

# Aircraft Aeroacoustics 0860395

## Project 1 - Fundamentals in Aeroacoustics

November 16, 2025

In this project, you will engage with fundamental aeroacoustic analysis, which will help you develop a basic understanding, as well as the tools and scripts you will be using later in the course. There are three sections to be answered in this project, as presented below. In part A (55 points of the project grade), you will learn about the sound pressure level and how it can be used to yield fast noise estimates of aerial systems. In part B (20 points of the project grade), you will be introduced to human noise perception effects and learn about common metrics. Finally, in part C (25 points of the project grade), you will compute the sound pressure level of a DJI rotor from a microphone measurement taken in an anechoic chamber, learning about common signal processing tools in acoustics. A total of 20 bonus points is available in this project (where a bolded **Bonus** text is presented); thus, the maximum grade available for this project is 120.

### Part A - Sound Pressure Level

**55 points (+ 10 bonus points)**

A sound disturbance from a compact or point source with a specific sound power,  $W$ , produces a spherical pressure wavefront that spreads out in time as it propagates away from the location of the original disturbance, a phenomenon known as ‘spherical spreading’ (see Fig. 1). Traditionally, in acoustics terminology, the sound power is denoted by the symbol  $W$ , but it should not be confused with the unit of power [W]. Nevertheless, the units of sound power are [W]. An assumption here is that spreading occurs in a free field, with no bounds or obstacles that could affect propagation. Sound power is the acoustic energy emitted per unit time and the rate at which the wavefront transfers sound energy. The intensity of the sound wave,  $I$ , is the time-averaged rate of energy flow per unit surface area or the sound power divided by the perpendicular area over which this power is spread (given in units of [W/m<sup>2</sup>]); i.e.,  $I = W/A$ . If the spherical pressure wave has a radius  $r$ , the original power of the wavefront is now spread out over a spherical surface of area  $A = 4\pi r^2$ . Therefore, the sound intensity can be shown to decrease inversely with the square of the distance ( $I \propto 1/r^2$ ).

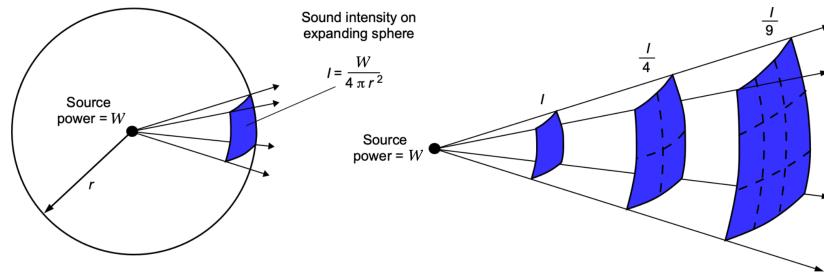
This result is known as the ‘inverse-square law of sound’. In the acoustic and geometric far field, sound waves propagate radially outward from a source and can be treated as locally **plane harmonic waves**. Thus, one can estimate the sound intensity in the far-field as  $I = p_{\text{rms}}^2 / \rho_o c_o$  (will be shown in class), where  $\rho_o$  and  $c_o$  are the background air density and speed of sound, respectively, and the root-mean-square (or “rms”) of the pressure is defined as:

$$p_{\text{rms}} = \sqrt{\frac{1}{2T} \int_{-T}^T [p(t) - p_o]^2 dt} = \sqrt{\frac{1}{2T} \int_{-T}^T [p'(t)]^2 dt} = \sqrt{\overline{[p'(t)]^2}}, \quad (1)$$

where  $T$  is the sample time period,  $p(t)$  is the pressure at a point in the fluid,  $p_o$  is the background pressure, and  $p'(t) = p(t) - p_o$  is the time-varying pressure perturbation. The intensity of sound and noise is quantified using the *decibel* (dB) scale, a logarithmic unit, named after Alexander Graham Bell [1]. The choice of a logarithmic scale better quantifies the wide range of sound pressures perceptible to the human ear, which has a logarithmic sensitivity to sound (see Section B). The corresponding relationship between the sound pressure level (SPL) and pressure is given below:

$$\text{SPL} = 10 \log_{10} \left( \frac{p_{\text{rms}}^2}{p_{\text{ref}}^2} \right) \quad (2)$$

where the units are stated as [dB(re  $p_{\text{ref}}$ )]. It is important to specify the reference sound pressure  $p_{\text{ref}}$  when quoting a result in decibels because different units are used in different applications. For almost all aeronautical applications, the standard is  $p_{\text{ref}} = 20 \mu\text{Pa}$ , which is the threshold of human hearing in air at 1,000 Hz. However, for applications of underwater acoustics, a common reference pressure of  $p_{\text{ref}} = 1 \mu\text{Pa}$  is used [2].



**Figure 1:** A sound disturbance produces a pressure wavefront that spreads spherically in time as it propagates away from the original disturbance.

- **A.1** The acoustic particle velocity  $u'_a$  of a time-harmonic plane wave can be estimated as  $p' = \rho_o c_o u'_a$ , where  $\rho_o$  and  $c_o$  are constants denoting the air density ( $\rho_o = 1.225 \text{ kg/m}^3$ ) and speed of sound ( $c_o = 340 \text{ m/s}$ ), respectively. Calculate the root-mean-square value of the acoustic velocity  $u_{a,\text{rms}}$  corresponding to a sound pressure level of 130 dB(re  $p_{\text{ref}} = 20 \mu\text{Pa}$ ). **(5 points)**.
- **A.2** The turbulence intensity  $Tu$  is a measure of the fluctuating velocity compared to the mean flow velocity  $U_\infty$ , defined as  $Tu = u_{\text{rms}}/U_\infty$ . Calculate the turbulent (fluctuating) velocity when  $Tu = 5\%$  for  $U_\infty = 30 \text{ m/s}$ . Compare  $u_{\text{rms}}$  with the acoustic velocity  $u_{a-\text{rms}}$  obtained in (A.1). **(5 points)**.

Assume two independent, plane harmonic, and stationary noise sources with pressure perturbations  $p'_1(t) = \mathcal{R}\{\hat{p}_1 e^{i(\omega_1 t + \phi_1)}\}$  and  $p'_2(t) = \mathcal{R}\{\hat{p}_2 e^{i(\omega_2 t + \phi_2)}\}$ , each has its own acoustic wave amplitude  $\hat{p}$ , frequency  $\omega$  in [rad/sec], and phase  $\phi$  in [rad].  $\mathcal{R}$  denotes the real component. Note that for a simple harmonic wave, the rms pressure relates to the pressure amplitude as  $p_{\text{rms}} = \hat{p}/\sqrt{2}$ . In linear acoustics, we can add multiple independent noise sources. For example, for the two noise sources above, we can invoke linear superposition to write:  $p'(t) = p'_1(t) + p'_2(t)$ .

- **A.3** Prove that when the two noise sources,  $p'_1(t)$  and  $p'_2(t)$ , are **incoherent (uncorrelated, randomly phased)** and summed linearly, their cross product term vanishes in the averaging process (over many samples) when computing  $p_{\text{rms}}$ , resulting in **(10 points)**:

$$p_{\text{rms}}^2 = p_{1,\text{rms}}^2 + p_{2,\text{rms}}^2 = \frac{1}{2} (\hat{p}_1^2 + \hat{p}_2^2). \quad (3)$$

- **A.4** Assume the two noise sources above,  $p'_1(t)$  and  $p'_2(t)$ , are **coherent (correlated)**; i.e., they share the same frequency  $\omega$ . When the two noise sources are randomly phased, prove that the rms of the pressure is now: **(10 points)**.

