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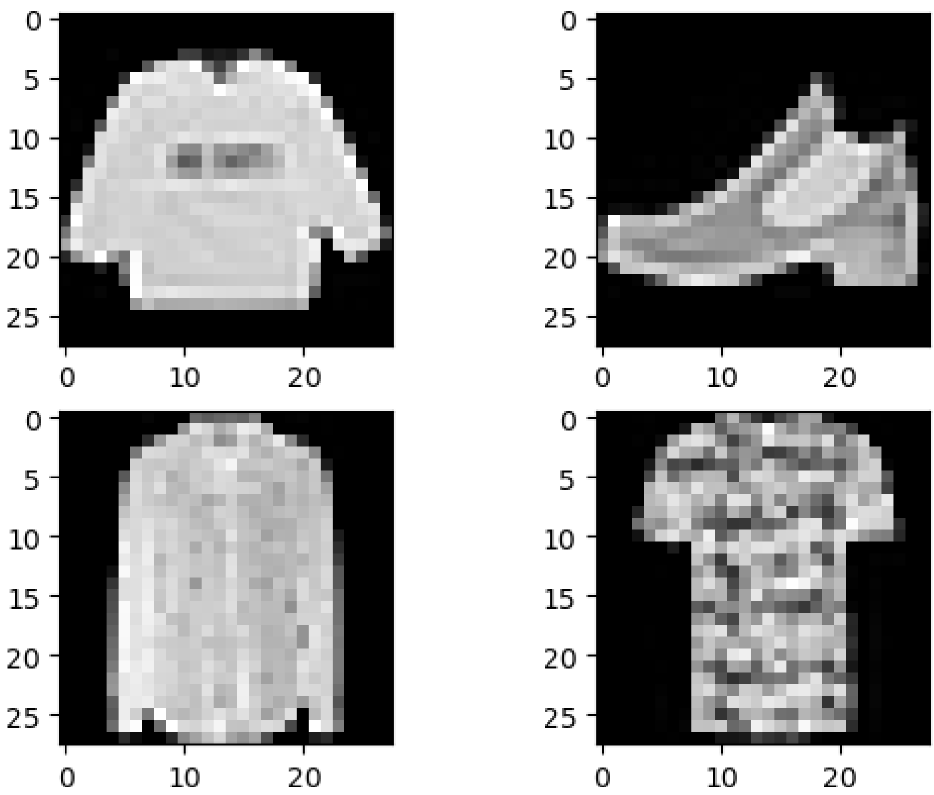
# Introduction

In [79]:
```
import torch
from AutoDecoder import AutoDecoder
from utils import create_dataloaders, plot_tsne
from evaluate import evaluate_model
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
import os
```

In [80]:
```
# Load the dataset (train and test splits are handled in the function)
train_ds, train_dl, test_ds, test_dl = create_dataloaders(data_path='dataset', ba

# Debugging:
print(f"Training dataset size: {len(train_ds)}")
print(f"Test dataset size: {len(test_ds)}")
print(f"Number of batches in training DataLoader: {len(train_dl)}")
print(f"Number of batches in test DataLoader: {len(test_dl)}")
```

```
Training dataset size: 1000
Test dataset size: 1000
Number of batches in training DataLoader: 16
Number of batches in test DataLoader: 16
```

In [81]:
```
#explore the training dataset
plt.figure(1)
num_samples = 4
for i in range(num_samples):
    plt.subplot(221+i)
    plt.imshow(train_ds[i][1], cmap='gray')
plt.show()

#lets see the size of the images
print(f"Image size: {train_ds[0][1].shape}")
```




```
Image size: torch.Size([28, 28])
```

In [82]:
```python
#lets have a look at the values of the pixels of the images
print(f"Min pixel value: {train_ds[0][1].min()}")
print(f"Max pixel value: {train_ds[0][1].max()}")

# TODO: consider if we need to normalize the images
# for i in range(len(train_ds)):
#     train_ds[i] = (train_ds[i][0], train_ds[i][1].float()/255)
```

```
Min pixel value: 0
Max pixel value: 255
```

