Q1

In [1]:
```
import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.graphics import tsaplots
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.tsa.holtwinters import SimpleExpSmoothing
from statsmodels.tsa.holtwinters import ExponentialSmoothing
from statsmodels.tsa.exponential_smoothing.ets import ETSModel
from prophet import Prophet
import warnings
warnings.filterwarnings('ignore')
```

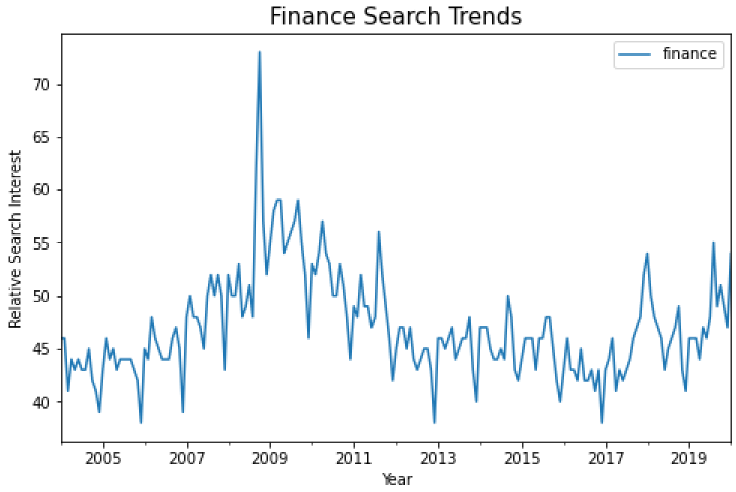
```
Importing plotly failed. Interactive plots will not work.
```

A

In [2]:
```
finance = pd.read_csv("finance_popularity.csv")
```

In [3]:
```
def preprocess_and_plot(df, col = "month", freq='MS', title = "Finance Search Trends",
                        xlabel = 'Year', ylabel = 'Relative Search Interest', start = "", end = ""):
    df[col] = pd.date_range(start = df[col][0] if start == "" else start,
                            end = df[col].to_numpy()[-1]  if end == "" else end, freq=freq)
    df.set_index(col, inplace = True)
    df.plot(figsize = (8,5))
    plt.title(title,fontsize=15)
    plt.xlabel(xlabel, fontsize=10)
    plt.ylabel(ylabel, fontsize=10)
    plt.show()
    return df
```

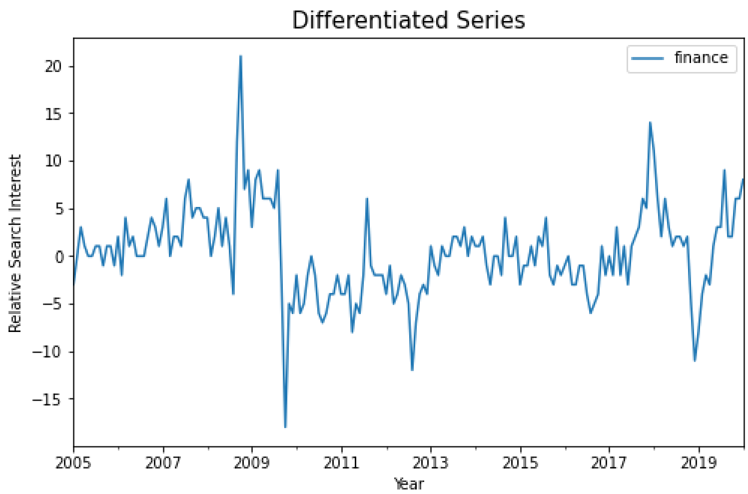
In [4]:
```
finance = preprocess_and_plot(finance)
```



there seems to be a seasonality of 12 months according to the local bottom then a local peak at the start of each year, thus the data is not stationary

B

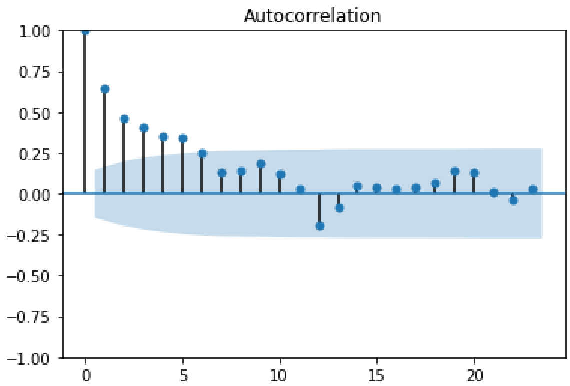
In [5]:
```
def plot_diff_series(df = finance, periods=12, xlabel= "Year", ylabel = 'Relative Search Interest'):
    df_diff=df.diff(periods=periods)
    df_diff=df_diff.dropna()

    df_diff.plot(figsize = (8,5))
    plt.title("Differentiated Series",fontsize=15)
    plt.xlabel(xlabel, fontsize=10)
    plt.ylabel(ylabel, fontsize=10)
    return df_diff
finance_year_diff = plot_diff_series()
```



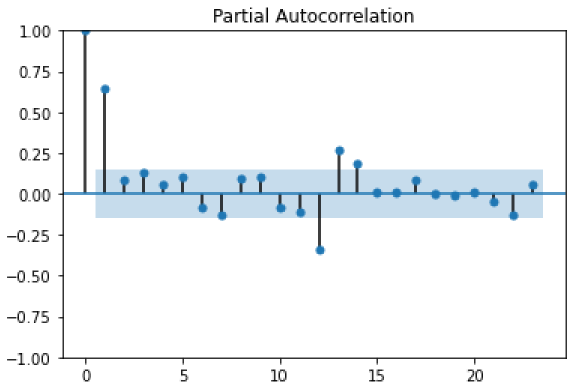
we removed the yearly seasonality and also we can see that there is no trend as the data is disributed around 0 therefore the data is stationary

C

In [6]:
```
tsaplots.plot_acf(finance_year_diff);
```



In [7]:
```
tsaplots.plot_pacf(finance_year_diff);
```



the pacf is around zero at 3 or more lags, thus we can assume that a good model would be $AR(2)$ or $SARIMA(2,0,0)(1,1,0)_{12}$

D

$SARIMA(2,0,0)(1,1,0)_{12}$

In [8]:
```
def fit_summarize_sarima(data ,order, seasonal_order = (0,0,0,0)):
    sarima_model = SARIMAX(data, order=order,seasonal_order=seasonal_order, freq="MS")
    sarima_model_fit = sarima_model.fit(disp=False)
    display(sarima_model_fit.summary())
fit_summarize_sarima(finance, (2, 0, 0), (1,1,0,12))
```

SARIMAX Results

Dep. Variable:	finance	**No. Observations:**	193
Model:	SARIMAX(2, 0, 0)x(1, 1, 0, 12)	**Log Likelihood**	-464.106
Date:	Wed, 14 Feb 2024	**AIC**	936.212
Time:	13:03:29	**BIC**	949.006
Sample:	01-01-2004	**HQIC**	941.399
	- 01-01-2020		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	0.5935	0.044	13.570	0.000	0.508	0.679
ar.L2	0.1923	0.063	3.075	0.002	0.070	0.315
ar.S.L12	-0.4594	0.044	-10.479	0.000	-0.545	-0.373
sigma2	9.6836	0.661	14.648	0.000	8.388	10.979

Ljung-Box (L1) (Q):	0.14	**Jarque-Bera (JB):**	175.39
Prob(Q):	0.71	**Prob(JB):**	0.00
Heteroskedasticity (H):	0.53	**Skew:**	0.76
Prob(H) (two-sided):	0.01	**Kurtosis:**	7.57

