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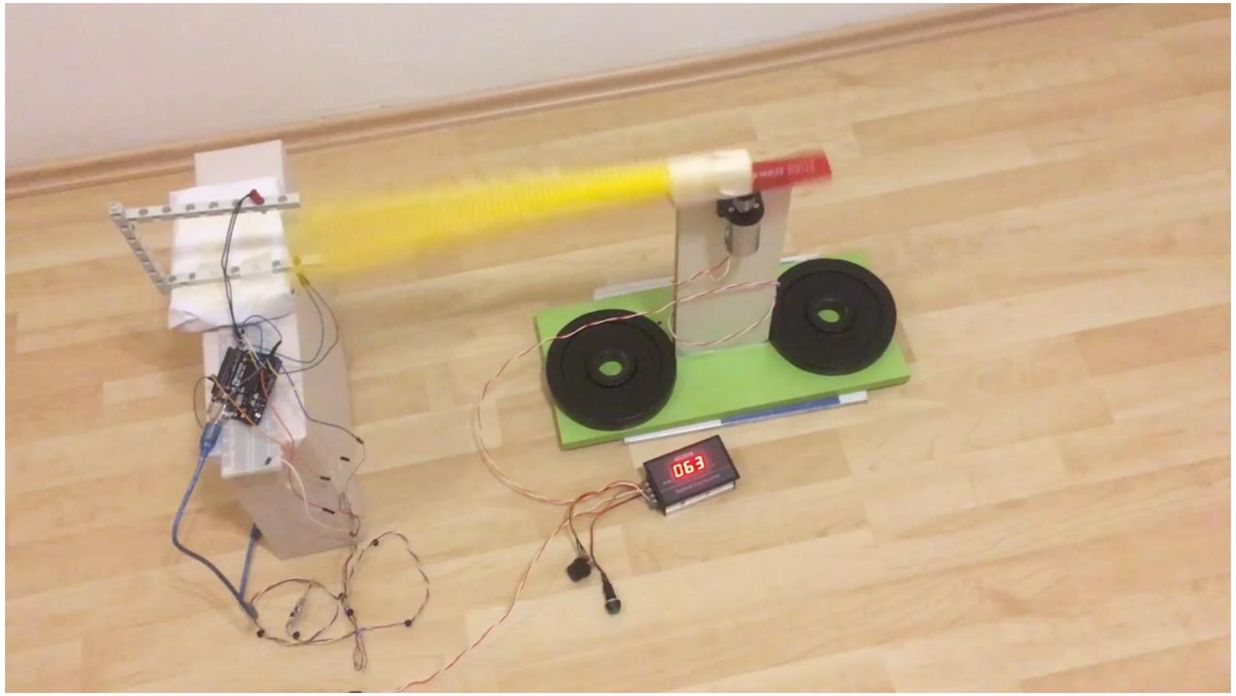
Rohan Walia presents Problem No. 3
Swinging Soundtube

33rd IYPT 2020 | Online

Problem statement

- A Sound Tube is a toy, consisting of a corrugated plastic tube, that you can spin around to produce sounds. Study the **characteristics of the sounds** produced by such toys, and how they are affected by the **relevant parameters**.



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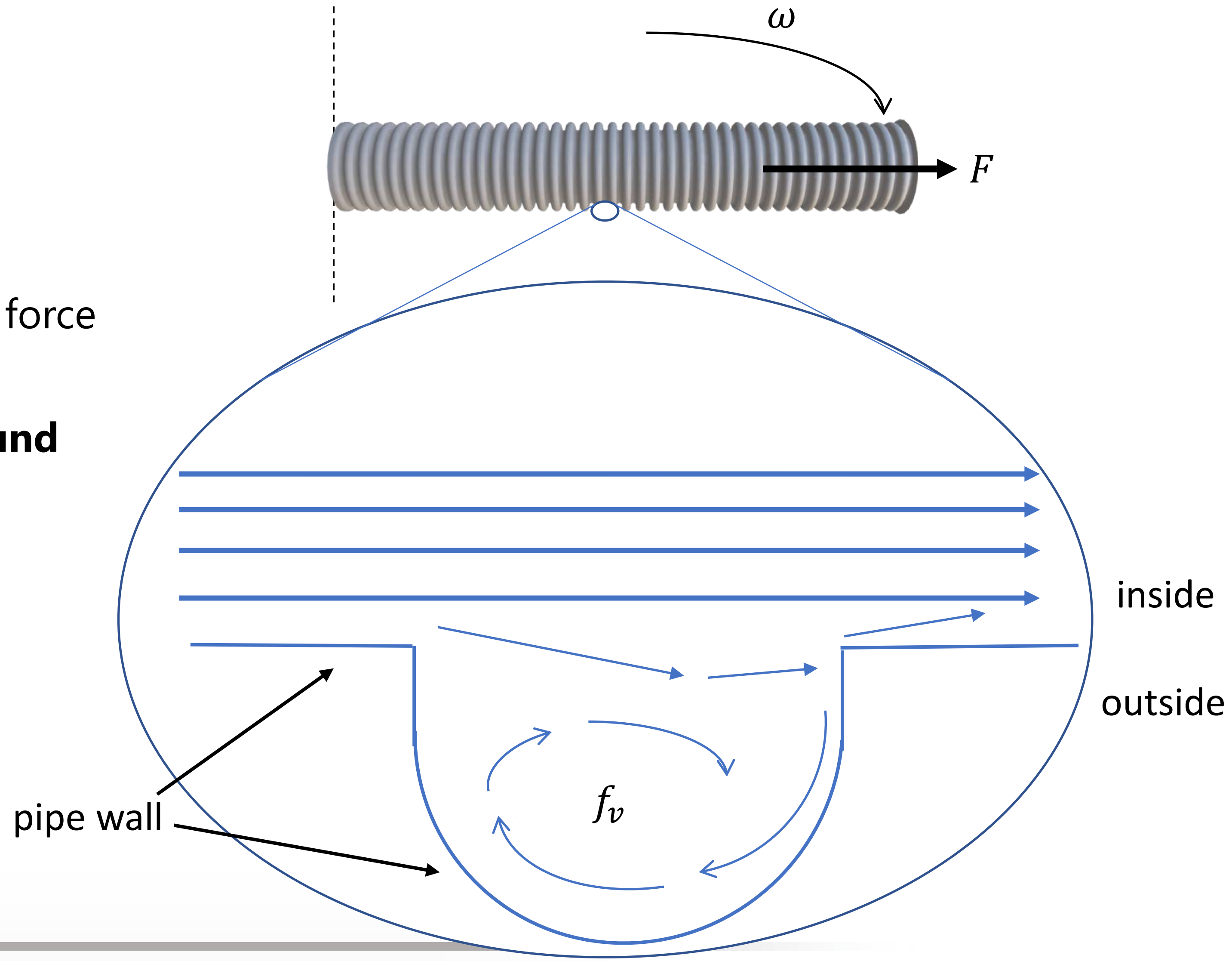
Basic concepts

- Rotation → inertial force → flow towards rotating end of tube
- Flow passes over corrugation
  - vortices in corrugation → periodic force
  - periodic force acts on pipe wall
- **Resonance → condition for audible sound**
  - $\boldsymbol{f_v = f_p}$



f_v...vortex frequency

f_p...pipe resonance frequency

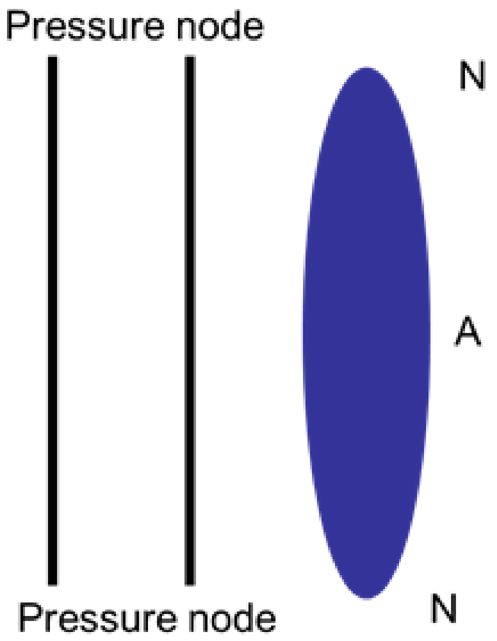
Theory Part Ⅰ：Determining f_v and f_p

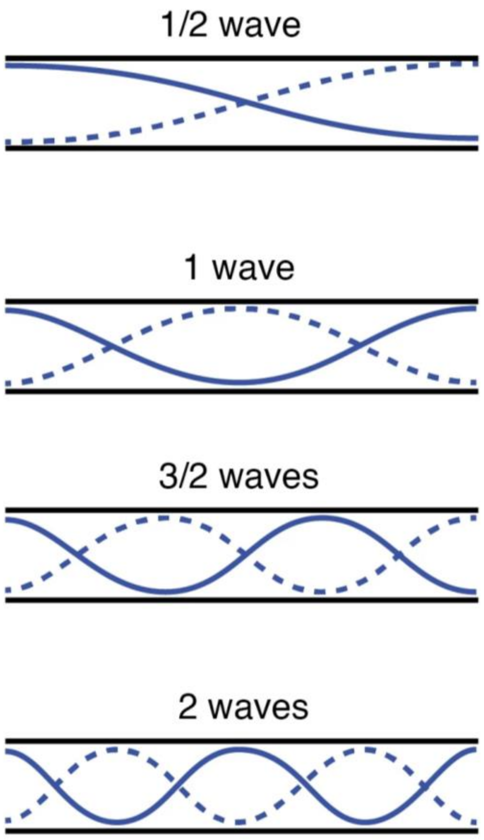
Resonance frequency of pipe

- Standing waves in open pipe:

$$f_p = \frac{n \cdot c}{2 \cdot L_A}$$

$$L_A = L_G + \underbrace{1.22 \cdot R}_{\text{End correction}}$$





n...mode of frequency

c...speed of sound

L_A...acoustic length of pipe

L_G...geometric length of pipe

R...radius of tube

Modifications for resonance frequency

- Corrugations lower f_p
- Additional factor α:

$$\alpha = \frac{1}{\left(1 + \frac{V}{A \cdot l_p}\right)^{0.5}} - M^2$$

- **<u>Resonance formula:</u>**

$$\boxed{f_p = \frac{n \cdot c}{2 \cdot (L_G + 1.22 \cdot R)} \cdot \alpha}$$

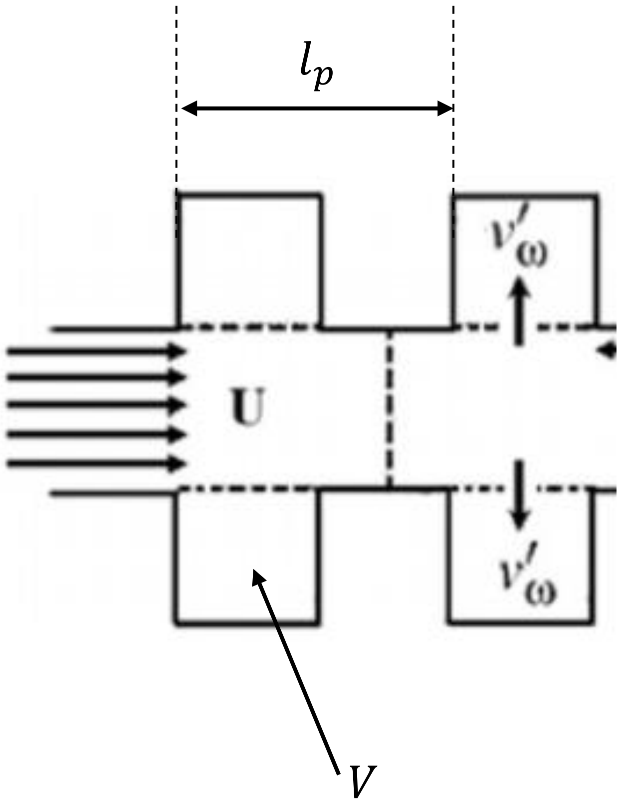
V...volume of a corrugation

l_p...pitch of corrugation

A...cross-sectional area

R...radius of tube

M...Mach number



Rajavel, B. & Prasad, M.G.. (2013). Acoustics of Corrugated Pipes: A Review. Applied Mechanics Reviews. 65. 050000. 10.1115/1.4025302.

Vortex frequency

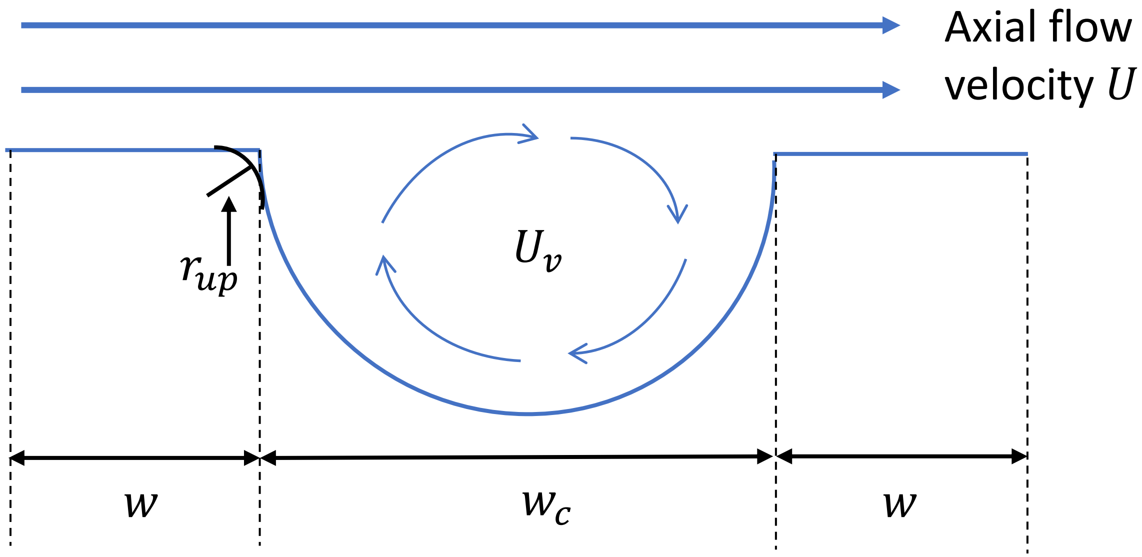
- With Strouhal number St:

$$\boxed{f_v = \frac{St \cdot U}{L_c}}$$

- Empirical L_c:

$$L_c = \frac{w^2}{w_c + r_{up}} \; with \; r_{up} \ll w_c$$

$$\boldsymbol{St} \cdot \boldsymbol{U} = \boldsymbol{U_v} \text{ and } U_v = f_v \cdot L_c$$



- Strouhal number obtained experimentally with hot wire anemometer

L_c...characteristic length/ U_v...vortex velocity
w_c...width of corrugation
w...width between corrugation

King et al. (2018). An improved characteristics length in strouhal number for internal flow induced acoustics in corrugated pipe.

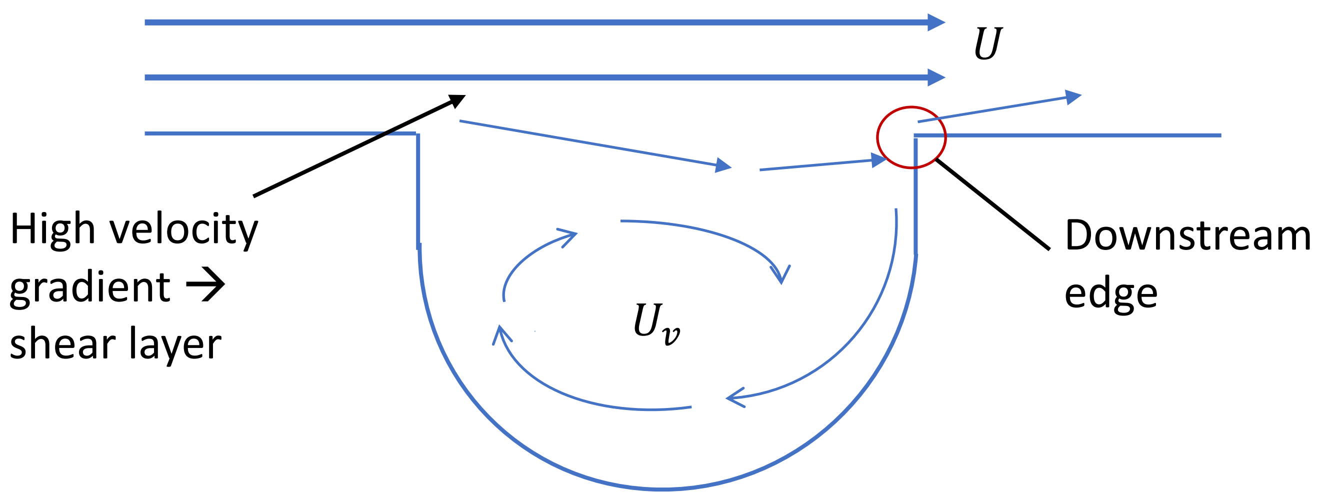
Determine U_{min} for audible sound

- Turbulence required for audible sound:

$$Re(U_v) > 2300$$

$$U_{min} = 2300 \cdot \frac{\mu}{\rho \cdot 2 \cdot R \cdot St}$$

$$f_{v\_min} = \frac{St \cdot U_{min} \cdot w_c}{w^2}$$



μ...dynamic viscosity of air
ρ...density of air
R...radius of tube
U_v...vortex velocity
w_c...width of corrugation
w...width between corrugation

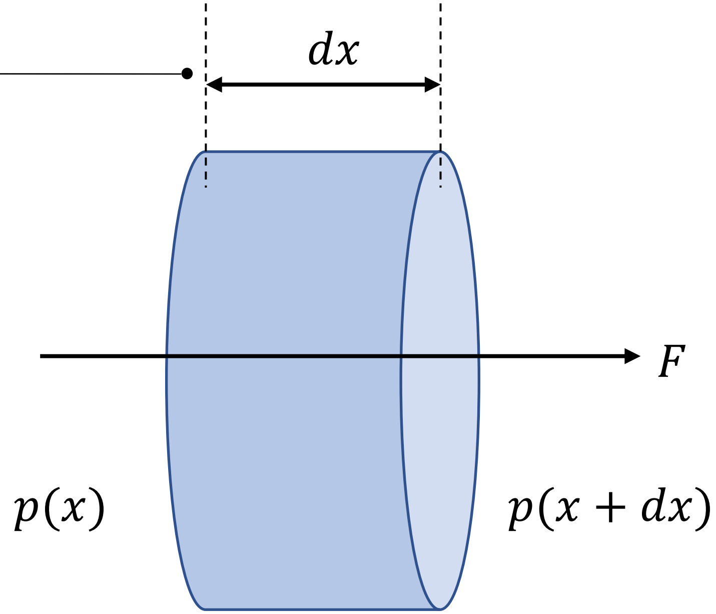
Relevant parameters

- Parameter affecting f_p
  - Length of tube: $f_p \rightarrow 1/L_G$
  - Radius of tube: $f_p \rightarrow 1/R$
  - Width between corrugations: $f_p \rightarrow \sqrt{w}$
- Paramters changing f_v
  - Width of corrugation: $f_v \rightarrow w_c$
  - Width between corrugations: $f_v \rightarrow 1/w^2$
  - Axial flow velocity and angular velocity
    →$U\ and\ \omega$