# AutoDecoder

## AutoDecoder Implementation Overview

In [83]:
```python
from AutoDecoder import Trainer
trainer = Trainer()
print(trainer.model)
```

```
AutoDecoder(
  (decoder): Sequential(
    (0): ConvTranspose2d(128, 256, kernel_size=(7, 7), stride=(1, 1), bias=False)
    (1): BatchNorm2d(256, eps=1e-05, momentum=0.1, affine=True, track_running_stat
s=True)
    (2): ReLU(inplace=True)
    (3): ConvTranspose2d(256, 128, kernel_size=(4, 4), stride=(2, 2), padding=(1,
1), bias=False)
    (4): BatchNorm2d(128, eps=1e-05, momentum=0.1, affine=True, track_running_stat
s=True)
    (5): ReLU(inplace=True)
    (6): ConvTranspose2d(128, 64, kernel_size=(4, 4), stride=(2, 2), padding=(1,
1), bias=False)
    (7): BatchNorm2d(64, eps=1e-05, momentum=0.1, affine=True, track_running_stats
=True)
    (8): ReLU(inplace=True)
    (9): ConvTranspose2d(64, 1, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1),
bias=False)
    (10): Sigmoid()
  )
)
```

# Architecture and Design Choices

## 1. **Decoder Architecture**

The AutoDecoder reconstructs images from a latent space representation using a fully convolutional structure with transposed convolution layers. The reason we are using transposed convolution layers is for upsampling, and also we want to imitate "miror imaging" of the Encoder (which is not present). Moreove, the transposed convolutions are used to effectively expand spatial dimensions, which is essential for reconstructing images from compressed latent vectors. In addition we reducre the number of channels across layers. This helps refine features during each upsampling step. Also, we took inspiration for the architecture of the decoder from the following paper - arXiv:1512.09300v2 here. Below is the architecture:

- **Input**: Latent vector of size $latent\_dim$, reshaped to $(\text{batch\_size}, \text{latent\_dim}, 1, 1)$.
- **Output**: Reconstructed images of size $28 \times 28$ with pixel values scaled to $[0, 255]$.

### Layer Details:

1. **Transposed Convolution Layers**:
    - **Layer 1**: Converts the latent vector into a feature map of size $7 \times 7$ with $256$ channels.
    - **Layer 2**: Upsamples the feature map to $14 \times 14$ with $128$ channels.
    - **Layer 3**: Further upsamples to $28 \times 28$ with 64 channels.
    - **Layer 4**: Outputs the final image with 1 channel (grayscale as the inputs) using a $3 \times 3$.
2. **Activation Functions**:
    - **ReLU**: Used in intermediate layers to introduce non-linearity.

- **Sigmoid**: Used in the output layer to normalize pixel values to $[0, 1]$. This output is multiplied by 255 in order for the values to be in the same scale as the data.

3. **Batch Normalization**:

   - Applied after each transposed convolution (except the last layer) to stabilize training and improve convergence.

4. **Upsampling**:

   - Achieved through transposed convolutions, which increase the spatial dimensions while learning meaningful patterns to fill in details. We remind that the input vector to the decoder is a vector in a smaller dimebsion so we need to "fill in" patterns in order to compare to the data.

---

## Model Parameters

1. **Kernel Size**:

   - Larger $7 \times 7$ kernel at the first layer captures broad features, while smaller kernels in later layers refine finer details.
   - We also used the following equations from the pytorch documnetation here:
   - $H_{\text{out}} = (H_{\text{in}} - 1) \times \text{stride}[0] - 2 \times \text{padding}[0] + \text{dilation}[0] \times (\text{kernel}\backslash\_\text{size}[0]$
   - $W_{\text{out}} = (W_{\text{in}} - 1) \times \text{stride}[1] - 2 \times \text{padding}[1] + \text{dilation}[1] \times (\text{kernel}\backslash\_\text{size}[1]$

2. **Strides and Padding**:

   - Configured such that it would ensure smooth upsampling.

3. **Model Depth**:

   - Four transposed convolution layers provide sufficient capacity to reconstruct $28 \times 28$ images without overfitting. Recall we took inspiration from the above paper.

4. **Latent Dimension**:

   - We needed to choose a dimnsion that is smaller than the dimension of the data $28 \times 28 = 784$. This is because the latent space represents a more compressed space of the data.
   - After experimentation where we observed the latents dimesion's effect the model's ability to generalize, we chose $latent\_dim = 128$.
   - The latent dimension gives a good balance between reducing the dimensionality of the latent space for compact representation and maintaining a higher dimensional size to enable the model to capture more complex features for better reconstruction.

---

## Training Parameters

1. **Optimizer**:

   - **Adam Optimizer**: Selected for its adaptive learning rate and momentum, ensuring stable and efficient optimization.

- Adaptive learning rate is when the learning rate for each parameter is adjusted individually based on how frequently it has been updated. Momentum uses past gradient information to smooth out updates and avoid oscillations.

2. **Learning Rate**:

   - Set to $0.005$, a starting value that gave stable convergence.