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

$AR(2)$

In [9]:
```
fit_summarize_sarima(finance, (2, 0, 0))
```

SARIMAX Results

Dep. Variable:	finance	**No. Observations:**	193
Model:	SARIMAX(2, 0, 0)	**Log Likelihood**	-514.193
Date:	Wed, 14 Feb 2024	**AIC**	1034.386
Time:	13:03:29	**BIC**	1044.174
Sample:	01-01-2004	**HQIC**	1038.350
	- 01-01-2020		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	0.8166	0.038	21.481	0.000	0.742	0.891
ar.L2	0.1813	0.039	4.674	0.000	0.105	0.257
sigma2	11.7378	0.776	15.125	0.000	10.217	13.259

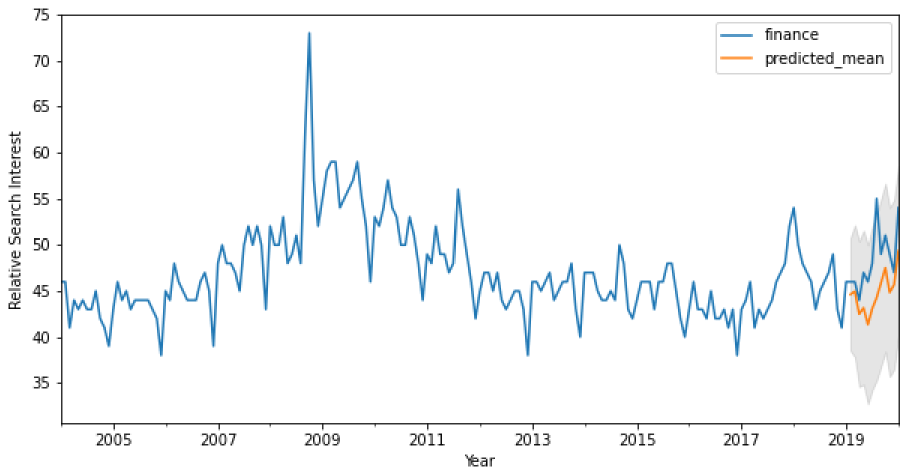
Ljung-Box (L1) (Q):	0.56	**Jarque-Bera (JB):**	66.56
Prob(Q):	0.45	**Prob(JB):**	0.00
Heteroskedasticity (H):	0.43	**Skew:**	0.35
Prob(H) (two-sided):	0.00	**Kurtosis:**	5.79

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

E

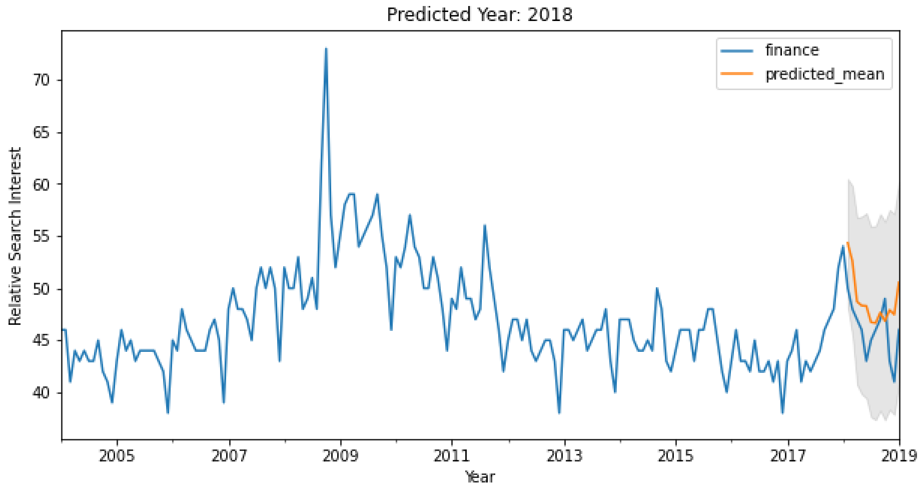
the $SARIMA(2,0,0)(1,1,0)_{12}$ model has a better AIC and BIC so we will choose this as our model, lets try to forcast the last year data (without fitting on it) to check how the model works.

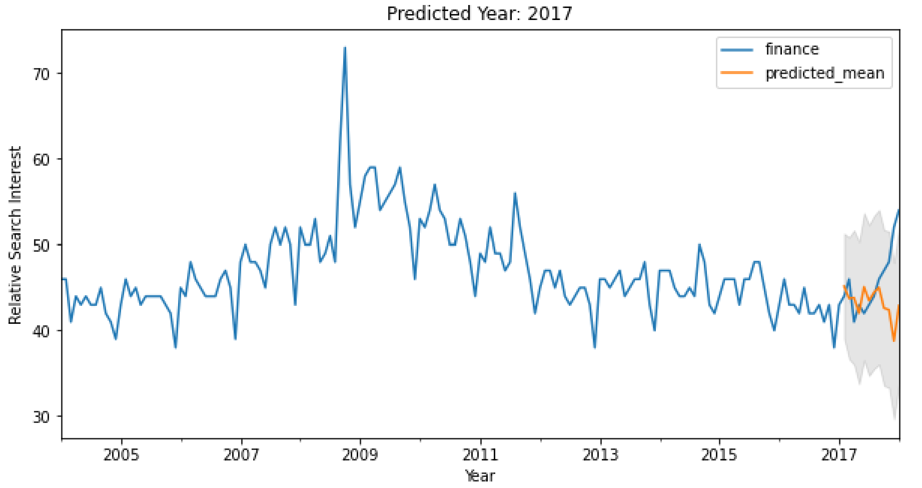
In [10]:
```
def forecast_with_sarimax(time_series, forecast_periods, order, seasonal_order, xlabel = 'Year',
                          ylabel = 'Relative Search Interest', freq='MS', plot=True, offset = 0, title =
"", forcast_next = False):
    if forcast_next:
        sarima_model = SARIMAX(time_series, order=order, seasonal_order=seasonal_order)
    else:
        sarima_model = SARIMAX(time_series[:-forecast_periods - offset], order=order, seasonal_order=seaso
nal_order)
    sarima_model_fit = sarima_model.fit(disp=False)

    next_periods_forecast = sarima_model_fit.get_forecast(steps=forecast_periods)
    next_periods_ci = next_periods_forecast.conf_int()

    if forcast_next:
        future_dates = pd.date_range(start=time_series.index[-1], periods=forecast_periods, freq=freq)
    else:
        future_dates = pd.date_range(start=time_series.index[-forecast_periods - offset], periods=forecast
_periods, freq=freq)
    next_periods_ci.index = future_dates

    futureDF_forecast = pd.DataFrame(next_periods_forecast.predicted_mean, index=future_dates)

    if plot:
        if offset == 0:
            ax = time_series.plot(label='Observed', figsize=(10, 5))
        else:
            ax = time_series[:-offset].plot(label='Observed', figsize=(10, 5))
        futureDF_forecast.plot(ax=ax, label='Forecast', legend=True)

        ax.fill_between(next_periods_ci.index,
                        next_periods_ci.iloc[:, 0],
                        next_periods_ci.iloc[:, 1], color='k', alpha=.1)
        ax.set_title(title)
        ax.set_xlabel(xlabel)
        ax.set_ylabel(ylabel)
        plt.legend()
        plt.show()
    return futureDF_forecast, next_periods_ci


future_finance_forecast, next_periods_ci = forecast_with_sarimax(finance, 12, order=(2, 0, 0),seasonal_ord
er=(1, 1, 0,12))
```