$$p_{\text{rms}}^2 = \frac{1}{2} [\hat{p}_1^2 + \hat{p}_2^2 + 2\hat{p}_1\hat{p}_2 \cos(\phi_2 - \phi_1)], \quad (4)$$

and that when the two noise sources are in-phase ( $\phi_1 = \phi_2$ ), the rms of the pressure can be written as:

$$p_{\text{rms}}^2 = (p_{1,\text{rms}} + p_{2,\text{rms}})^2. \quad (5)$$

- **A.5** A common, and quite simple, method to estimate the total noise emitted from a drone's rotors is by combining the noise spectra of its  $n$  rotors as independent (uncorrelated) far-field acoustic sources, assuming  $n$  identical power and incoherent noise sources. Given the sound pressure level (SPL) of a single rotor noise source,  $\text{SPL}_i = 10\log_{10}(p_{i,\text{rms}}^2/p_{\text{ref}}^2)$ , prove that the total SPL of a drone with  $n$  rotors (identical sound sources with the same power) is given by:  $\text{SPL}_{\text{tot}} = 10\log_{10} n + \text{SPL}_i$ . **(5 points)**.

- **A.6** A DJI drone has four identical rotors. During acoustic measurements in the anechoic chamber, you found the sound pressure level of a single rotor is  $SPL = 60 \text{ dB} (\text{re } p_{\text{ref}} = 20 \mu\text{Pa})$ . The DJI company is now considering increasing the number of rotors to improve the drone's performance, but they are unsure of the acoustic impact of such a design change. Help the DJI company evaluate the SPL of a DJI drone with 4, 6, and 8 identical rotors, by assuming the drone rotors are independent far-field acoustic sources with identical power and are considered as the major noise source of the drone. (**5 points**).
- **A.7** How would your answer in (A.6) change if the rotors are considered as **coherent (correlated)** noise sources and in-phase? (**5 points**).
- **A.8** Given a certain noise level of the drone  $SPL_1$  measured at a distance  $r_1$  from the drone, the overall attenuated sound pressure level of the drone at the observer location  $r_2$  is  $SPL_2$ , and is computed by considering geometrical attenuation effect, as expressed below:

$$SPL_2 = SPL_1 - 10 \log_{10} \left( \frac{r_2^2}{r_1^2} \right), \quad (6)$$

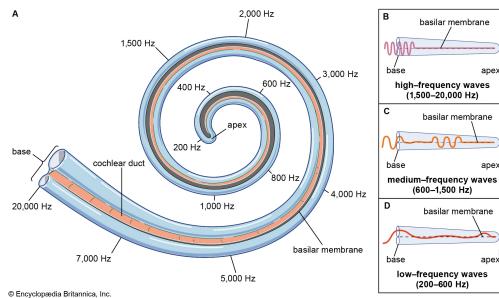
Assume the drone noise levels you computed in (A.6) are estimated at a distance  $r_1 = 1.5 \text{ m}$ . For each configuration of the DJI drone (with 4, 6, and 8 rotors), compute what shall be the distance the drone needs to hover in place above a stationary observer, such that the SPL at the observer location matches the value of a single rotor measured in the anechoic chamber, that is  $60 \text{ dB} (\text{re } p_{\text{ref}} = 20 \mu\text{Pa})$ . What will be the resulting distances of the drone from the observer (for the three drone configurations), assuming the rotor noise sources are coherent? Please discuss the differences you found. (**10 points**).

- **A.9 Bonus** In theory, would it be possible to reduce the drone SPL to 0 dB? If so, please describe the conditions/assumptions under which this can be met. Please discuss whether this can be achieved in practice, if the sound field disappears locally or globally, and what the limitations are. (**10 points**).

## Part B - Human Noise Perception

20 points

The ear responds to the pressure fluctuations of sound waves to cause the sensation of hearing. When calculating noise levels and designing quiet systems, it is always important to keep the end goal in mind. This is most often the level of annoyance experienced by a human observer. The human ear has a remarkable dynamic range and can hear sound waves with amplitudes as low as  $20 \mu\text{Pa}$  and as high as  $200 \text{ Pa}$  before encountering the threshold of pain (see schematics of the human ear cochlea in Fig. 2). The human ear exhibits a nonlinear spectral sensitivity in terms of frequency versus amplitude (due to the varying physical properties of the basilar membrane within the cochlea). For example, humans do not perceive low frequencies (less than  $20 \text{ Hz}$ ) or high-frequency sounds (more than  $20 \text{ kHz}$ ). The frequencies of most concern to humans usually lie between about  $350 \text{ Hz}$  and  $10 \text{ kHz}$ , roughly corresponding to wavelengths between  $1 \text{ m}$  and  $3 \text{ cm}$  in air. The ear is most sensitive to sounds between  $1 \text{ kHz}$  and  $4 \text{ kHz}$ , which is unsurprisingly the frequency range of human speech. Because of the logarithmic sensitivity of the human ear, we measure sound in SPL using a decibel scale.



