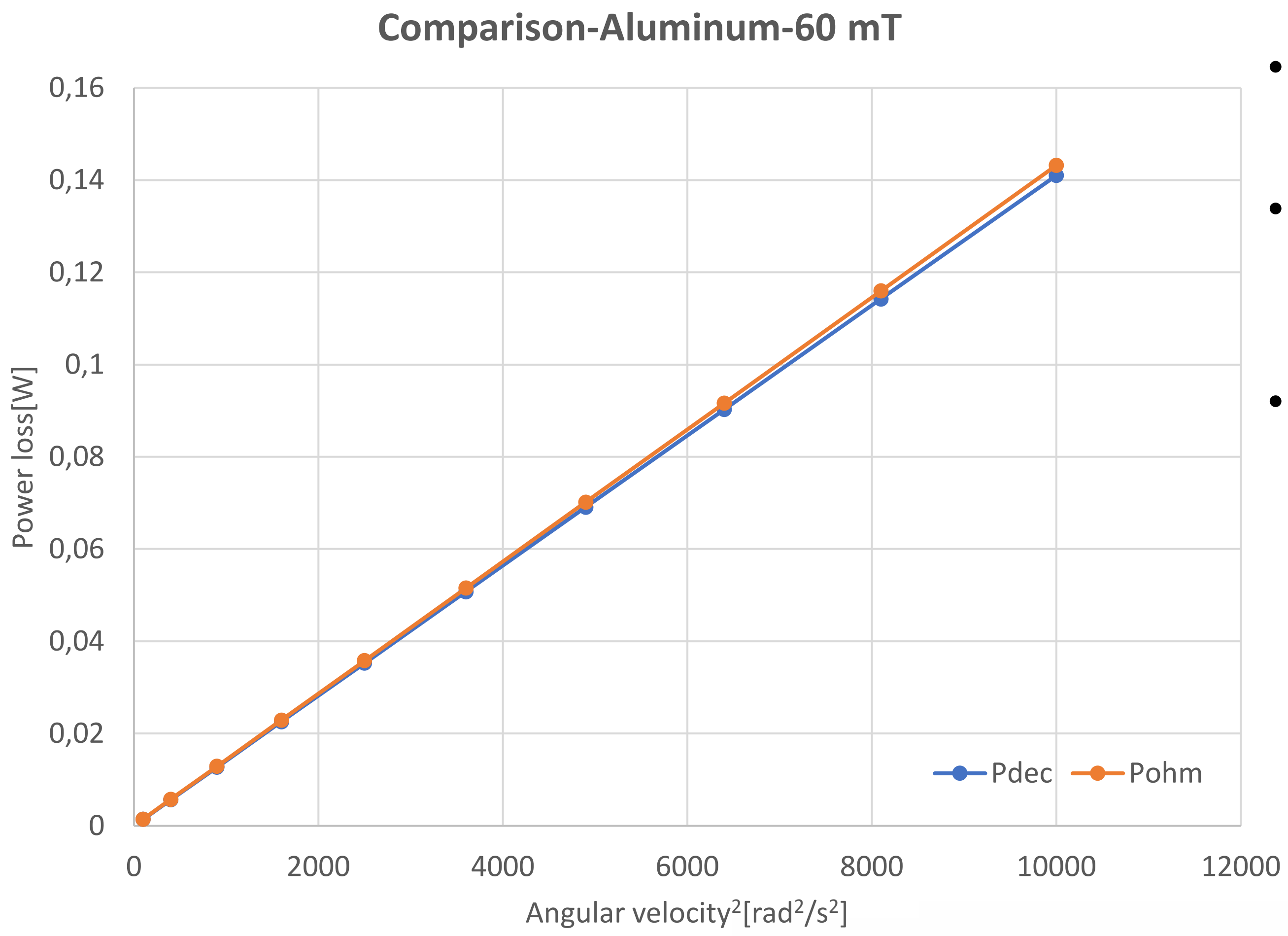
$x[m]$

$\boldsymbol{j}[\mathrm{A/cm^2}]$

$P_{Ohm}=\int dA\ j^2\ d\ \rho$



Power Loss given by 2 theories



- P_{ohm}
  - Heat loss through resistance
- P_{dec}
  - Kinetic energy loss per second due to torque on eddy currents in magnetic field
- **identical** (except for small numerical discrepancies)

Due to energy conservation $P_{dec} = P_{Ohm}$ ✔

Relevant parameters

<u>Parameters maintaining the system ***geometry***</u>

- Angular velocity
  - $\omega^2 \rightarrow P$
- Magnetic field strength
  - $B^2 \rightarrow P$
- Resistance of plate
  - Resistivity of material
    - $1/\rho \rightarrow P$
  - Thickness of plate
    - $d \rightarrow P$

Variations of P_{dec} described by **master formula**:

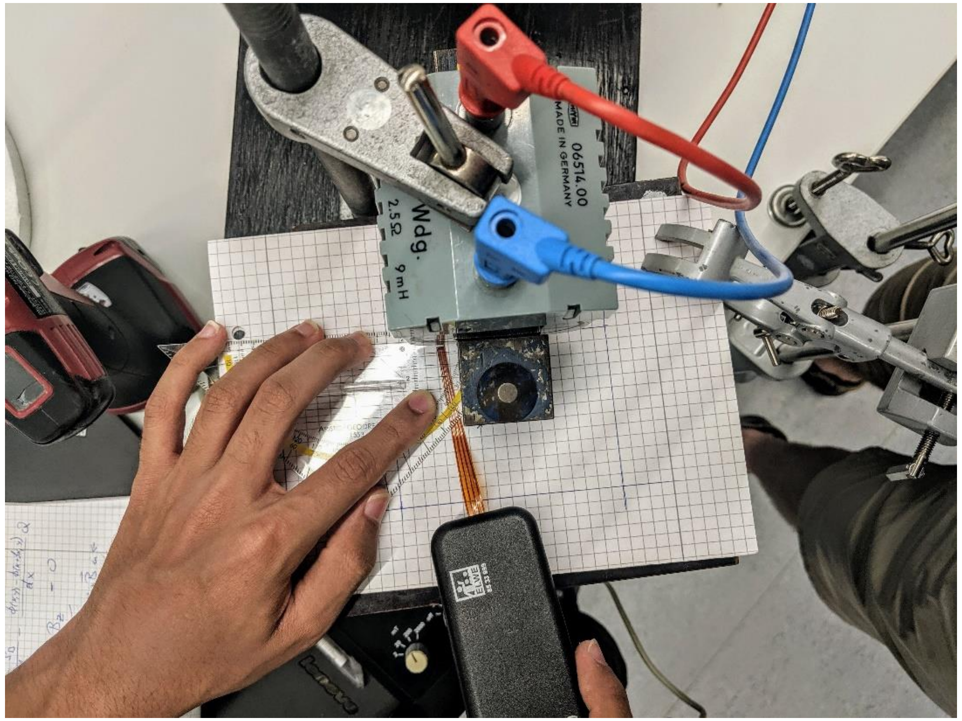
$$\boxed{P_{dec}(B_{max}, \omega, \rho, d) = P_{dec}(B_{max}^0, \omega^0, \rho^0, d^0)\ \frac{(B_{max}\omega)^2 \rho^0 d}{(B_{max}^0 \omega^0)^2 \rho d^0}}$$

<u>Parameters changing the system ***geometry***</u>

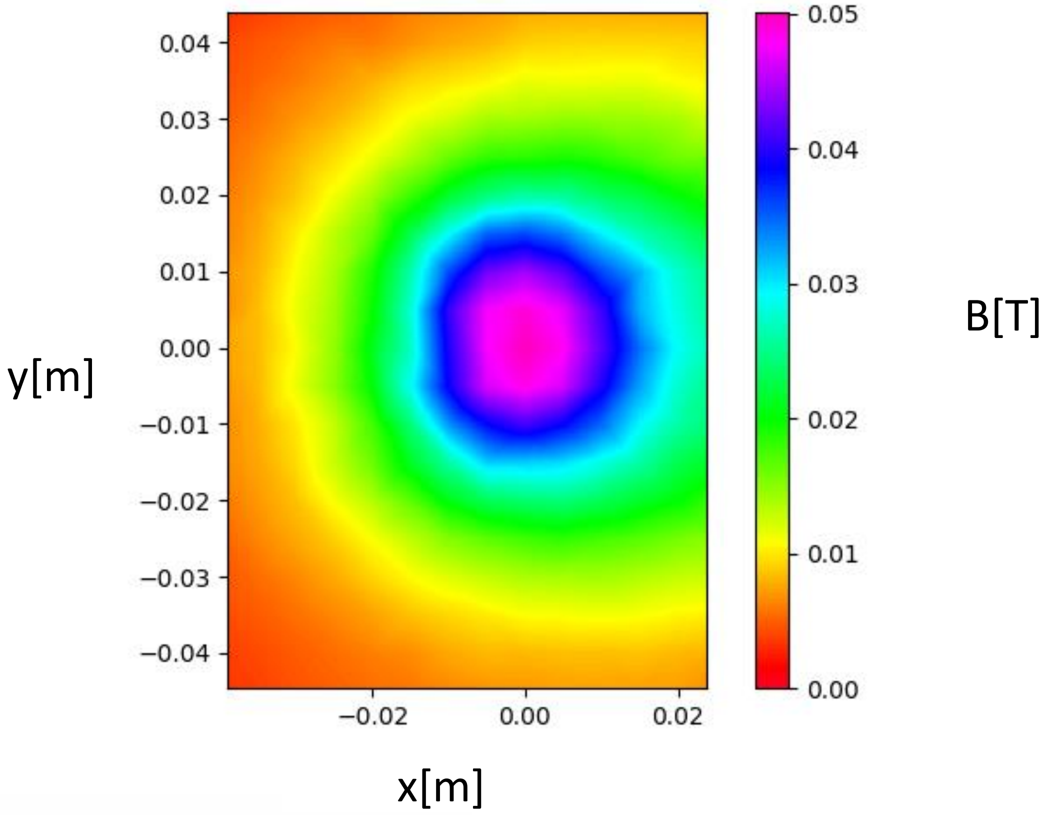
- Position of magnet (at which radius)
- Size of magnet (width in direction of radius)

Impact on P_{dec} & P_{Ohm} is less obvious (however, well described by our numerical model)