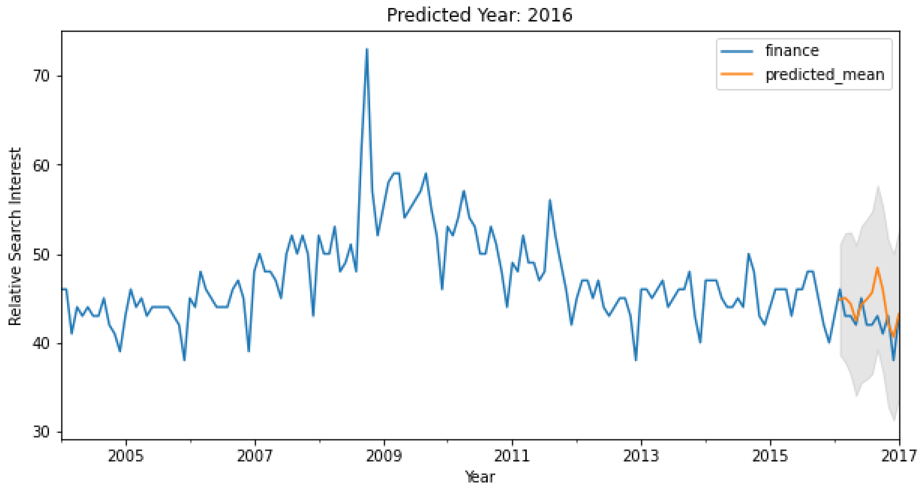


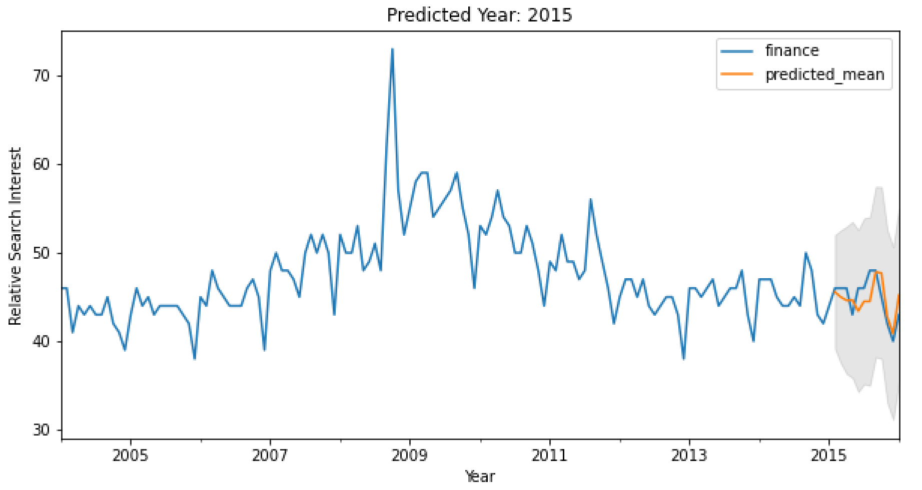
a one data sample for checking the model is rather low, lets create a loop to try several years

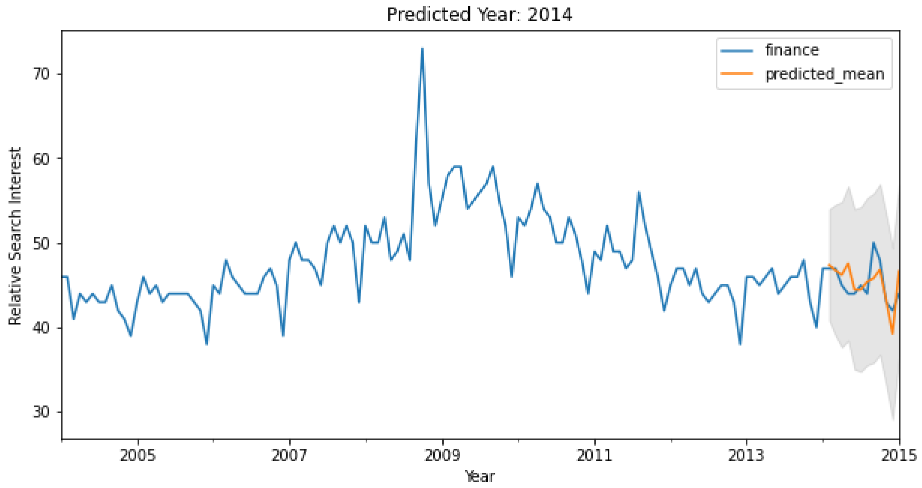
In [11]:
```
FORECAST = 12
OFFSET = 12
for year, OFFSET in zip(range(2018, 2010, -1), range(12, 9*12,12)):
    future_finance_forecast, next_periods_ci = forecast_with_sarimax(finance, FORECAST, order=(2, 0, 0),se
asonal_order=(1, 1, 0,12),
                                                                    offset = OFFSET, title = f"Predicted Y
ear: {year}")
```

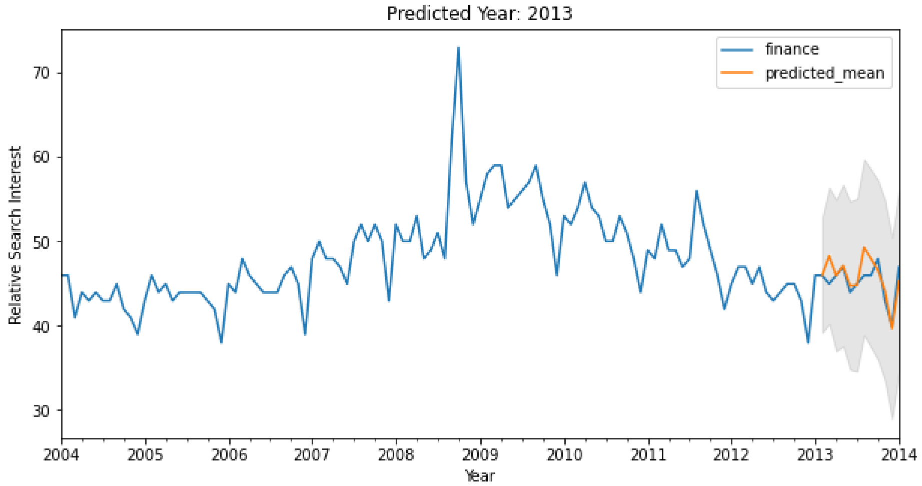

Predicted Year: 2018
finance
predicted_mean
Relative Search Interest
Year
40
45
50
55
60
65
70
2005
2007
2009
2011
2013
2015
2017
2019

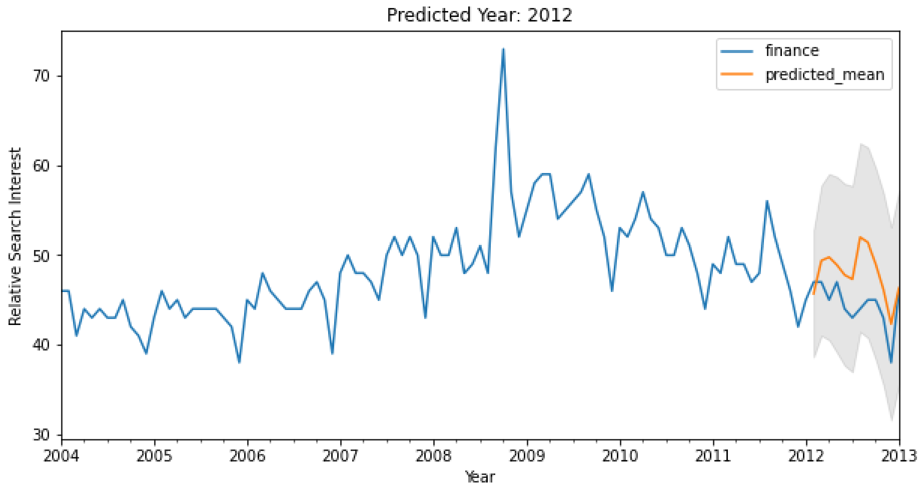

Predicted Year: 2017
finance
predicted_mean
Relative Search Interest
Year
30
40
50
60
70
2005
2007
2009
2011
2013
2015
2017