3. **Loss Function**:

   - **Reconstruction Loss**: Gave by the staff to use. This loss measures the discrepancy between the original and reconstructed images, ensuring the AutoDecoder prioritizes accurate image reconstruction.

4. **Batch Size**:

   - $batch\_size = 64$: Provided a balance between computational efficiency and stable gradient updates.

5. **Epochs**:

   - Trained for 100 epochs in order to minimize reconstruction loss.

In [84]:
```
trainer.train_and_evaluate()
```

```
Epoch 1/100
Epoch [1/100], Loss: 0.5514
Epoch 2/100
Epoch [2/100], Loss: 0.4765
Epoch 3/100
Epoch [3/100], Loss: 0.4364
Epoch 4/100
Epoch [4/100], Loss: 0.3958
Epoch 5/100
Epoch [5/100], Loss: 0.3686
Epoch 6/100
Epoch [6/100], Loss: 0.3540
Epoch 7/100
Epoch [7/100], Loss: 0.3425
Epoch 8/100
Epoch [8/100], Loss: 0.3382
Epoch 9/100
Epoch [9/100], Loss: 0.3452
Epoch 10/100
Epoch [10/100], Loss: 0.3416
Epoch 11/100
Epoch [11/100], Loss: 0.3330
Epoch 12/100
Epoch [12/100], Loss: 0.3307
Epoch 13/100
Epoch [13/100], Loss: 0.3145
Epoch 14/100
Epoch [14/100], Loss: 0.3085
Epoch 15/100
Epoch [15/100], Loss: 0.3058
Epoch 16/100
Epoch [16/100], Loss: 0.3014
Epoch 17/100
Epoch [17/100], Loss: 0.2885
Epoch 18/100
Epoch [18/100], Loss: 0.2712
Epoch 19/100
Epoch [19/100], Loss: 0.2625
Epoch 20/100
Epoch [20/100], Loss: 0.2511
Epoch 21/100
Epoch [21/100], Loss: 0.2146
Epoch 22/100
Epoch [22/100], Loss: 0.1768
Epoch 23/100
Epoch [23/100], Loss: 0.1915
Epoch 24/100
Epoch [24/100], Loss: 0.1700
Epoch 25/100
Epoch [25/100], Loss: 0.1483
Epoch 26/100
Epoch [26/100], Loss: 0.1401
Epoch 27/100
Epoch [27/100], Loss: 0.1422
Epoch 28/100
Epoch [28/100], Loss: 0.1390
Epoch 29/100
Epoch [29/100], Loss: 0.1266
Epoch 30/100
Epoch [30/100], Loss: 0.1256
Epoch 31/100
```

```
Epoch [31/100], Loss: 0.1227
Epoch 32/100
Epoch [32/100], Loss: 0.1128
Epoch 33/100
Epoch [33/100], Loss: 0.1163
Epoch 34/100
Epoch [34/100], Loss: 0.1138
Epoch 35/100
Epoch [35/100], Loss: 0.1224
Epoch 36/100
Epoch [36/100], Loss: 0.1213
Epoch 37/100
Epoch [37/100], Loss: 0.0957
Epoch 38/100
Epoch [38/100], Loss: 0.0901
Epoch 39/100
Epoch [39/100], Loss: 0.0939
Epoch 40/100
Epoch [40/100], Loss: 0.1010
Epoch 41/100
Epoch [41/100], Loss: 0.0977
Epoch 42/100
Epoch [42/100], Loss: 0.0904
Epoch 43/100
Epoch [43/100], Loss: 0.0923
Epoch 44/100
Epoch [44/100], Loss: 0.0890
Epoch 45/100
Epoch [45/100], Loss: 0.0843
Epoch 46/100
Epoch [46/100], Loss: 0.0839
Epoch 47/100
Epoch [47/100], Loss: 0.0854
Epoch 48/100
Epoch [48/100], Loss: 0.0821
Epoch 49/100
Epoch [49/100], Loss: 0.0794
Epoch 50/100
Epoch [50/100], Loss: 0.0820
Epoch 51/100
Epoch [51/100], Loss: 0.0797
Epoch 52/100
Epoch [52/100], Loss: 0.0809
Epoch 53/100
Epoch [53/100], Loss: 0.0881
Epoch 54/100
Epoch [54/100], Loss: 0.0962
Epoch 55/100
Epoch [55/100], Loss: 0.1037
Epoch 56/100
Epoch [56/100], Loss: 0.1036
Epoch 57/100
Epoch [57/100], Loss: 0.0857
Epoch 58/100
Epoch [58/100], Loss: 0.0717
Epoch 59/100
Epoch [59/100], Loss: 0.0675
Epoch 60/100
Epoch [60/100], Loss: 0.0676
Epoch 61/100
Epoch [61/100], Loss: 0.0678
```

```
Epoch 62/100
Epoch [62/100], Loss: 0.0699
Epoch 63/100
Epoch [63/100], Loss: 0.0687
Epoch 64/100
Epoch [64/100], Loss: 0.0654
Epoch 65/100
Epoch [65/100], Loss: 0.0632
Epoch 66/100
Epoch [66/100], Loss: 0.0625
Epoch 67/100
Epoch [67/100], Loss: 0.0632
Epoch 68/100
Epoch [68/100], Loss: 0.0645
Epoch 69/100
Epoch [69/100], Loss: 0.0629
Epoch 70/100
Epoch [70/100], Loss: 0.0599
Epoch 71/100
Epoch [71/100], Loss: 0.0579
Epoch 72/100
Epoch [72/100], Loss: 0.0573
Epoch 73/100
Epoch [73/100], Loss: 0.0586
Epoch 74/100
Epoch [74/100], Loss: 0.0604
Epoch 75/100
Epoch [75/100], Loss: 0.0577
Epoch 76/100
Epoch [76/100], Loss: 0.0579
Epoch 77/100
Epoch [77/100], Loss: 0.0578
Epoch 78/100
Epoch [78/100], Loss: 0.0597
Epoch 79/100
Epoch [79/100], Loss: 0.0604
Epoch 80/100
Epoch [80/100], Loss: 0.0574
Epoch 81/100
Epoch [81/100], Loss: 0.0569
Epoch 82/100
Epoch [82/100], Loss: 0.0592
Epoch 83/100
Epoch [83/100], Loss: 0.0622
Epoch 84/100
Epoch [84/100], Loss: 0.0600
Epoch 85/100
Epoch [85/100], Loss: 0.0574
Epoch 86/100
Epoch [86/100], Loss: 0.0567
Epoch 87/100
Epoch [87/100], Loss: 0.0565
Epoch 88/100
Epoch [88/100], Loss: 0.0540
Epoch 89/100
Epoch [89/100], Loss: 0.0494
Epoch 90/100
Epoch [90/100], Loss: 0.0491
Epoch 91/100
Epoch [91/100], Loss: 0.0464
Epoch 92/100
```

```
Epoch [92/100], Loss: 0.0472
Epoch 93/100
Epoch [93/100], Loss: 0.0523
Epoch 94/100
Epoch [94/100], Loss: 0.0536
Epoch 95/100
Epoch [95/100], Loss: 0.0522
Epoch 96/100
Epoch [96/100], Loss: 0.0520
Epoch 97/100
Epoch [97/100], Loss: 0.0505
Epoch 98/100
Epoch [98/100], Loss: 0.0459
Epoch 99/100
Epoch [99/100], Loss: 0.0448
Epoch 100/100
Epoch [100/100], Loss: 0.0439
Training Loss: 0.0439
Test Loss: 0.1270
Training and Evaluation Complete!
```