Measuring B Field profile of the magnet

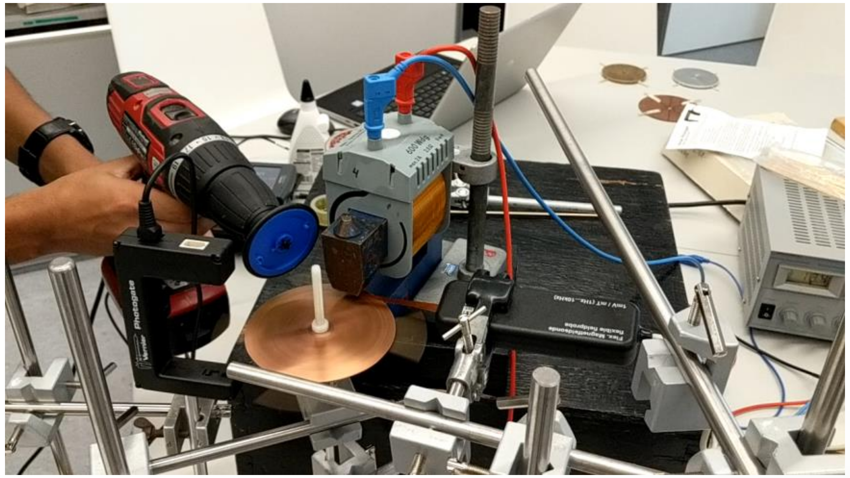


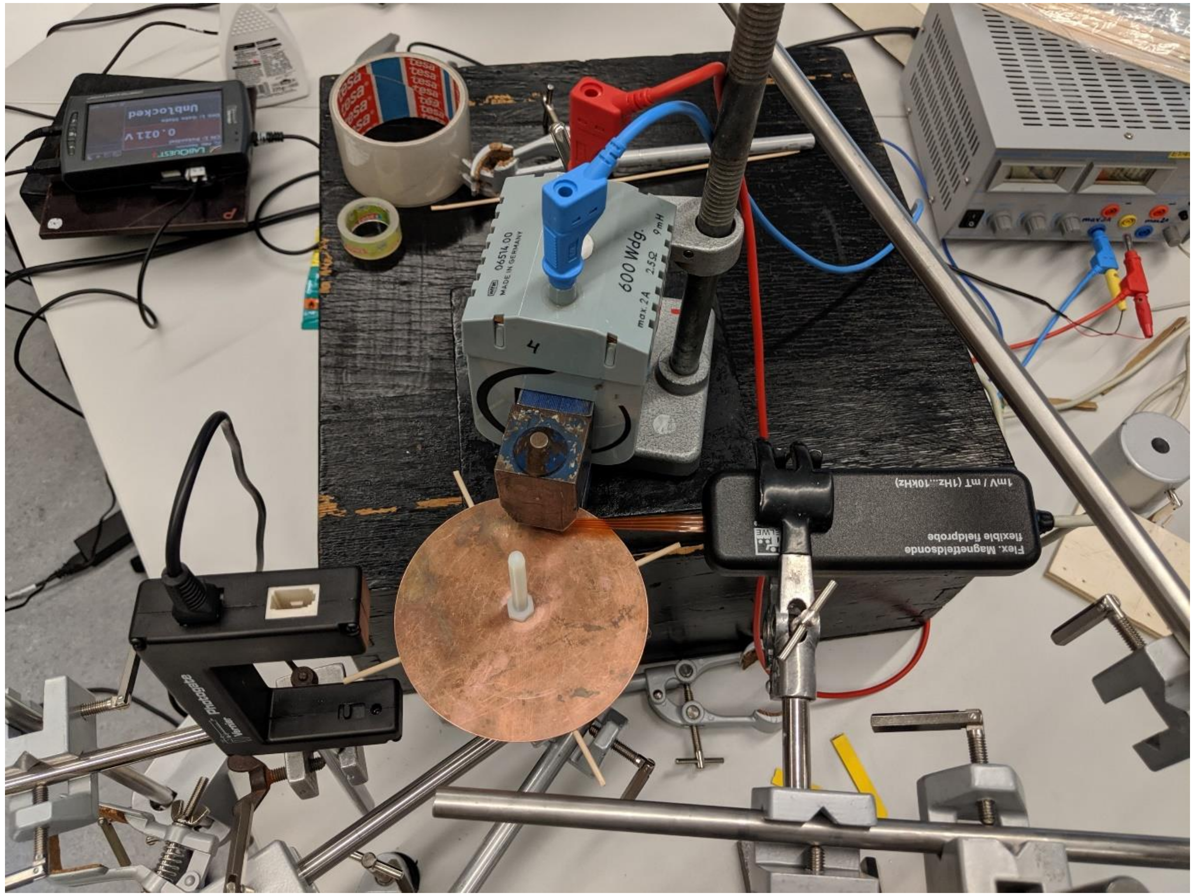
Obtained with **300** values



Experimental setup

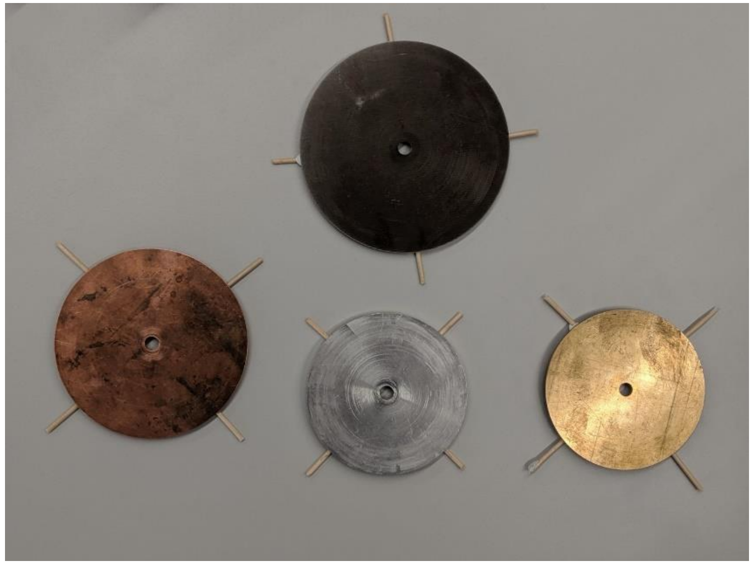
- Electromagnet
    - Strength with Teslameter
- Frequency with photogate
    - Wooden stick attached
- Spin on plastic pin (low friction)
- Speed with cordless screw-driver

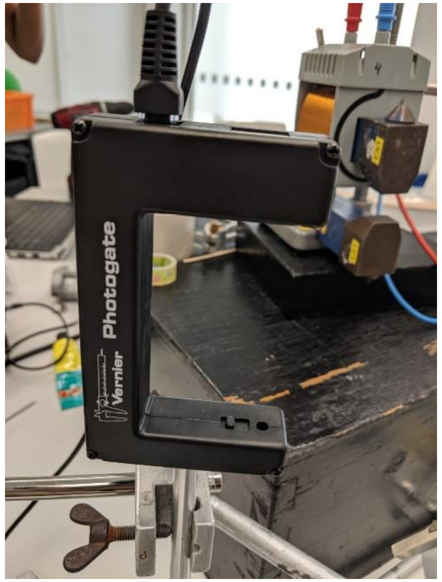




Parameter variation

- 10 magnetic field strengths
  - 0-100 mT
- 2 Radii
  - 1.5 cm
  - 3.5 cm
- 3 Materials
  - Copper
  - Brass
  - Aluminum
- 2 Thicknesses
  - 35 µm
  - 1 mm





Experimental Power Loss

$$P_{dec} = T \cdot \omega = I_m \cdot \frac{d\omega}{dt} \cdot \omega$$

Using:

$$I_m = \frac{m \cdot r^2}{2}$$

Friction term P_f from a trend line

Yields:

$$\boxed{P_{dec} = \frac{m \cdot r^2}{2} \cdot \frac{d\omega}{dt} \cdot \omega - Pf}$$