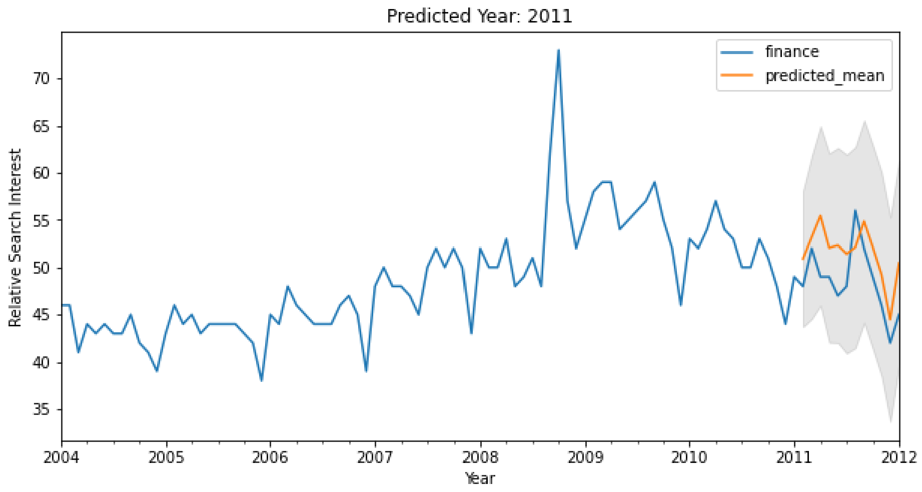

Predicted Year: 2016
finance
predicted_mean
Relative Search Interest
Year
30
40
50
60
70
2005
2007
2009
2011
2013
2015
2017


Predicted Year: 2015
finance
predicted_mean
Relative Search Interest
Year
30
40
50
60
70
2005
2007
2009
2011
2013
2015


Predicted Year: 2014
finance
predicted_mean
Relative Search Interest
Year


Predicted Year: 2013
finance
predicted_mean
Relative Search Interest
Year


Predicted Year: 2012
finance
predicted_mean
Relative Search Interest
Year


Predicted Year: 2011
finance
predicted_mean
Relative Search Interest
Year

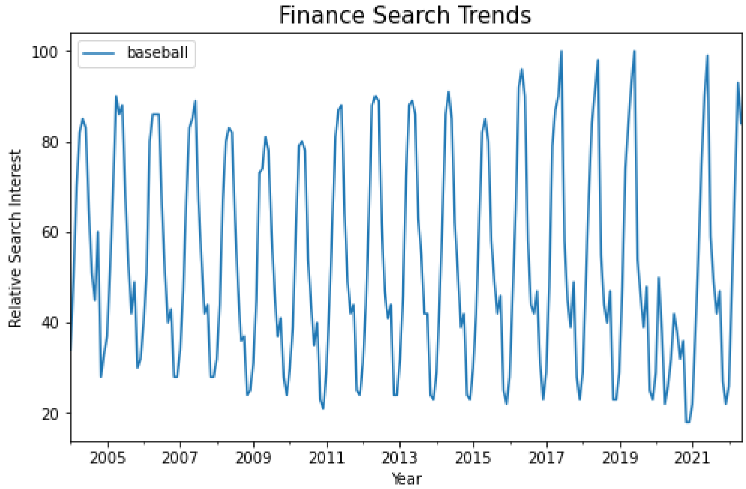
the model seem to work quite well, i tried some different models but did not get much better AIC, BIC and plots

Q2

A

In [12]:
```
baseball = pd.read_csv("baseball_popularity.csv")
```

In [13]:
```
baseball = preprocess_and_plot(baseball)
```

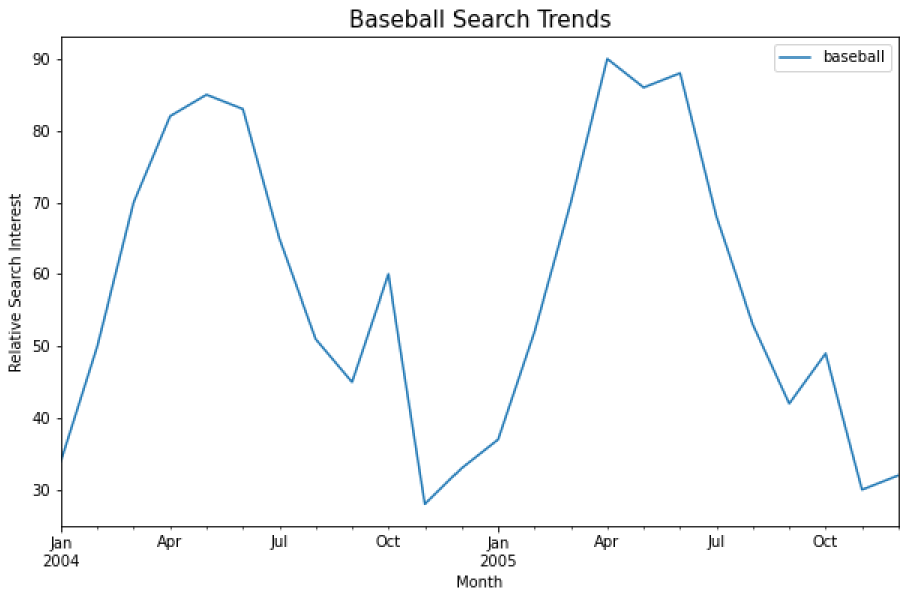


this time series also looks seasonal without a trend, thus it is not stationary. baseball season usually starts at march\april and ends at october, so we can expect a 12 month seasonality

taking a look at the first 2 years

In [14]:
```
baseball[:24].plot(figsize = (10,6))
plt.title("Baseball Search Trends",fontsize=15)
plt.xlabel('Month', fontsize=10)
plt.ylabel('Relative Search Interest', fontsize=10)
```

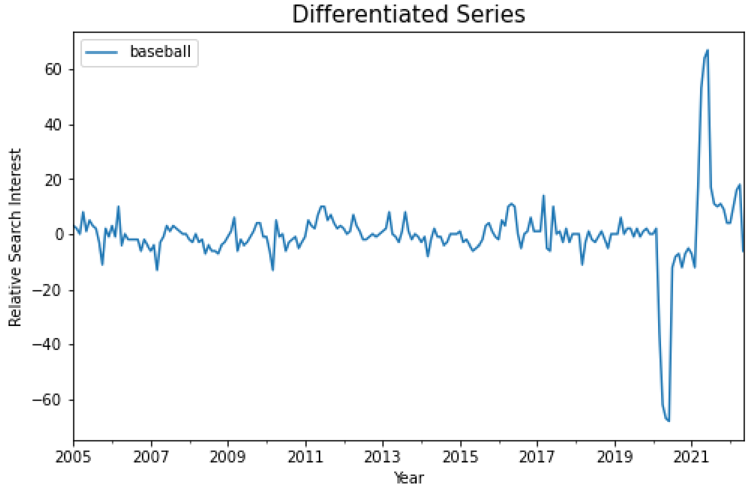
Out[14]:
```
Text(0, 0.5, 'Relative Search Interest')
```



we can see a peak at the start of the season that a slowly decays, until another local peak at october probably due to the finals.

B

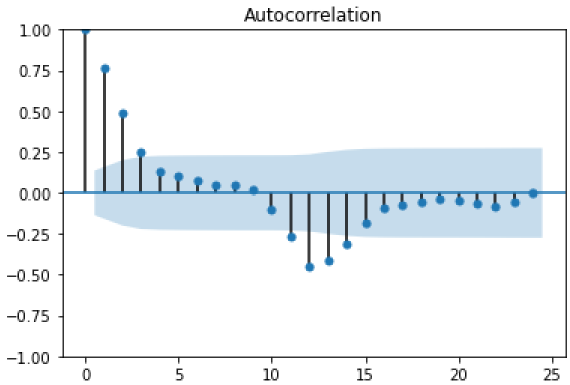
In [15]:
```
baseball_year_diff = plot_diff_series(baseball)
```



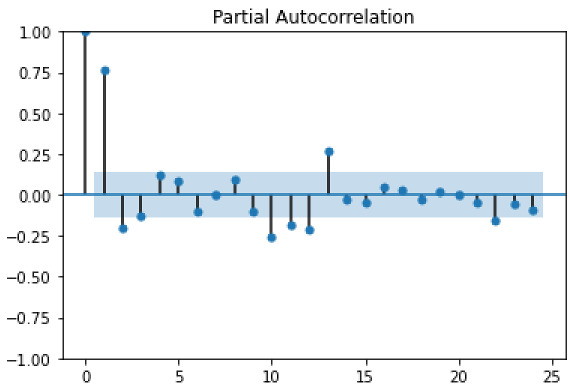
looks stationary, except for the major noise at 2020-2021 (corona)?

