Exercise 3 (for submitting)

This exercise deals with numerical computations of optimal stopping and the corresponding optimal strategy.

1. Let $s > 0$ be a parameter. Let $n = 10000$ be the time horizon and consider an investor who owns an asset. The asset price at time $k = 0, 1, ..., 10000$ is a random variable given by

$$Y_k = e^{-\frac{k}{10^5}} \max\left(0, 10 - s\left(1 + 0.007 \sum_{i=1}^{k} \xi_i\right)\right), \quad k = 0, 1, ..., 10000$$

where $\xi_1, ..., \xi_{10000}$ are i.i.d. random variables which take the values ± 1 with probability 0.5. For $k = 0$ we have $Y_0 = \max(0, 10 - s)$.

i. Write a code that computes (for a given s) the optimal stopping value

$$V(s) = \max_{\tau} \mathbb{E}[Y_\tau].$$

Your function should be called as follows. R users: define the function

```
vmax1 = function(s),
```

Python users: define

```
def vmax1(s).
```

The input is the s parameter as specified above, and the output value should be $V(s)$.

ii. The decision whether to sell the asset (i.e. to stop) at time k is based only on the value of $\sum_{i=1}^{k} \xi_i$. Observe that this sum can take only values in the set $\{-k, 1-k, ..., 0, 1, ..., k-1, k\}$. Write a code that for a given time k gives all the values of $\sum_{i=1}^{k} \xi_i$ for which we should sell the asset immediately at time k.

R users: define the function

```
vmax2 = function(k),
```

Python users: define

```
def vmax2(k).
```