**Figure 2:** The analysis of sound frequencies by the basilar membrane. (A) The fibers of the basilar membrane become progressively wider and more flexible from the base of the cochlea to the apex. As a result, each area of the basilar membrane vibrates preferentially to a particular sound frequency. (B) High-frequency sound waves cause maximum vibration of the area of the basilar membrane nearest to the base of the cochlea; (C) medium-frequency waves affect the center of the membrane; (D) low-frequency waves preferentially stimulate the apex of the basilar membrane [<https://www.britannica.com/science/ear/Transmission-of-sound-within-the-inner-ear>].

Often, the audible frequency range of noise (from 20 Hz to 20 kHz) is filtered into octave bands [1], which appear as equal intervals on a logarithmic scale. This is to account for the fact that the human ear discriminates sound frequencies in a logarithmic manner. A band is said to be an *octave* in width when the upper band frequency  $f_u$  is twice the lower band frequency  $f_l$ ; i.e.,  $f_u/f_l = 2$ . Since octaves are not linear scales, higher-frequency bands are wider than lower-frequency bands. The frequency interval over which measurements are made is called the bandwidth. The bandwidth may be described by the lower frequency of the interval  $f_l$  and the upper frequency of the interval  $f_u$ , where the mid-band (logarithmic center) frequency is  $f_c = \sqrt{f_u f_l}$ , which is always less than the arithmetic mean frequency,  $0.5(f_l + f_u)$ . For example, an octave has a mid-band frequency of  $f_c = \sqrt{2}f_l = f_u/\sqrt{2}$ , and when applied to the audio spectrum, can divide it into up to 11 octave bands, where  $20 \text{ Hz} \lesssim f_c \lesssim 20 \text{ kHz}$ . In some cases, a more refined division of the frequency range is used, such as one-third octave band, in which  $f_u/f_l = 2^{1/3}$ . For the third-octave band spectrum, the mid-band frequencies in [Hz] are given by the relation:

$$f_c = 1000 \times 2^{(n-30)/3} \quad (7)$$

where  $n$  is known as the band number [3]. The frequency content of noise is therefore often characterized by summing up the sound into one-third octave bands (a total of 32 bands) that appear as equal intervals on a logarithmic scale. Thus, the one-third band extends from  $f_l = f_c \times 2^{-1/6}$  to  $f_u = f_c \times 2^{1/6}$ . A plot of SPL in each band against the mid-band center frequency is called the one-third-octave band spectrum. Please note, the third-octave, tenth-octave, and twelfth-octave bands are sub-intervals of one octave. An octave is spanned by three 1/3-octave bands and 12 1/12-octave bands. Table 1 presents a summary of commonly used octave bands as well as the general form.

**Table 1:** Summary of logarithmic octave bands [1].

Frequency band	1 octave	1/3 octave	1/10 octave	1/n octave
Upper frequency/ lower frequency, $f_u/f_l$	2	$2^{1/3}$	$2^{1/10}$	$2^{1/n}$
Center frequency, $f_c = \sqrt{f_u f_l}$	$\sqrt{2}f_l$	$2^{1/6}f_l$	$2^{1/20}f_l$	$2^{1/2n}f_l$
Bandwidth, $(f_u - f_l)/f_c$	0.7071	0.2315	0.0693	$(2^{1/n} - 1)2^{-1/2n}$

- **B.1** Write a MATLAB code to calculate the lower, center, and upper bands of 1-octave, 1/3-octave, and 1/12-octave spectra. Start your calculation from the center frequency  $f_c = 1000 \text{ Hz}$ . **(5 points)**