C

In [16]:
```
tsaplots.plot_acf(baseball_year_diff);
```



In [17]:
```
tsaplots.plot_pacf(baseball_year_diff);
```



the graphs suggests a strong correlation between the last 2 lags and maybe the third lag but also for the lags a year ago, so we can assume that a good model would be $AR(2)$ or $AR(3)$ $SARIMA(2,0,0)(1,1,0)_{12}$ or $SARIMA(2,0,0)(2,1,0)_{12}$, or even maybe Holt Winter's Exponential Smoothing with 12 months seasonality

D

$$SARIMA(2,0,0)(1,1,0)_{12}$$

In [18]:
```
fit_summarize_sarima(baseball, (2,0,0), (1,1,0,12))
```

SARIMAX Results

Dep. Variable:	baseball	**No. Observations:**	221
Model:	SARIMAX(2, 0, 0)x(1, 1, 0, 12)	**Log Likelihood**	-696.006
Date:	Wed, 14 Feb 2024	**AIC**	1400.013
Time:	13:03:32	**BIC**	1413.382
Sample:	01-01-2004	**HQIC**	1405.418
	- 05-01-2022		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	0.9349	0.034	27.439	0.000	0.868	1.002
ar.L2	-0.2219	0.041	-5.461	0.000	-0.302	-0.142
ar.S.L12	-0.5296	0.035	-15.308	0.000	-0.597	-0.462
sigma2	44.6508	2.200	20.292	0.000	40.338	48.963

Ljung-Box (L1) (Q):	0.03	**Jarque-Bera (JB):**	759.63
Prob(Q):	0.87	**Prob(JB):**	0.00
Heteroskedasticity (H):	5.23	**Skew:**	-0.07
Prob(H) (two-sided):	0.00	**Kurtosis:**	12.34

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

$SARIMA(2,0,0)(2,1,0)_{12}$

In [19]:
```
fit_summarize_sarima(baseball, (2,0,0), (2,1,0,12))
```

SARIMAX Results

Dep. Variable:	baseball	**No. Observations:**	221
Model:	SARIMAX(2, 0, 0)x(2, 1, 0, 12)	**Log Likelihood**	-690.039
Date:	Wed, 14 Feb 2024	**AIC**	1390.078
Time:	13:03:33	**BIC**	1406.790
Sample:	01-01-2004	**HQIC**	1396.835
	- 05-01-2022		
Covariance Type:	opg		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
ar.L1	0.8810	0.036	24.285	0.000	0.810	0.952
ar.L2	-0.1692	0.045	-3.794	0.000	-0.257	-0.082
ar.S.L12	-0.6692	0.057	-11.748	0.000	-0.781	-0.558
ar.S.L24	-0.3245	0.067	-4.821	0.000	-0.456	-0.193
sigma2	41.7372	1.940	21.519	0.000	37.936	45.539

Ljung-Box (L1) (Q):	0.02	**Jarque-Bera (JB):**	1060.91
Prob(Q):	0.89	**Prob(JB):**	0.00
Heteroskedasticity (H):	5.87	**Skew:**	-0.60
Prob(H) (two-sided):	0.00	**Kurtosis:**	13.97

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

$AR(2)$

In [20]:
```
fit_summarize_sarima(baseball, (2,0,0))
```

SARIMAX Results

Dep. Variable:	baseball	**No. Observations:**	221
Model:	SARIMAX(2, 0, 0)	**Log Likelihood**	-885.737
Date:	Wed, 14 Feb 2024	**AIC**	1777.474
Time:	13:03:33	**BIC**	1787.669
Sample:	01-01-2004	**HQIC**	1781.591
	- 05-01-2022		
Covariance Type:	opg		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
ar.L1	1.3758	0.075	18.277	0.000	1.228	1.523
ar.L2	-0.4219	0.083	-5.105	0.000	-0.584	-0.260
sigma2	174.7732	19.362	9.027	0.000	136.825	212.722

Ljung-Box (L1) (Q):	0.70	**Jarque-Bera (JB):**	62.03
Prob(Q):	0.40	**Prob(JB):**	0.00
Heteroskedasticity (H):	1.81	**Skew:**	-1.11
Prob(H) (two-sided):	0.01	**Kurtosis:**	4.34

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

$AR(3)$

In [21]:
```
fit_summarize_sarima(baseball, (3,0,0))
```

SARIMAX Results

Dep. Variable:	baseball	**No. Observations:**	221
Model:	SARIMAX(3, 0, 0)	**Log Likelihood**	-885.468
Date:	Wed, 14 Feb 2024	**AIC**	1778.937
Time:	13:03:33	**BIC**	1792.529
Sample:	01-01-2004	**HQIC**	1784.425
	- 05-01-2022		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ar.L1	1.3550	0.081	16.675	0.000	1.196	1.514
ar.L2	-0.3535	0.148	-2.392	0.017	-0.643	-0.064
ar.L3	-0.0499	0.101	-0.496	0.620	-0.247	0.147
sigma2	174.2810	19.538	8.920	0.000	135.987	212.575

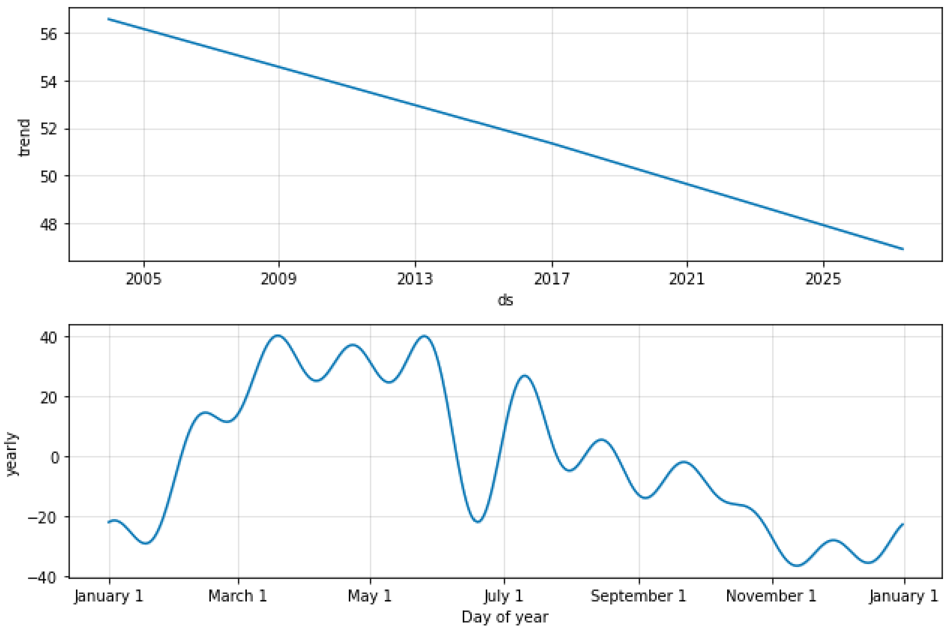
Ljung-Box (L1) (Q):	0.13	**Jarque-Bera (JB):**	62.61
Prob(Q):	0.72	**Prob(JB):**	0.00
Heteroskedasticity (H):	1.83	**Skew:**	-1.11
Prob(H) (two-sided):	0.01	**Kurtosis:**	4.37