P[W]…power loss

T[Nm]…torque

ω[rad/s]…angular velocity opposing

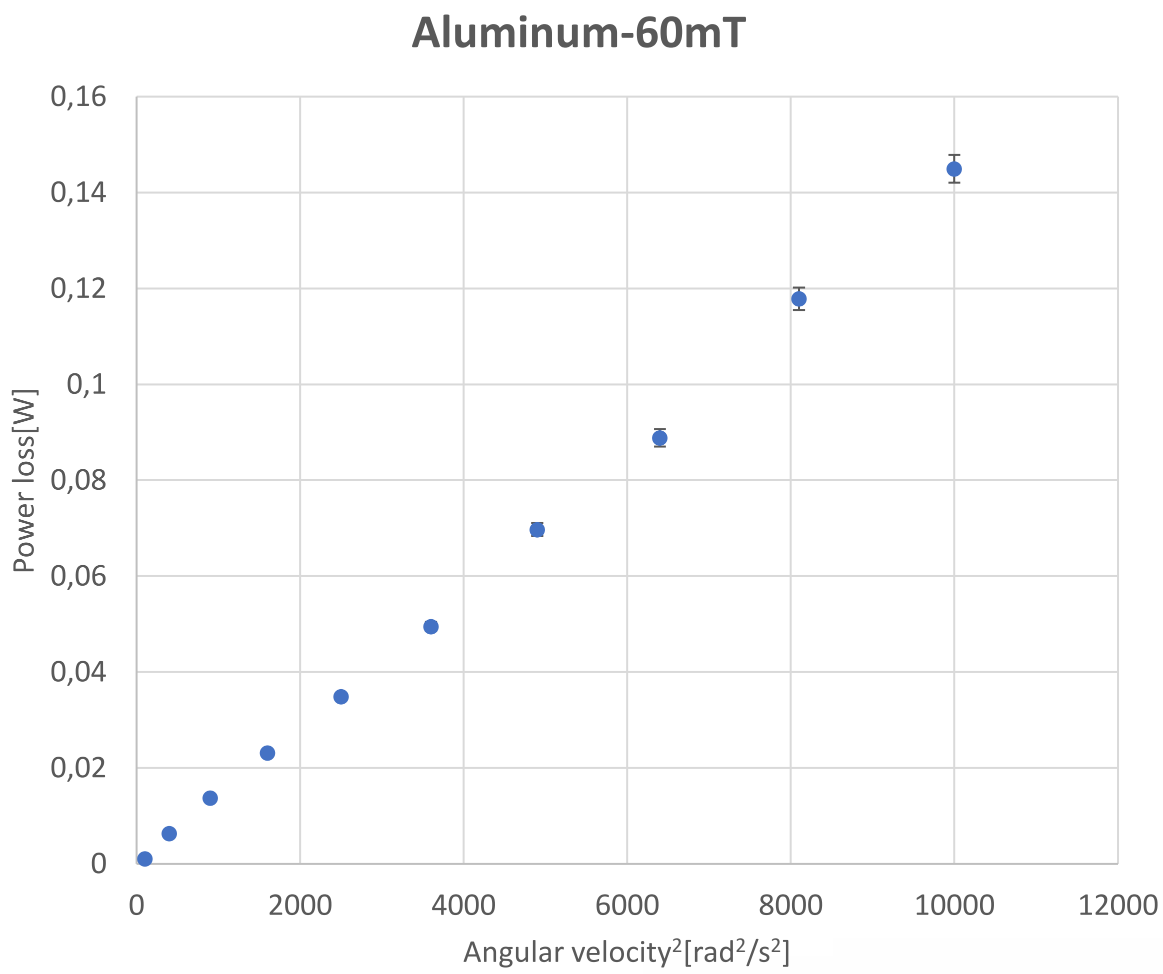
- obtained by experiment

I_m…[kgm^2]…moment of inertia of homogenous disk

m[kg]…mass of plate

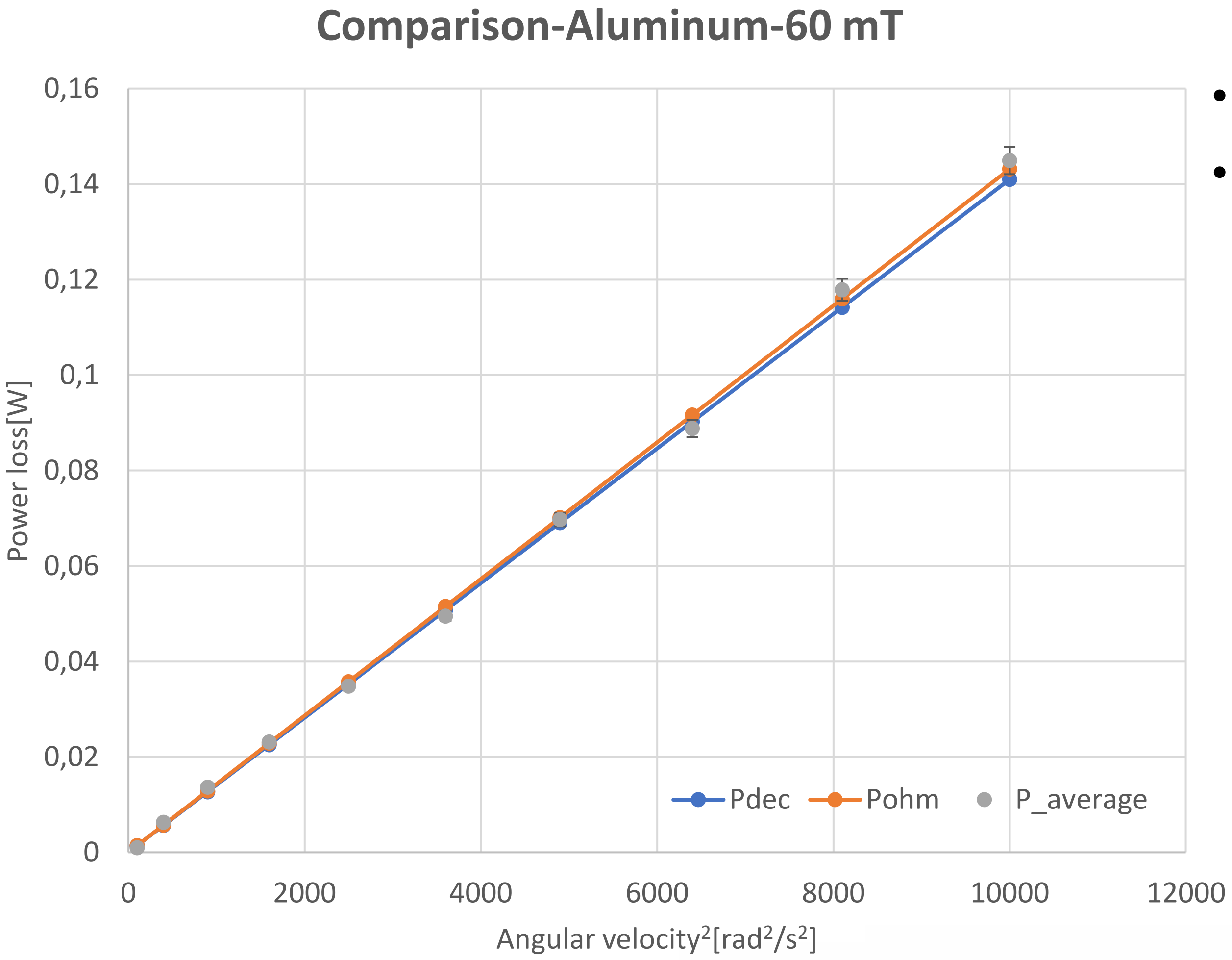
r[m]…radius of plate

Power Loss of Experiment



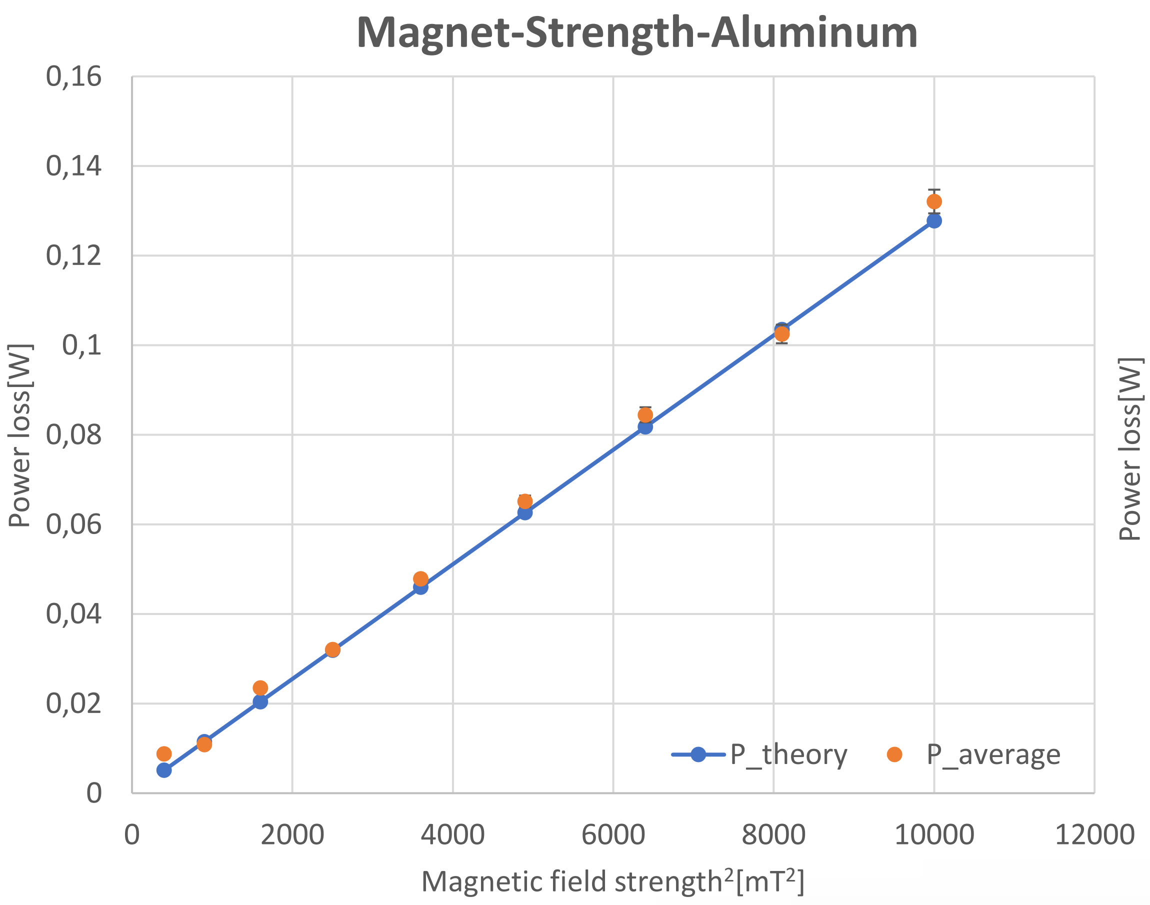
- Angular velocity instead of time as plate slows down
- Angular velocity is a function of time
- Easier to compare→no time delays
- Aluminum-60 mT
    - Average of 3 runs
    - Square relationship

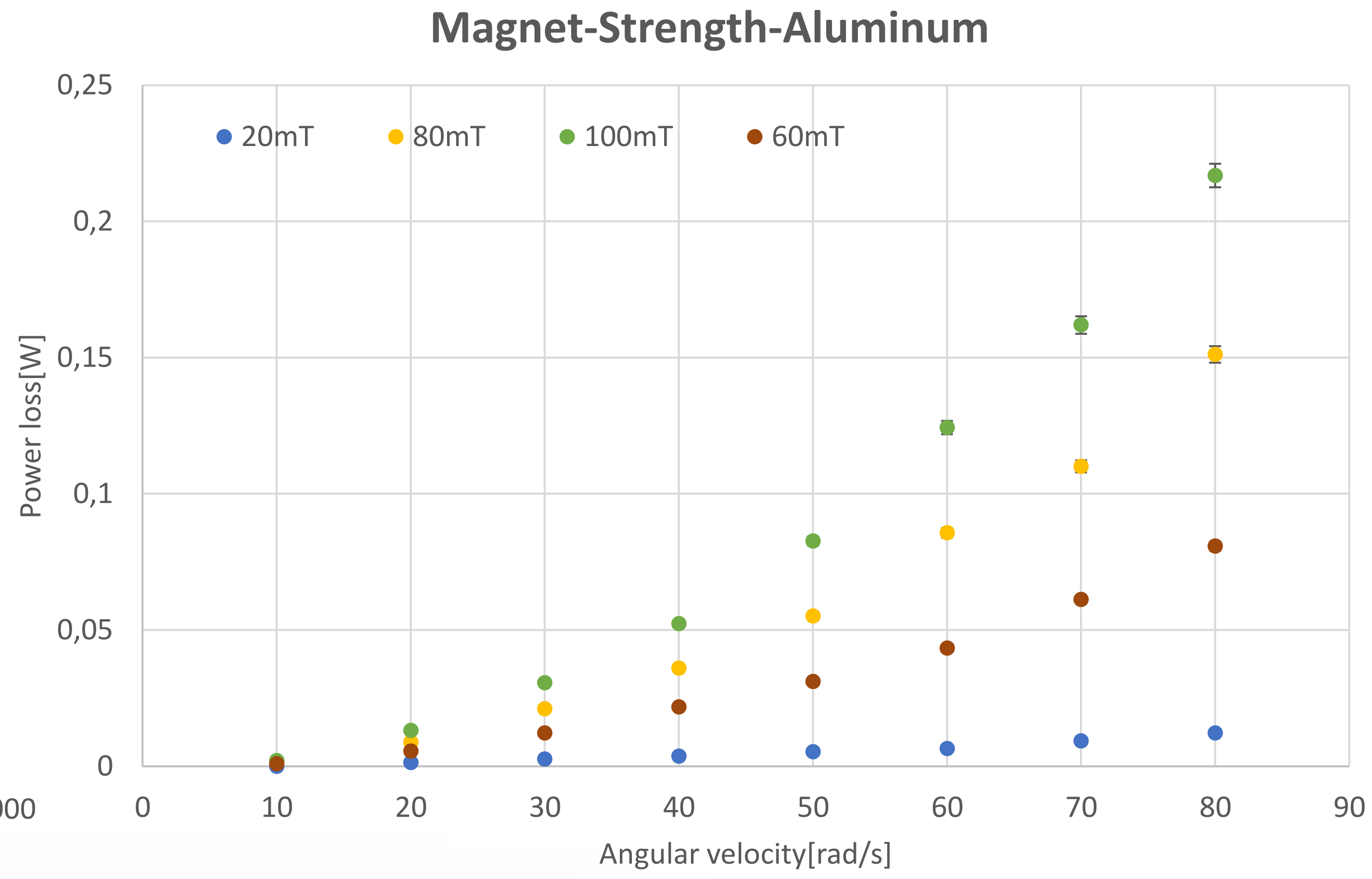
Comparison of 3 Power Losses



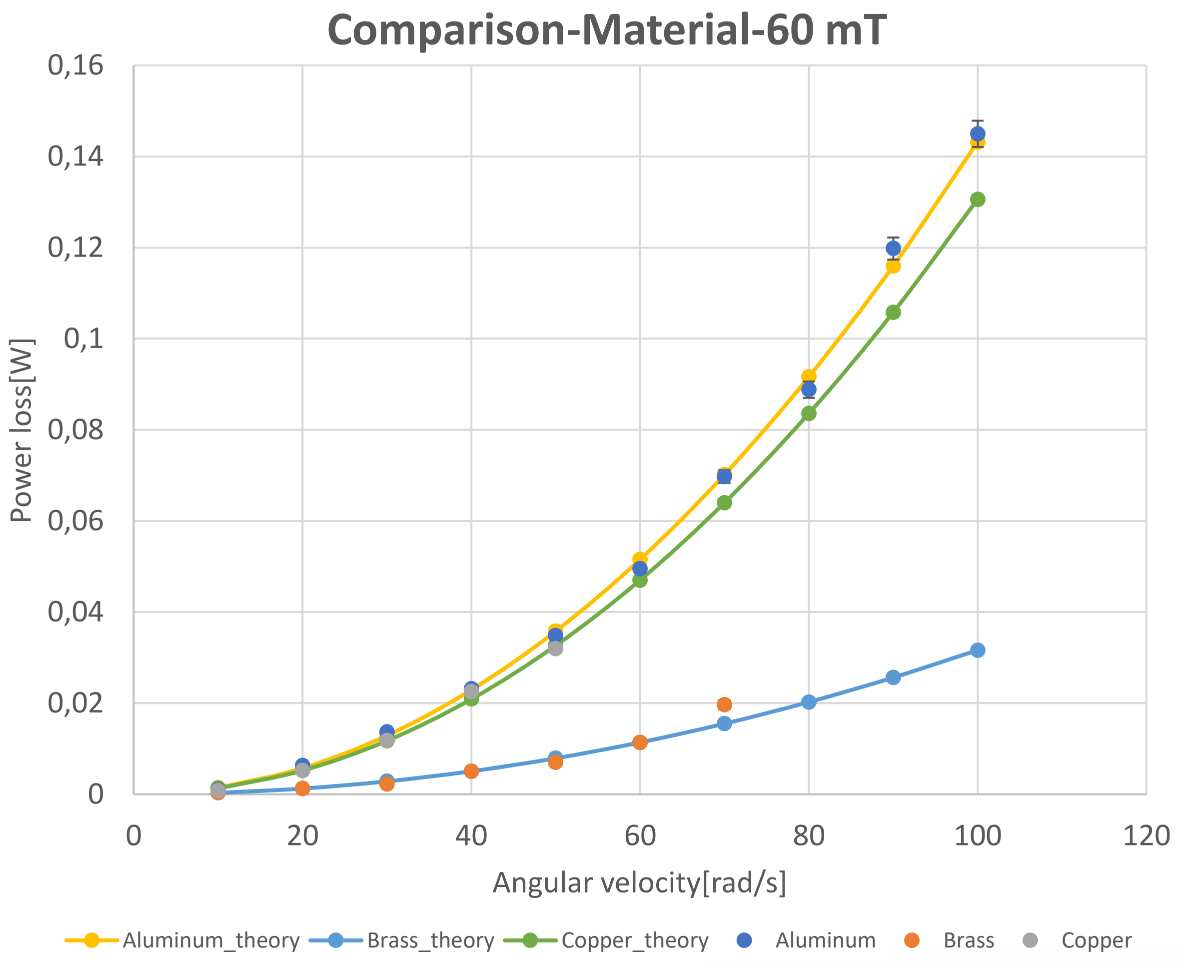
- Good correspondance
- Discrepancies
  - →error estimation

