$d \in N$
$\mathcal{D} = \{w | w \in \{0,1\}^d\}$
p_i
$\mathcal{DB}_i \subset \mathcal{D}$
$|\mathcal{DB}_i| = n \in N$
$\mathcal{P} = \{p_0, p_1, p_2, \ldots, p_{|\mathcal{P}|}\}$
$\mathcal{IS} = \mathcal{DB}_0 \cap \mathcal{DB}_1 \cap \mathcal{DB}_2 \cap \ldots \cap \mathcal{DB}_{|\mathcal{P}|}$
p_0

$\mathcal{DB}_{p_i}, k, m, \lambda$
$p_0 k H p_i \in \mathcal{P} \setminus p_0$

$p_i \in \mathcal{P}$

$GBF_{\mathcal{DB}_i} = BuildGBF(\mathcal{DB}_i, H, k, m, \lambda) H$
$BF_{\mathcal{DB}_i} = BuildBF(\mathcal{DB}_i, H, k, m)$
$|\mathcal{P}| GBF^0_{\mathcal{DB}_i} \oplus GBF^1_{\mathcal{DB}_i} \oplus \ldots \oplus GBF^{|\mathcal{P}|}_{\mathcal{DB}_i} = \mathcal{DB}_i{}^a$

$p_i \in \mathcal{P}$

$p_j \in \mathcal{P} \setminus p_i$
$p_i p_j m \lambda (x_{(i,j,b,0)}, x_{(i,j,b,1)}) x_{(i,j,b,0)} \lambda x_{(i,j,b,1)} = GBF^j_{\mathcal{DB}_i}[b] b = 0 \ldots m$
$p_i BF_i b = 0 \ldots m p_i GBF^{\star i}_{\mathcal{DB}_j} GBF^{\star i}_{\mathcal{DB}_j}[b] = x_{(j,i,b,BF_i[b])}$

$p_i \in \mathcal{P}$

$GBF^{\star i}_{\mathcal{DB}_{intersec}} = GBF^{\star i}_{\mathcal{DB}_0} \oplus GBF^{\star i}_{\mathcal{DB}_1} \oplus \ldots \oplus GBF^{\star i}_{\mathcal{DB}_{|\mathcal{P}|}}$
$GBF^{\star i}_{\mathcal{DB}_{intersec}} p_0$
p_0
$GBF_{intersec} = GBF^{\star 0}_{\mathcal{DB}_{intersec}} \oplus GBF^{\star 1}_{\mathcal{DB}_{intersec}} \oplus \ldots \oplus GBF^{\star |\mathcal{P}|}_{\mathcal{DB}_{intersec}}$
$item \in \mathcal{DB}_0 GBFQuery(GBF_{intersec}, item) = 1 item \in \mathcal{IS}$
$p_0 \mathcal{IS} p_1 \ldots p_{|\mathcal{P}|}$
$p_0 \mathcal{IS}$

[a] It should be noted that every $GBF^{\circ}_{\mathcal{DB}_i}$ is equivalent to random String in length of $\lambda * m$.

π
IDEAL:
$\mathcal{DB}$
$\mathcal{IS}$
p_0
REAL hybrid:
π
Definition:
$IDEAL^{p_i}_{view}$
p_i
$(\mathcal{DB}_i)$
Definition:
$S \subseteq \mathcal{P}$
$Sim^{\pi}(IDEAL^S_{view})$
$p \in S$
π
$p_{\in} S$
Claim:
$S \subseteq \mathcal{P}$
$IDEAL^S_{view}$
$Sim^{\pi}(IDEAL^S_{view})$
$\mathcal{P}$
$S \mathcal{P}$
$\mathcal{IS}$
$IDEAL^S_{view}$
$\mathcal{IS}$
$REAL_{view} S$
Sim^{π}
$\mathcal{IS}$
Sim^{π}
H
$p_i \notin S$
$\mathcal{DB}_i$
$IDEAL^S_{view}$
Sim^{π}