

Exercises: Basic Python, Binary search and Recursion

Easy Exercise 1:

Implement a function that finds the sum, average, min and max for the numbers in an unsorted list

```
def sumAverageMinMax(myList):
    mySum=0
    min=myList[0]
    max=myList[0]
    for x in myList:
        mySum = mySum + x
        if x<min:
            min=x
        if x>max:
            max=x
    average = mySum/len(myList)
    return mySum, min, max, average
```

Easy Exercise 2:

Implement a function that finds the frequency of all numbers in a sorted list.

```
def frequencyPrint(myList):
    index = 0
    number = myList[0]
    freq=0
    while index < len(myList):
        if myList[index] == number:
            freq = freq +1
        else:
            print ("Found {} of {}".format(freq, myList[index-1]))
            freq=1
            number=myList[index]

        index = index +1
    print("Found {} of {}".format(freq, myList[index-1]))
```

Exercise 1: What is the sum?

Implement a recursive function that returns the sum of all digits in a positive integer

(e.g. `sum_digits(222)` → $2+2+2 = 6$)

```
def sum_digits(content):
    content_str = str(content)
    content_len = len(content_str)
    if content_len == 1:
        return content
    else:
        middle = content_len // 2
        left = content_str[0:middle]
        right = content_str[middle:content_len]
        return (sum_digits(int(left)) + sum_digits(int(right)))
```

```
sum_digits(222)
```

1. What is the base case
2. What is the time complexity?

d = number of digits in the original number

We can conclude that the total number of recursive calls is bounded by a constant $\cdot d$
Since asymptotic analysis requires us to remove constants, the time complexity is $O(d)$

Exercise 2: Unique vowels

Implement a recursive function that returns all unique vowels

(e.g. `unique_vowels("Recursion is great!")` → `['e', 'u', 'i', 'o', 'a']`)

```
def is_vowel(letter):
    vowels = {"a", "e", "i", "o", "u"}
    return letter in vowels

def recursive_vowels(content, vowels=None):
    if vowels == None:
        vowels = set()
    content_len = len(content)
    if content_len == 1:
        content_lower = content.lower()
        if is_vowel(content_lower):
            vowels.add(content_lower)
        return vowels
    else:
        print(f"We have {vowels}")
        middle = content_len // 2
        recursive_vowels(content[0:middle], vowels)
        recursive_vowels(content[middle:content_len], vowels)
        return vowels

print(recursive_vowels("coffee store")) # {'o', 'e'}
print(recursive_vowels("Aa")) # {'a'}
```

1. What is the base case

When `content_len == 1` (the string is one character)

2. What is the time complexity?

$O(n \log(n))$: There are $\log n$ levels of calls and we do n work at each level due to slicing of the arrays (all elements are moved from the original array to the new smaller array)

Can be improved to $O(n)$ if indexes are used instead of slicing (no copying of elements)

Exercise 3: Coffee Inheritance

Create a Coffee class with the attributes:

- name
- price

Create two subclasses:

- Espresso
- Latte

Each subclass should:

- Have its own default price
- Have a to_string function for printing its content

```
class Coffee:
    def __init__(self, name, cost):
        self.name = name
        self.cost = cost

class Espresso(Coffee):
    def __init__(self):
        super().__init__(name = "espresso", cost = 15)

    def to_string(self):
        return f"Espresso, 1 shot: {self.cost} DKK"

class Latte(Coffee):
    def __init__(self):
        super().__init__(name = "latte", cost = 35)

    def to_string(self):
        return f"Latte, whole milk: {self.cost} DKK"

my_latte = Latte()
print(my_latte.to_string())
```

Exercise 4: Creating a CoffeeStore

Implement a CoffeeStore class that holds a coffee_menu with a to_string method that could return this:

```
1  Menu:
2  - Espresso (small): 18 kr
3  - Latte (medium): 24 kr
4  - Cappuccino (large): 28 kr
```

Exercise 4:

Creating a Coffee store

```
class CoffeeStore:
    def __init__(self):
        self.menu = [Espresso(), Latte()]

    def to_string(self):
        current_content = "MENU:"
        for drink in self.menu:
            current_content = current_content + f"\n {drink.to_string()}"

        return current_content

my_store = CoffeeStore()
print(my_store.to_string())
```

Exercise 5: Binary search

Implement a function that uses binary search on a sorted list of ints, to find out how many times an int occurs in the list:

Hint: find the index of left-most occurrence and right-most and calculate the distance between these

```
1 count_occurrences([1, 2, 2, 2, 3, 4], 2) # Returns 3
2 count_occurrences([1, 3, 5, 7], 4)      # Returns 0
```

A solution that use recursion to find the element and then the first and last index – very complex, an example of recursion that is difficult to understand.

```
def binary_search(content, item, index_type, low=0, high=None):
    if high == None:
        high = len(content) - 1

    if low > high:
        return -1

    middle = (low + high) // 2
    if content[middle] == item:
        index = -1
        if index_type == "left":
            index = binary_search(content, item, index_type, low, high=middle - 1)
        else:
            index = binary_search(content, item, index_type, low = middle + 1, high=high)
        return index if index != -1 else middle

    elif content[middle] < item:
        return binary_search(content, item, index_type, low = middle + 1, high=high)
    else:
        return binary_search(content, item, index_type, low, high=middle - 1)

def count_occurrences(content, item):
    left_index = binary_search(content, item, index_type = "left")
    right_index = binary_search(content, item, index_type = "right")
    return right_index - left_index + 1 if left_index != -1 else 0

print(count_occurrences([1,2,2,2,3], 2))
```

A solution uses recursion to find the element and then iteration to find the first and last element (much simpler).

```
def numberOfoccurrences(elm, mylist, low=0, high=None):
    if high is None:
        high = len(mylist) - 1

    if low > high:
        return 0

    mid = (low+high)//2

    if elm == mylist[mid]:
        low = mid
        high = mid
        while low>0 and mylist[low-1] == elm:
            low=low-1
        while high<len(mylist)-1 and mylist[high+1] == elm:
            high=high+1
        return high-low+1
    elif elm < mylist[mid]:
        return numberOfoccurrences(elm, mylist, low, mid-1)
    else:
        return numberOfoccurrences(elm, mylist, mid+1, high)
```