Different Magnetic Field Strengths



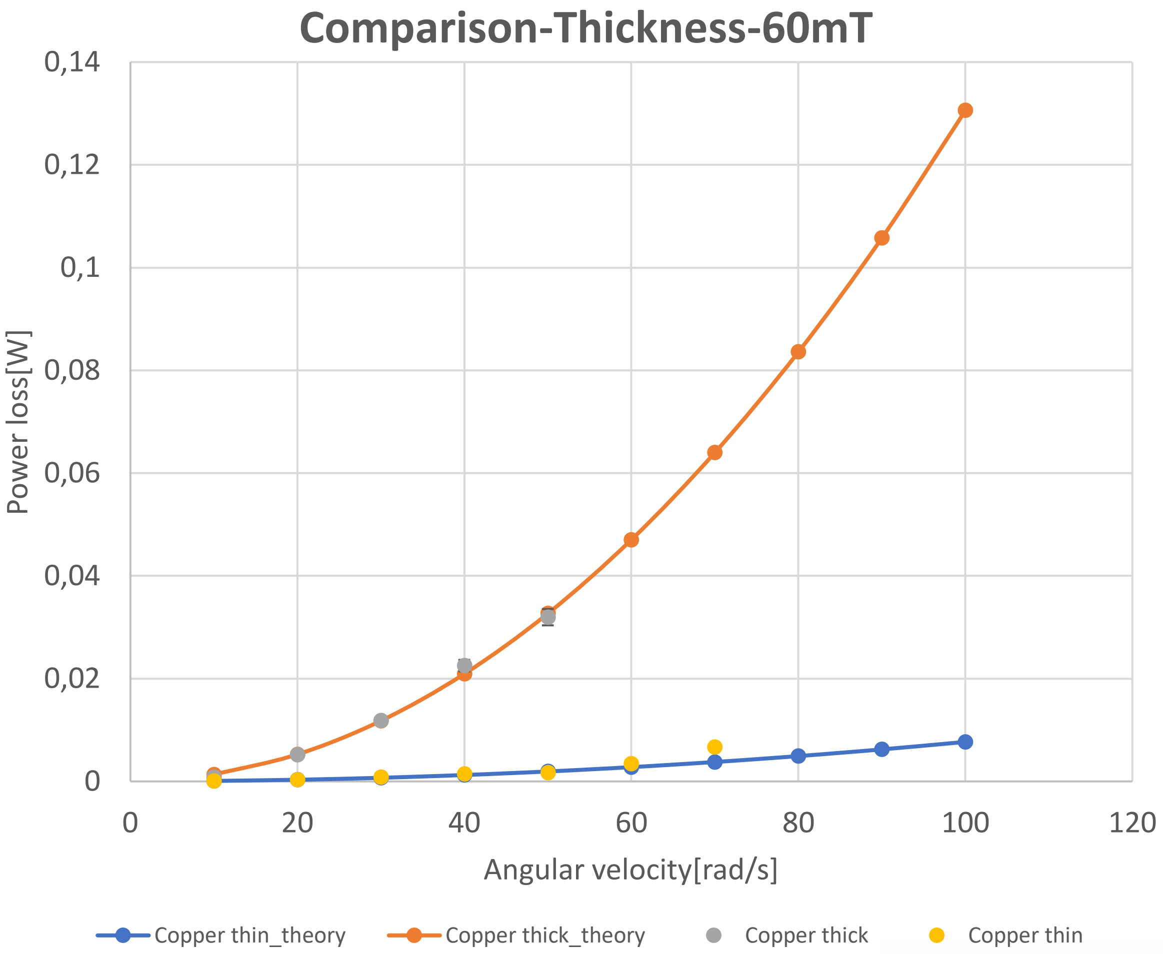


Different Materials



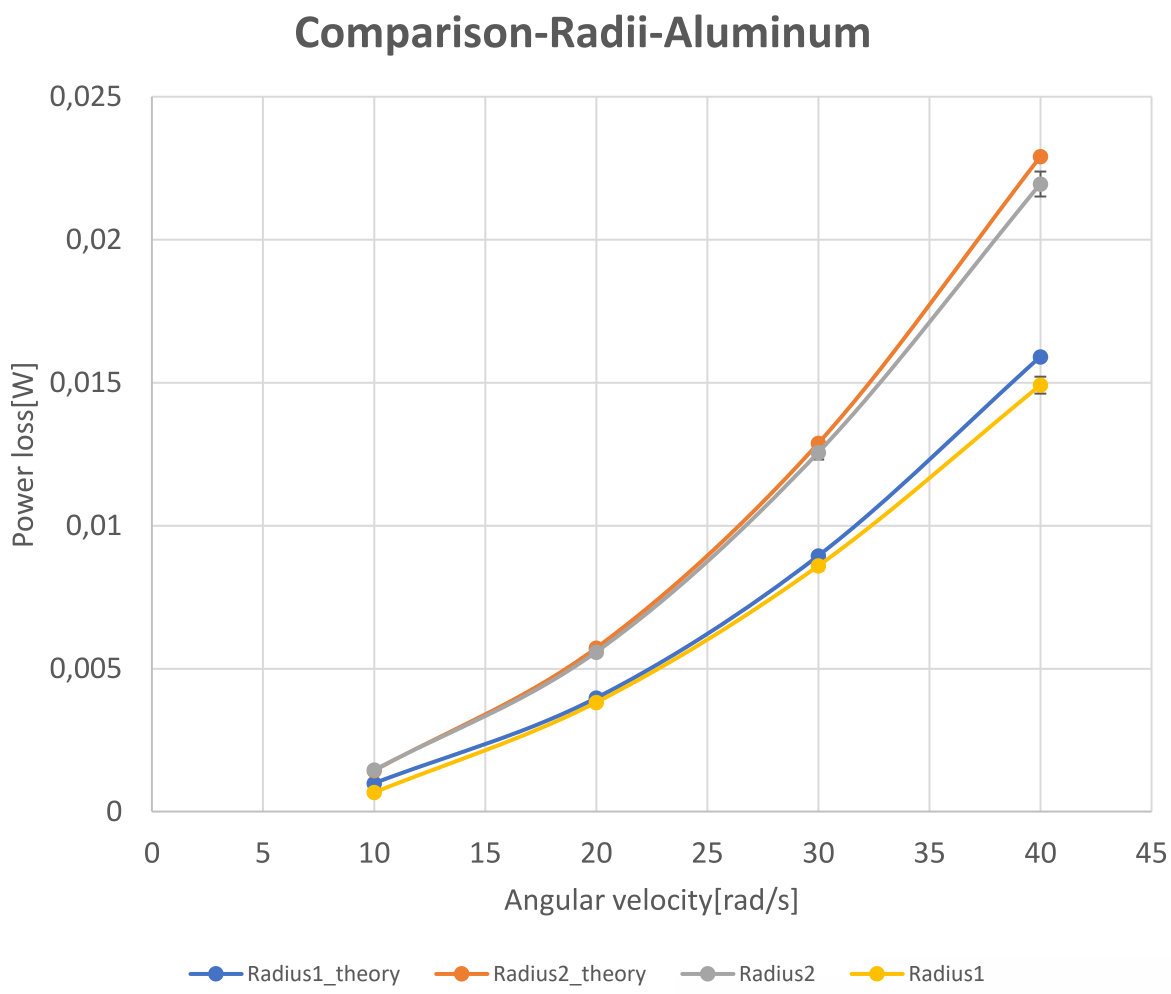
- Copper, Aluminum, Brass
- 60 mT
- Copper
  - Resistivity: $1.68*10^{-8}$
  - Thickness: 1mm
  - Radius: 4.9 cm
- Aluminum
  - Resistivity: $2.65*10^{-8}$
  - Thickness: 4 mm
  - Radius: 4.2 cm
- Brass
  - Resistivity: $0.9*10^{-7}$
  - Thickness: 3mm
  - Radius: 4.2 cm

Different Thicknesses



- Copper
- 60 mT
- Thick: 1mm
- Thinn: 35 μm

Different Radii



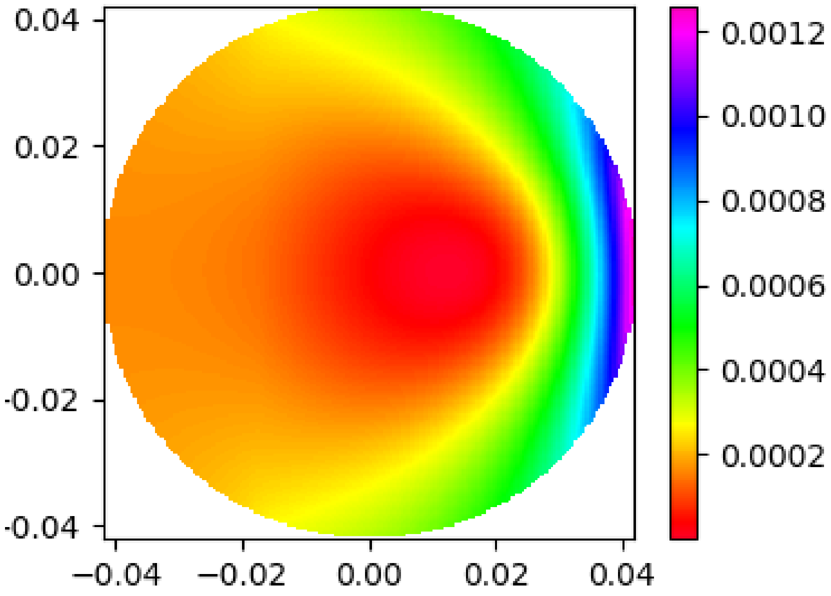
- Aluminum
- 50 mT
- Radius 1: 3.5 cm
- Radius 2: 1.5 cm