## TSNE Plot

Plotting TSNE plot.

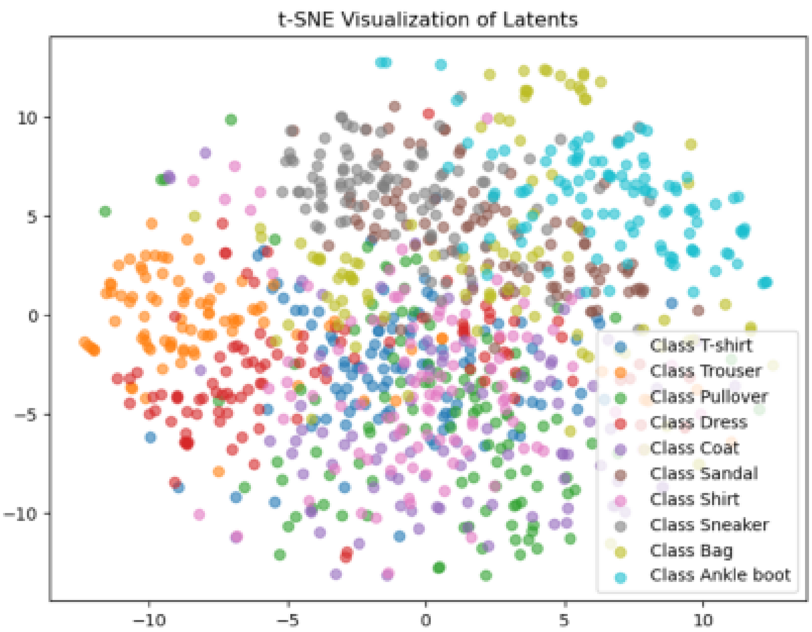
In [85]:

```python
trainer.plot_tsne()
```

```
Generating t-SNE plot...
<Figure size 800x600 with 0 Axes>
```