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

Prophet

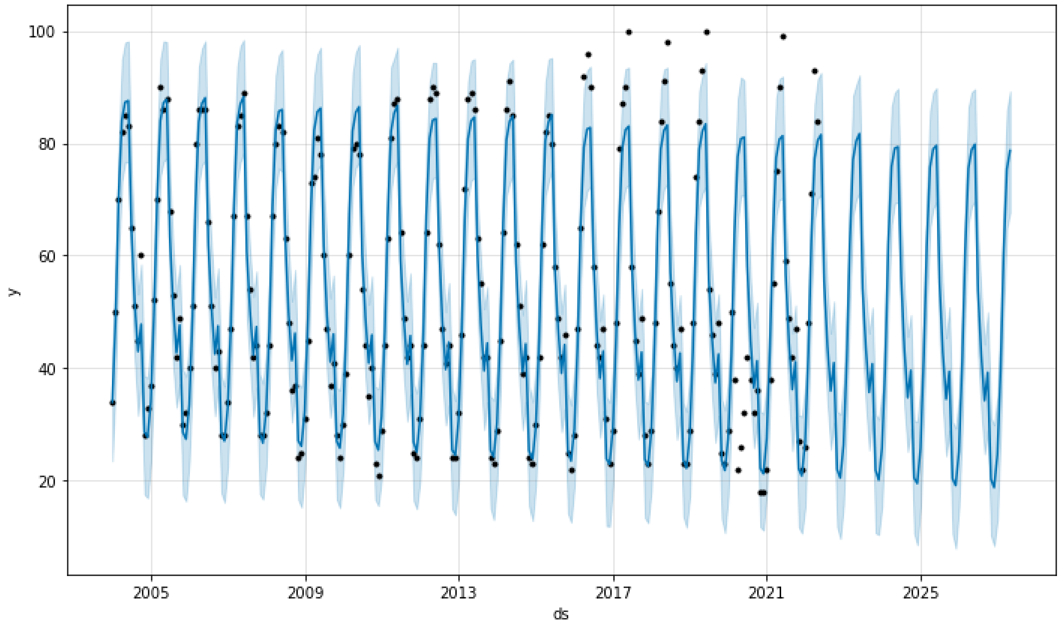
In [22]:
```
baseball_prophet = baseball.reset_index().rename({"month": "ds", "baseball": "y"}, axis =1)
m = Prophet()
m.fit(baseball_prophet)
future_dates = m.make_future_dataframe(periods=60, freq='MS')
forecast = m.predict(future_dates)
forecast[['ds', 'yhat', 'yhat_lower', 'yhat_upper']]
m.plot_components(forecast);
```

```
13:03:33 - cmdstanpy - INFO - Chain [1] start processing
13:03:33 - cmdstanpy - INFO - Chain [1] done processing
```



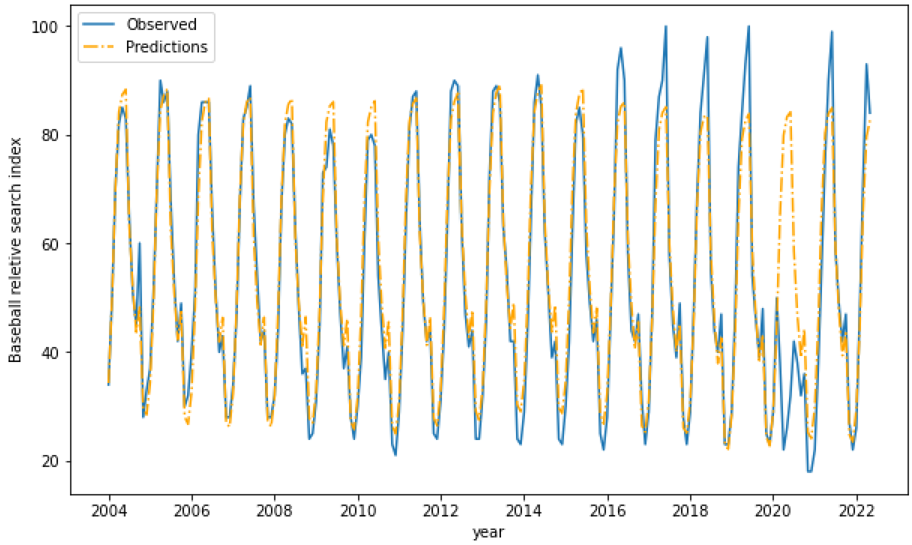
we can see that there is indeed yearly seasonality April-June seem to be the time when people usually search baseball. we also see a linear decreasing trend

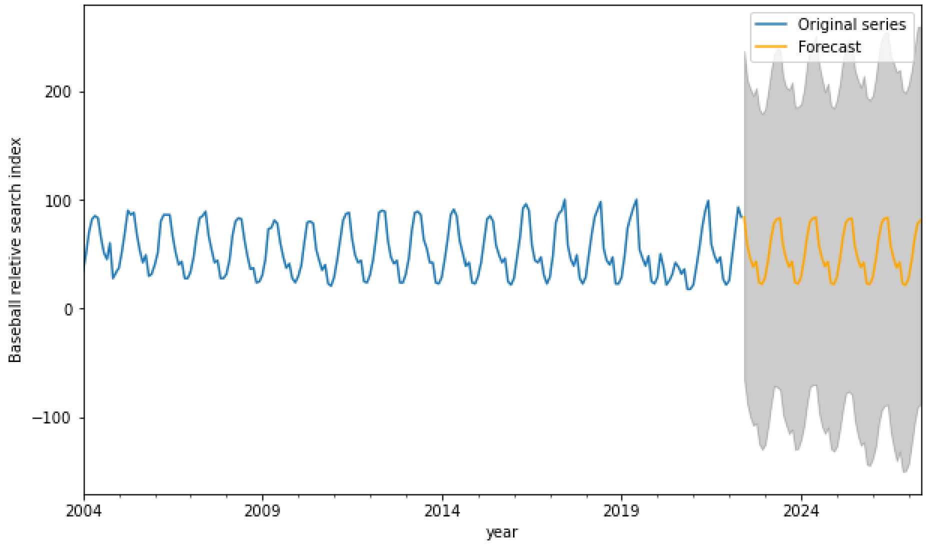
In [23]:
```
m.plot(forecast);
```



HWES

In [24]:
```
import matplotlib.pyplot as plt

def plot_observed_predictions(data, preds, forecast_len, ylabel='', xlabel=''):
    fig, ax = plt.subplots(figsize=(10, 6))
    ax.plot(data.index, data, label="Observed")
    ax.plot(data.index, preds[:-forecast_len], color="orange", ls = "-.", label="Predictions")
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)
    plt.legend()
    plt.show()

def plot_forecast_with_confidence_intervals(data, preds, upper_ci, lower_ci, ylabel='', xlabel=''):
    ax = data.plot(label="Original series", figsize=(10, 6))
    preds.plot(label="Forecast", color="orange", ax=ax)
    ax.fill_between(upper_ci.index, upper_ci, lower_ci, color='k', alpha=.2)
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)
    plt.legend()
    plt.show()

def simulate_ets_predictions(data, column_name, seasonal='add', trend='add', seasonal_periods=12, n_repeti
tions=500,
                             forecast_len=60, plot_obs_pred=True, ylabel='', xlabel="", plot_org_forecast=
True):
    ets_model = ETSModel(
        endog=data[column_name],
        seasonal=seasonal,
        trend=trend,
        seasonal_periods=seasonal_periods)
    ets_result = ets_model.fit()

    # Simulate predictions.
    n_steps_prediction = data[column_name].shape[0]

    df_simul = ets_result.simulate(
        nsimulations=n_steps_prediction+forecast_len,
        repetitions=n_repetitions,
        anchor='start',
    )
    preds = df_simul.mean(axis=1)
    # Calculate confidence intervals.
    upper_ci = df_simul.quantile(q=0.95, axis='columns')
    lower_ci = df_simul.quantile(q=0.05, axis='columns')

    if plot_obs_pred:
        plot_observed_predictions(data[column_name], preds, forecast_len, ylabel, xlabel)

    if plot_org_forecast:
        plot_forecast_with_confidence_intervals(data[column_name], preds[-forecast_len:],
                                                upper_ci[-forecast_len:], lower_ci[-forecast_len:], ylabe
l, xlabel)

    return preds, lower_ci, upper_ci

preds, lower_ci, upper_ci = simulate_ets_predictions(baseball, "baseball",
                                                     ylabel = 'Baseball reletive search index', xlabel="ye
ar")
```