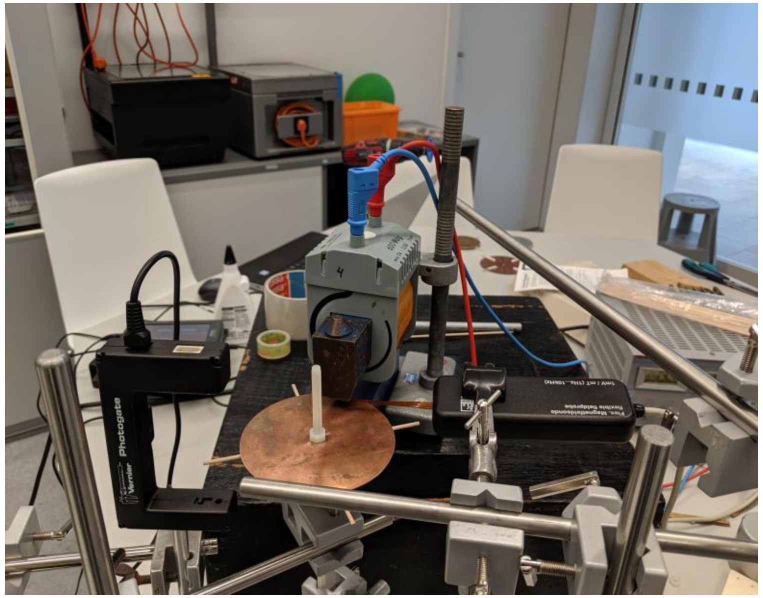
Error estimation

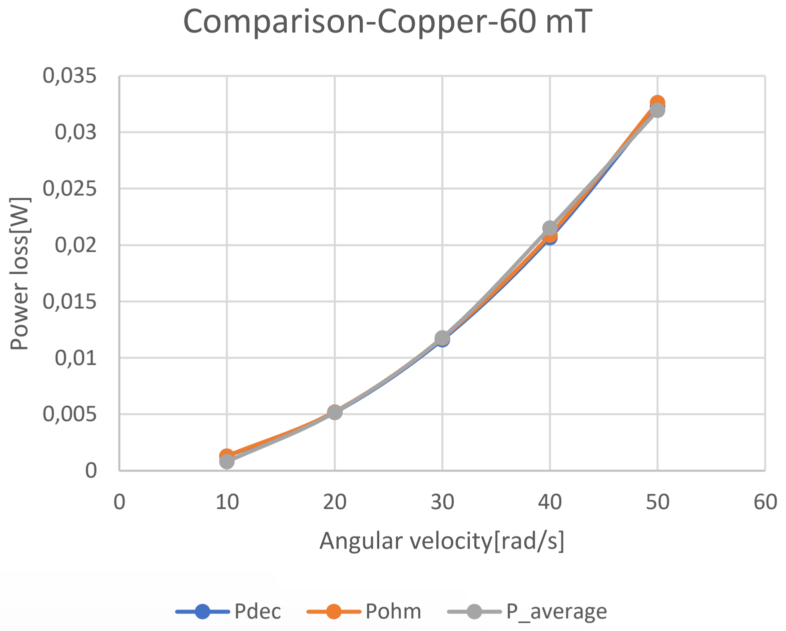
- Moment of inertia of wooden sticks
  - Negligible
- Drag of rotational pivot & air
  - Approximated with trend line
  - Negligible at high B
- B profile
  - Finer grid for more accuracy
- Heat created
  - Negligible
- Higher resolution with more sticks
  - 4 suffice
- Light wavering of plate in vertical direction

Conclusion

- Theory
  - Simulation
  - Energy loss
  - Force
- Experiment
  - Paramter variation
  - Setup
- Comparison
  - Theory proved to be right
  - Relationships identified for:
    - ω, B, d, ρ
    - Numerical solutions for others







Thank you for your attention!

Appendix

Teslameter-Comparison-Aluminum

- Find out B with P at certain parameters
- Unable to do that
    - No uniform magnetic field
    - Need magnetic B profile
- Magnetic field strength calculation:
    - Invert the formula
    - Apply on power loss of experiment
- Example:
- Aluminum, 100 rad/s
- Magnetic field strength measured: 50 mT
- Magnetic field strength from gyroscope teslameter: 51,4 mT

$$\boxed{P_{dec}(B_{max}, \omega, \rho, d)=P_{dec}(B^0_{max}, \omega^0, \rho^0, d^0)\ \frac{(B_{max}\omega)^2\rho^0 d}{(B^0_{max}\omega^0)^2\rho d^0}}$$

