Assignment 4

1. **a.** $\{f : [T1 \rightarrow T2], g : [T1 \rightarrow T2], a : T1\} \vdash (f\ (g\ a)) : T2$
   This typing statement is false because if a is from type $T1$, so $(g\ a)$ will return the type $T2$, and from that we can see that f in the expression $(f\ (g\ a))$ will get the type $T2$ instead of the type $T1$.
   b. $\{x: T1,\ y: T2, f: [T2 \rightarrow T1]\} \vdash (f\ y) : T1$
   This typing statement is true. y type is $T2$, so f return value's type will be $T1$ as the typing statement shows.
   c. $\{f : [T1 \rightarrow T2]\} \vdash \big(lambda\ (x)(f\ x)\big): [T1 \rightarrow T2]$
   This typing statement is true. We can infer this from the following type inference:

Expression	**Var**
$(lambda\ (x)\ (f\ x))$	T_0'
$(f\ x)$	T_1'
f	T_f'
x	T_x'

   Now we can build the following type equations:

Expression	**Var**
$(lambda\ (x)\ (f\ x))$	$T_0' = [T_x' \rightarrow T_1']$
$(f\ x)$	$T_f' = [T_x' \rightarrow T_1']$

   Now from the substitution $[T1 \rightarrow T2]$ we will be able to see that $T1 \coloneqq T_x',\ T2 \coloneqq T_1'$ and because of that we can see that:
   $\big(lambda\ (x)\ (f\ x)\big) : T_0' = [T_x' \rightarrow T_1'] = [T1 \rightarrow T2] \Rightarrow \big(lambda\ (x)\ (f\ x)\big): [T1 \rightarrow T2]$
   d. $\{f: [T1 * T2 \rightarrow T3]\} \vdash \big(lambda\ (x)\ (f\ x\ 100)\big) : [T1 \rightarrow T3]$
   This typing statement is false. We can infer this from the following type inference:

Expression	**Var**
$(lambda\ (x)\ (f\ x\ 100))$	T_0'
$(f\ x\ 100)$	T_1'
f	T_f'
x	T_x'
100	T_{num100}'

   Now we can build the following type equations:

Expression	**Var**
$(lambda\ (x)\ (f\ x\ 100))$	$T_0' = [T_x' \rightarrow T_1']$
$(f\ x\ 100)$	$T_f' = [T_x' * T_{num100}' \rightarrow T_1']$
100	$T_{num100}' = Number$

   Now from the substitution $\{T1 * T2 \rightarrow T3]$ we will be able to see that:
   $[T_x' * T_{num100}' \rightarrow T_1'] = [T1 * T2 \rightarrow T3] \Rightarrow\ \ T_{num100}' = Number = T2$
   So, we get that $T2 = Number$ in contradiction because we got 2 atomic variables that are equal but not the same.

2. **a.** $((lambda\ (x1)\ (+\ x1\ 1))\ 4)$

STAGE 1: Renamed bound variables: $((lambda\ (x)\ (+\ x\ 1))\ 4)$

STAGE 2: Assign type variables to all sub-exps:

Expression	Var
$((lambda\ (x)\ (+\ x\ 1))\ 4)$	T_0
$(lambda\ (x)\ (+\ x\ 1))$	T_1
$(+\ x\ 1)$	T_2
$+$	T_+
x	T_x
1	T_{num1}
4	T_{num4}

STAGE 3: Construct type equations:

Expression	Equation
$((lambda\ (x)\ (+\ x\ 1))\ 4)$	$T_1 = [T_{num4} \rightarrow T_0]$
$(lambda\ (x)\ (+\ x\ 1))$	$T_1 = [T_x \rightarrow T_2]$
$(+\ x\ 1)$	$T_+ = [T_x * T_{num1} \rightarrow T_2]$
$+$	$T_+ = [Number * Number \rightarrow Number]$
1	$T_{num1} = Number$
4	$T_{num4} = Number$

STAGE 4: Solving the equations:

Equation	Substitution
$T_1 = [T_{num4} \rightarrow T_0]$	$\{\}$
$T_1 = [T_x \rightarrow T_2]$	
$T_+ = [T_x * T_{num1} \rightarrow T_2]$	
$T_+ = [Number * Number \rightarrow Number]$	
$T_{num1} = Number$	
$T_{num4} = Number$	

Equation	Substitution
$T_1 = [T_x \rightarrow T_2]$	$\{T_1 \coloneqq [T_{num4} \rightarrow T_0]\}$
$T_+ = [T_x * T_{num1} \rightarrow T_2]$	
$T_+ = [Number * Number \rightarrow Number]$	
$T_{num1} = Number$	
$T_{num4} = Number$	