Because the frequency response of human hearing is nonlinear, psychoacoustic weightings have been established to measure and compare sound pressures. To obtain a measure of the perceived loudness of a sound, the most commonly used metric is the dB(A) level [4, 5]. The A-weighted sound pressure (see Fig. 3), denoted as dBA or dB(A), was designed to mimic the human ear's sensitivity to different frequencies at lower sound levels. The weighting attenuates the contributions of low and high frequencies while emphasizing the mid-range around 1 kHz, where human hearing is most sensitive. Note especially that the ear does not respond well to frequencies below 20 Hz, even though low-frequency sounds are sometimes identified as most irritating. Since dB(A) accounts for the ear's sensitivity and is easy to measure directly with a sound level meter, it is widely used in noise control applications. It has also been found to be a good measure of annoyance, and so most noise ordinances are defined using this unit with corrections for duration, pure tones, and day/night levels.

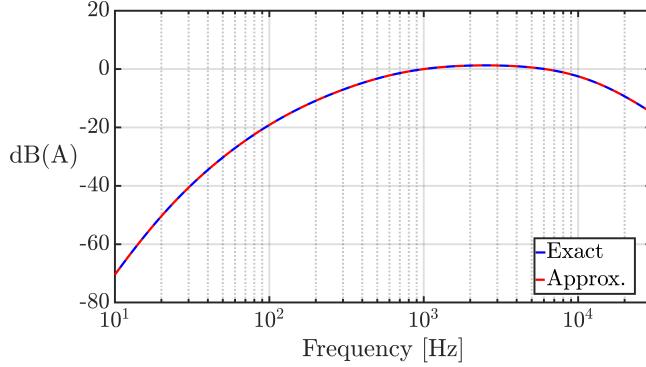
The dB(A) weighting,  $A(f)$ , is measured in [dB] units and is computed using the weighting function  $R_A(f)$  (applied to the amplitude spectrum and not the intensity spectrum of the unweighted sound level), as follows:

$$R_A(f) = \frac{a_1^2 f^4}{(f^2 + a_2^2)\sqrt{(f^2 + a_3^2)(f^2 + a_4^2)}(f^2 + a_1^2)} \quad (8)$$

$$A(f) = 20\log_{10}(R_A(f)) - 20\log_{10}(R_A(1000))$$

where  $a_1 = 12194$ ,  $a_2 = 20.6$ ,  $a_3 = 107.7$ , and  $a_4 = 737.9$ . As a close approximation, one can also write  $A(f) \approx 20\log_{10}(R_A(f)) + 2.00$ .

- **B.2** A propeller emits a strong tone at 400 Hz, and the frequency is found to increase linearly with the propeller RPM. Using the weighting shown in Fig. 3, show that the dB(A) level will be reduced by 6 dB if the speed of the propeller is reduced by a factor of 2 and the source level remains the same. What is the result if the speed is doubled? (**10 points**).
- **B.3** It is often found that the source level of a propeller is a function of RPM and the SPL is given by the empirical formula  $SPL = C + 60\log_{10}(\text{RPM})$ , where  $C$  is a constant. How does this alter the results obtained in (B.2)? (**5 points**).



**Figure 3:** The dB(A) weighting scale, representing typical human ear sensitivity.

## Part C - Signal Processing in Acoustics

**25 points (+ 10 bonus points)**

Fourier transforms are perhaps the most important mathematical tool of aeroacoustic analysis. Except where otherwise noted, we define the Fourier transform of a time history as:

$$\tilde{p}(\omega) = \frac{1}{2\pi} \int_{-T}^{T} p'(t) e^{i\omega t} dt \quad (9)$$

where  $T$  tends to infinity, and the inverse Fourier transform is:

$$p'(t) = \int_{-\infty}^{\infty} \tilde{p}(\omega) e^{-i\omega t} d\omega \quad (10)$$