Case of many magnets

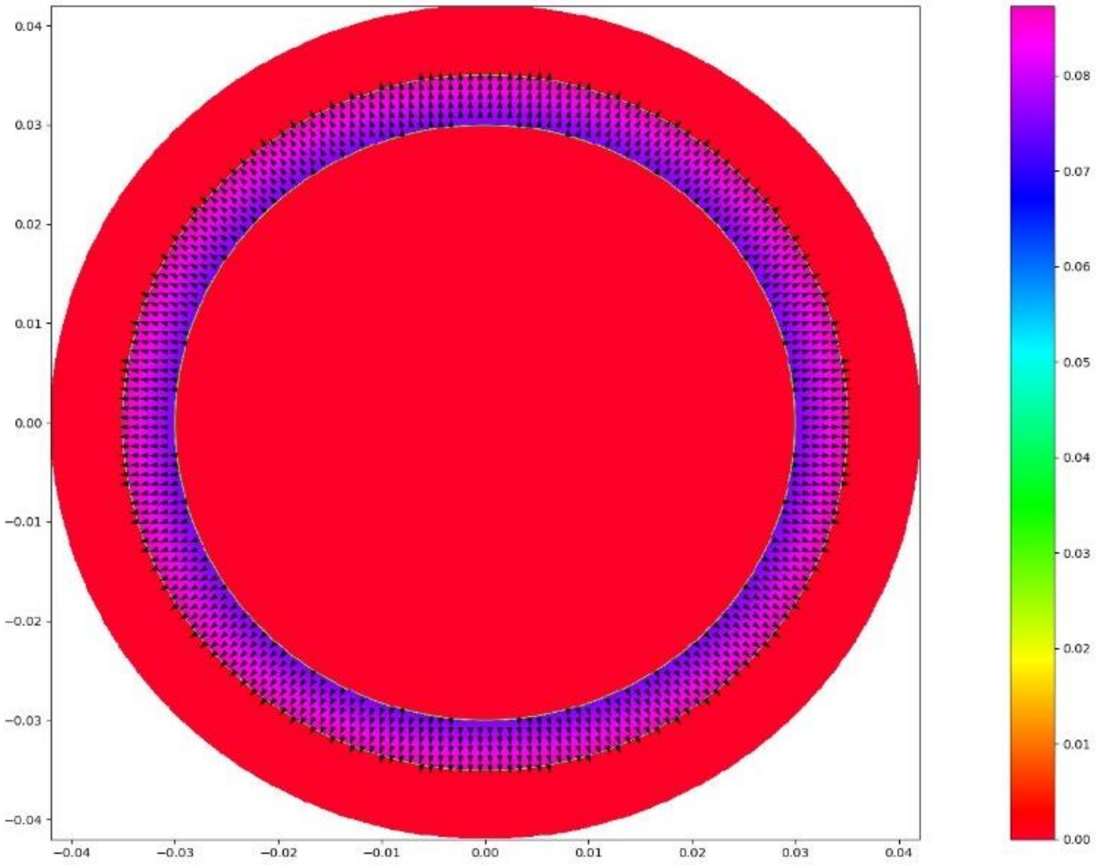
Magnets on different places on plate

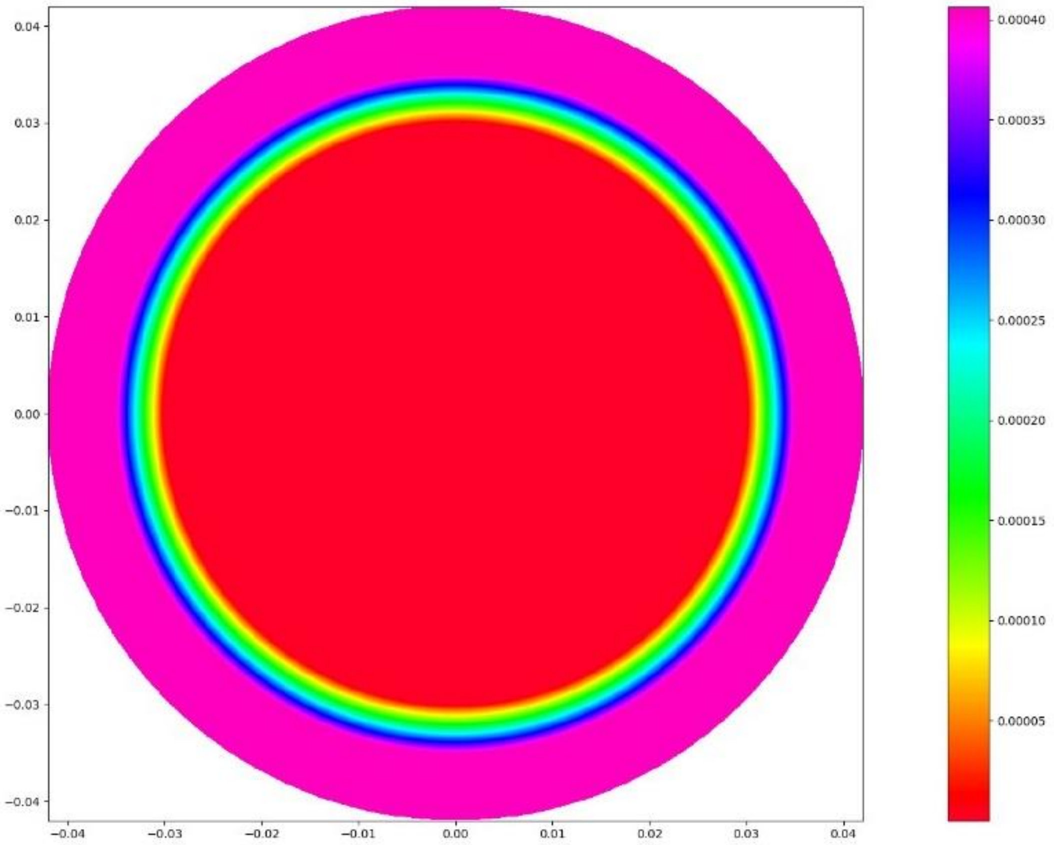
- Deceleration rate adds up

Magnets next to each other

- No effect

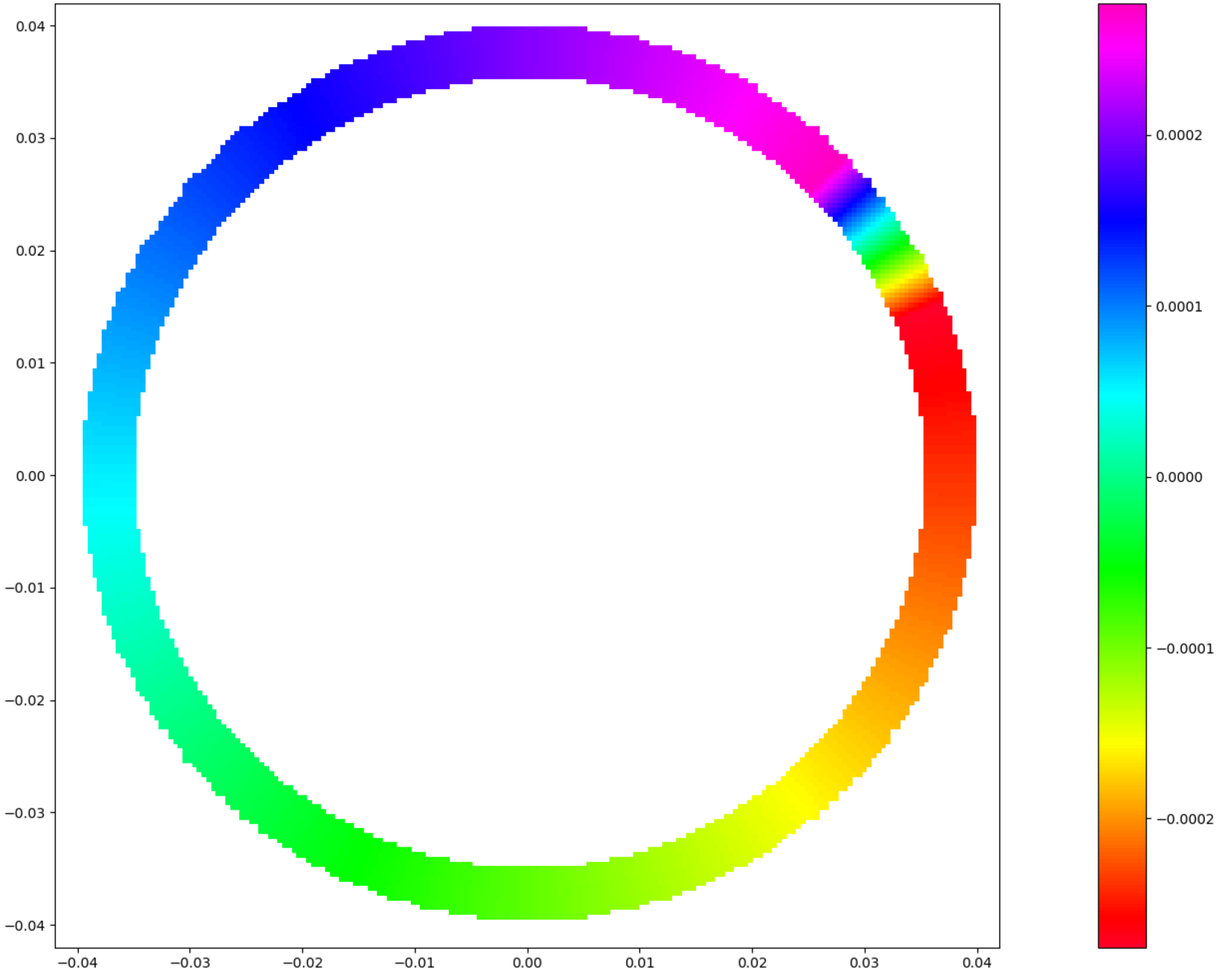
Special case-whole plate under magnet





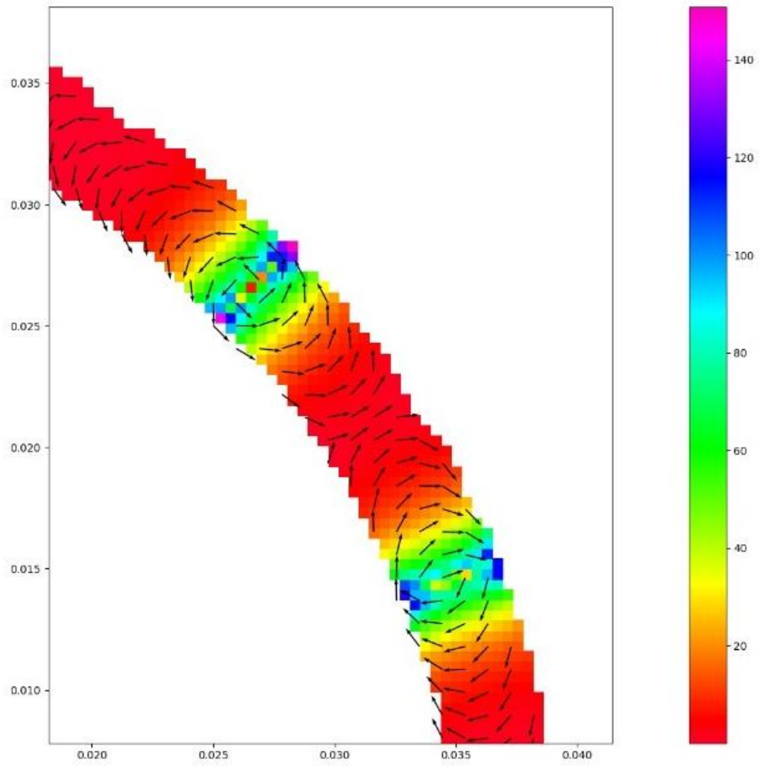
Special case-electric field exerted on ring structure

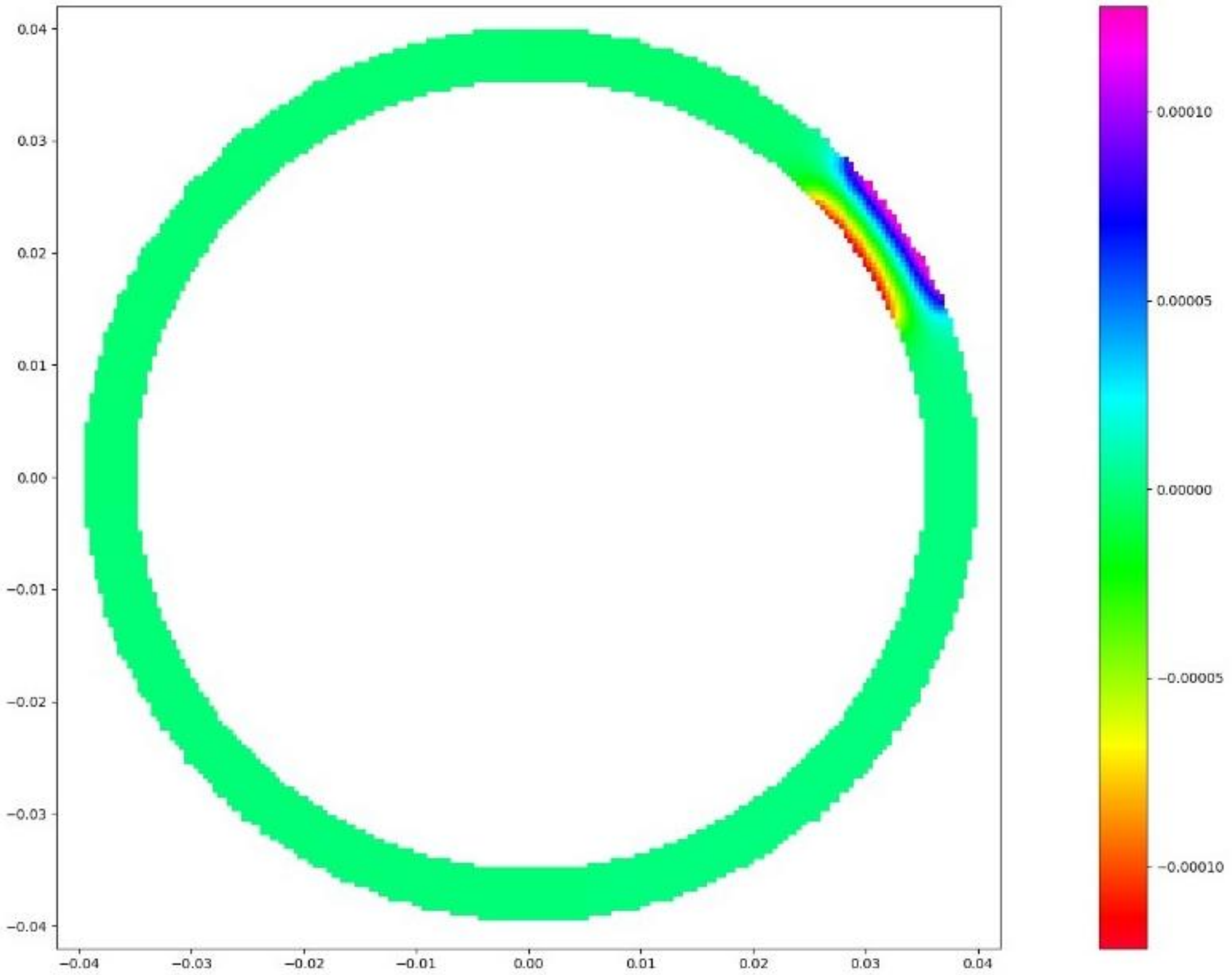
- Electric field only on small part
- Electrical potential in ring → electrical field opposing external external field
- Causing currents to flow
- Power loss → $P = U^2 \cdot R$