Equation	Substitution
$T_+ = [T_x * T_{num1} \rightarrow T_2]$	$\{T_1 \coloneqq [T_{num4} \rightarrow T_0]\}$
$T_+ = [Number * Number \rightarrow Number]$	
$T_{num1} = Number$	
$T_{num4} = Number$	
$T_x = T_{num4}$	
$T_2 = T_0$	

Equation	Substitution
$T_+ = [Number * Number \rightarrow Number]$	$\{T_1 \coloneqq [T_{num4} \rightarrow T_0]$
$T_{num1} = Number$	$T_+ := [T_x * T_{num1} \rightarrow T_2]\}$
$T_{num4} = Number$	
$T_x = T_{num4}$	
$T_2 = T_0$	

Equation	Substitution
$T_{num1} = Number$	$\{T_1 \coloneqq [T_{num4} \rightarrow T_0]$
$T_{num4} = Number$	$T_+ := [T_x * T_{num1} \rightarrow T_2]\}$
$T_x = T_{num4}$	
$T_2 = T_0$	
$T_x = Number$	
$T_{num1} = Number$	
$T_2 = Number$	

Equation	Substitution
$T_{num4} = Number$	$\{T_1 \coloneqq [T_4 \rightarrow T_0]$
$T_x = T_{num4}$	$T_+ \coloneqq [T_x * Number \rightarrow T_2]$
$T_2 = T_0$	$T_{num1} := Number\}$
$T_x = Number$	
$T_{num1} = Number$	
$T_2 = Number$	

Equation	Substitution
$T_x = T_{num4}$	$\{T_1 \coloneqq [Number \rightarrow T_0]$
$T_2 = T_0$	$T_+ \coloneqq [T_x * Number \rightarrow T_2]$
$T_x = Number$	$T_{num1} \coloneqq Number$
$T_{num1} = Number$	$T_{num4} = Number\}$
$T_2 = Number$	

Equation	Substitution
$T_x = Number$	$\{T_1 \coloneqq [Number \rightarrow T_0]$
$T_2 = T_0$	$T_+ \coloneqq [T_x * Number \rightarrow T_2]$
$T_x = Number$	$T_{num1} \coloneqq Number$
$T_{num1} = Number$	$T_{num4} = Number\}$
$T_2 = Number$	

Equation	Substitution
$T_2 = T_0$	$\{T_1 \coloneqq [Number \to T_0]$
$T_x = Number$	$T_+ \coloneqq [Number * Number \to T_2]$
$T_{num1} = Number$	$T_{num1} \coloneqq Number$
$T_2 = Number$	$T_{num4} = Number$
	$T_x = Number\}$

Equation	Substitution
$T_x = Number$	$\{T_1 \coloneqq [Number \to T_0]$
$T_{num1} = Number$	$T_+ \coloneqq [Number * Number \to T_0]$
$T_2 = Number$	$T_{num1} \coloneqq Number$
	$T_{num4} = Number$
	$T_x = Number$
	$T_2 = T_0\}$

Equation	Substitution
$Number = Number$	$\{T_1 \coloneqq [Number \to T_0]$
$T_{num1} = Number$	$T_+ \coloneqq [Number * Number \to T_0]$
$T_2 = Number$	$T_{num1} \coloneqq Number$
	$T_{num4} := Number$
	$T_x := Number$
	$T_2 := T_0\}$

Equation	Substitution
$T_{num1} = Number$	$\{T_1 \coloneqq [Number \to T_0]$
$T_2 = Number$	$T_+ \coloneqq [Number * Number \to T_0]$
	$T_{num1} \coloneqq Number$
	$T_{num4} := Number$
	$T_x := Number$
	$T_2 := T_0\}$

Equation	Substitution
$Number = Number$	$\{T_1 \coloneqq [Number \to T_0]$
$T_2 = Number$	$T_+ \coloneqq [Number * Number \to T_0]$
	$T_{num1} \coloneqq Number$
	$T_{num4} := Number$
	$T_x := Number$
	$T_2 := T_0\}$

Equation	Substitution
$T_2 = Number$	$\{T_1 \coloneqq [Number \to T_0]$
	$T_+ \coloneqq [Number * Number \to T_0]$
	$T_{num1} \coloneqq Number$
	$T_{num4} := Number$
	$T_x := Number$
	$T_2 := T_0\}$

