

## Exercise 1 (for submitting)

1. i. Use the bisection method to find a positive root of the equation

$$x = 2\sin x.$$

accurate to six significant digits.

- ii. Use the Newton method to find a positive root of the equation

$$x = 2\sin x.$$

- iii. Which method is faster?

2. Your dog chewed your calculator and damaged the division key! Design an algorithm that, of course, does not rely on division which computes  $1/\sqrt{2}$  up to six significant digits.

3. Consider the function  $f(x) = e^x - x - 1$ . Prove that  $x = 0$  is the unique root of  $f$ .

- i. Write a code which implement the Newton method.

- ii. Write a code which implement the following modification of the Newton method:

$$x_{n+1} = x_n - 2f(x_n)/f'(x_n).$$

- iii. Which code perform better? Try to give a theoretical explanation of your numerical results.

4. Write a code which implement the Newton method for the function  $f(x) = \arctan(x)$ . Try to characterize those initial values  $x_1$  for which the iteration method converges to the unique root  $x = 0$ .