Special case-magnetic field exerted on ring structure

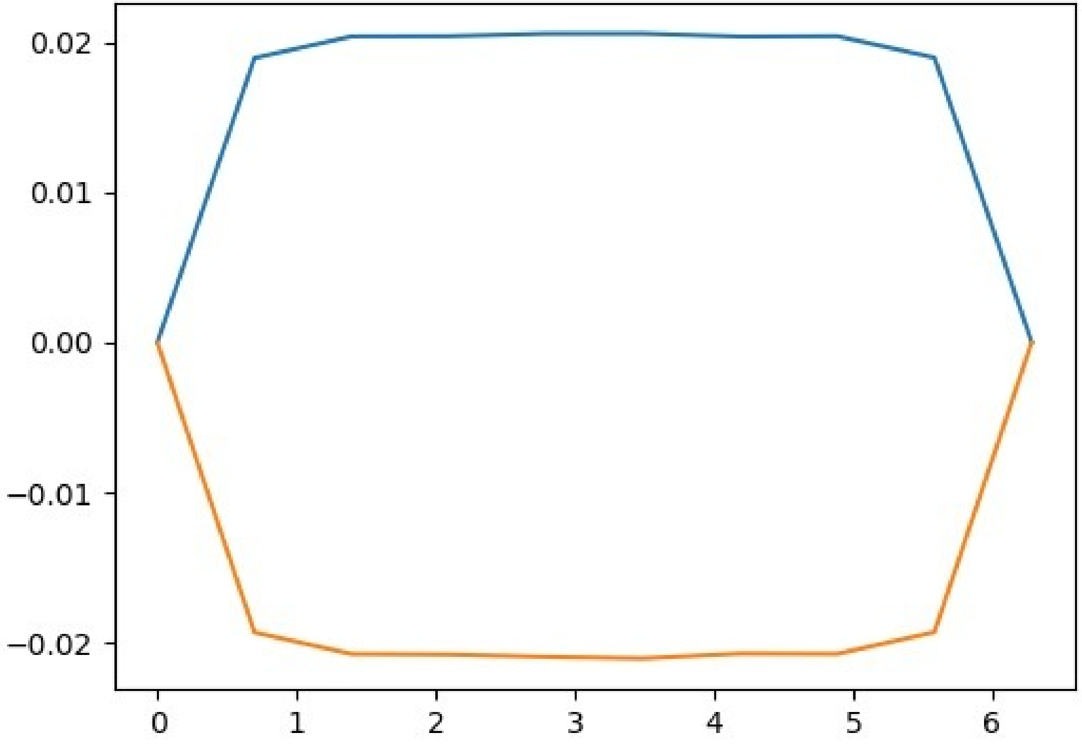
- Same as with plate



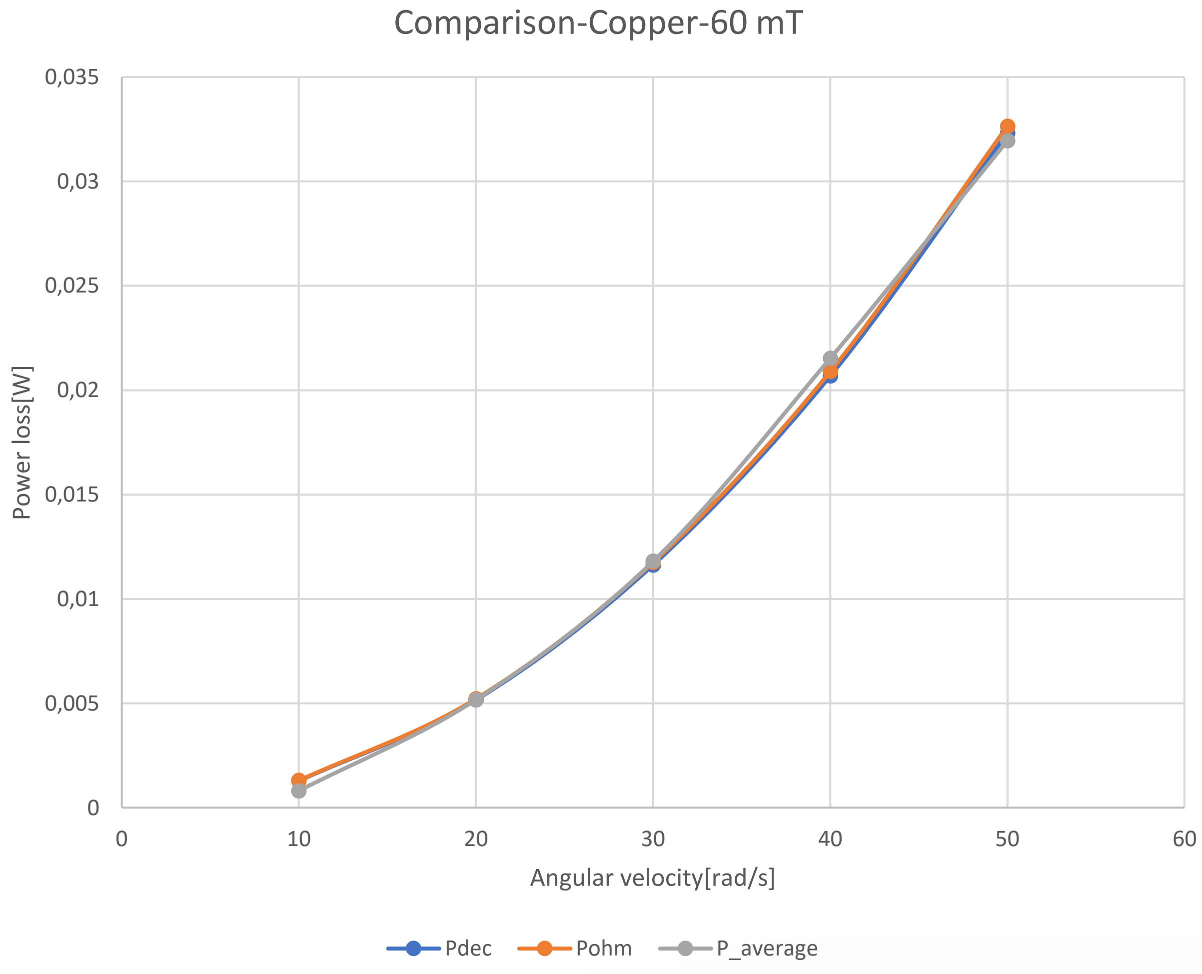


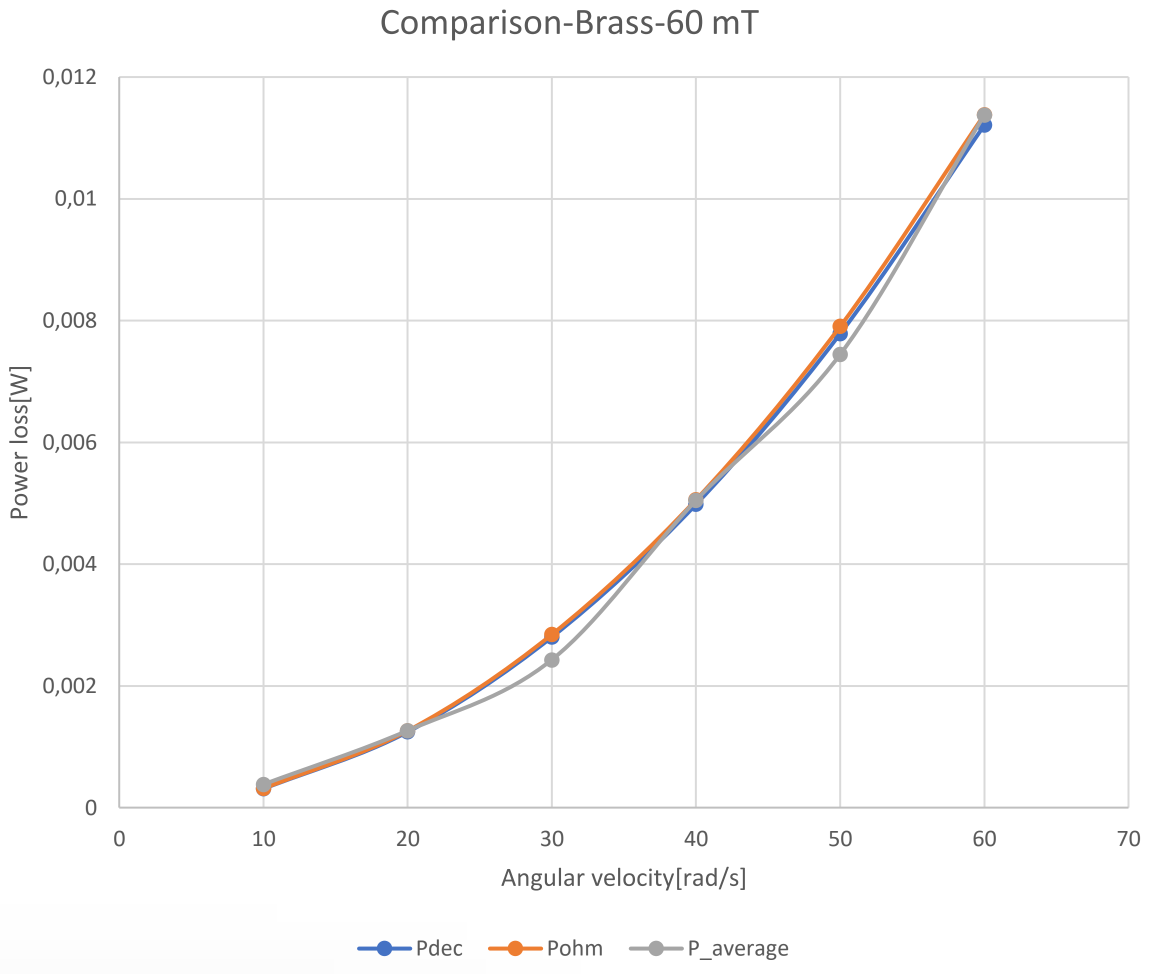
Special case-magnetic field with different angles

- High angles to full plate
  - No place for the charges to flow back
- Low angles to no magnet
  - Charges do not build up that fast
- Between extreme cases power loss is quite constant
  - →angle does not have effect



Comparison of 3 Power Losses-Copper, Brass





Angular velocity as a function of time

$$P = P_0 \cdot \frac{\omega^2}{\omega_0^2}$$

$$P = \alpha \cdot I_m \cdot \omega$$

$$\alpha = \frac{P_0 \cdot \frac{\omega^2}{\omega_0^2}}{I_m \cdot \omega} = \frac{P_0}{I_m \cdot \omega_0^2} \cdot \omega$$

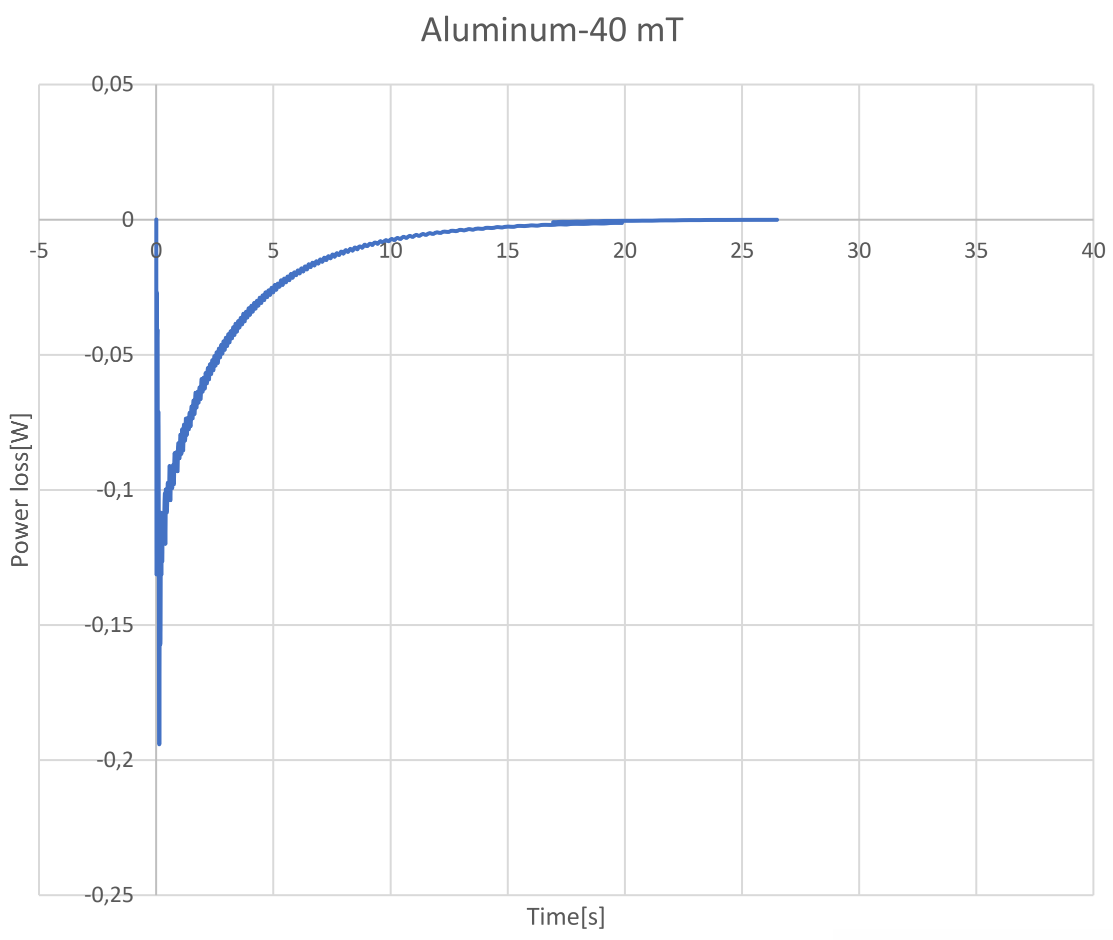
$$\omega(t) = \omega_0 \cdot (\frac{P_0}{I_m \cdot \omega_0^2})^t$$

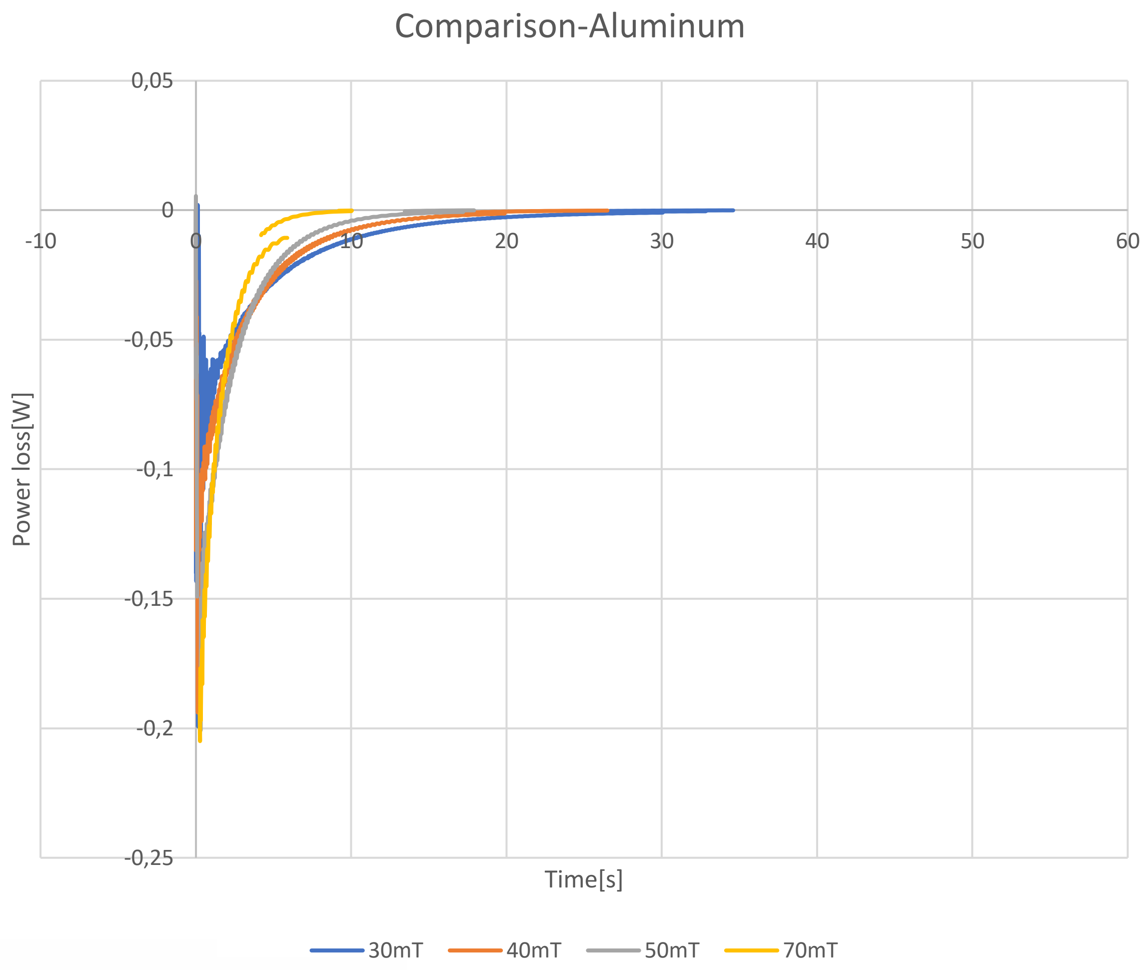
ω over time is exponential → power over time is exponential