Equation	Substitution
	$\{T_1 \coloneqq [Number \to Number]$ $T_+ \coloneqq [Number * Number \to Number]$ $T_{num1} \coloneqq Number$ $T_{num4} \coloneqq Number$ $T_x =: Number$ $T_2 \coloneqq Number\}$

$$Exp: ((lambda\ ([x1: Number])\ (+[x1: Number]\ 1))\ 4)$$

b. $((lambda\ (f1\ x1)\ (f1\ x1\ 1))\ 4\ +)$

STAGE 1: Renamed bound variables: $((lambda\ (f\ x)\ (+\ x\ 1))\ 4)$

STAGE 2: Assign type variables to all sub-exps:

Expression	Var
$((lambda\ (f\ x)\ (f\ x\ 1))\ 4\ +)$	T_0
$(lambda\ (f\ x)\ (f\ x\ 1))$	T_1
$(f\ x\ 1)$	T_2
f	T_f
x	T_x
1	T_{num1}
4	T_{num4}
$+$	T_+

STAGE 3: Construct type equations:

Expression	Equation
$((lambda\ (f\ x)\ (f\ x\ 1))\ 4\ +)$	$T_1 = [T_{num4} * T_+ \to T_0]$
$(lambda\ (f\ x)\ (f\ x\ 1))$	$T_1 = [T_f * T_x \to T_2]$
$(f\ x\ 1)$	$T_f = [T_x * T_{num1} \to T_2]$
4	$T_{num4} = Number$
$+$	$T_+ = [Number * Number \to Number]$

STAGE 4: Solving the equations:

Equation	Substitution
$T_1 = [T_{num4} * T_+ \to T_0]$	$\{\}$
$T_1 = [T_f * T_x \to T_2]$	
$T_f = [T_x * T_{num1} \to T_2]$	
$T_{num4} = Number$	
$T_+ = [Number * Number \to Number]$	

Equation	Substitution
$T_1 = [T_f * T_x \rightarrow T_2]$	$\{T_1 = [T_{num4} * T_+ \rightarrow T_0]\}$
$T_f = [T_x * T_{num1} \rightarrow T_2]$	
$T_{num4} = Number$	
$T_+ = [Number * Number \rightarrow Number]$	

Equation	Substitution
$T_f = [T_x * T_{num1} \rightarrow T_2]$	$\{T_1 = [T_{num4} * T_+ \rightarrow T_0]\}$
$T_{num4} = Number$	
$T_+ = [Number * Number \rightarrow Number]$	
$T_f = T_{num4}$	
$T_x = T_+$	
$T_2 = T_0$	

Equation	Substitution
$T_{num4} = Number$	$\{T_1 = [T_{num4} * T_+ \rightarrow T_0]$
$T_+ = [Number * Number \rightarrow Number]$	$T_f = [T_x * T_{num1} \rightarrow T_2]\}$
$T_f = T_{num4}$	
$T_x = T_+$	
$T_2 = T_0$	

Equation	Substitution
$T_+ = [Number * Number \rightarrow Number]$	$\{T_1 = [Number * T_+ \rightarrow T_0]$
$T_f = T_{num4}$	$T_f = [T_x * T_{num1} \rightarrow T_2]$
$T_x = T_+$	$T_{num4} = Number\}$
$T_2 = T_0$	

Equation	Substitution
$T_f = T_{num4}$	$\{T_1 = [Number * [Number * Number \rightarrow Number] \rightarrow T_0]$
$T_x = T_+$	$T_f = [T_x * T_{num1} \rightarrow T_2]$
$T_2 = T_0$	$T_{num4} = Number$
	$T_+ = [Number * Number \rightarrow Number]\}$

Now we get that $Number = [T_x * T_{num1} \rightarrow T_2]$ so we can see that the expression in this question is not valid.

Question 2.2 (b):

The wrapped function returns Promise<R> because first, we need to create a function that returns the value of the key in the store which is in type R. Second, because we return a value of a store (which means Promise), we wrote the word 'async' before this function which we return (when the word 'async' comes before a function' it's means that this function return Promise), we get that the wrapped function returns Promise <R>.

Question 3.1:

Typing Rule define:
For every: type environment _Tenv,
variable _x1
expressions _e1 and
type expressions _S1, _U1:

If _Tenv ○ [_x1 : _S1] |-- _e1 : _S1

Then _Tenv ○ [_x1 : _S1] |-- (define _x1 _e1) : (void)

Typing Rule set:
For every: type environment _Tenv,
variable _x1
expressions _e1 and
type expressions _S1:
If _Tenv |-- _x1 : _S1
_Tenv |-- _e1: _S1
Then _Tenv |-- (set _x1 _e1) : (void)