where  $\omega$  is the angular frequency. Generally, Eq. 9 is considered as the continuous-time Fourier transform (CTFT) of an analog signal  $y(t)$ , mapping it from the *continuous-time domain* to the *continuous frequency domain*. Since microphone data is recorded digitally (either experimentally or numerically), when analyzing acoustic data, one has to use the discrete Fourier transform (DFT). The DFT of a digital signal  $y[k]$  maps the discrete-time domain to the discrete frequency domain. Note that the DFT is symmetric with respect to the *Nyquist rate*. This also allows the DFT to be visually interpreted in the  $[0, F_s/2]$  band, where  $F_s$  is the sampling rate in [Hz]. The DFT is defined as:

$$A_m = \sum_{n=1}^{N} a_n e^{-\frac{2\pi i(n-1)(m-1)}{N}} \quad (11)$$

for  $m = 1$  to  $N$ . The inverse DFT (IDFT) is defined as:

$$a_n = \frac{1}{N} \sum_{m=1}^N A_m e^{\frac{2\pi i(n-1)(m-1)}{N}} \quad (12)$$

for  $n = 1$  to  $N$ . Eqs. 11-12 are rarely computed explicitly as written since this is an expensive calculation requiring  $O(N^2)$  operations. Instead, the *Fast Fourier transform* algorithm [6] is used, which takes advantage of efficiencies that become possible when  $N$  is a composite number, and particularly when it is a power of 2. This reduces the computational effort to  $O(N\log N)$  - a huge saving when large data sets are involved! In MATLAB, DFT and IDFT are done using the `fft()` and `ifft()` functions.

- **C.1** Write a MATLAB script to compute the Fourier transform of:

$$f(x) = 10\sin(t) \quad (13)$$

using the built-in `fft()` function. Validate your results with the analytical solution, and plot the two functions. Take the inverse Fourier transform to retrieve the  $f(x)$  (both using the built-in `ifft()` function and analytically). Study the two cases, corresponding to a low and high frequency resolution, and motivate your choice. (**15 points**).

The Fourier transform  $\tilde{p}(\omega)$  is not by itself a very useful measure since, just like  $p'(t)$ , it will vary stochastically. The appropriate average measure of the frequency content is given by the **autospectral density** of  $p'$  (often referred to as the autospectrum, the power spectrum, or just the spectrum), defined as:

$$S_{pp}(\omega) = \frac{1}{2\pi} \int_{-T}^T R_{pp}(\tau) e^{i\omega\tau} d\tau \quad (14)$$

where  $R_{pp}(\tau)$  is the time-delay auto-correlation function, defined as:

$$R_{pp}(\tau) = E[p'(t)p'(t + \tau)] \quad (15)$$

where  $E$  is the expected value. This is the mean of the value of a stochastic variable taken over many repeated realizations of the same flow. In each realization, we imagine running the flow under conditions identical in all respects, except for the stochastic behavior. Thus, we can imagine obtaining multiple independent samples of our flow quantity  $p'$  at the same defined position and time. These independent samples can now be averaged to obtain a mean that remains dependent on time:

$$E[p'(t)] = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N p'_n(t) \quad (16)$$

where  $p'_n(t)$  is the  $n$ -th sample of  $p'$ , and  $N$  is the total number of samples taken. The expected value required in Eq. 15 was obtained by calculating the product  $p'(t)p'(t+\tau)$  for every time instant at which the signal was measured (assuming a sampling rate of  $F_s$ ) and then taking the mean value of those numbers. As can be seen from Eq. 15,  $R_{pp}(\tau)$  is the average of the signal multiplied by itself at a later time. For a time stationary signal, the expected value of  $p'(t)p'(t+\tau)$  will not depend on  $t$ . The inverse Fourier transform relates the spectrum back to  $R_{pp}(\tau)$ :

$$R_{pp}(\tau) = \int_{-\infty}^{\infty} S_{pp}(\omega) e^{-i\omega\tau} d\omega \quad (17)$$

It is noteworthy that by definition, the time-delay auto-correlation function (see Eq. 15) is even; i.e., it is symmetric about  $\tau = 0$  because  $E[a(t)a(t+\tau)] = E[a(t-\tau)a(t)] = E[a(t)a(t-\tau)]$ . That means that the autospectral density of  $p'$ ,  $S_{pp}(\omega)$ , is a real and even function of frequency.