In [86]:

```python
file_name = 'tsne_plot_AD'
image_path = f'{file_name}.png'
img = mpimg.imread(image_path)
plt.imshow(img)
plt.axis('off')  # Hide axes
plt.show()
```




We can see from the plot that there are som classes are organized well, but overall the classes arent seperated well. This is probably due to the fact that the model didn't learn a continuous space but rather a mapping from latent vectors to a reconstructed image similar to the data.

## Testing Samples

In [9]:
```
trainer.test_samples()
```

```
Generating random samples...
Saving images...
Images saved to output_images/random_latents_images.png
Saving images...
Images saved to output_images/test_set_latents_images.png
```

In [10]:
```
image_paths = ['output_images/random_latents_images.png', 'output_images/test_set

fig, axs = plt.subplots(1, len(image_paths), figsize=(15, 5))  # 1 row, len(image

for i, image_path in enumerate(image_paths):
    img = mpimg.imread(image_path)
    axs[i].imshow(img)
    axs[i].axis('off')  # Hide axes

plt.show()
```


## Comparing Random & Test Images

**Results Comparission**

- We can see that the decoded test set latents gave far better results than the decoded latents from $N(0, I)$.
- The decoder learned to take a mapping of a certain image in the latent space and to construct it to an image that is close to the original one. The test latents were fitted to a close mapping in the latent space of similar images. Then the Decoder can take the test latent and reconstruct an image that is similar to the images in the data.
- On the other hand, latent vectors that were sampled randomly from $N(0, I)$ were not fitted and therefore the decoder did not learn how to construct images similiar to the data images from their mappings in the latent space.
- The decoder learned to reconstruct specific latent mappings to reconstructions closely resembling the original images. Test latents, optimized to align with the learned latent space, enabled the decoder to produce accurate reconstructions. However, randomly sampled latents do not align with the learnt latent mappings, resulting in less coherent and detailed images. This reflects the model's reliance on a latent space shaped by training (not continuous) and its limited ability to generalize to unseen latent points.

# Variational Auto Decoder - VAD

## VariationalAutoDecoder Implementation Overview

The Variational AutoDecoder is designed to reconstruct images by learning distributions in the latent space, unlike a standard AutoDecoder, which uses fixed latent vectors. This approach leverages learned distributions, making the architecture more versatile for tasks involving random sampling from the latent space.

In this design, two key parameters are initialized for each sample: the mean (μ) and standard deviation (σ) of the latent distribution. These trainable parameters enable the model to learn a unique distribution for each sample, improving generalization when generating new latents. The decoder consists of a sequence of linear layers followed by transposed convolutional layers.

The reparameterization trick was used to scale and shift Gaussian noise using the learned μ and σ for each sample. This approach ensures differentiability, enabling backpropagation through the random sampling process.

Laslty,in thhe training process an ELBO loss function was used, which combines reconstruction loss, quantifying the difference between the original and reconstructed images, and a KL divergence term that ensures the learned distribution remains close to a standard normal distribution. This balance between reconstruction accuracy and distribution regularization is essential for effective training and generalization.

In [1]:
```python
from VariationalAutoDecoder import VariationalAutoDecoderNormal, VariationalAutoD
import numpy as np
import torch
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
```

In [2]:
```python
learning_rate = 0.02
latent_dim = 60
model_normal = VariationalAutoDecoderNormal(latent_dim=latent_dim,lr=learning_rat
print(model_normal.decoder)
```

```
VAD_Decoder(
  (decoder): Sequential(
    (0): Linear(in_features=60, out_features=12544, bias=True)
    (1): ReLU(inplace=True)
    (2): Unflatten(dim=1, unflattened_size=(256, 7, 7))
    (3): ConvTranspose2d(256, 128, kernel_size=(4, 4), stride=(2, 2), padding=(1,
1), bias=False)
    (4): BatchNorm2d(128, eps=1e-05, momentum=0.1, affine=True, track_running_stat
s=True)
    (5): ReLU(inplace=True)
    (6): ConvTranspose2d(128, 64, kernel_size=(4, 4), stride=(2, 2), padding=(1,
1), bias=False)
    (7): BatchNorm2d(64, eps=1e-05, momentum=0.1, affine=True, track_running_stats
=True)
    (8): ReLU(inplace=True)
    (9): ConvTranspose2d(64, 1, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1),
bias=False)
    (10): Sigmoid()
  )
)
```