$$\mathrm{P} = P_0 \cdot \frac{\omega^2}{\omega_0^2}$$

$$\boxed{P = P_0 \cdot (\frac{P_0}{I_m \cdot \omega_0^2})^{2t}}$$

Power loss over time





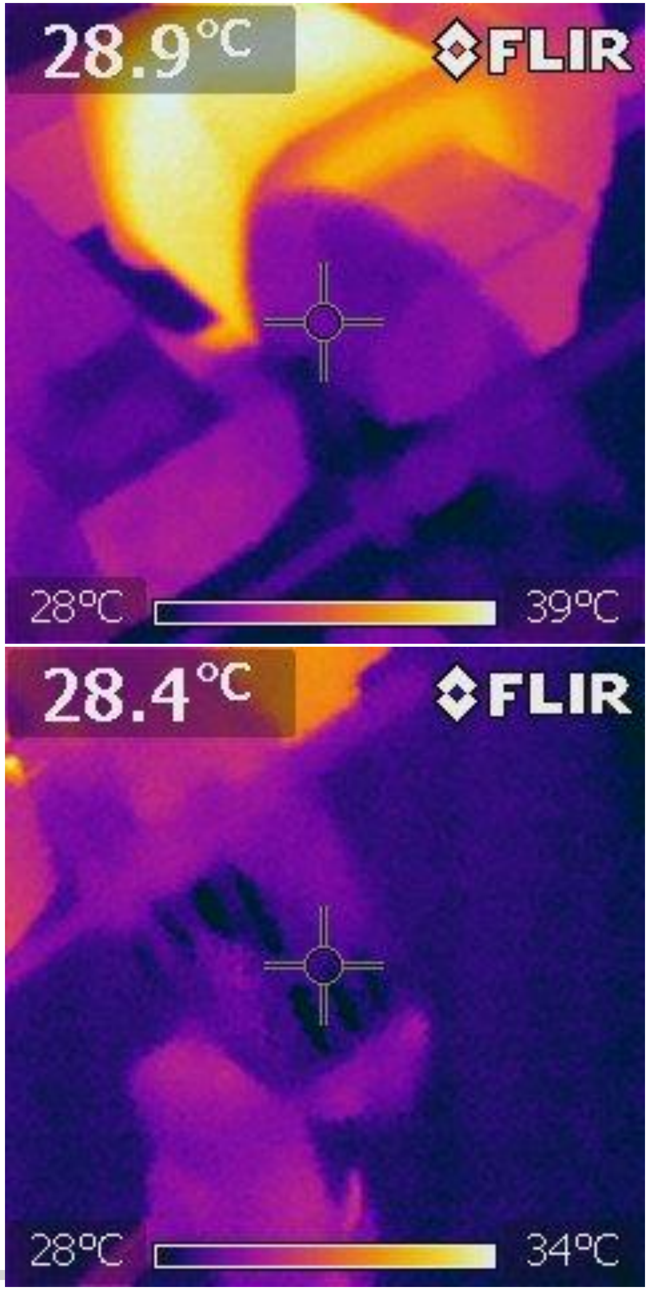
Joule heating-Copper

Temperature increase relatively small

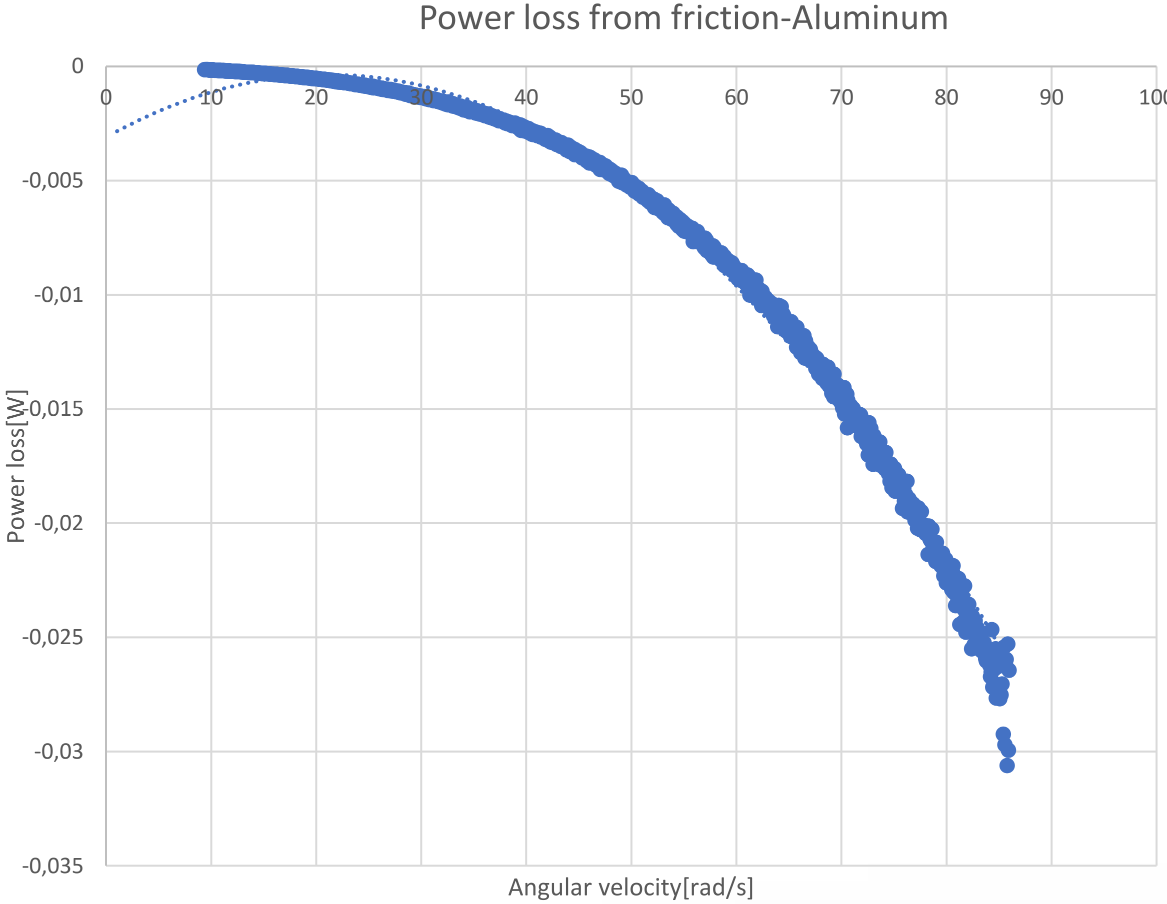
- Small effect
- About 0.5°C-1°C

Only effects resistance

- Depends on material properties
- Not considerable

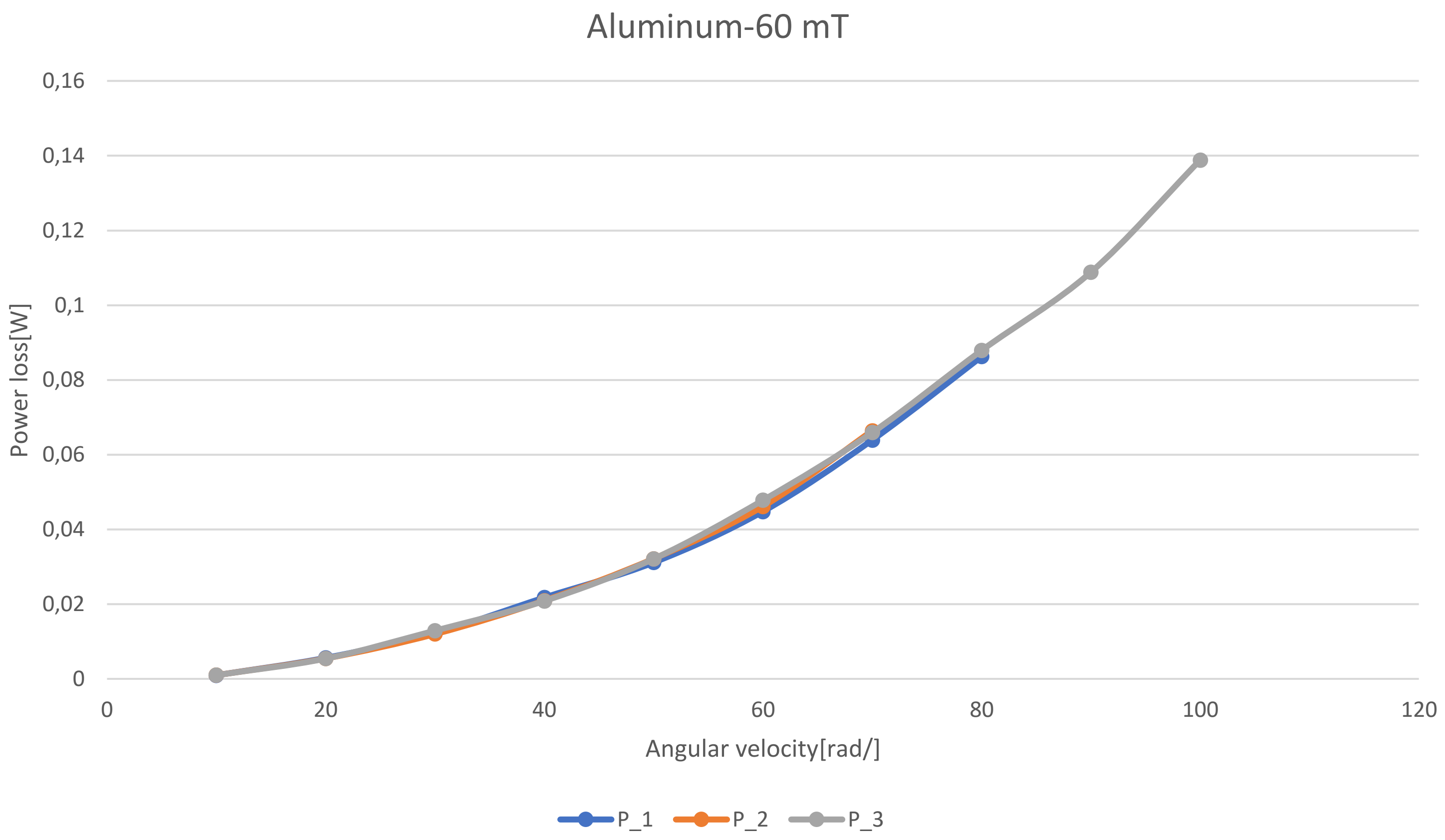


Trend line for friction term P_f



- Value taken for different angular velocities
- Assumed with a square function
- Subtracted from total power loss
- Power loss from magnetic field obtained

3 runs



- Average deviation of 2%

OUTLINE

Introduction

- Problem statement
- Basic concepts

Theory

- Theoretical power loss
  - Derivation
  - Resistance & current
- Relevant parameters

Experiment

- Experimental Setup
- Parameter variation

Comparison

- Result of experiment
- Magnetic field
- Frequency
- Thickness
- Radius

Conclusion

- Teslameter-comparison
- Error estimation