Relations towards vortex frequency less obvious but integrated in numerical model

Theory Part II :
Calculating U along a smooth pipe

U along pipe element dx



- Centrifugal force:

$$F = m \cdot \omega^2 \cdot x$$

- Resultung dp integrated over dx :

$$\int_{p(x)}^{p(x+dx)} dp = \omega^2 \cdot \frac{\rho}{2} \cdot (2x \cdot dx + (dx)^2)$$

- Darcy-Weisbach equation for friction:

$$\Delta p = f_D \cdot \frac{\rho}{2} \cdot \frac{U^2}{2 \cdot R} \cdot dx$$

- Subtacting two pressures = pressure difference at inlet:

$$(\boldsymbol{I})\ \mathbf{U} = \boldsymbol{\omega} \cdot \sqrt{\frac{\mathbf{2x} \cdot \boldsymbol{dx} + (\boldsymbol{dx})^2}{\mathbf{1} + \mathbf{f_D} \cdot \frac{\boldsymbol{dx}}{\mathbf{2}\boldsymbol{R}}}}$$

$$where\ dx = L_G$$

ω...angular frequency
f_D...Darcy friction factor
ρ...density of air
R...radius of tube
w_c...width of corrugation
w...width between corrugation

Solving for U with f_D

- Expression for f_D dependent on Re:

$$f_D = \left(\frac{1}{0.838 \cdot W(0.629 \cdot Re)}\right)^2$$

- Re can be given in terms of U:

$$(II) \quad \mathbf{f_D} = \left(\frac{\mathbf{1}}{\mathbf{0.838 \cdot W\left(0.629 \cdot \frac{U \cdot \rho \cdot 2R}{\mu}\right)}}\right)^{\mathbf{2}}$$

- Equation system (I) and (II) solved numerically with python for the unknown variables U and f_D

$$(I)\ \boxed{\mathbf{U}} = \boldsymbol{\omega} \cdot \sqrt{\frac{\mathbf{2x} \cdot \boldsymbol{dx} + (\boldsymbol{dx})^2}{\mathbf{1} + \boxed{\mathbf{f_D}} \cdot \frac{\boldsymbol{dx}}{\boldsymbol{2R}}}}$$

$$(II) \quad \boxed{\mathbf{f_D}} = \left(\frac{\mathbf{1}}{\mathbf{0.838 \cdot W\left(0.629 \cdot \frac{\boxed{U} \cdot \rho \cdot 2R}{\mu}\right)}}\right)^{\mathbf{2}}$$

Theory Part Ⅲ：Flow over corrugation

Compressible Navier-Stokes equations

- Continuity equation:

$$\boxed{\frac{\partial \boldsymbol{\rho}}{\partial \boldsymbol{t}} + \boldsymbol{\nabla} \cdot (\boldsymbol{\rho}\vec{\mathbf{u}}) = \mathbf{0}}$$

- Momentum equation:

$$\boxed{\boldsymbol{\rho}\left((\vec{\mathbf{u}} \cdot \boldsymbol{\nabla})\vec{\mathbf{u}} + \frac{\partial \vec{\mathbf{u}}}{\partial \mathbf{t}}\right)} = \boxed{-\boldsymbol{\nabla}\mathbf{p}} + \boxed{\boldsymbol{\mu}\boldsymbol{\nabla}^2\vec{\mathbf{u}}} + \boxed{(\boldsymbol{\mu} + \boldsymbol{\xi}) \cdot \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \vec{\mathbf{u}})} + \boxed{\vec{\boldsymbol{f}}}$$

- **Basis of Reynolds-averaged Navier-Stokes equation**

$$\dot m_{in} \rightarrow \boxed{\frac{\partial\rho}{\partial t}} \rightarrow \dot m_{out}$$

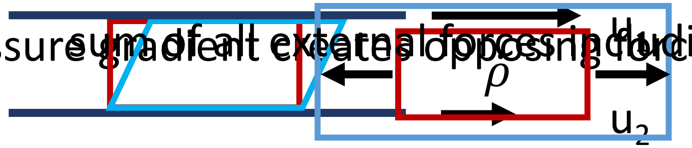
$$\frac{\dot m_{in} - \dot m_{out}}{dA} = \rho\vec{u} \rightarrow \frac{\partial\rho}{\partial t}$$

Newtons second law applied on a fluid:

$$\boxed{m \cdot \vec{a}} = \boxed{\sum \vec{F}}$$

$$\sum \vec{F} = \boxed{F_{pressure}} + \boxed{F_{viscosity}} + \boxed{F_{external}}$$

$$\rho \cdot \dot{\vec{u}} \quad \text{where} \quad \dot{\vec{u}} = (\vec{u} \cdot \nabla)\vec{u} + \frac{\delta \vec{u}}{\delta t}$$



Pressure gradient creates opposing force

sum of all external forces including gravity

$u_1 > u_2$ → velocity gradient causes shear stress

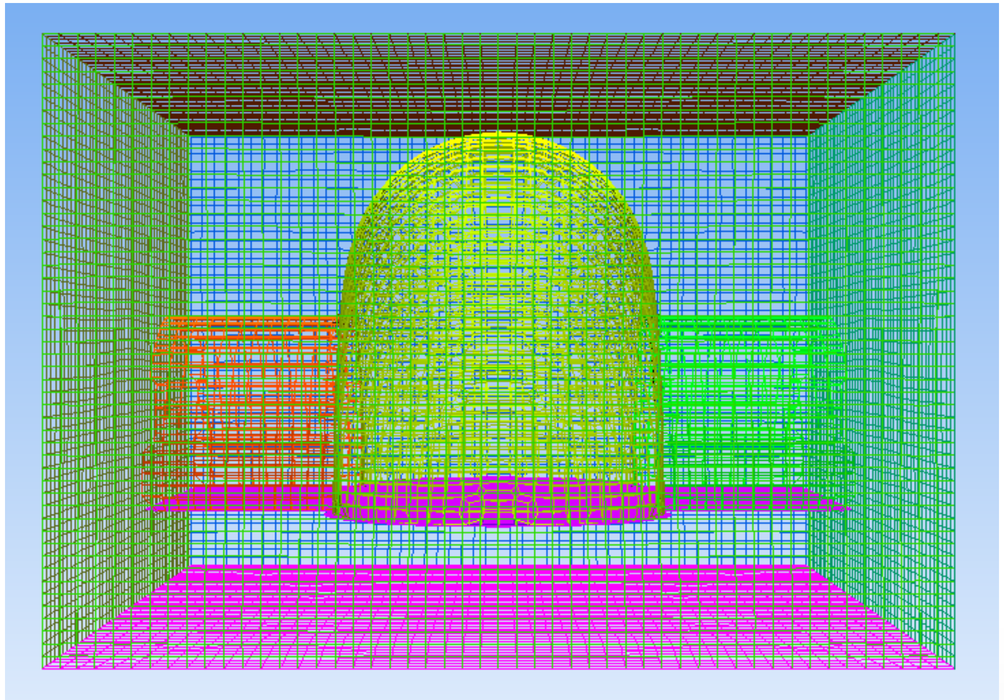
change in volume causes density change

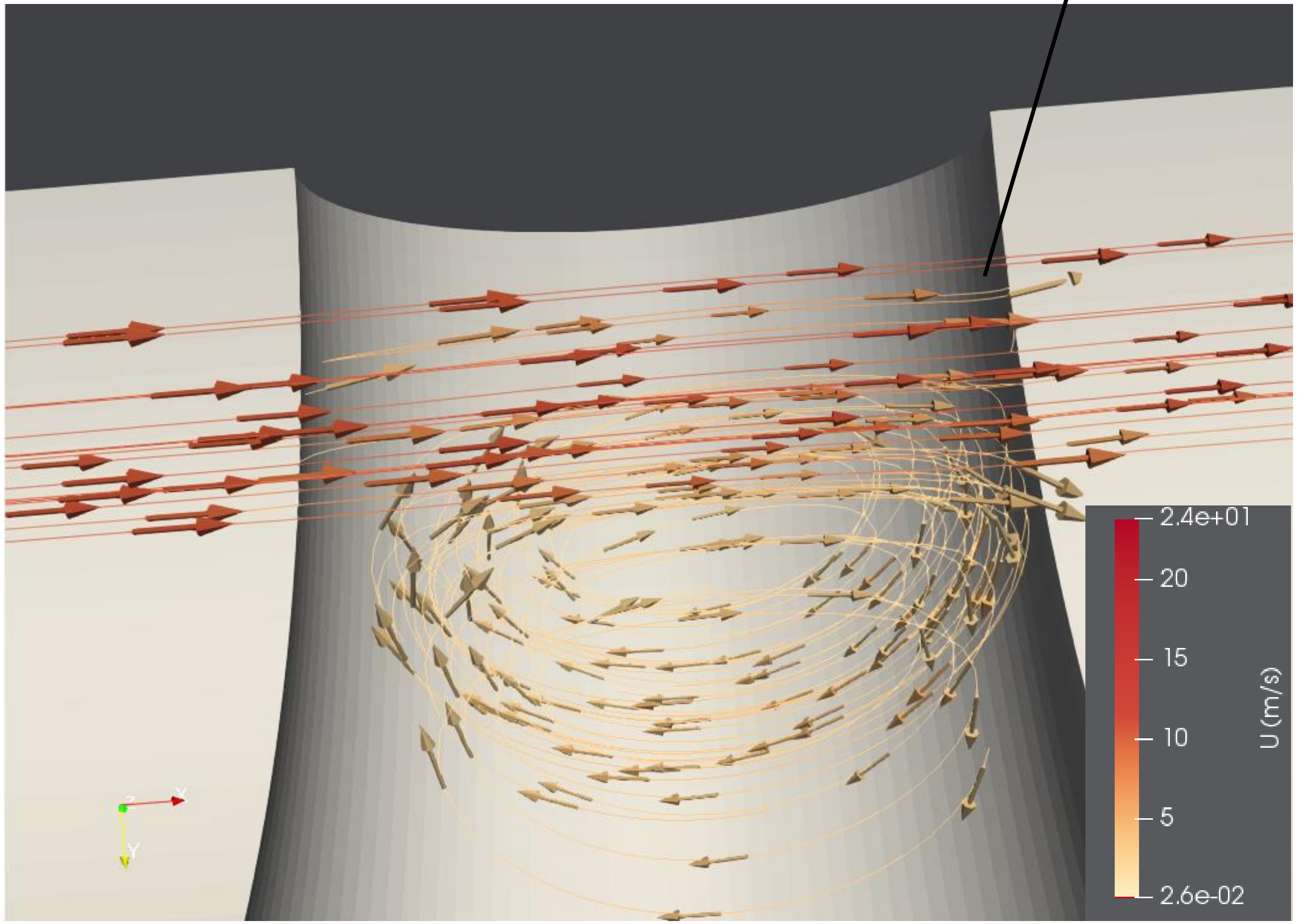
Vortex model with CFD

Downstream edge

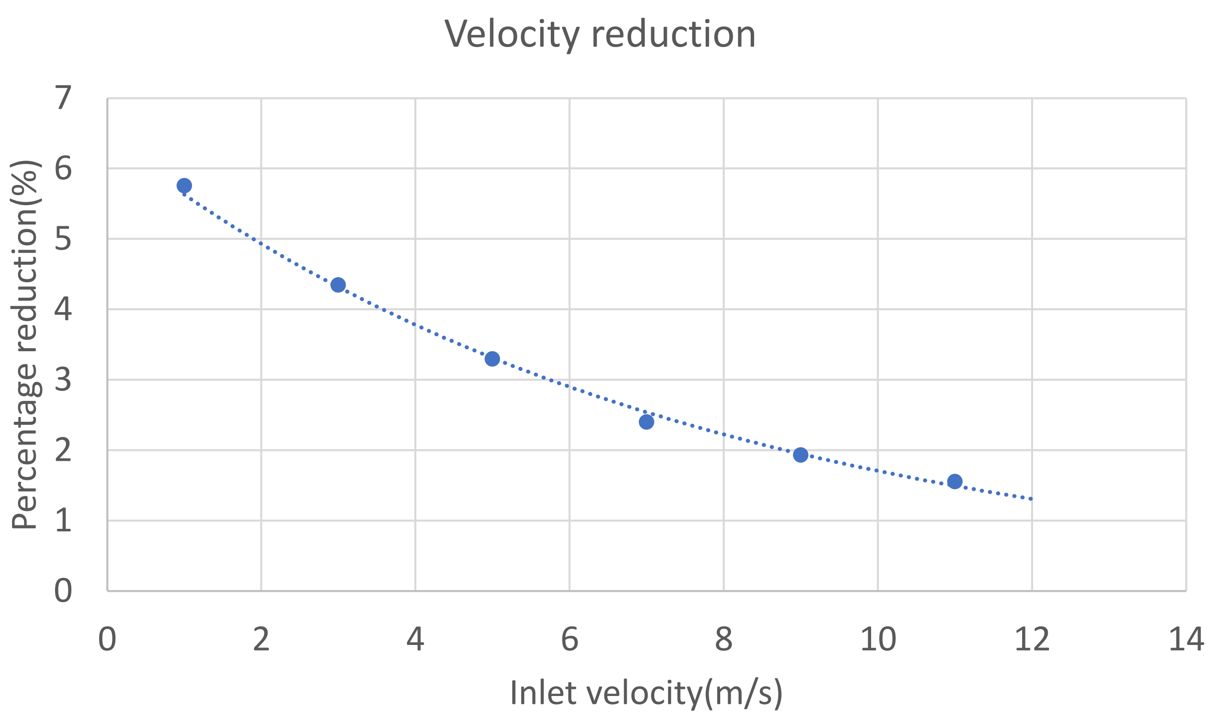
Simulation-setup:

- k-ω-SST model for RANS
- rhoPimple algorithm used





Inflow-outflow relation

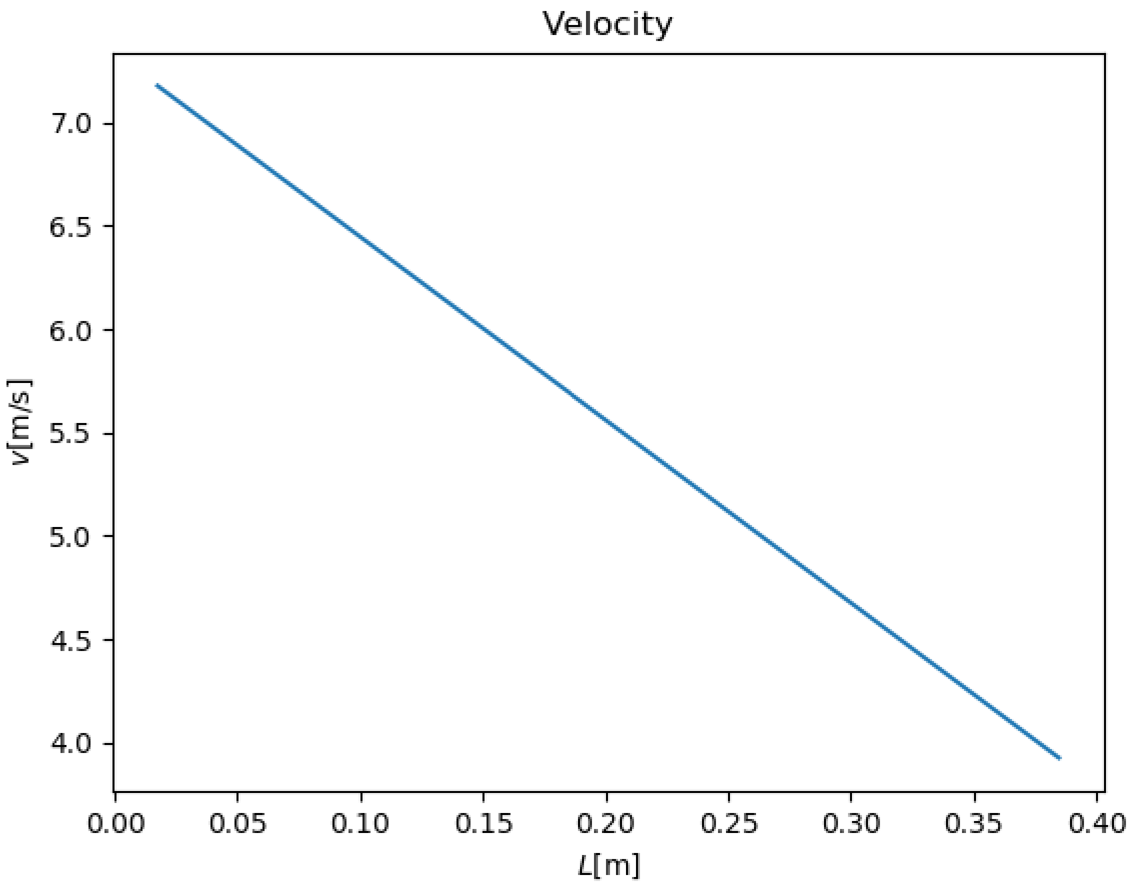


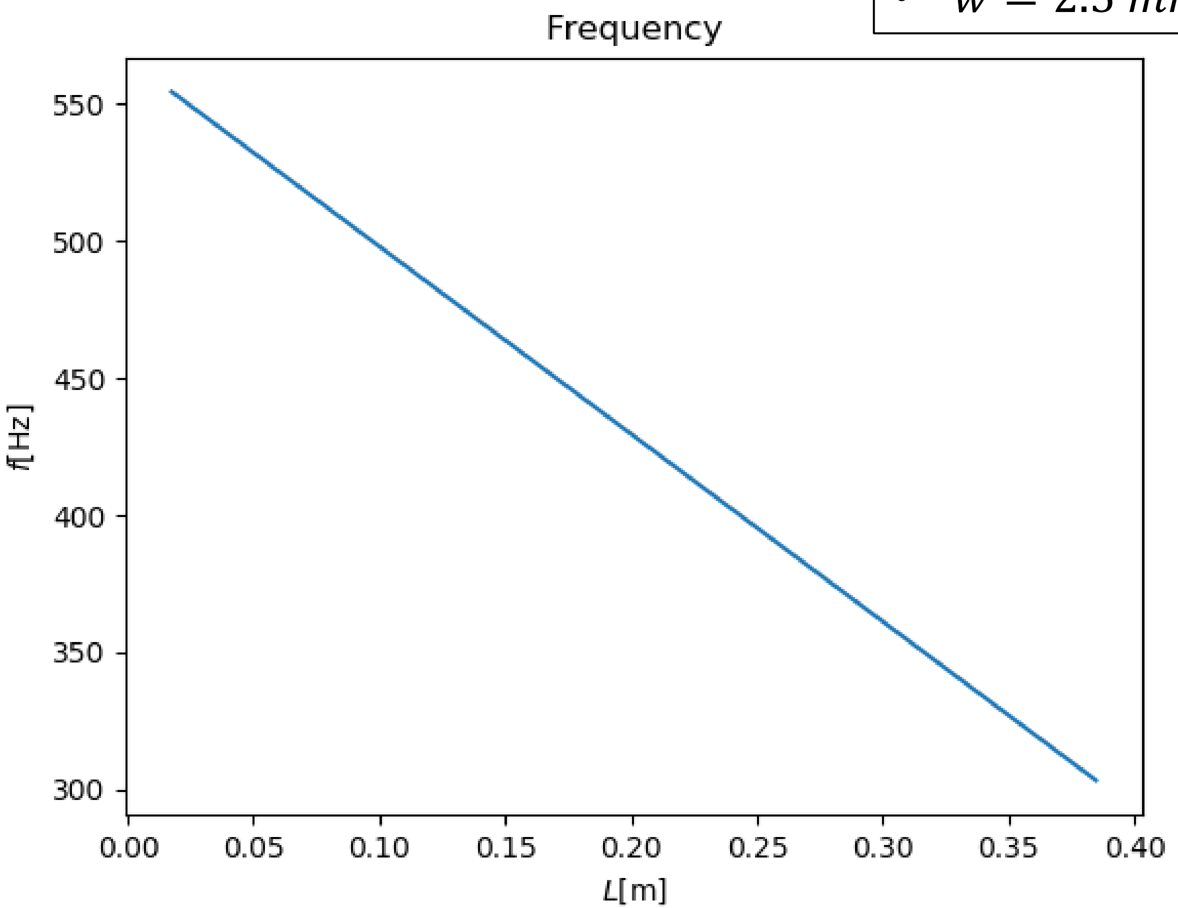
- Vortex model in simFlow
  - Vortices reduce velocity more
  - Laminar → little vortex formation
- Used rhoPimple solver to account for compressibility
- Fit through vortex model implemented into theory of smooth pipes

