Advanced Model Predictive Control — Fall 2021
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# Solution 4

## Nonlinear robust MPC - Part II

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# 1 Exercise

### Nonlinear Robust MPC

1. Implementation of nonlinear robust MPC in the `Nonlinear_RMPC.m` file.

   a. Implement the computation of the RPI set and the tightenings, which we derived in the recition in the `compute_tightening` method.

   b. Compute the constraint tightenings for different choices of $\rho$ and observe how the tightenings and $\Omega$ change. Choose a $\rho$ for the remainder of the exercise.

   c. Consider the nonlinear robust MPC problem

$$\min_{V,z_0} \quad \sum_{i=0}^{N-1} z_i^T Q z_i + v_i^T R v_i \tag{1a}$$

$$\text{s.t.} \quad \forall i = 0, \dots, N-1, \tag{1b}$$

$$z_{i+1} = f(z_i, v_i), \tag{1c}$$

$$[A_x]_j z_i \leq [b_x]_j - \|P^{-\frac{1}{2}}[A_x]_j^\top\|_2 \delta, \qquad j \in [1, n_x], \tag{1d}$$

$$[A_u]_j v_i \leq [b_u]_j - \|P^{-\frac{1}{2}} K^\top [A_x]_j^\top\|_2 \delta, \qquad j \in [1, n_u], \tag{1e}$$

$$z_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{1f}$$

$$\|x(k) - z_0\|_P^2 \leq \delta^2, \tag{1g}$$

   where $\delta = \frac{\bar{\alpha}_w}{1-\rho} = \max_{w \in \mathcal{W}} \|w\|_P \cdot \frac{1}{1-\rho}$.
   Implement (1) in the provided `Nonlinear_RMPC.m` file.

   *Note:* The control parameters, e.g. $Q$ and $R$, are loaded by the `Controller` class (super class) constructor. Therefore, you can access them with `obj.params.Q`. However, the system object is directly passed to the constructor of the `Nonlinear_MPC` class. This means you can access system properties, like e.g. the state constraints, directly through the `sys` object, i.e. `sys.X`.

2. Implementation of better tightening computations.

   a. Consider the optimization problem

$$
\begin{aligned}
\min_{E,Y,\beta_j,\gamma_j,\omega} \quad & \frac{1}{2(1-\rho)}\left((n_x+n_u)\omega + \sum_{j=1}^{n_x}\beta_j + \sum_{j=1}^{n_u}\gamma_j\right) && \text{(2a)}\\
\text{s.t.} \quad & E \succeq I, && \text{(2b)}\\
& \begin{bmatrix} \rho^2 E & (A_1E+BY)^\top \\ A_1E+BY & E \end{bmatrix} \succeq 0, && \text{(2c)}\\
& \begin{bmatrix} \rho^2 E & (A_2E+BY)^\top \\ A_2E+BY & E \end{bmatrix} \succeq 0, && \text{(2d)}\\
& \begin{bmatrix} \beta_j & [A_x]_jE \\ E^\top[A_x]_j^\top & E \end{bmatrix} \succeq 0,\ j\in[1,n_x], && \text{(2e)}\\
& \begin{bmatrix} \gamma_j & [A_u]_jY \\ Y^\top[A_u]_j^\top & E \end{bmatrix} \succeq 0,\ j\in[1,n_u], && \text{(2f)}\\
& \begin{bmatrix} \omega & v_w^\top \\ v_w & E \end{bmatrix} \succeq 0,\ \forall v_w\in\mathcal{V}(\mathcal{W}). && \text{(2g)}
\end{aligned}
$$

   Implement (2) in the `compute_min_tightening` method in the `Nonlinear_RMPC.m` file.

   b. Compute the constraint tightenings for different choices of $\rho$ and observe how the tightenings and $\Omega$ change. Choose a $\rho$ and simulate the system with the nonlinear robust MPC controller.

# 2 Solution

**Nonlinear Robust MPC**

1./2. The MATLAB code for these questions can be found on Moodle.