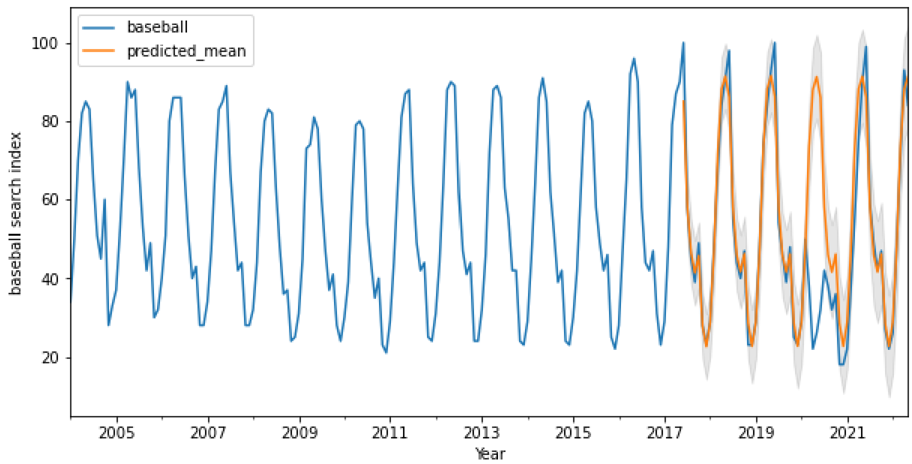


E

$SARIMA(2,0,0)(2,1,0)_{12}$ has the best AIC and BIC (excluding prophet and HWES)

prophet and HWES have a smilar fit based on the graphs

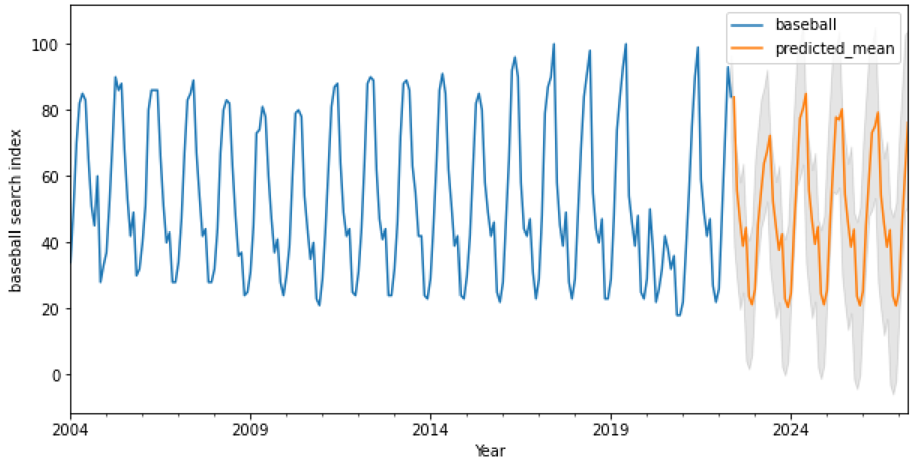
In [25]:
```
_,_ = forecast_with_sarimax(baseball, 60, order=(2, 0, 0),seasonal_order=(2, 1, 0,12), ylabel = "baseball search index")
```



$SARIMA(2,0,0)(2,1,0)_{12}$ also seem like a good fit by the graph and a little better than the HWES and PROPHET so we will chose this as our model

F

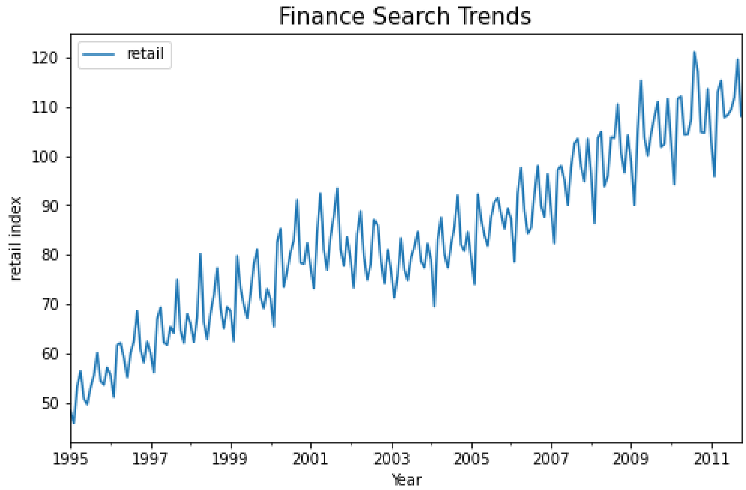
In [26]:
```
future_finance_forecast, next_periods_ci = forecast_with_sarimax(baseball, 60, order=(2, 0, 0),seasonal_order=(2, 1, 0,12),
                                                                 ylabel = "baseball search index", forcast_next = True)
```



Q3

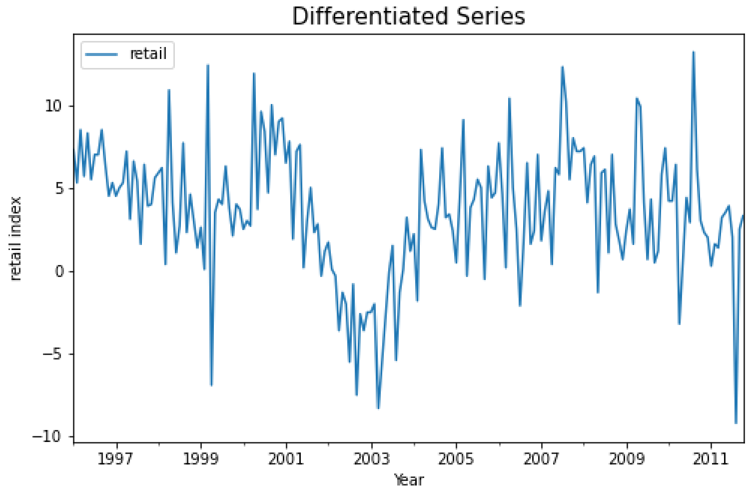
A

In [27]:
```
retail = pd.read_csv("retail.csv")
retail = preprocess_and_plot(retail, start = '1995-01-01', end = '2011-10-01',ylabel = "retail index")
```



we can see an uptrend with some seasonlity, lets fit hte sarima model and check how it fits the data