In general, the spectrum of a time history  $p'(t)$  can be physically interpreted as revealing the contributions to the mean square fluctuation  $\overline{p'^2}$  at each frequency. This can be demonstrated very simply by using the inverse transform relationship, Eq. 17, which, for zero-time delay  $\tau$  becomes:

$$R_{pp}(0) = \overline{p'^2} = p_{\text{rms}}^2 = \int_{-\infty}^{\infty} S_{pp}(\omega) d\omega \quad (18)$$

The result in Eq. 18 is known as **Parseval's theorem**, which states that the “power” in the time and frequency domains is the same and that the spectrum divides up that power by frequency. (Note that the term “power” is loosely used in spectral analysis to refer to the mean square). It is in this sense that  $S_{pp}(\omega)$  is a spectral density function, with units of  $p'^2$  per radian-per-second; i.e., [Pa<sup>2</sup>/rad·Hz].

Please note that the mathematical definition of the spectral density includes both positive and negative frequencies. In the context of one-dimensional spectra, these mean the same thing. However, note from Eq. 18 that the energy is spread over both the positive and negative domains. Thus, the  $S_{pp}(\omega)$  spectrum is referred to as *double-sided*, which is the norm in mathematical analysis. However, for predictions or measurements of sound spectra, it is normal to consider them to exist only for positive frequencies and to double the spectral values. Therefore, the spectrum should be treated as *single-sided*, for which we introduce the symbol  $G_{pp}(\omega) = 2S_{pp}(\omega)$  for  $\omega > 0$  ( $G_{pp}(\omega < 0) = 0$ )

The fact that  $S_{pp}(\omega)$  (and by extension  $G_{pp}(\omega)$ ) is a density function means that we expect it to integrate into a defined physical quantity. This limits the ways we can scale it and the frequency variable it depends on. For example, if we wanted to express our spectrum as a function of frequency  $f$  [Hz], then we

would write  $S_{pp}(f) = 2\pi S_{pp}(\omega)$  since we are expecting that:

$$\overline{p'^2} = p_{\text{rms}}^2 = \int_{-\infty}^{\infty} S_{pp}(f) df \quad (19)$$

as  $d\omega/df = 2\pi$ . By taking only the *single-sided* spectrum, one can write:

$$\overline{p'^2} = p_{\text{rms}}^2 = \int_0^{\infty} G_{pp}(f) df \quad (20)$$

Here,  $G_{pp}(f)$  is the *single-sided spectral density* spectrum, given in units of [Pa<sup>2</sup>/Hz]. While one can compute the spectral density using the DFT, a rather more common method to estimate  $G_{pp}(f)$  is by using **Welch's method** [7]. This is a statistical technique for estimating a signal's spectral density by dividing the time series data into overlapping segments, applying a window to each segment, and then averaging the resulting periodograms. This averaging process reduces the variance of the estimate, yielding a smoother, more reliable power spectrum than simple periodogram methods (e.g., DFT). In MATLAB, Welch's method can be used using the ***pwelch()*** function to estimate the spectral density spectrum (by default) of a time-dependent signal.

Using the spectral density definition, one can present acoustic pressure spectra in [dB] using the *narrow band* sound pressure level, as follows:

$$L_p(f) = 10\log_{10} \left( \frac{G_{pp}(f)\Delta f}{p_{\text{ref}}^2} \right) \quad [\text{dB} (\text{re } p_{\text{ref}} = 20\mu\text{Pa})] \quad (21)$$

Alternatively, one can compute the power spectral density (PSD) in [dB/Hz] according to:

$$PSD(f) = 10\log_{10} \left( \frac{G_{pp}(f)}{p_{\text{ref}}^2} \right) \quad \left[ \frac{\text{dB}}{\text{Hz}} \right] \quad (22)$$