## Architecture and Design Choices

### 1. Decoder Architecture

The Decoder architecture in the VAD is very similar to the AutoDecoder architecture: We first use a fully connected layer for the decoder to learn how to upsample a vector from the the latent space. We then use RELU to introduce non-linearity and after that we unflattent the result vector in order for it to be in proper dimensions for the convolutional layers.

Then we use fully convolutional structure with transposed convolution layers. The reason we are using transposed convolution layers was stated in the AD architecture above. Below is the architecture:

- **Input**: Latent vector of size $latent\_dim$.
- **Output**: Reconstructed images of size $28 \times 28$ with pixel values scaled to $[0, 255]$.

### Layer Details

1. **Fully Connected Layer and Reshaping**:

   - **Layer 0**: Maps the latent vector into a feature vector of size $256 \times 7 \times 7 = 12544$.

- **Reshape**: Converts the output into a feature map with dimensions $7 \times 7$ and $256$ channels, preparing it for convolutional decoding.

2. **Transposed Convolution Layers**:

- **Layer 1**: Upsamples the feature map from $7 \times 7$ to $14 \times 14$ with $128$ channels.
- **Layer 2**: Further upsamples the feature map to $28 \times 28$ with $64$ channels.
- **Layer 3**: Outputs the final image with $1$ channel (grayscale, as the inputs) using a $3 \times 3$ kernel.

3. **Activation Functions**:

- **ReLU**: Applied after each intermediate layer to introduce non-linearity and allow the model to learn complex relationships.
- **Sigmoid**: Applied in the output layer to normalize pixel values to the range $[0, 1]$, and then multiplied by $255$ aligning with the expected intensity values of the data.

4. **Batch Normalization**:

- Included after each transposed convolution layer (except the last) to stabilize learning by normalizing activations, improving convergence and reducing overfitting.

5. **Upsampling**:

- Performed using transposed convolutions, which increase the spatial dimensions of the feature maps while learning patterns to generate meaningful reconstructions.
- This step is crucial as the input vector to the decoder is a compressed representation, and the decoder must "expand" and "fill in" details to recreate an image comparable to the original data.

---

## Model Parameters

1. **Kernel Size**:

- A $4 \times 4$ kernel is used in the first two transposed convolution layers to upsample and capture broad patterns efficiently. The final layer uses a smaller $3 \times 3$ kernel to refine details in the reconstructed image.
- The output dimensions after each layer were calculated using the equations from the PyTorch documentation here:
  - $H_{\text{out}} = (H_{\text{in}} - 1) \times \text{stride}[0] - 2 \times \text{padding}[0] + \text{dilation}[0] \times (\text{kernel}\backslash\_\text{siz}$
  - $W_{\text{out}} = (W_{\text{in}} - 1) \times \text{stride}[1] - 2 \times \text{padding}[1] + \text{dilation}[1] \times (\text{kernel}\backslash\_\text{si}$

2. **Stride**:

- A stride of $2 \times 2$ in the first two layers doubles the spatial dimensions of the feature maps, enabling effective upsampling.

3. **Padding**:

- Padding of $1 \times 1$ ensures the output dimensions grow proportionally while maintaining spatial consistency during upsampling.

4. **Model Depth**:

    - Three transposed convolution layers provide sufficient capacity to reconstruct $28 \times 28$ images without overfitting. Recall we took inspiration from the above paper.

5. **Latent Dimension**:

    - After experimentation where we observed the latents dimesion's effect on the VAD loss and the model's ability to generalize, we chose $latent\_dim = 60$.
    - The latent dimension gives a good balance between reducing the dimensionality of the latent space for compact representation and maintaining a higher dimensional size to enable the model to capture more complex features for better reconstruction.

---

## Training Parameters

### Overview:

The parametes that were trained in the model except for the decoder, were the log of the variance and the mean for each training sample. Training the log of the variance helps ensure numerical stability and precision during the training process.

After experimanting we found that the following hyper-parameters balance well between the reconstruction of images and the orginaization of a learned continuos latent space.