simFlow

Axial flow velocity profile along pipe

- $L_G = 0.445\ m$
- $R = 0.0125\ m$
- $f_\omega = 1.88\ Hz$
- $w_c = 4\ mm$
- $w = 2.5\ mm$





Experiment: 2 different setups proposed

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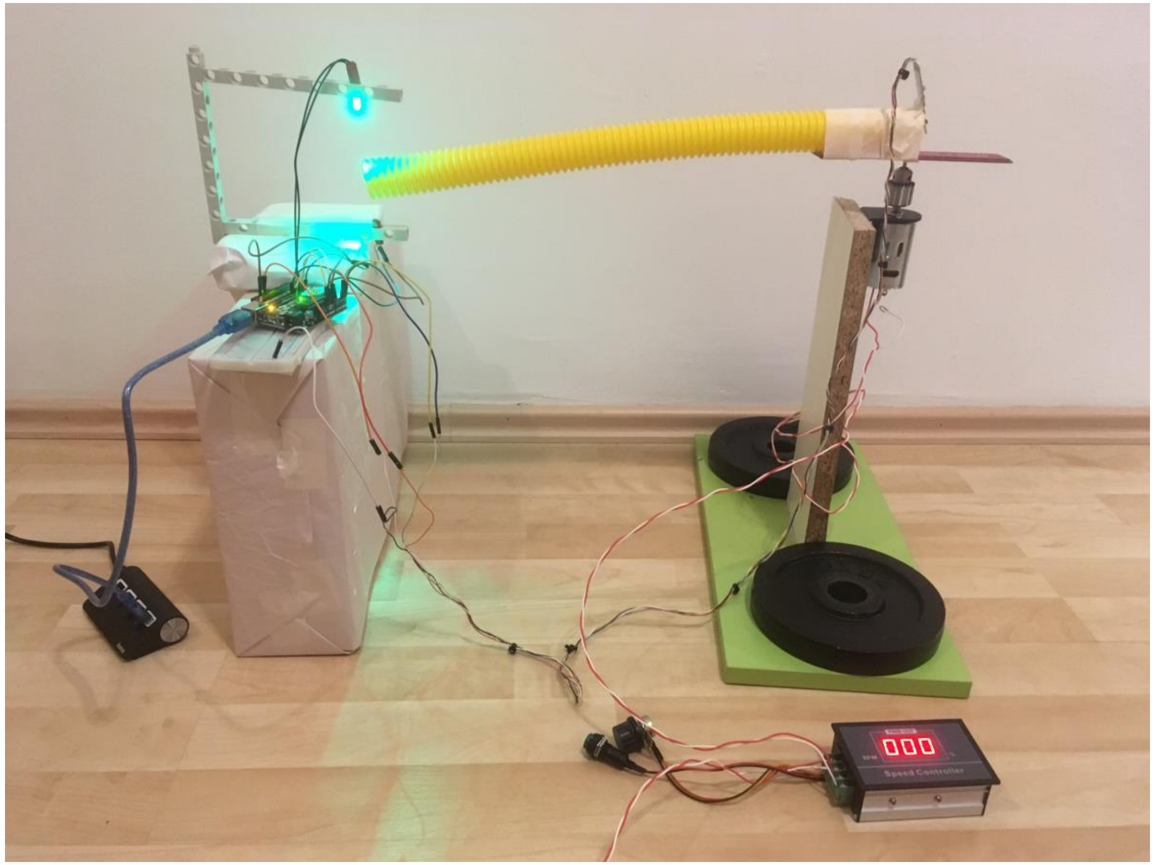
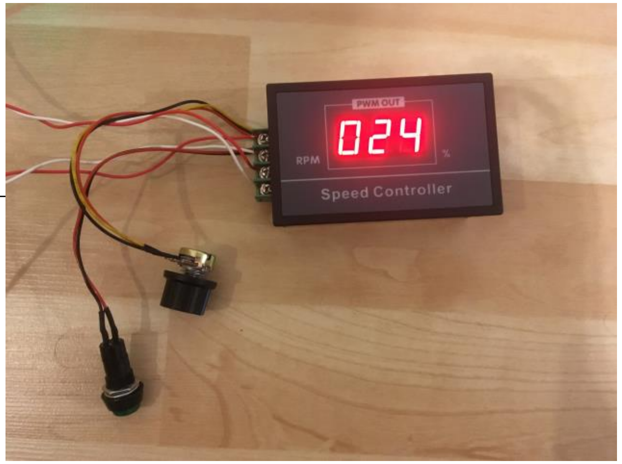
Rotating setup

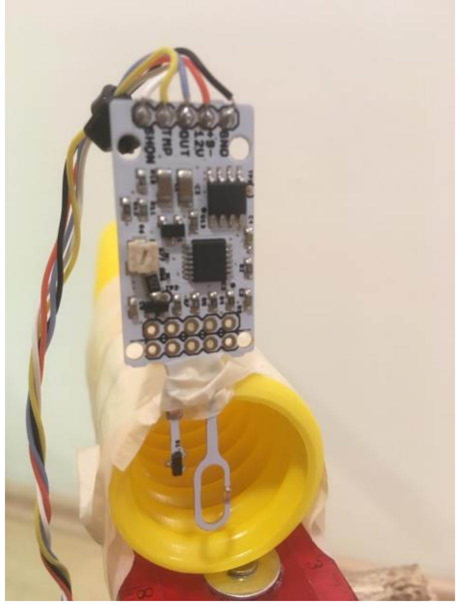




RPM controller

75 W DC motor

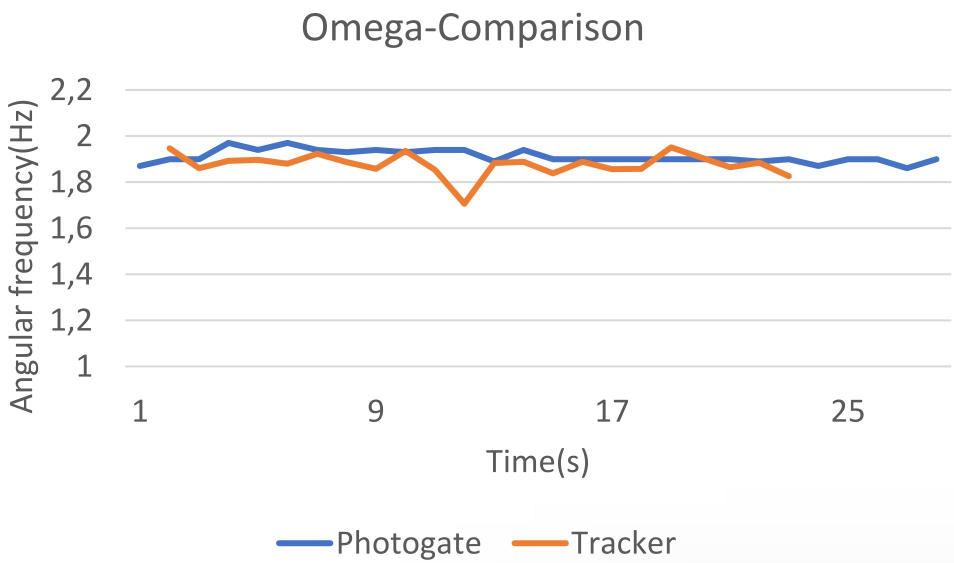
Microphone

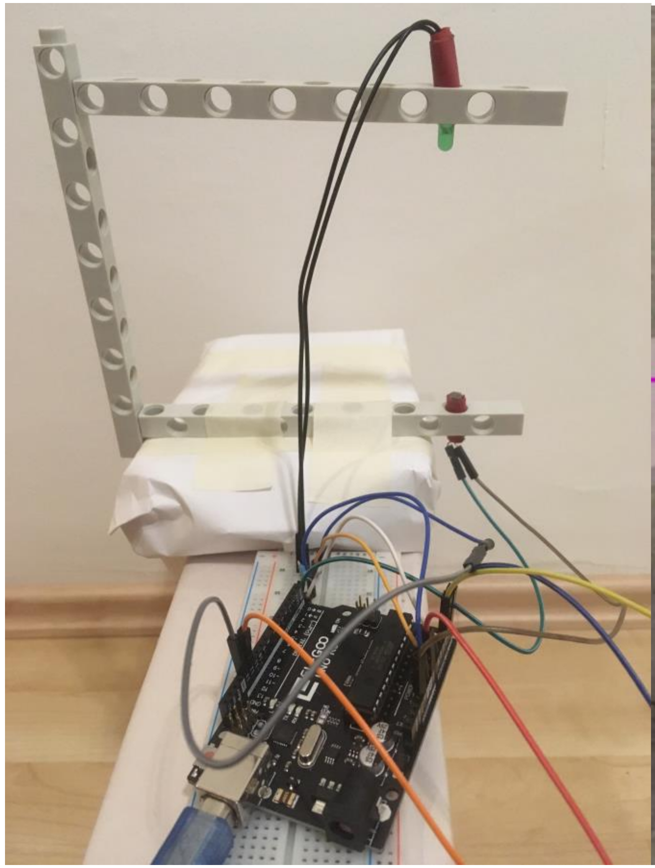


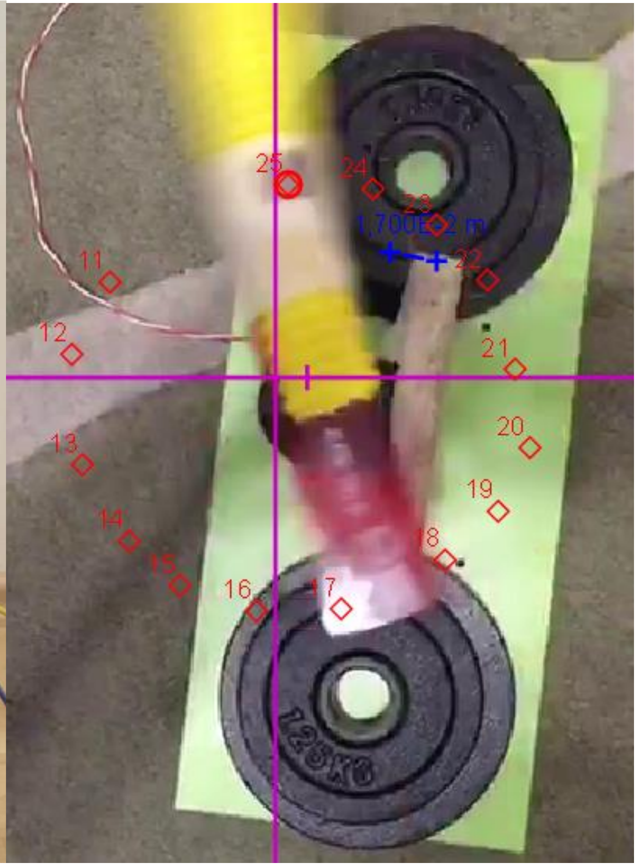
Hot wire anemometer

Measuring angular velocity

- Photogate with Arduino
  - Time between two interruptions → T
  - $\omega = \frac{2\pi}{T}$
- Dark point on tube was tracked
  - $\omega = \frac{\theta}{t}$





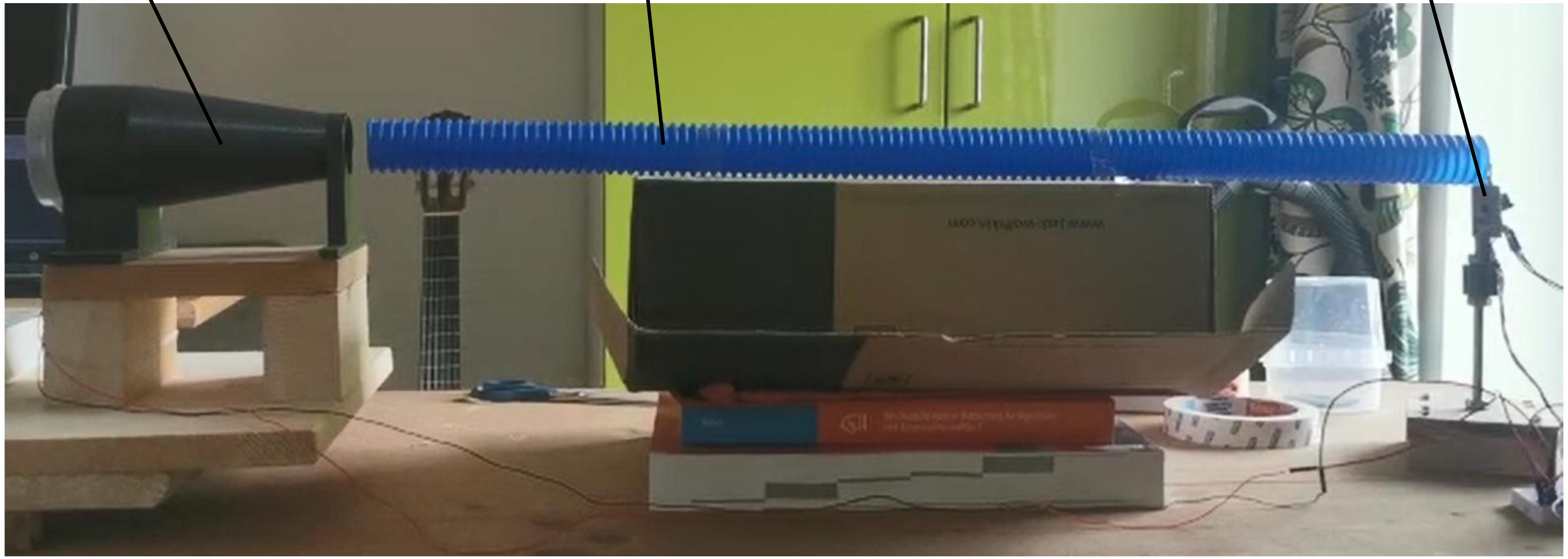


Blowing setup

PWM of motor fan

tube fixed to plateau
at fixed distance

Sound sensor & hot
wire anemometer

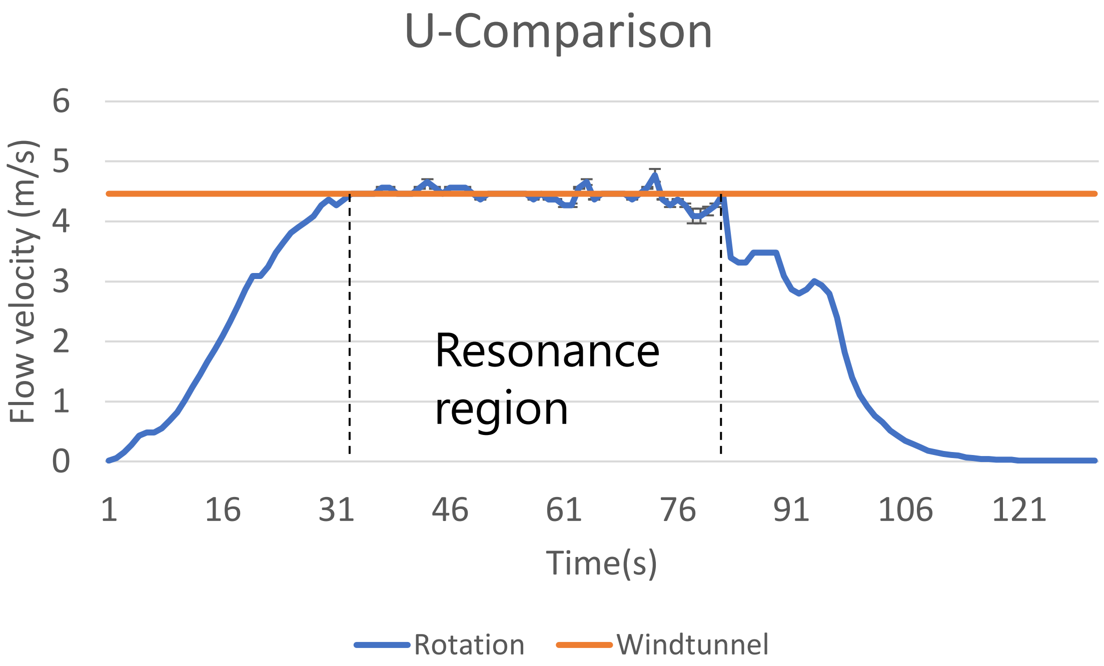


Experimental determination of St

- If tube started resonating:

$$St = \frac{f_v \cdot L_c}{U} \ and \ f_v = f_p$$

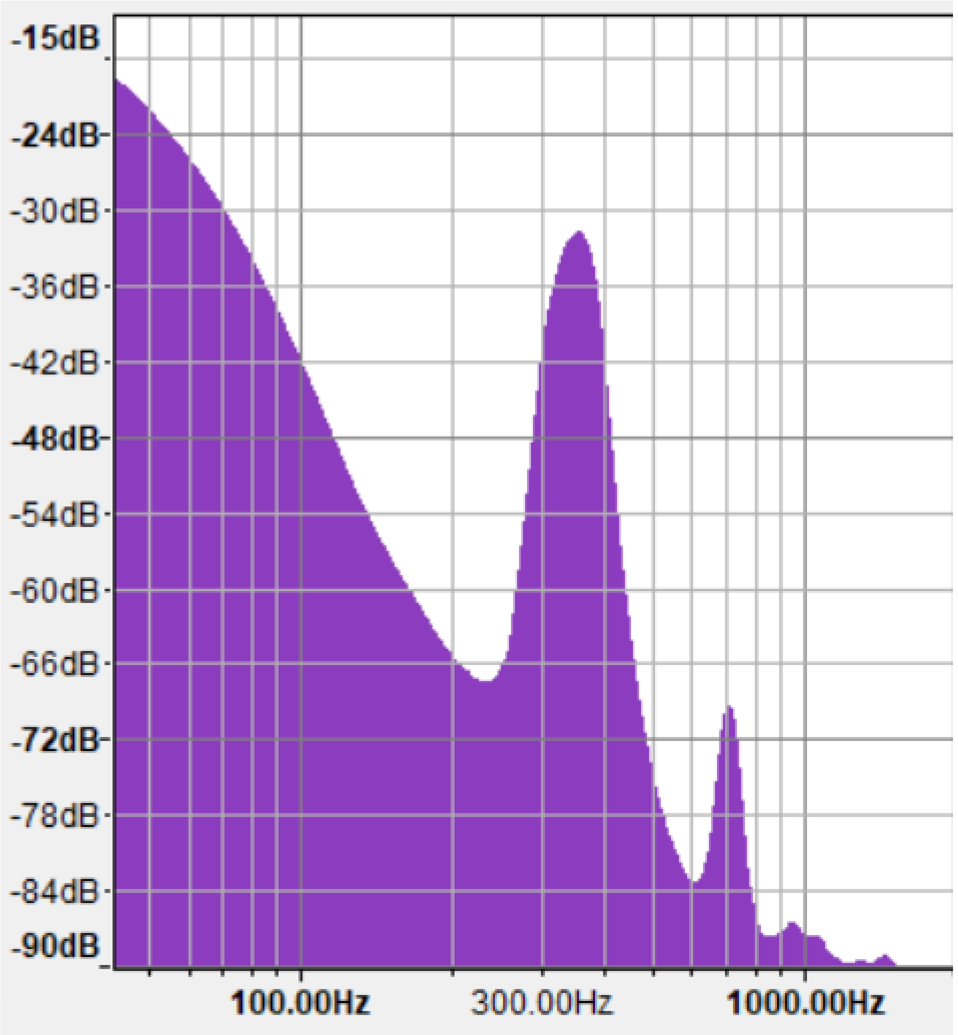
- f_v is the frequency of the sound recorded and U ist the flow velocity measured with the anemometer
- Attached to inlet of tube
  - While tube rotating data gathered
- Placed at inlet and outlet of tube
  - More precise measurements possible



- In above example experimental results:
  - Rotation: St=0.3208
  - Blowing: St=0.3222
  - All St values in range 0.3 to 0.4

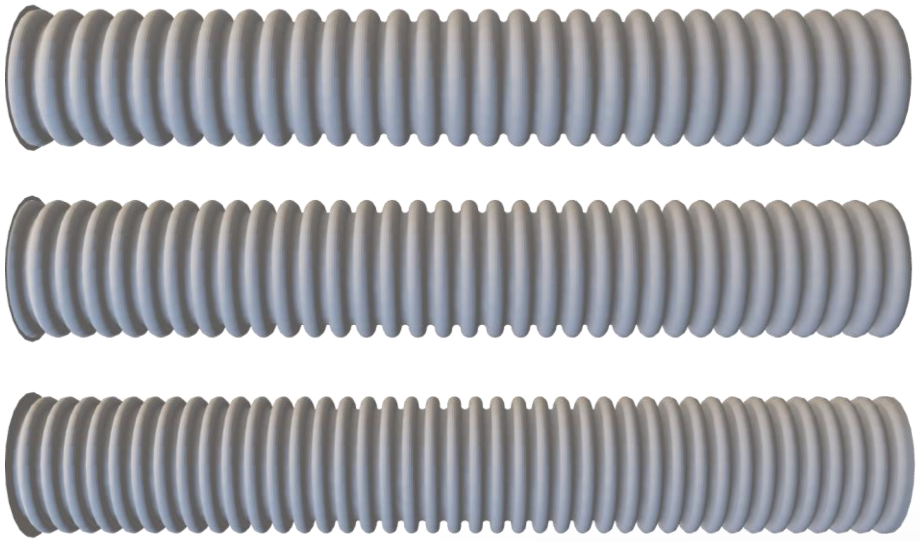
Audio analysis with Audacity

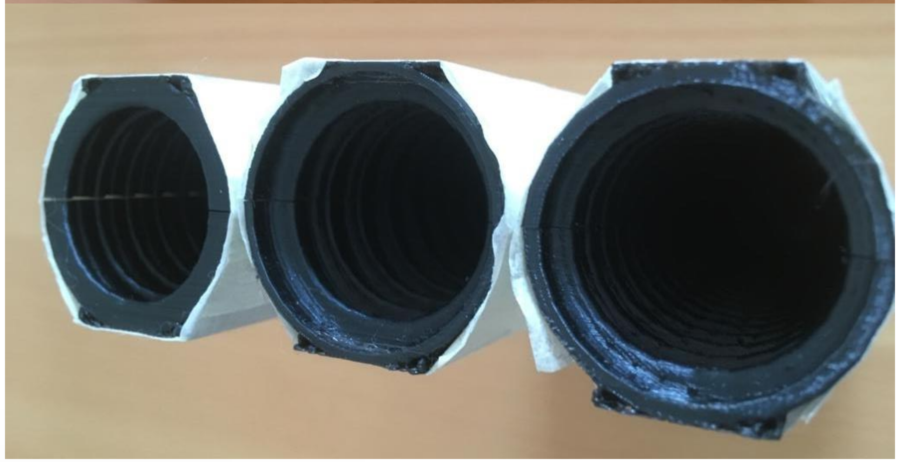
- Plot spectrum → gives FFT of input sound and shows each frequency with distinct power level
- First and second harmonic can be obtained easily



Parameter variation

- 7 different lengths of tube
- 4 different radii of tube
- 3 different widths of corrugations → tubes were modelled and 3D-printed
- U varied over whole range of PWM of fan





Results: Comparison of experiment and theory

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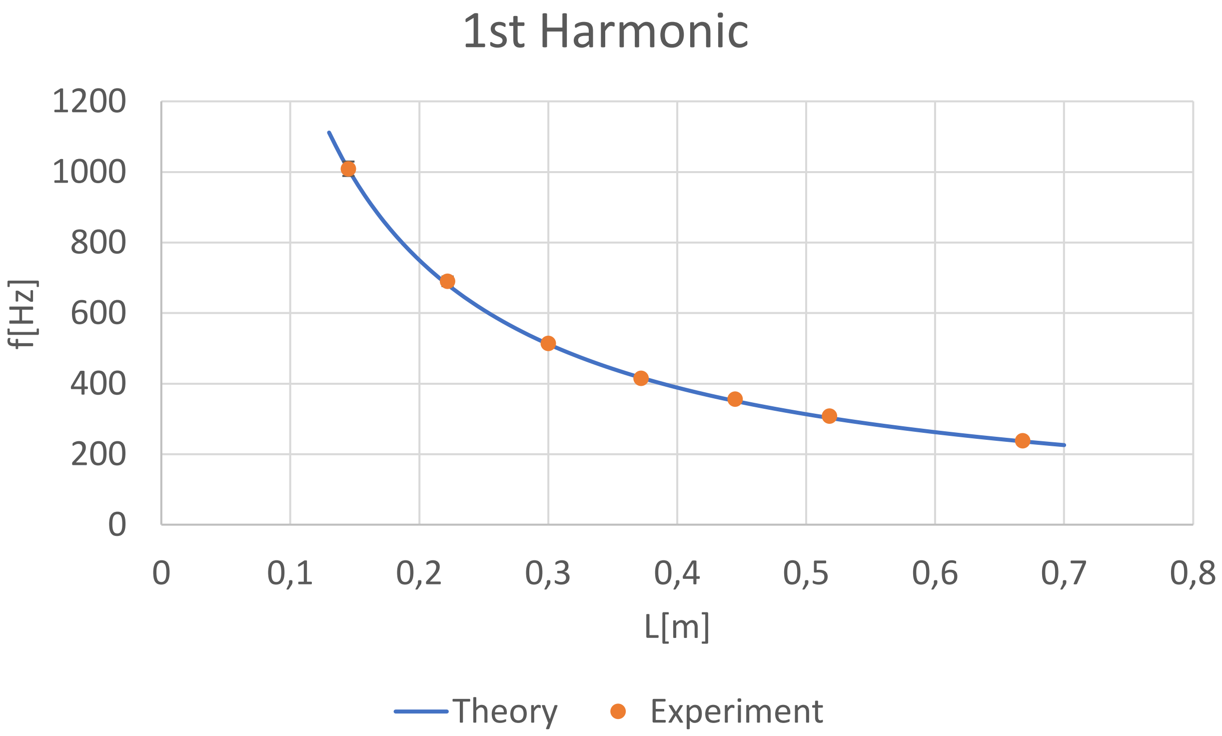
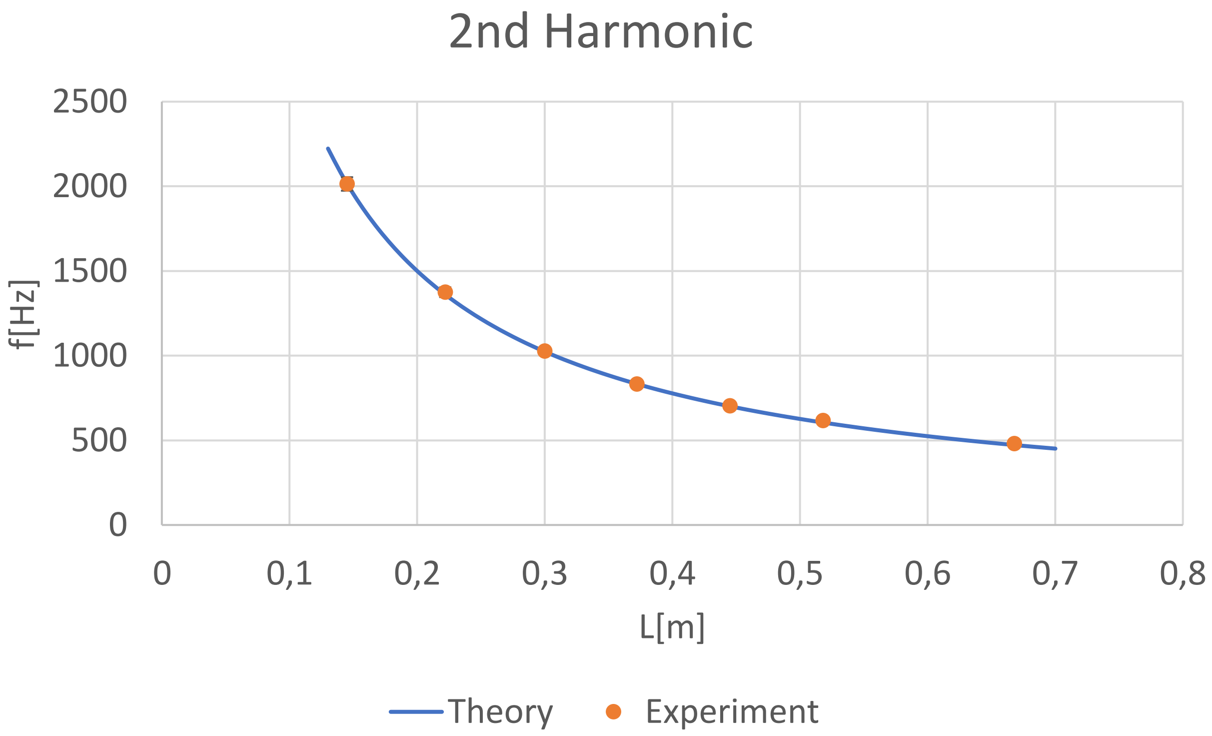
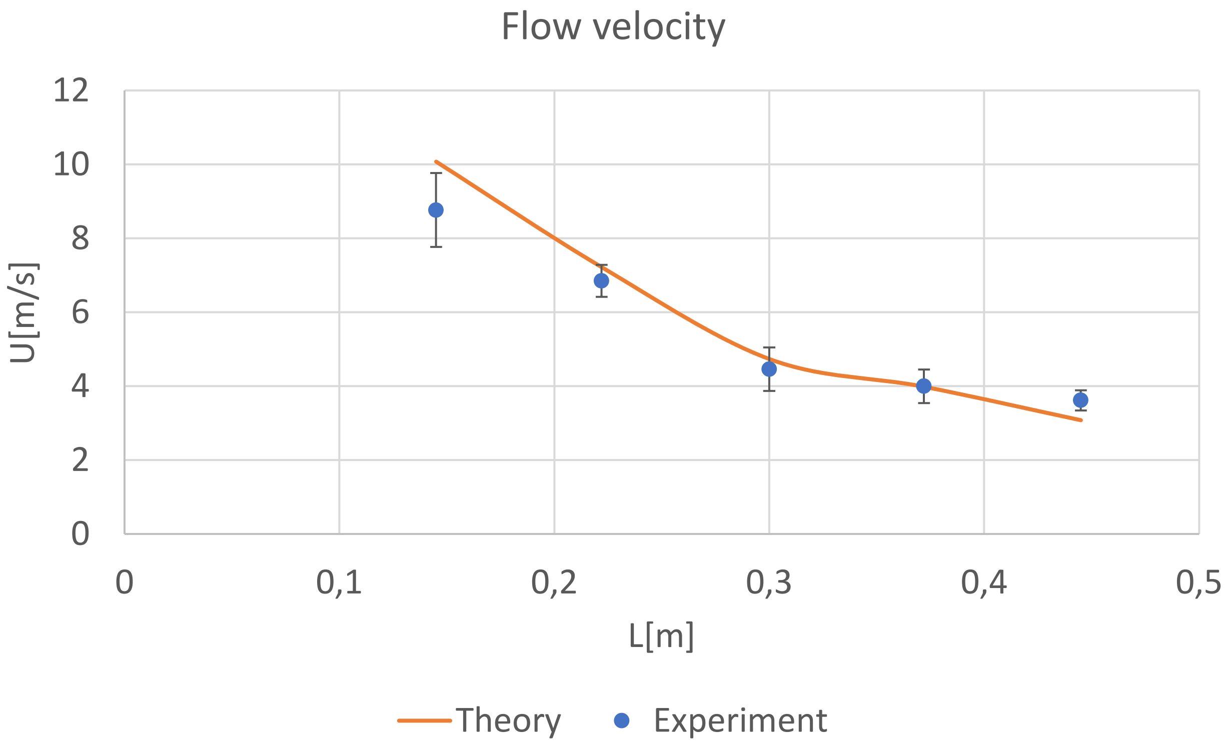
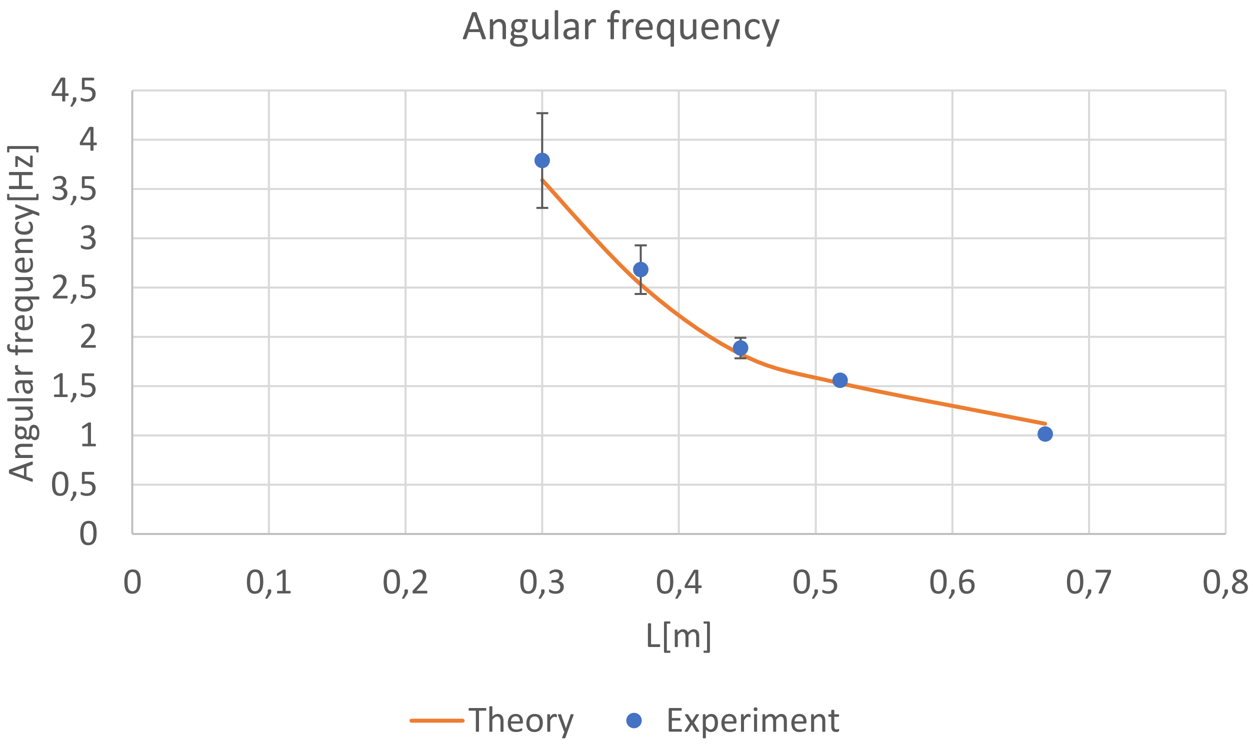
Different lengths - f_p



