Advanced Model Predictive Control — Fall 2021
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Solution 2

Introduction to Code Framework & Nominal Nonlinear MPC

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1 Exercise

Nominal Nonlinear MPC

1. Make yourself familiar with the MATLAB code base.
2. Consider the nominal nonlinear MPC problem

$$\min_{x,u} \quad l_f(x_N) + \sum_{i=0}^{N-1} l(x_i, u_i) \tag{1a}$$
$$\text{s.t.} \quad \forall i = 0, \dots, N-1, \tag{1b}$$
$$x_{i+1} = f(x_i, u_i), \tag{1c}$$
$$x_i \in \mathcal{X},\ u_i \in \mathcal{U}, \tag{1d}$$
$$x_N \in \mathcal{X}_f,\ x_0 = x(k). \tag{1e}$$

Implement (1) in the provided `Nonlinear_MPC.m` file, using the following choices of cost function, nonlinear segway dynamics, constraints, and terminal ingredients:

$$\begin{aligned} l(x,u) &= x^\top Q\, x + u^\top R\, u, \\ f(x,u) &= \begin{bmatrix} x_1 + \delta t \cdot x_2 \\ x_2 + \delta t \left[-k x_1 - c x_2 + {}^{g}/_{l} \cdot \sin x_1 + u\right] \end{bmatrix}, \\ \mathcal{X} &= \{x \mid A_x x \le b_x\}, \\ \mathcal{U} &= \{u \mid A_u u \le b_u\}, \\ l_f(x) &= 0, \\ \mathcal{X}_f &= \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}. \end{aligned}$$

Note: The control parameters, e.g. Q and R, are loaded by the `Controller` class (super class) constructor. Therefore, you can access them with `obj.params.Q`. However, the system object is directly passed to the constructor of the `Nonlinear_MPC` class. This means you can access system properties, like e.g. the state constraints, directly through the `sys` object, i.e. `sys.X`.

3. Consider now the same nonlinear segway system but with additive disturbances.
   a. Run the cell labelled "Exercise 3a" in `main.m` and observe how the initial state and the disturbance affect the feasibilty of the closed-loop trajectories.
   b. Run the cell labelled "Exercise 3b" in `main.m` with different choices of initial states and disturbance sizes. Observe how these two parameters affect the closed-loop trajectories and the cost decrease.

2 Solution

Nominal Nonlinear MPC

2./3. The MATLAB code for these questions can be found on Moodle.