1. **Optimizer**:

    - **Adam Optimizer**: Selected for its adaptive learning rate and momentum, ensuring stable and efficient optimization.
    - Adaptive learning rate is when the learning rate for each parameter is adjusted individually based on how frequently it has been updated. Momentum uses past gradient information to smooth out updates and avoid oscillations.

2. **Learning Rate**:

    - Set to $0.02$, a starting value that gave stable convergence.

3. **Loss Function**:

    - The loss function combines two key components: the **reconstruction loss** and the **KL-Divergence loss**. These terms work together to balance reconstruction accuracy and the organization of the latent space.

    - **Reconstruction Loss**:

        - The reconstruction term in the loss function measures how accurately the decoder can generate an image that is similar to the original input. This guides the model in outputing new images that are similar to the input images during training.
        - We use Mean Squared Error (MSE) for the reconstruction term.

- **Purpose**: MSE measures the pixel-wise difference between the original input image and the reconstructed image, providing an evaluation of reconstruction quality. It penalizes larger errors more heavily, ensuring that even small differences in pixel values are minimized.
- **Why MSE?**: MSE aligns well with our goal of producing outputs visually close to the input. It was also used in the tutorials robust loss for tasks involving grayscale images.

- **KL-Divergence Loss**:

    - The KL divergence term in the loss measures how closely the learned latent space distribution matches the target distribution, which is a standard normal distribution. Its purpose is to guide the model in generating better samples from the latent space. This enables us to sample from the prior target distibution, decode the samples, and obtain reconstructions that are consistent with the distribution $P(X)$.
    - **Purpose**: This term ensures that the latent space remains structured and meaningful, allowing for smooth interpolation. Without it, the latent space might become disorganized, leading to poor generalization when decoding random samples. Also, this term sgives aregularization effect.

- **Total Loss**:

    - The total loss is computed as the weighted sum of the two terms: $\text{Loss} = \text{Reconstruction Loss (MSE)} + \beta \cdot \text{KL-Divergence Loss}$. Where $\beta$ controls the trade-off between the reconstruction of the image and latent space regularization.
    - A good balance ensures that the decoder reconstructs images accurately while also allowing meaningful latent space manipulation for generative tasks.

4. **Batch Size**:

    - $batch\_size = 64$: Provided a balance between computational efficiency and stable gradient updates.

5. **Epochs**:

    - After experimenting with fixed hyper-parameters, we observed that as we increased the epochs the KL-divergence term increased resulting in a disorganized latent space. On the other hand the reconsruction quality becomes better.

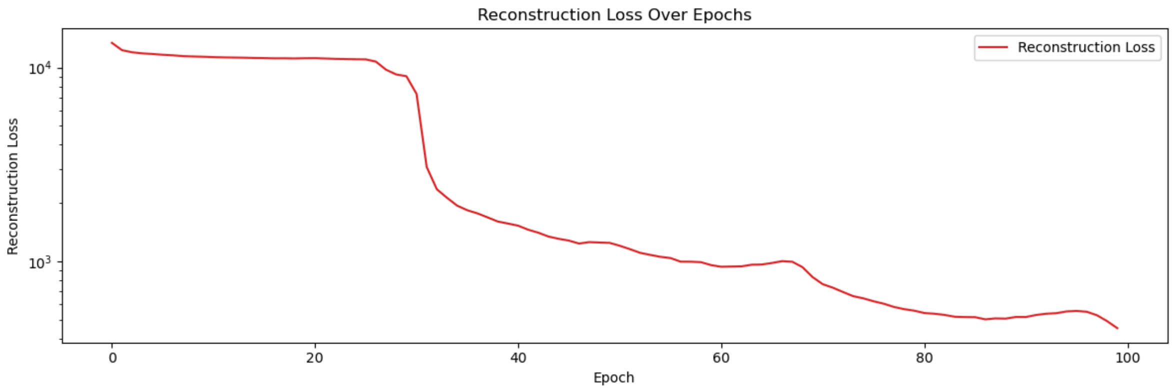
Training VAD With Normal Distribution.