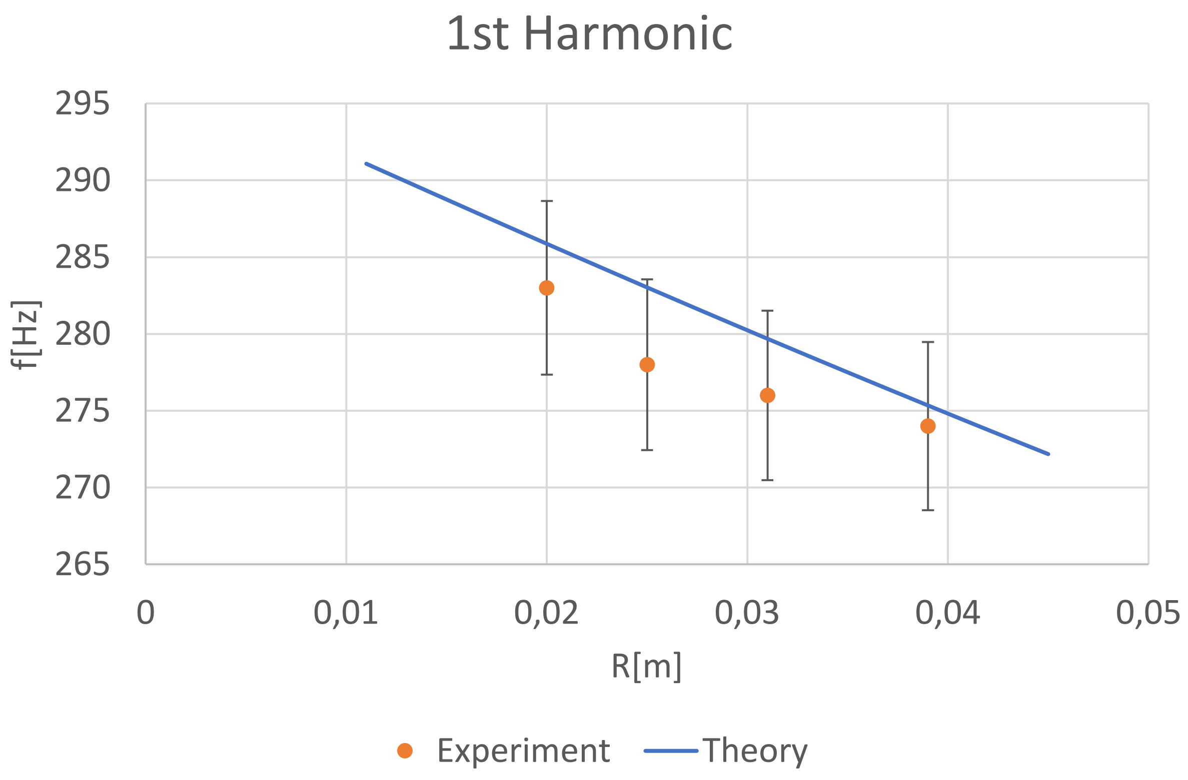


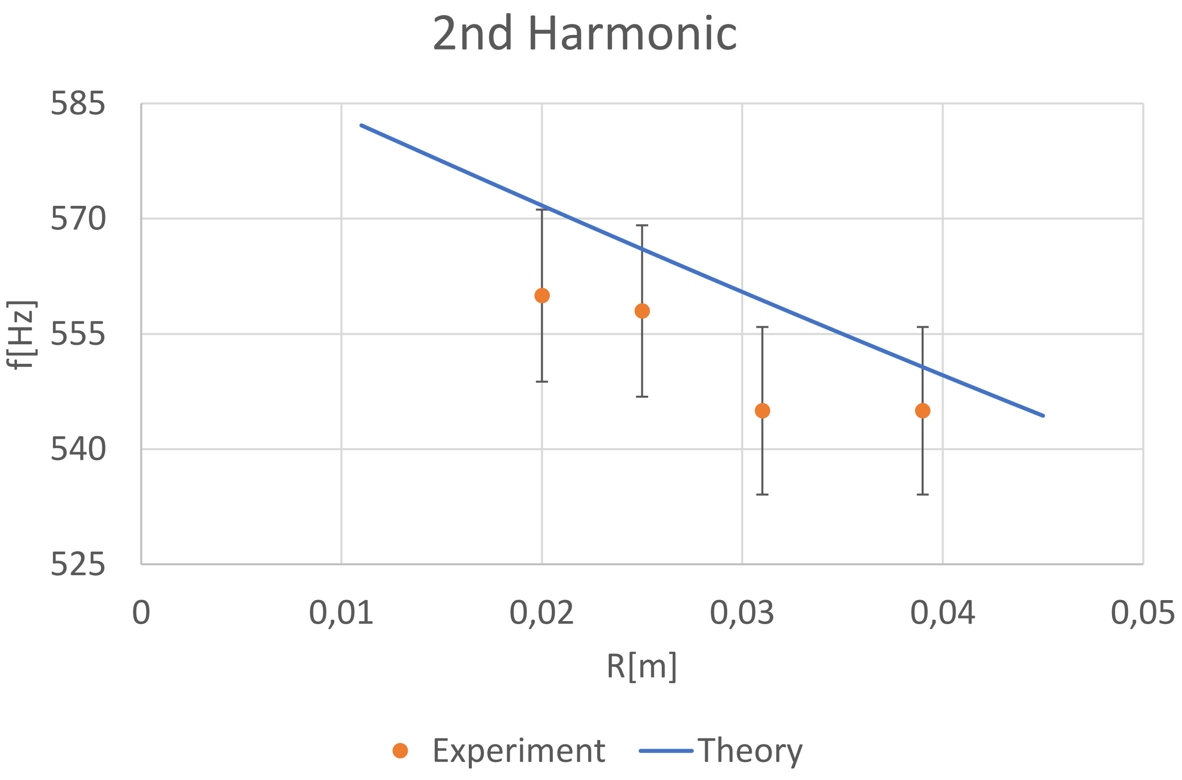
Different lengths - U and ω





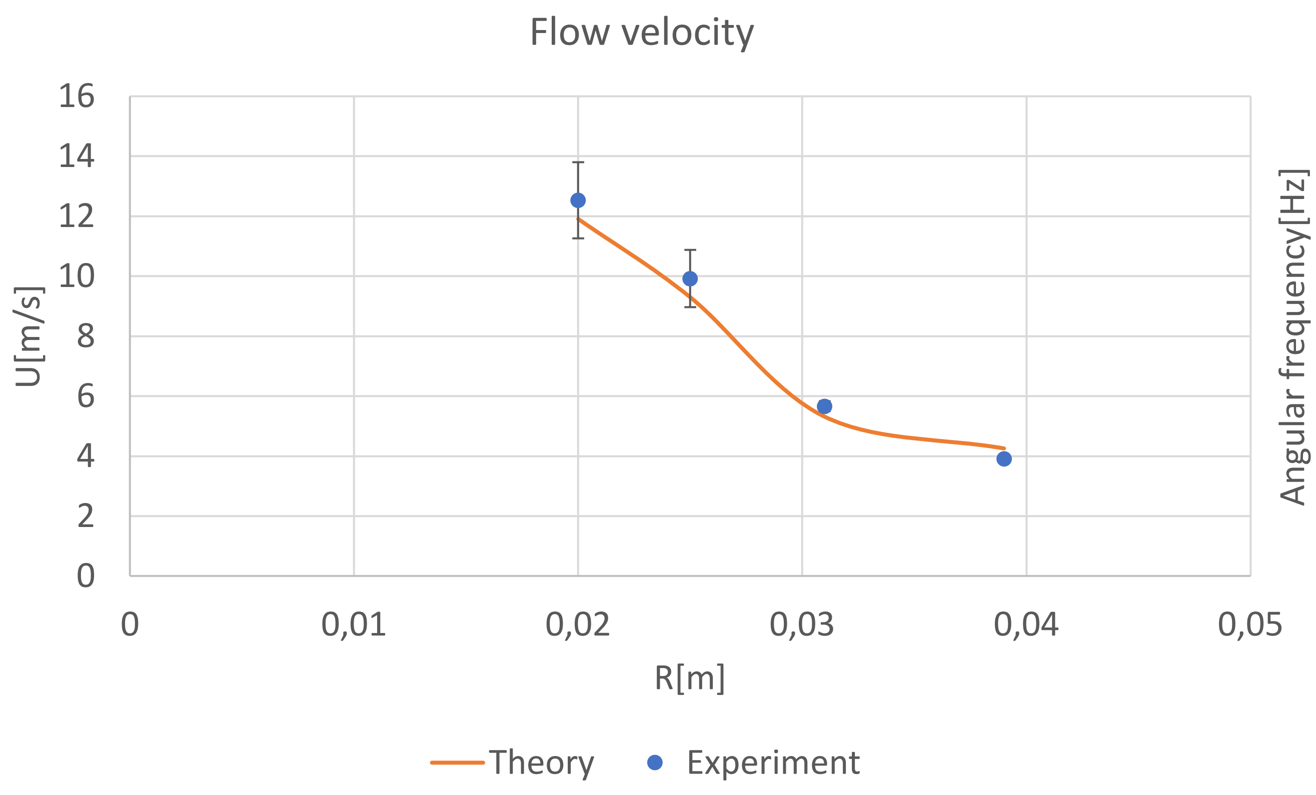
Different radii - f_p

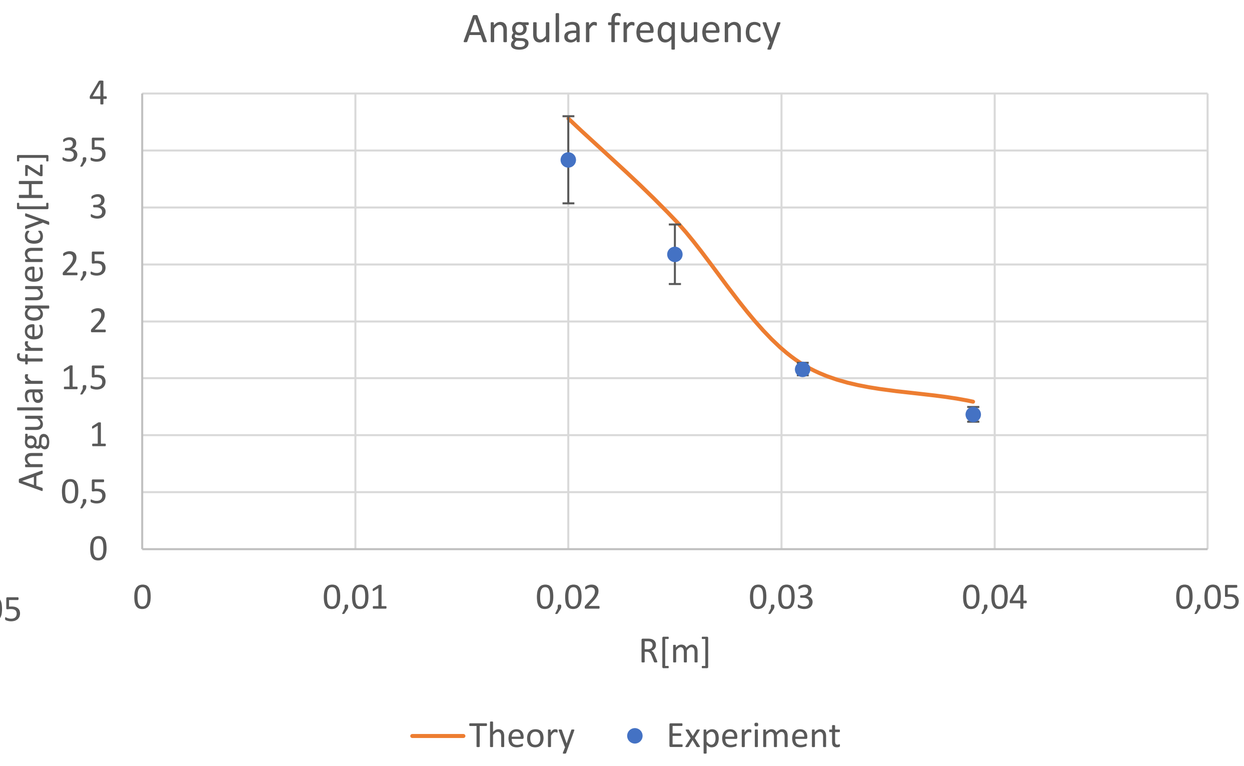




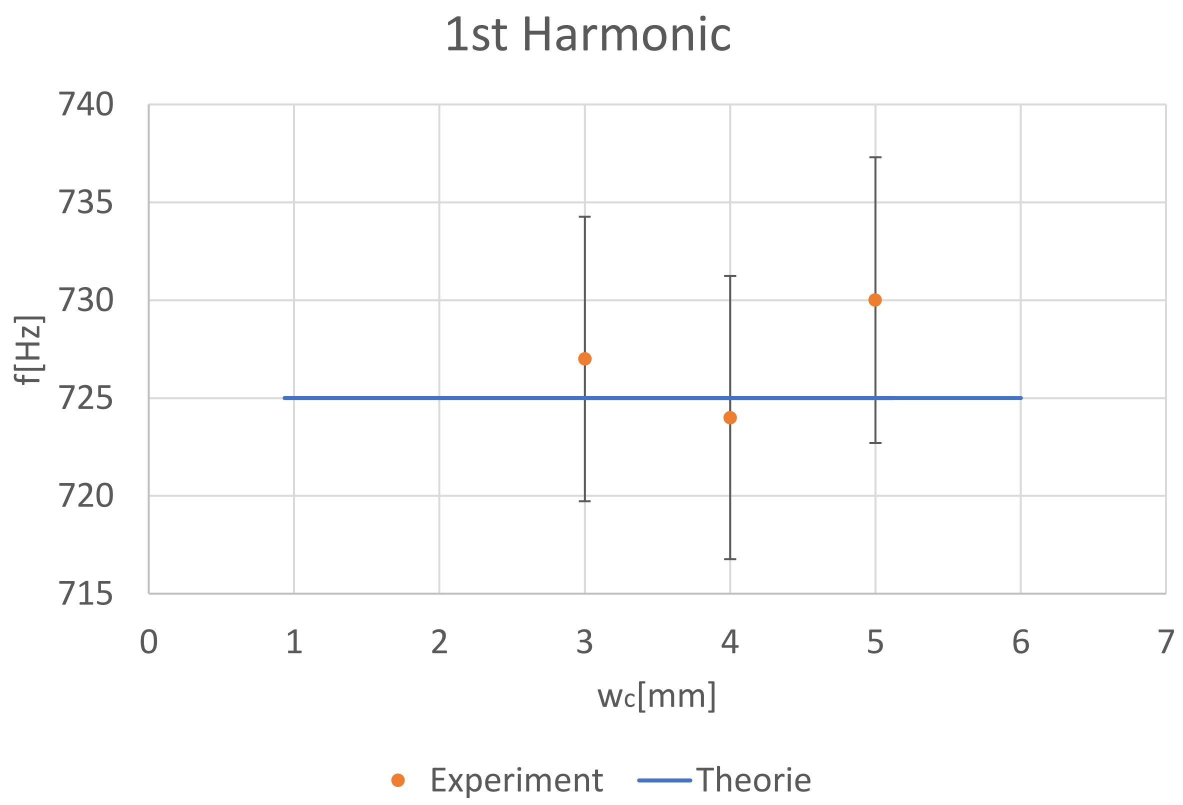
Different radii - U and ω

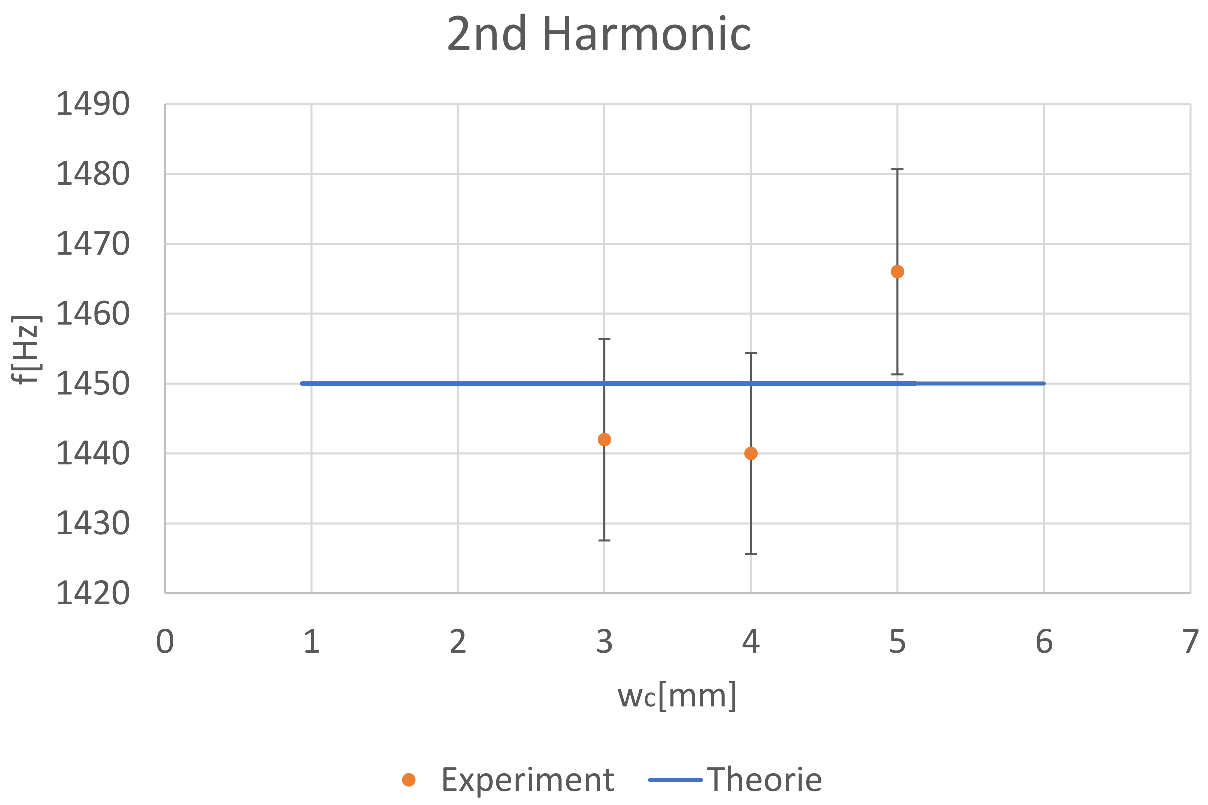
- All tubes cut down to 58 cm





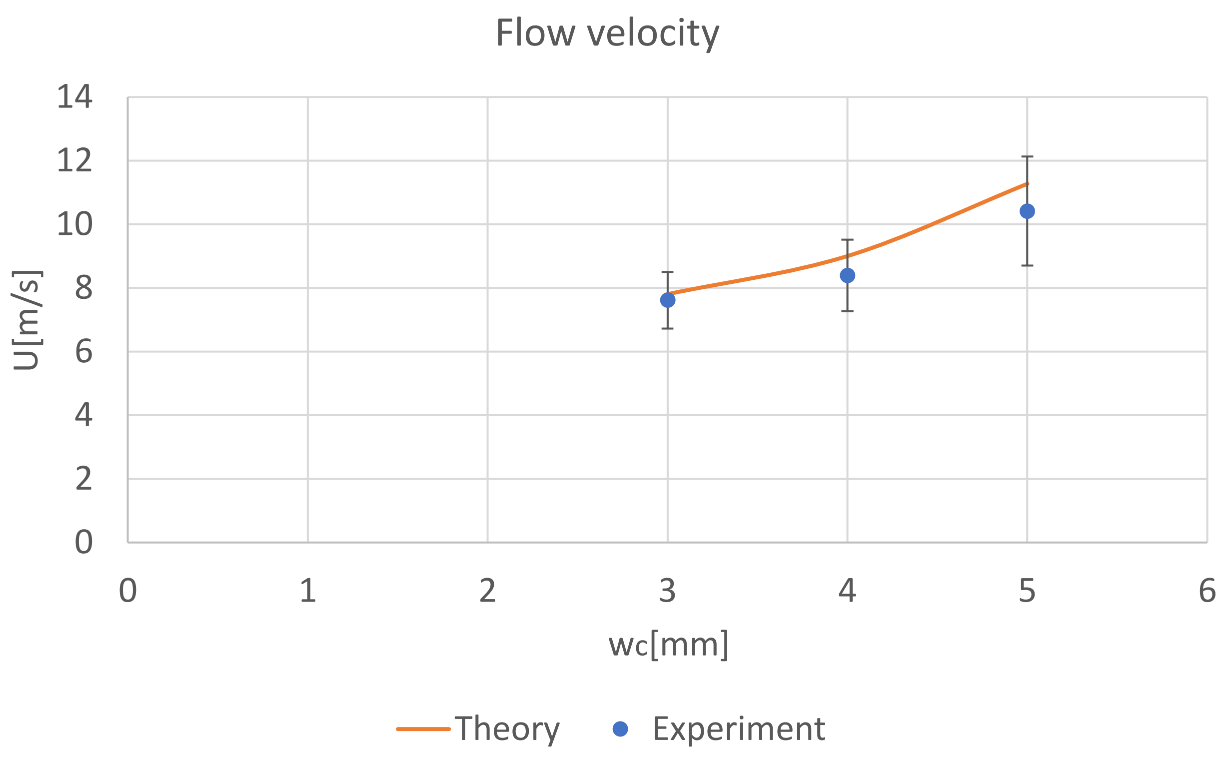
Different corrugation widths - f_p

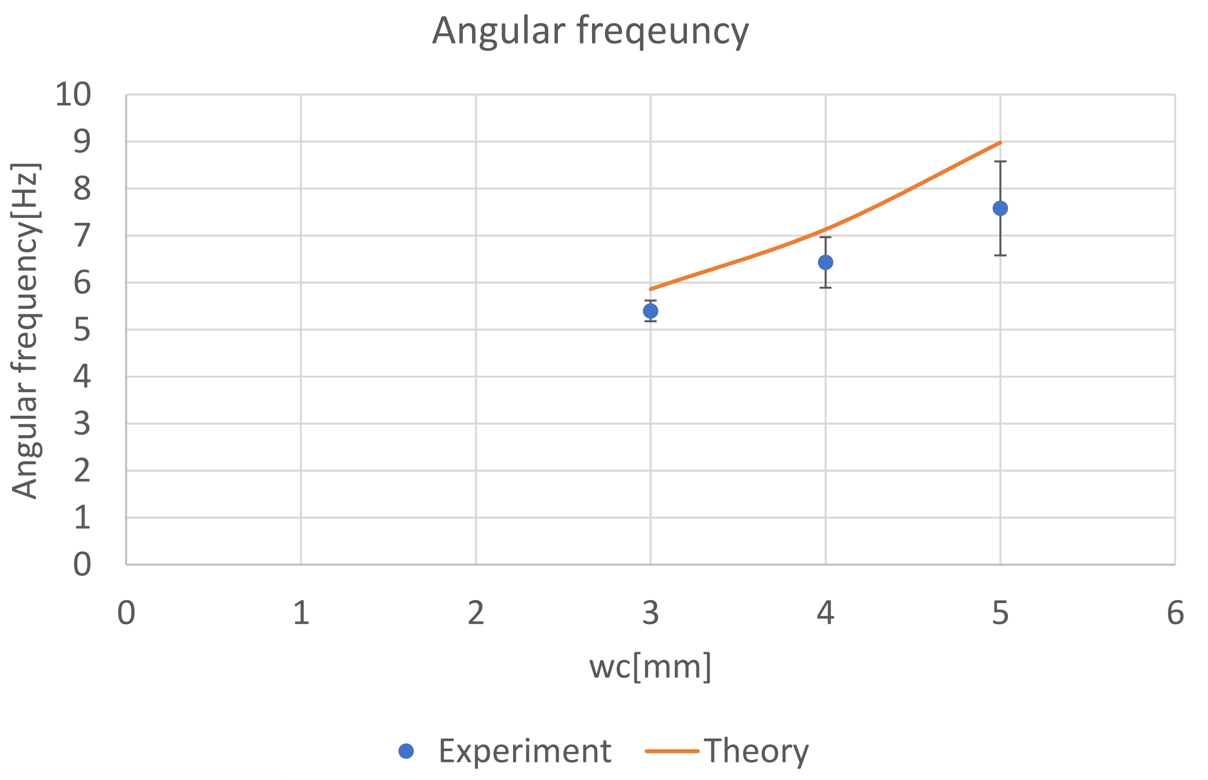




Different corrugation widths - U and ω

- All tubes made with 20 cm length and radius 1.25 cm

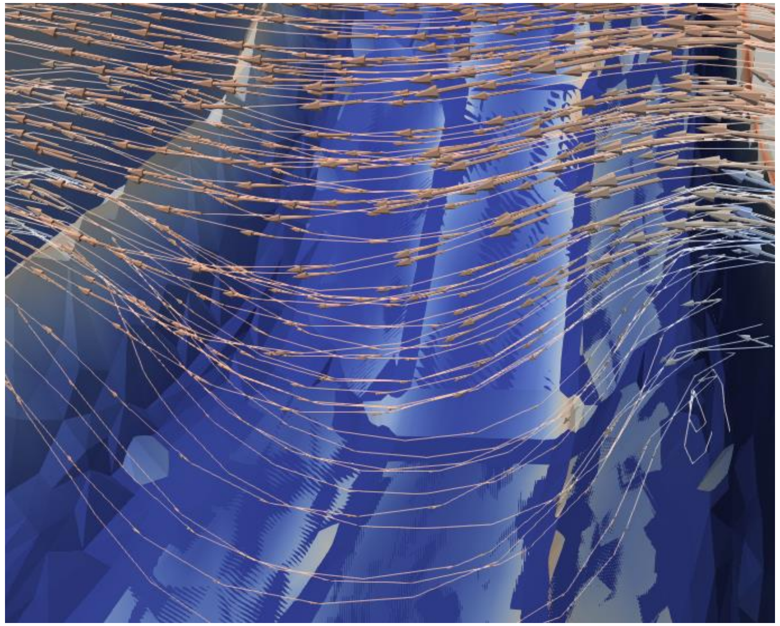




Summary

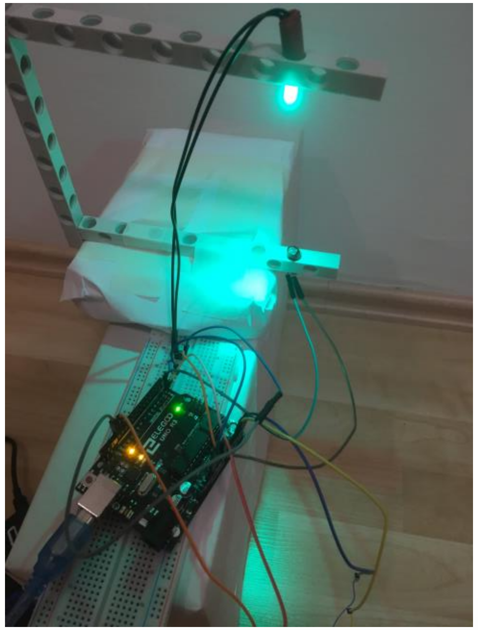
Theory

- f_v and f_p investigated → relevant paramters identified
- Numerical simulation in python
- Vortex model with CFD in simFlow



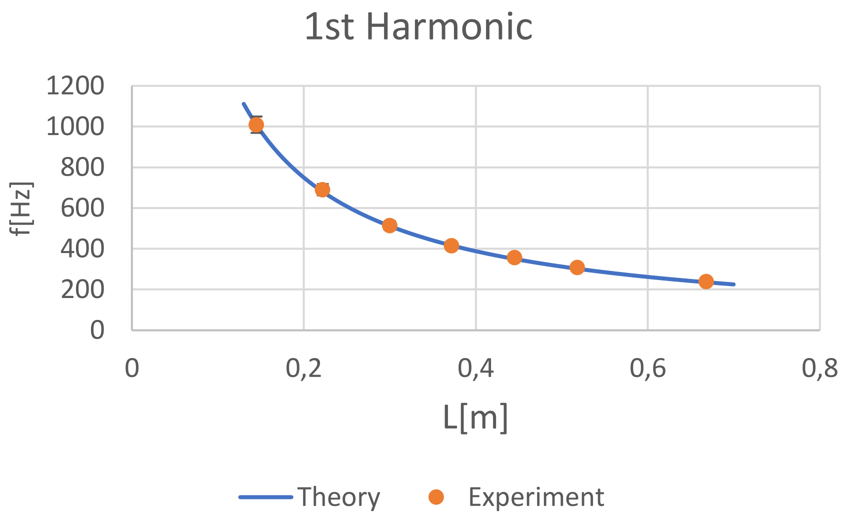
Experiment

- Sufficient parameter variation
- 2 accurate setups
  - Rotating
  - Blowing
- Measurement of St value



Comparison