In [28]:
```
_ = plot_diff_series(retail, ylabel = "retail index")
```



In [29]:
```
fit_summarize_sarima(retail, (0,1,1), (0,1,1,12))
```

SARIMAX Results

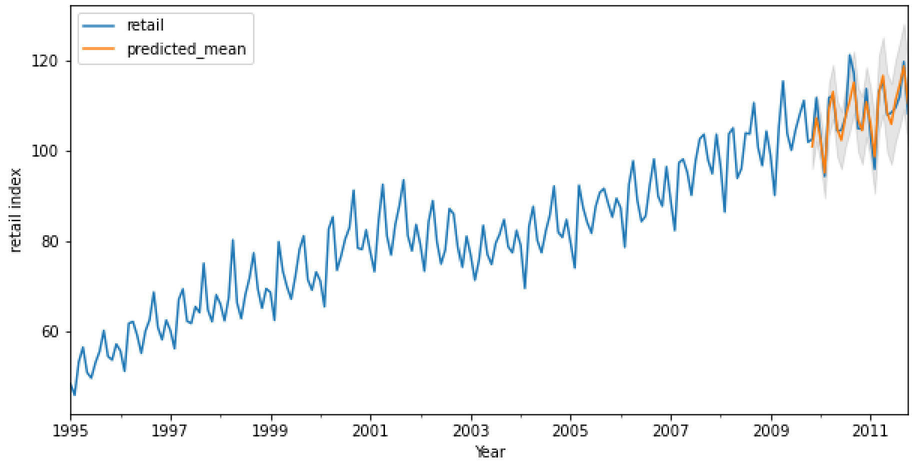
Dep. Variable:	retail	**No. Observations:**	202
Model:	SARIMAX(0, 1, 1)x(0, 1, 1, 12)	**Log Likelihood**	-453.669
Date:	Wed, 14 Feb 2024	**AIC**	913.338
Time:	13:03:36	**BIC**	923.063
Sample:	01-01-1995	**HQIC**	917.278
	- 10-01-2011		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
ma.L1	-0.7060	0.052	-13.462	0.000	-0.809	-0.603
ma.S.L12	-0.7814	0.065	-11.967	0.000	-0.909	-0.653
sigma2	6.6805	0.569	11.732	0.000	5.564	7.797

Ljung-Box (L1) (Q):	0.91	**Jarque-Bera (JB):**	23.46
Prob(Q):	0.34	**Prob(JB):**	0.00
Heteroskedasticity (H):	1.16	**Skew:**	0.47
Prob(H) (two-sided):	0.56	**Kurtosis:**	4.45

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

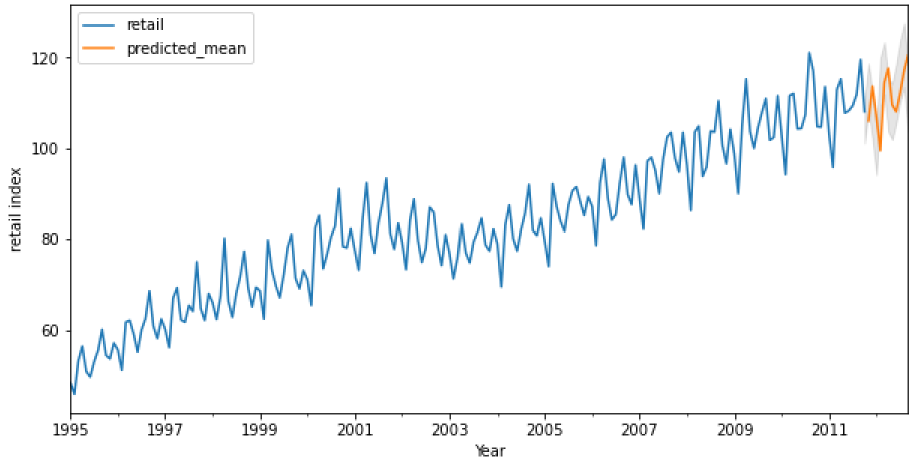
In [30]:
```
future_finance_forecast, next_periods_ci = forecast_with_sarimax(retail, 24, order=(0, 1, 1),seasonal_orde
r=(0, 1, 1,12), ylabel = "retail index")
```



by the predicted line we can see that the model is a very good fit to the data

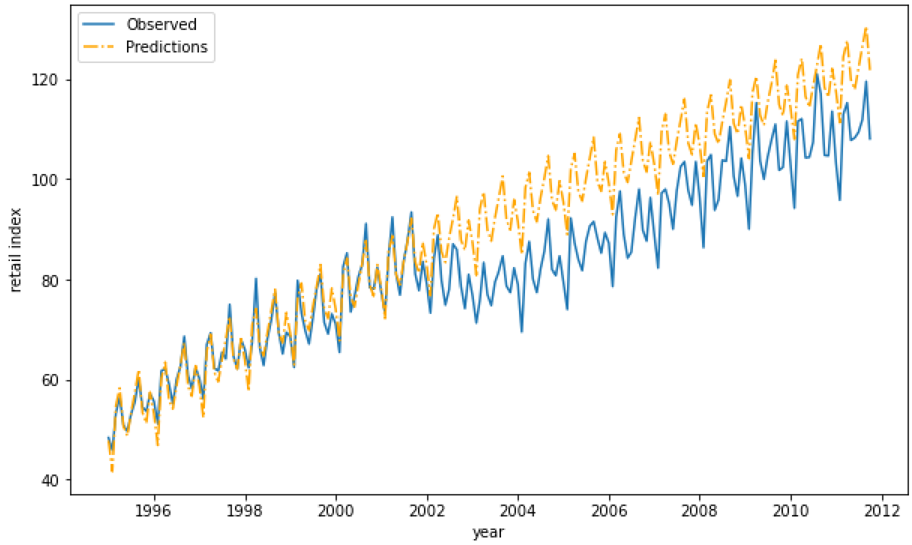
B

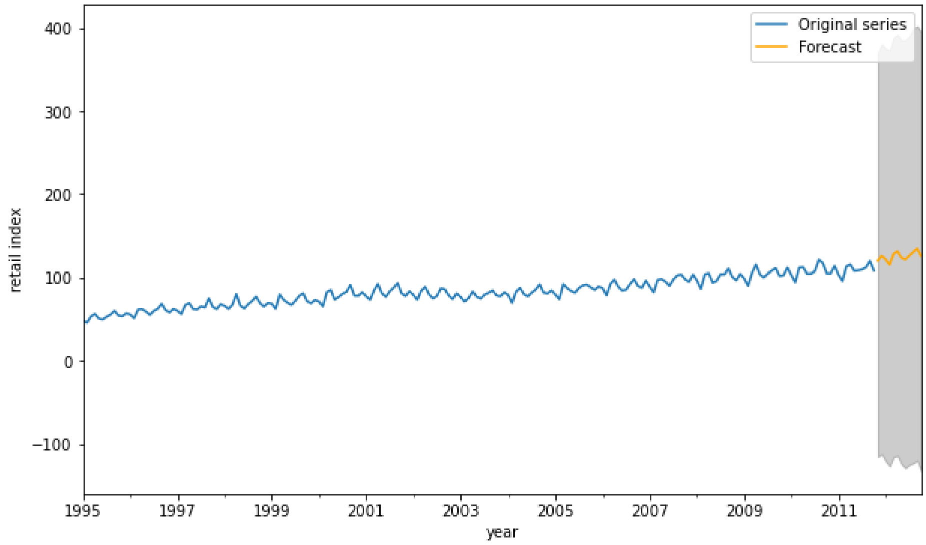
In [31]:
```
future_finance_forecast, next_periods_ci = forecast_with_sarimax(retail, 12, order=(0, 1, 10),seasonal_ord
er=(0, 1, 1,12),
                                                                              forcast_next = True, ylabel = "retail ind
ex")
```



C

In [32]:
```
preds, lower_ci, upper_ci = simulate_ets_predictions(retail, "retail",
                                                     ylabel = 'retail index', xlabel="year", forecast_len
= 12)
```





the ES does not work well here because the trend changes, prophet will probably be better.

In [33]:
```
retail_prophet = retail.reset_index().rename({"month": "ds", "retail": "y"}, axis =1)
m = Prophet()
m.fit(retail_prophet)
future_dates = m.make_future_dataframe(periods=60, freq='MS')
forecast = m.predict(future_dates)
forecast[['ds', 'yhat', 'yhat_lower', 'yhat_upper']]
m.plot_components(forecast);
m.plot(forecast);
```

```
13:03:38 - cmdstanpy - INFO - Chain [1] start processing
13:03:38 - cmdstanpy - INFO - Chain [1] done processing
```