Exercise 1 (no need to submit)

1. Prove that the convergence of the sequence $x_n = (1 + 1/n)^n$, $n \in \mathbb{N}$ is worse than linear.

2. Use the bisection method to find a positive root of the equation

$$x = 2sinx.$$

accurate to two significant digits. Use a hand calculator!

3. Let $a > 0$. Bailey's iteration for calculating $\sqrt{a}$ a is obtained by the iterative scheme:

$$x_{n+1} = \frac{x_n(x_n^2 + 3a)}{3x_n^2 + a}, \quad n \geq 1.$$

Show that this iteration is of order at least three.

4. Your dog chewed your calculator and damaged the division key! To compute reciprocals (i.e., one-over a given number R) without division, we can solve $x = 1/R$ by finding a root of a certain function f with Newton's method. Design such an algorithm (that, of course, does not rely on division).

5. Show that if A is any positive number, then the sequence defined by

$$x_{n+1} = \frac{x_n}{2} + \frac{A}{x_n}, \quad n \geq 1$$

converge to $\sqrt{A}$ whenever $x_1 > 0$.
b. What happens if $x_1 < 0$.