- Relationships found for:
  - L, R, w, w_c
- For ω and U numerical solution found



Sources

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Apendix 1 – Theory

Doppler effect

- Standard formula:

$$f = \left(\frac{c}{c+v}\right) \cdot f_0$$

- Adapted for this case:

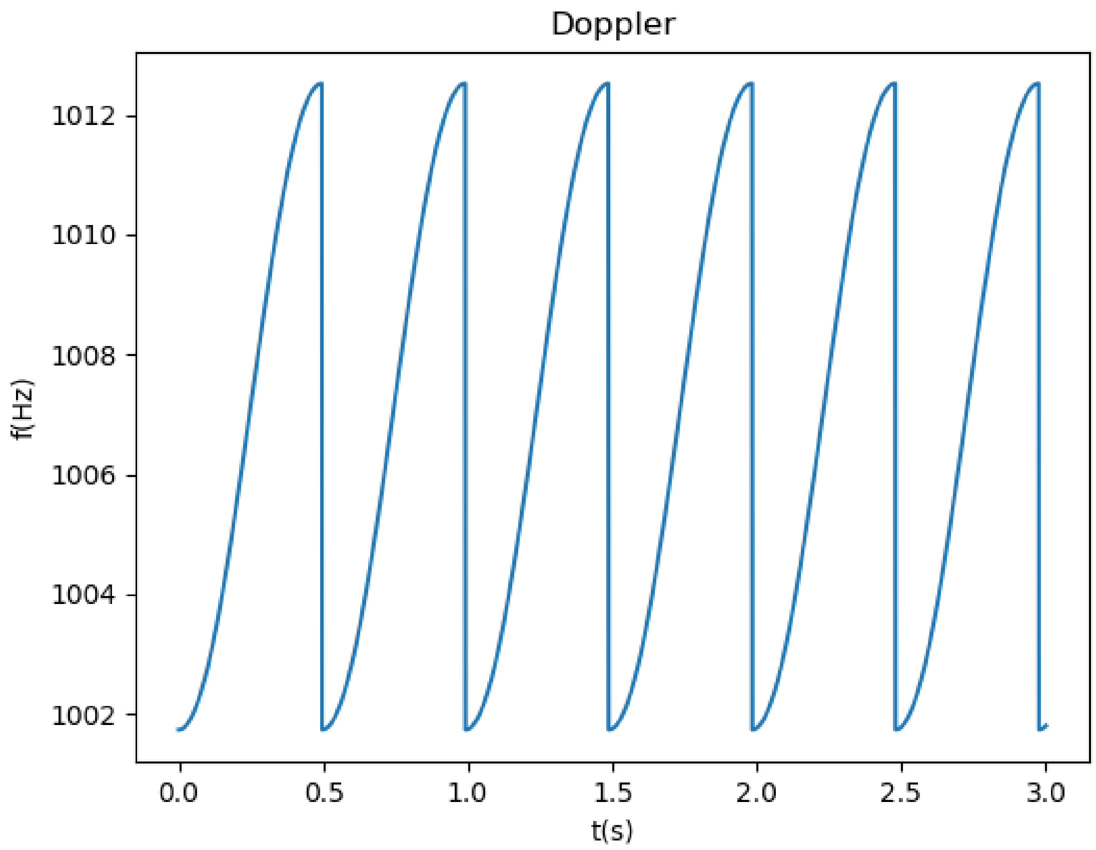
$$v = L_G \cdot \omega \cdot \cos\left(\frac{\omega \cdot t}{2}\right)$$

- Boundaries can be calculated:

$$t = 0 \;\rightarrow\; f_{min} = \left(\frac{c}{c + L_G \cdot \omega}\right) \cdot f_0$$

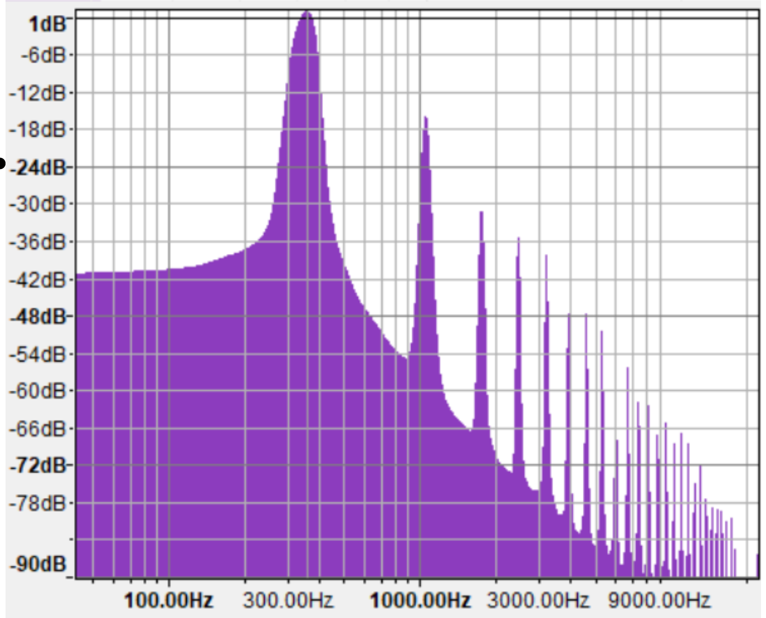
$$t = T \;\rightarrow\; f_{max} = \left(\frac{c}{c - L_G \cdot \omega}\right) \cdot f_0$$

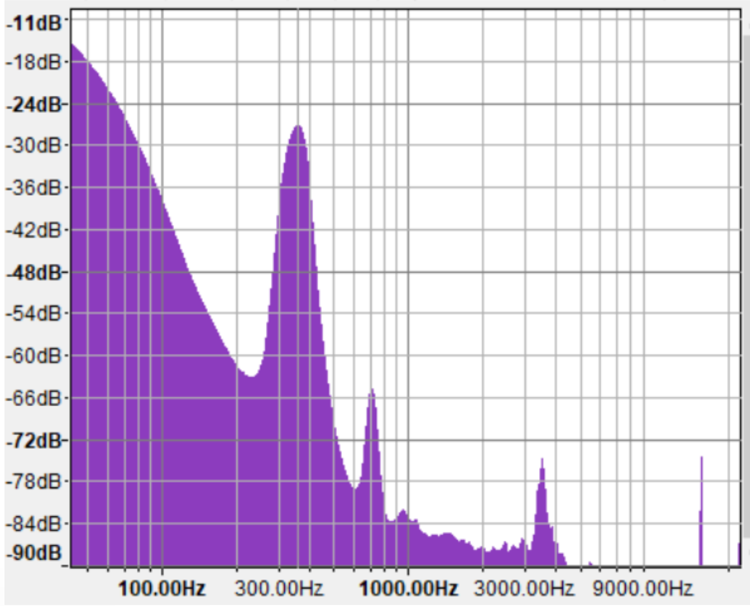
- Generally low variations in frequency



Simulating sound

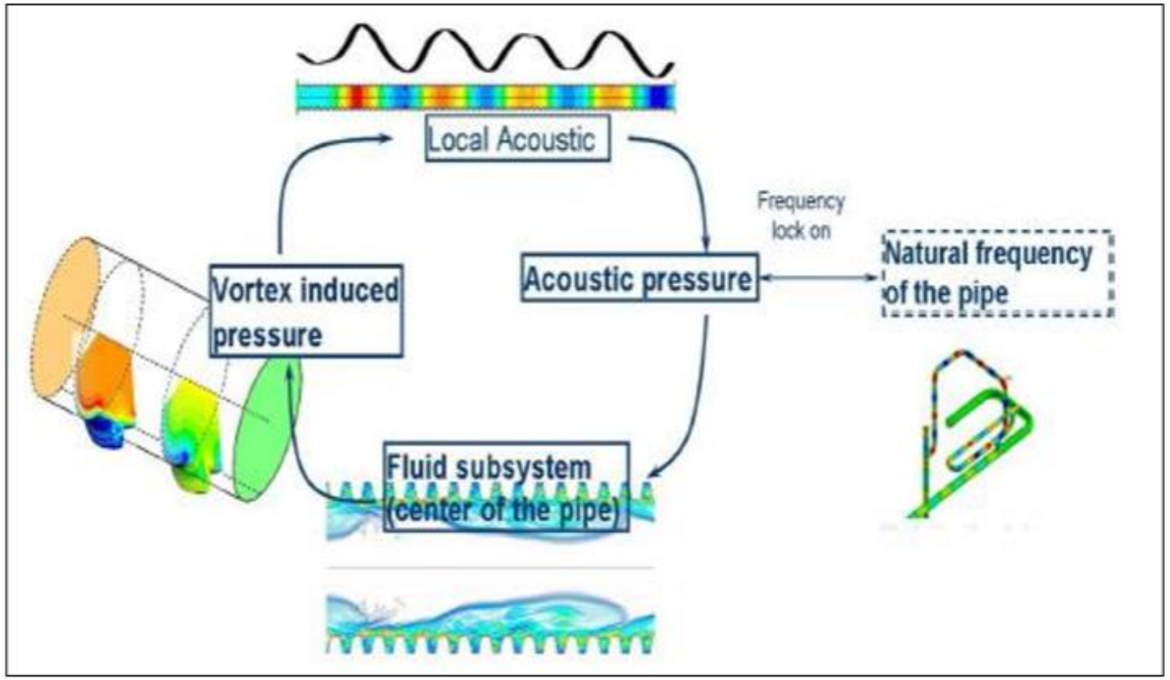
- Able to predict which sound a tube would make for given paramters:
  - $L_G = 0.5\ m$
  - $R = 0.0125\ m$
  - $f_\omega = 2\ Hz$
  - $w_c = 4\ mm$
  - $w = 2.5\ mm$
- Theoretical sound:
- Experimental sound:





Feedback loop in acoustic system

- Phenomenon can also be seperated into two subsystems
  - Fluid subsystem
  - Acoustic subsystem
- Fluid substystem
  - Vortices with instabilties
  - Induce local pressure differences
- Acoustic subsystem
  - Standing waves
  - Acousic pressure over entire pipe
- If frequencies of subsystems coinicde → acousitc pressure will encourage vortex shedding, vortex shedding will increase acoustic pressure



Reynolds-averaged NS equations

- Decomposing fluid velocity into mean and fluctuating part:

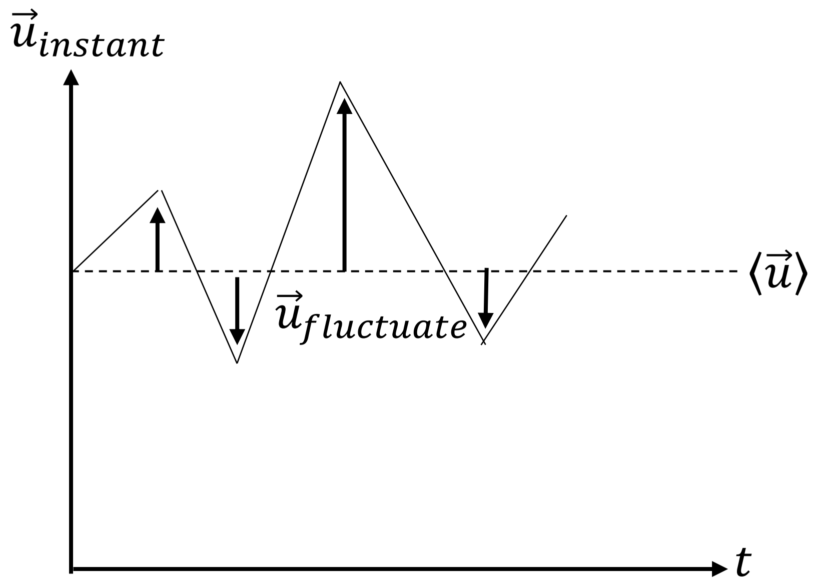
$$\vec{u}_{instant}(x,t) = \langle\vec{u}\rangle(x) + \vec{u}_{fluctuate}(x,t)$$

- Time averaging each part of continuity and momentum equation:

$$\langle\langle\vec{u}\rangle\rangle = \langle\vec{u}\rangle$$

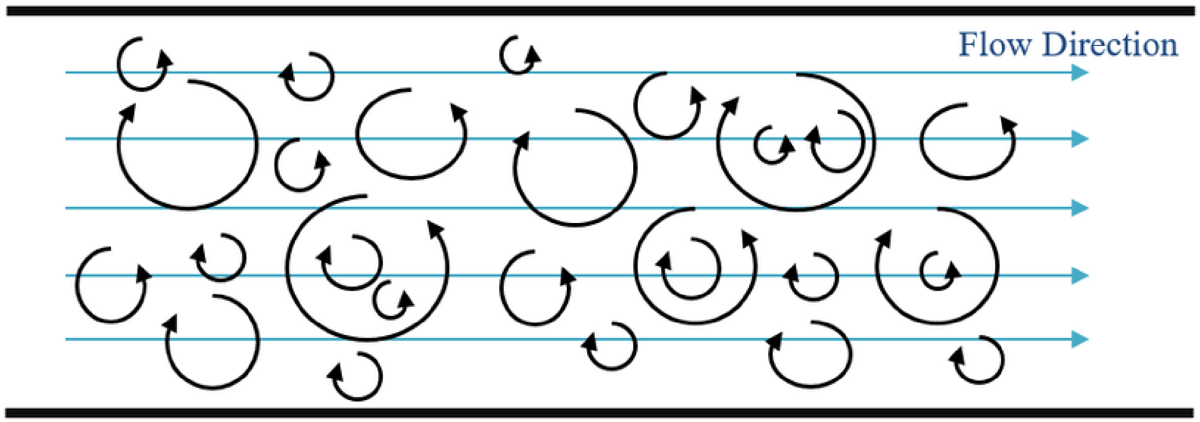
$$\langle\vec{u}_{fluctuate}\rangle = 0$$

- Averaging results in appearance of Reynolds stress term



Turbulence model and algorithm

- k-ω-SST model
  - Two-equation model → turbulent properties of flow
  - Transport variables → k and ω
- Turbulent kinetic energy k → energy in the turbulence per unit mass
- Specific turbulence dissipation rate ω → scale of turbulence
- SST (shear stress transport) → ideal for boundary layers and free stream
- Pimple algorithm → incompressible, transient, turbulent
- RhoPimple algorithm → similar to Pimple but compressible



$$\omega = \frac{\epsilon}{k}$$

ϵ ... *turbulence dissipation rate*

k-epsilon model in simFlow

- Solving RANS equations
  - Reynolds stress expressed with known quantities
  - With eddy viscosity and Boussinesq hypothesis
- Eddy viscosity from mixing length → showing size of eddies formed in flow
- Transport equations
  - Turbulent kinetic energy k
  - Turbulence dissipation rate ϵ
  - Takes into account convection, diffusion over time

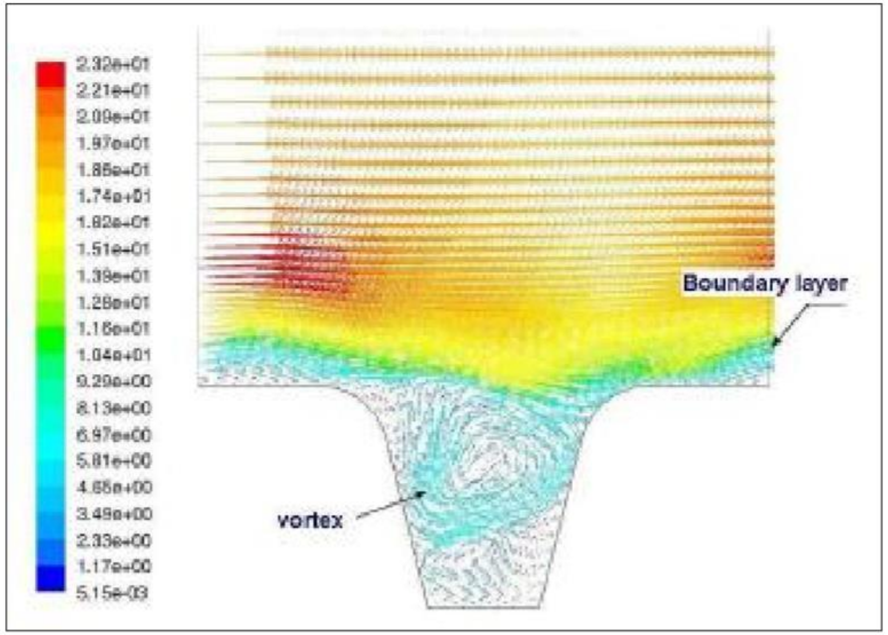
- Older models use mixing length: $\mu_t = \rho k^{1/2} l_m$
- As eddy viscosity isn't static improved models use turbulent factors:

$$\mu_t = C_\mu \frac{\rho k^2}{\epsilon}$$

- Equation system can be completed and velocity values for each direction obtained
- Limitations in viscous sublayer where damping functions have to be introduced

St changing with corrugation dimensions

- c_{max} does not have effect on St values as thickness of shear layer not really affected
- r_{down} has minor effects on St values as thickness of shear layer narrows at larger down stream edges → U_v becomes higher
- r_{up} has bigger effect on St values as thickness of shear layer is dependent on seperation of flow at upstream edge → larger r_{up} gives us lower U_v
- w_c has bigger effect on St values as thickness of shear layer also depends on opening angle of the corrugation → larger w_c gives us thicker shear layer and higher U_v



Amplitude changing with corrugation dimensions

- Depends on how thick the shear layer is and how strong the force is acting on the wall from instabilities
- c_{max} has almost no effect on amplitude
- r_{down} increases the amplitude a bit
- r_{up} decreases the amplitude a bit
- w_c reduces amplitude significantly as secondary and tertiary vortices start forming in the corrugation → opposing primary vortex and force exerted on wall

Reynolds number Re

- The Reynolds number is a measure of the turbulence in a system
- Defined as: $Re = \frac{\rho \cdot U \cdot L}{\mu}$
- $\rho = 1.204\ kg/m^3$
- $\mu = 1.825 \cdot 10^{-5} Pa \cdot s$
- L is in case of a pipe equal to the pipe diameter

μ...viscosity of air

L...characterisitc linear dimension

ρ...density of air

R...radius of tube

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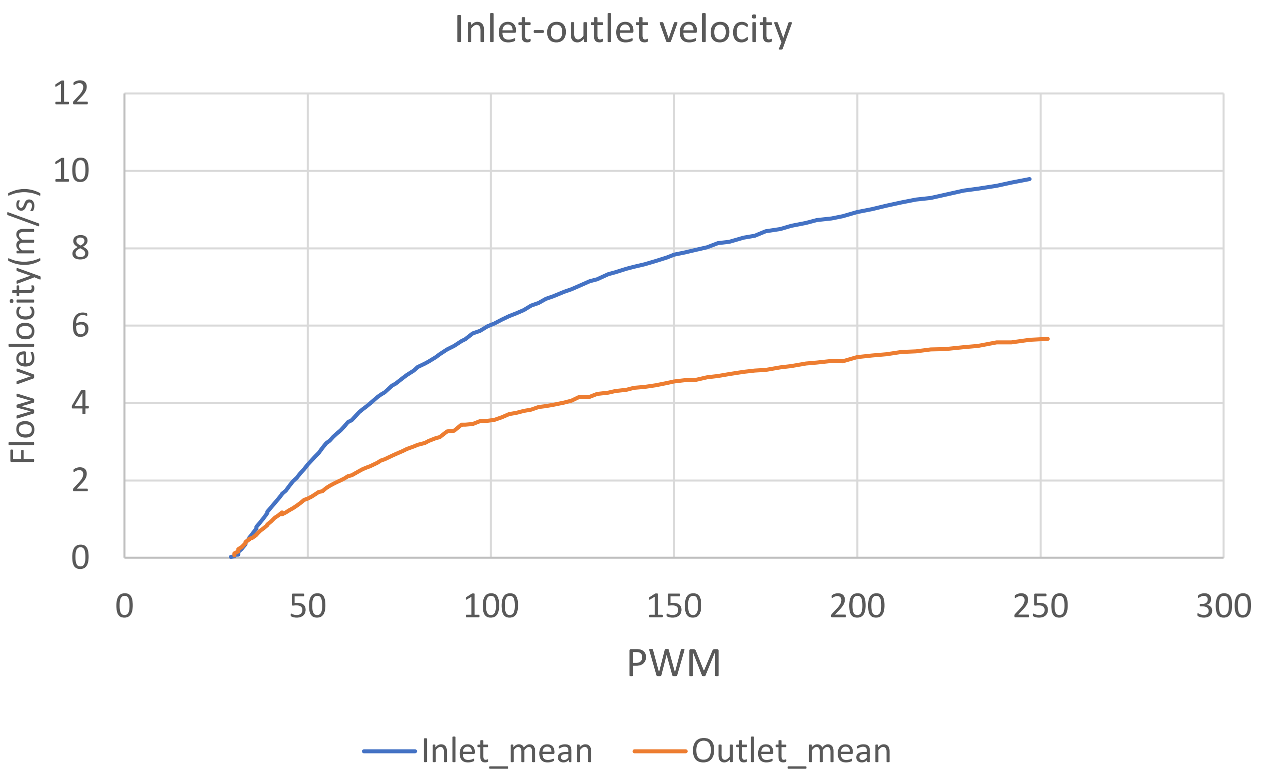
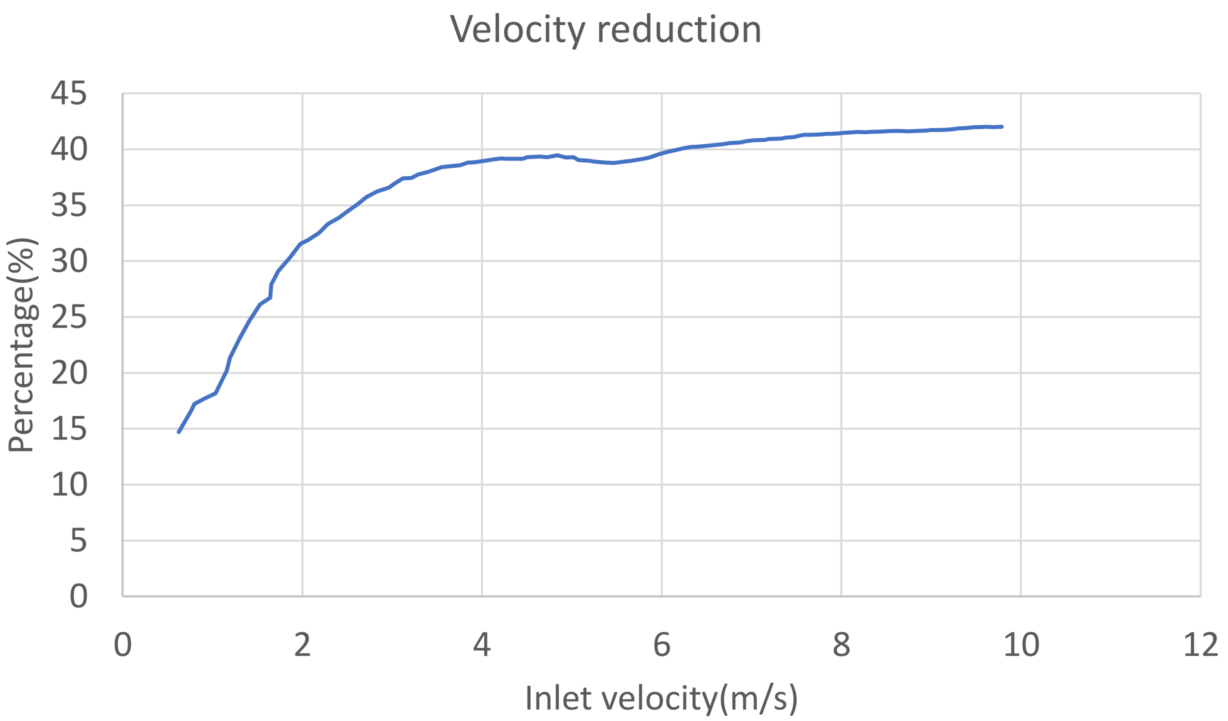
Apendix 2 – Experiments