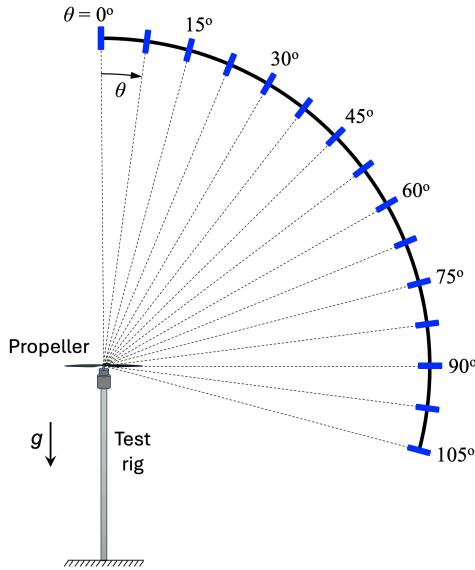
As discussed in Section B, the 1/3-octave spectrum divides the spectrum into frequency bands, with each band being  $2^{1/3}$  times the size of the preceding band. In this case, the SPL in the  $n$ -th band is given by:

$$L_p(n) = 10\log_{10} \left( \frac{1}{p_{\text{ref}}^2} \int_{f_l^{(n)}}^{f_u^{(n)}} G_{pp}(f) df \right) \quad [\text{dB} (\text{re } p_{\text{ref}} = 20\mu\text{Pa})] \quad (23)$$

where the lower and upper frequency limits of each band are defined in terms of the mid-frequency  $f_c$  according to Table 1. Please note that integrating  $G_{pp}(f)$  across the full frequency domain yields the overall sound pressure level (OASPL) of the measured signal, accordingly:

$$\text{OASPL} = 10\log_{10} \left( \frac{1}{p_{\text{ref}}^2} \int_0^{\infty} G_{pp}(f) df \right) = 10\log_{10} \left( \frac{p_{\text{rms}}^2}{p_{\text{ref}}^2} \right) \quad [\text{dB}] \quad (24)$$

- **C.2** The acoustic emission of a DJI rotor (DJI 9450) operating in hover conditions at  $\omega = 6,700$  [rev/min] was measured in the anechoic chamber by a microphone array (see Fig. 4). The microphone array was placed at a radial distance of  $r = 1.5$  m from the rotor hub (center) and at an azimuth angle varying from  $\theta = 0^\circ$  to  $\theta = 105^\circ$ . Download the .zip file with the microphone data recorded in the experiments from this [LINK](#). **Please read the README.txt file!** Write a MATLAB script to compute the SPL spectrum of the noise signal measured by the microphone placed at  $\theta = 105^\circ$  using both the `fft()` and `pwelch()` functions. When using the `pwelch()` function, apply 50% window overlap and a Hanning window, and use a frequency resolution of  $\Delta f = 1.235$  Hz. Compare the noise spectrum obtained from each method and discuss the results in terms of the spectrum shape and whether Parseval's theorem is being satisfied. What technique (DFT or Welch's) would you recommend using when analyzing acoustic data of rotors? **(10 points)**.
- **C.3 Bonus** Compute the SPL spectra for all microphones using the method you find to work best in (C.2). Discuss the effect of directivity ( $\theta$ ) on the noise spectrum. What could be the source of such an effect? **(10 points)**.



**Figure 4:** Schematic of the rotor rig and microphone array

## References

- [1] Robert D. Blevins. *Formulas for dynamics, acoustics, and vibration*. John Wiley & Sons, Ltd, 2016.
- [2] Stewart Glegg and William Devenport. *Aeroacoustics of low Mach number flows*. Elsevier, 2<sup>nd</sup> edition, 2024.
- [3] Acoustic Society of America. *Specification for octave-band and fractional-octave-band analog and digital filters, ANSI S1.11-2004*. Acoustical Society of America, Melville, NY, 2004.
- [4] Acoustic Society of America. *Design response of weighting networks for acoustical measurements, ANSI/ASA S1.42*. Acoustical Society of America, Melville, NY, 2020.
- [5] Wikipedia. A-weighting. Technical report, 2025. <https://en.wikipedia.org/wiki/A-weighting>.
- [6] J. W. Cooley and J. W. Tukey. An algorithm for the machine calculation of complex Fourier series. *Math. Comput.*, 19:297–301, 1965.
- [7] P. Welch. The use of fast Fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms. *IEEE Transactions on Audio and Electroacoustics*, 15(2):70–73, 1967.