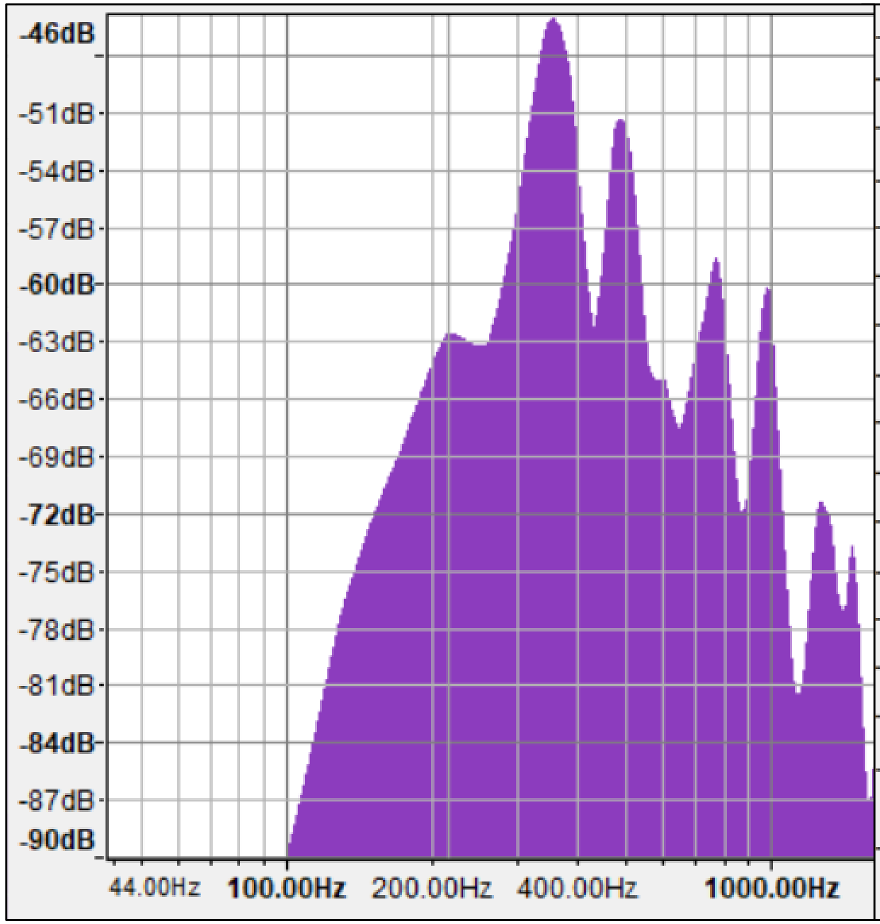
Velocity reduction over pipe



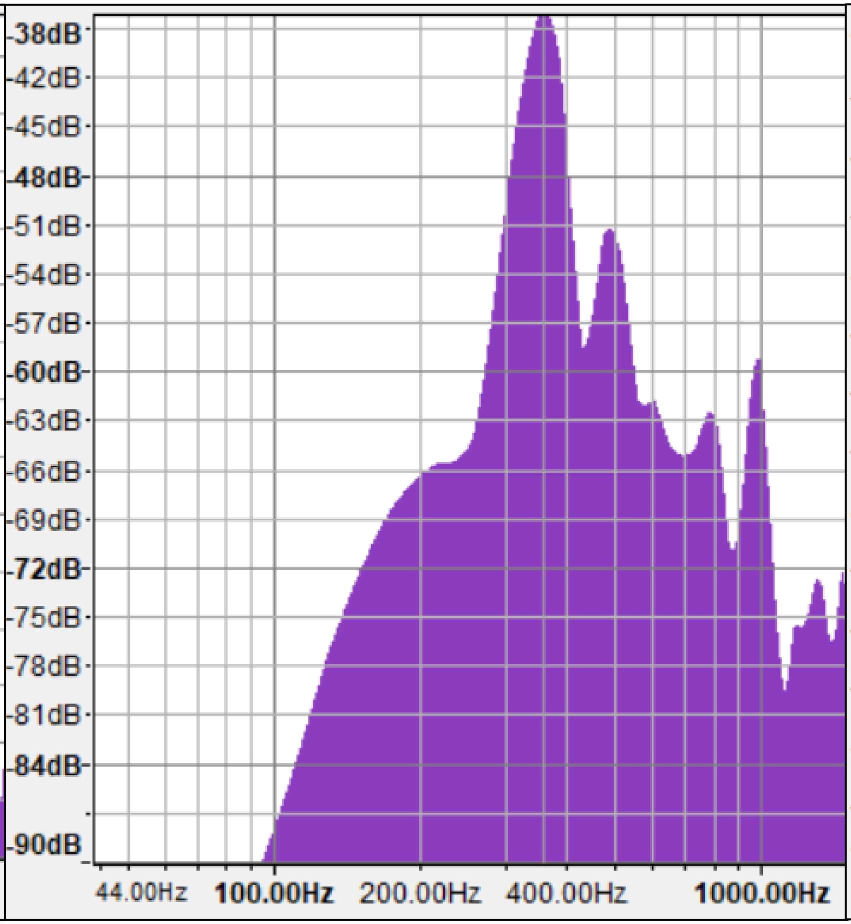


Frequency spectra for different U

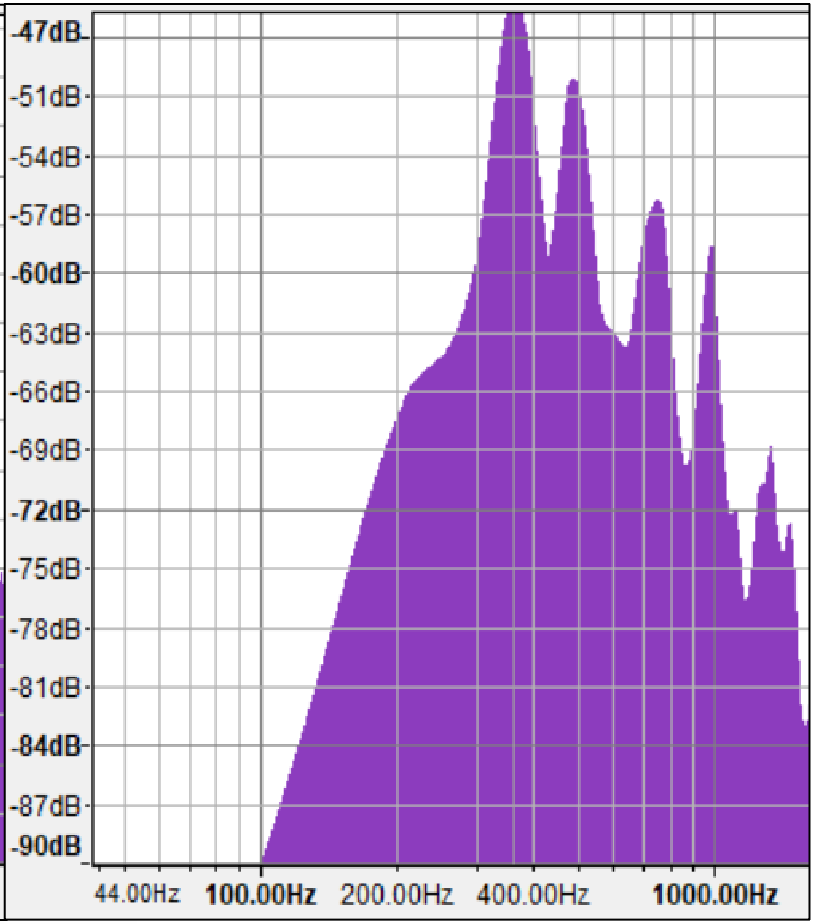
U=4.28 m/s with -46 dB



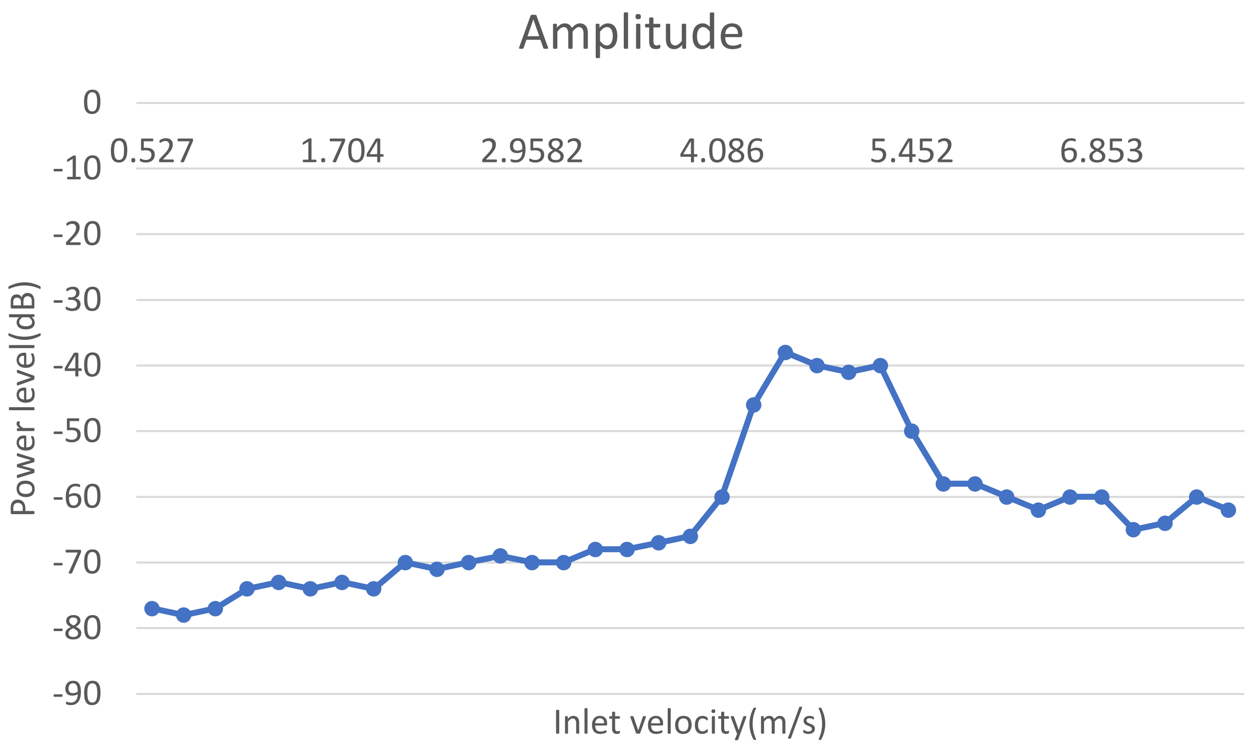
U=4.47 m/s with -38 dB



U=5.45 m/s with -47 dB



Amplitude development over U

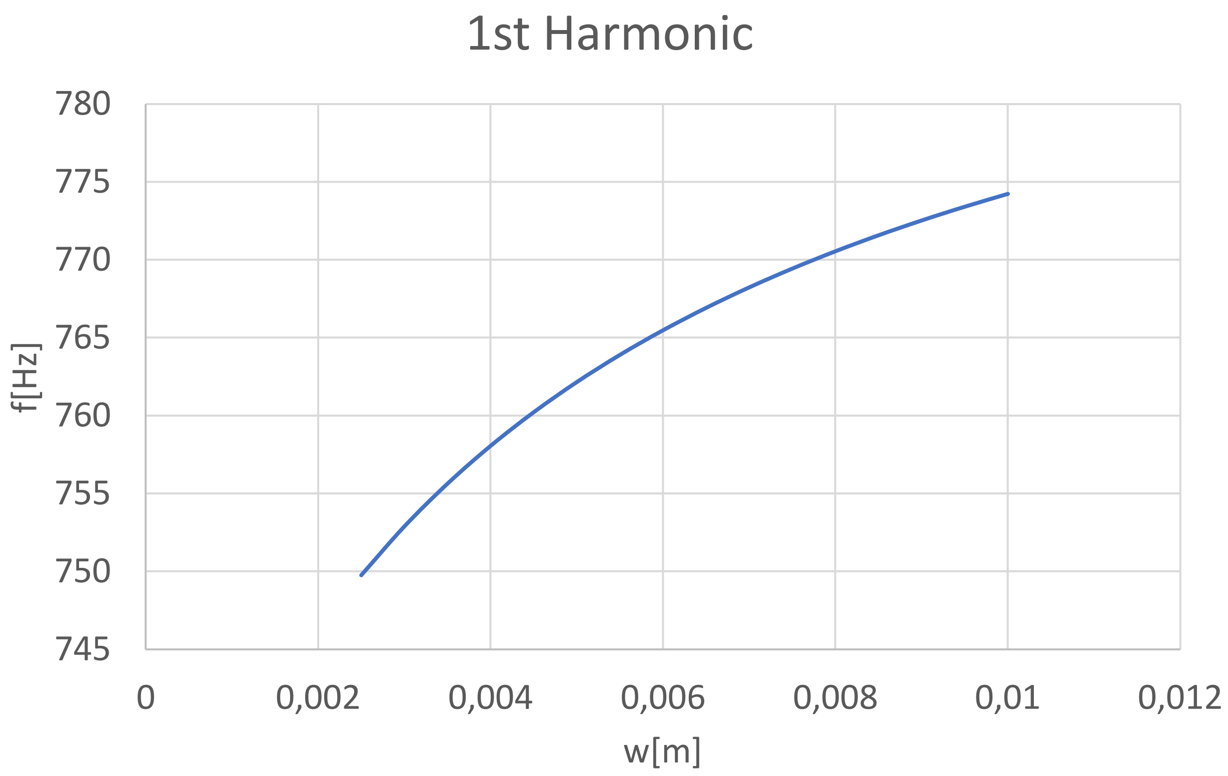


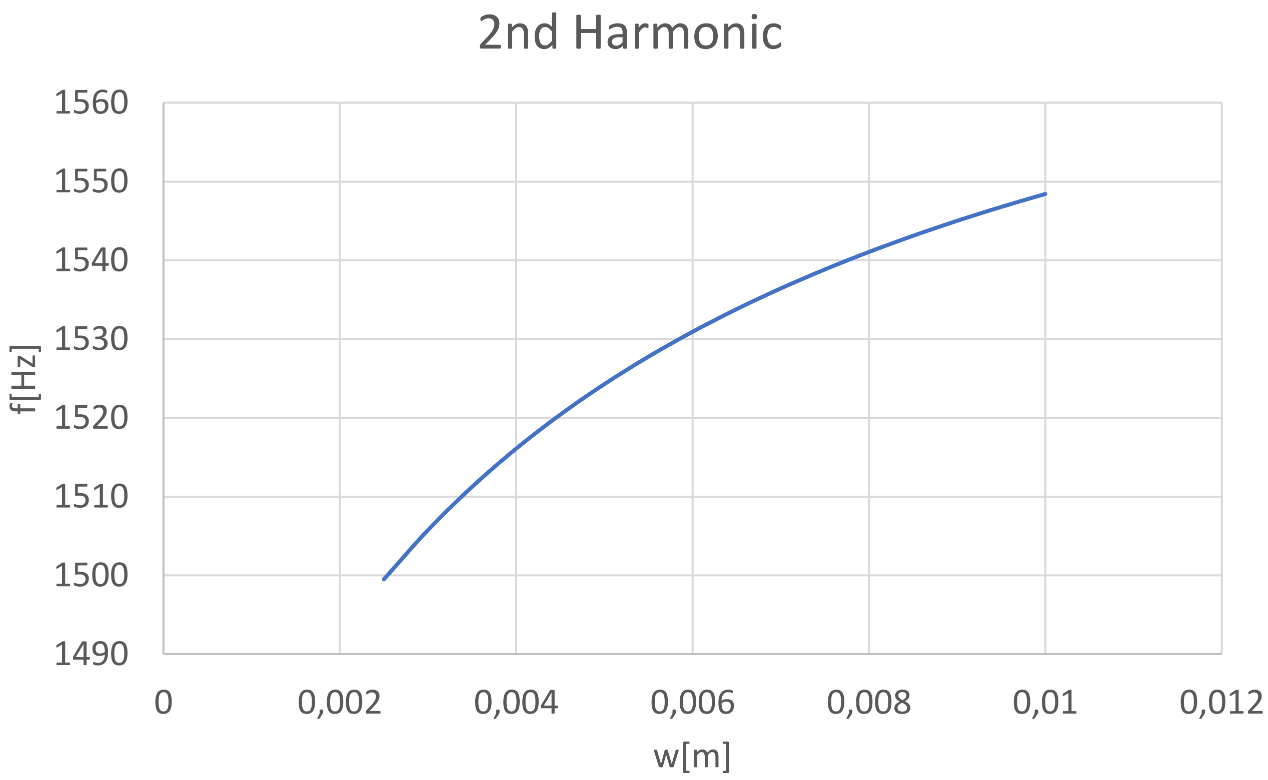
- Region of increased resonance → higher amplitude
- $f_v \approx f_p$ in this region

Error estimation

- Rotating setup → bending and wiggling of the tube at rotating end
- Small temeperature variations affecting speed of sound
- Determination of St values with CFD

Different distance between corrugation - f_p





Different width between corrugation - ω

- Lost access to our 3D-printer
- Only theoretical model possible

Smooth tubes giving no significant sound

- Without corrugations there is only very little turbulence and a very high fluid velocity is needed to hear a very weak fundamental tone of smooth pipe
- The corrugations and the vortices enhance the resonance freqeuncy making it a lot louder and clearer to hear
- We also tried this and in our case we didn‘t hear a sound ourself but on the audio spectrum a very small peak was found

Calculating error

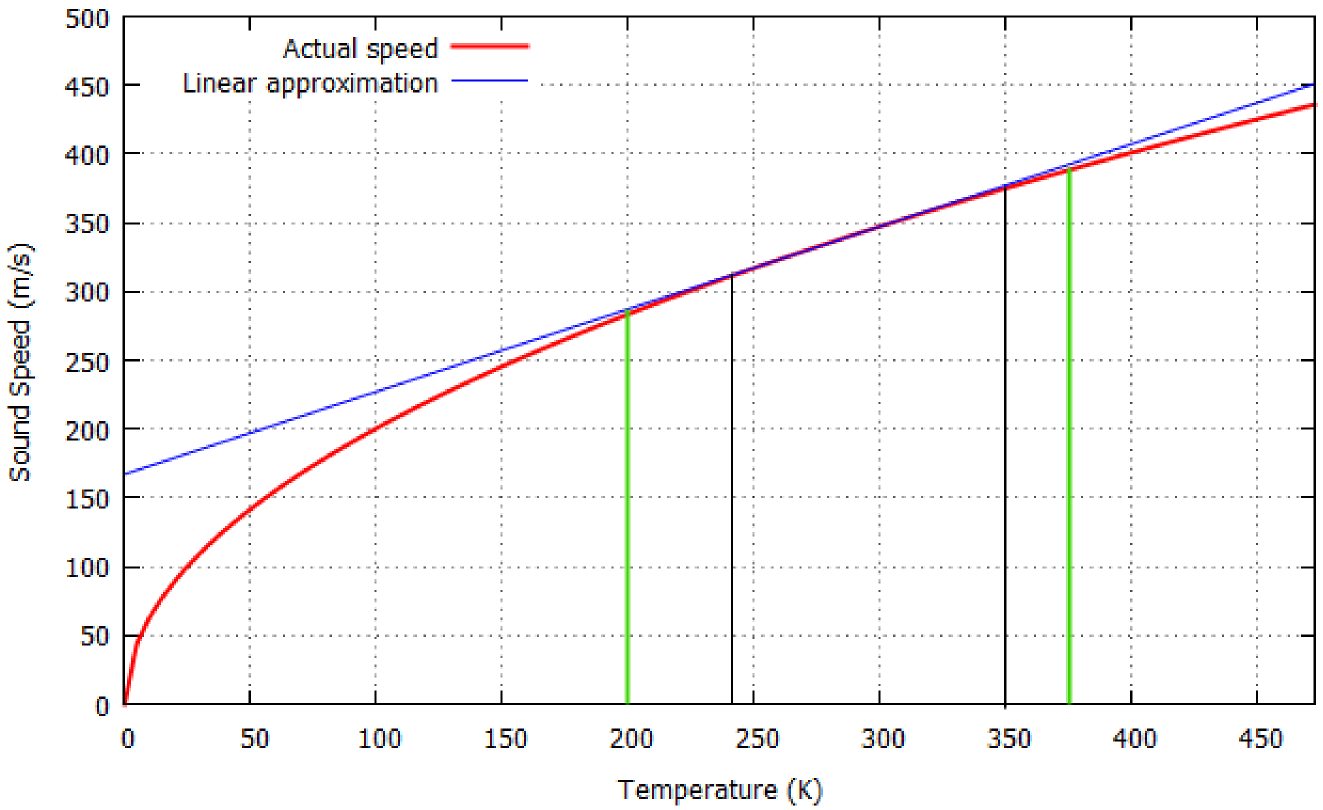
- We calculated standard deviation via:
- $s = \sqrt{\frac{1}{N-1} \cdot \sum_{i=1}^{N} (x_i - \bar{x})^2}$
- x_i are the observed values and $\bar{x}$ is their mean value
- There were always 3 runs for each measurement of sound frequencies and angular frequencies
- There were always 5 runs for each St value obtained for corrugations dimension

Dependancy of c on temperature

- Dependancy of speed of sound:

$$c = \sqrt{\frac{\gamma \cdot R \cdot T}{M}}$$

$$c \propto \sqrt{T}$$



γ...adiabatic index

R...molar gas constant

T...temperature of air in Kelvin

M...molar mass of air