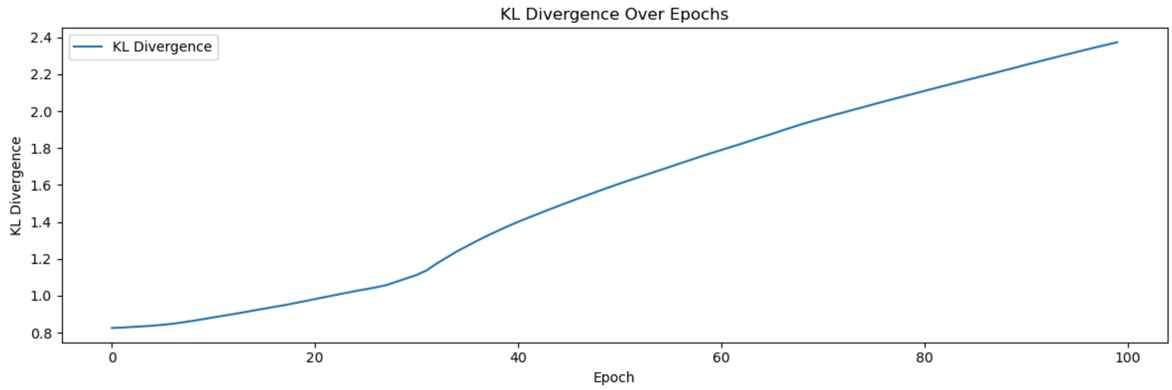
In [8]:
```
epochs = 100
beta = 1
learning_rate = 0.02
latent_dim = 60
model_normal = VariationalAutoDecoderNormal(latent_dim=latent_dim,lr=learning_rat
train_loss,_,_ = model_normal.fit_and_train(num_epochs=epochs, beta=beta)
print(f'Training loss: {train_loss:.4f}')
```

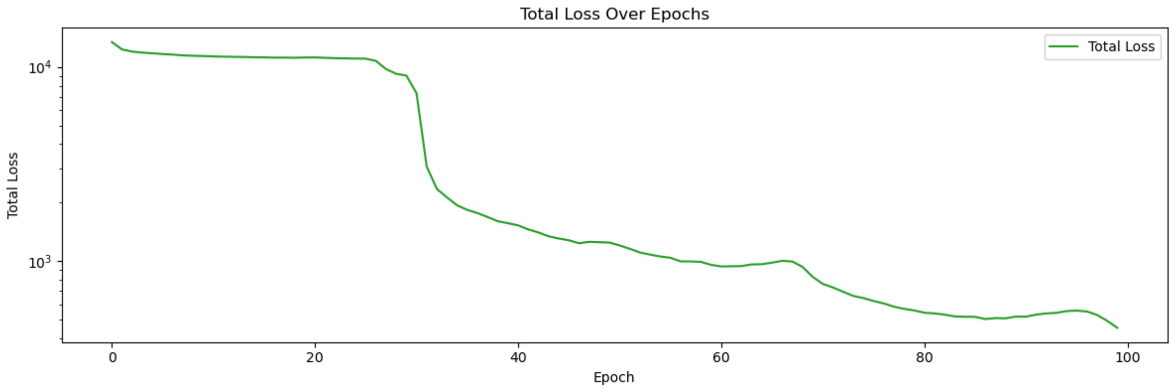
```
test_loss = model_normal.test_vad(num_epochs=epochs, learning_rate=learning_rate)
print(f'Test loss: {test_loss:.4f}')
```

```
Training the VAD model...
Epoch 1, Loss: 13412.0275
Epoch 2, Loss: 12295.4293
Epoch 3, Loss: 11987.2966
Epoch 4, Loss: 11835.2003
Epoch 5, Loss: 11748.7730
Epoch 6, Loss: 11639.7417
Epoch 7, Loss: 11566.9484
Epoch 8, Loss: 11454.3896
Epoch 9, Loss: 11405.9931
Epoch 10, Loss: 11369.3024
Epoch 11, Loss: 11318.7924
Epoch 12, Loss: 11283.7548
Epoch 13, Loss: 11261.3706
Epoch 14, Loss: 11238.8860
Epoch 15, Loss: 11206.0403
Epoch 16, Loss: 11191.6157
Epoch 17, Loss: 11154.7577
Epoch 18, Loss: 11161.7433
Epoch 19, Loss: 11139.7089
Epoch 20, Loss: 11171.9247
Epoch 21, Loss: 11181.9141
Epoch 22, Loss: 11133.9845
Epoch 23, Loss: 11085.0201
Epoch 24, Loss: 11061.1423
Epoch 25, Loss: 11035.4640
Epoch 26, Loss: 11017.7168
Epoch 27, Loss: 10731.7625
Epoch 28, Loss: 9740.6093
Epoch 29, Loss: 9209.7742
Epoch 30, Loss: 9022.1972
Epoch 31, Loss: 7311.8362
Epoch 32, Loss: 3067.3180
Epoch 33, Loss: 2353.8813
Epoch 34, Loss: 2126.5519
Epoch 35, Loss: 1937.2821
Epoch 36, Loss: 1834.2201
Epoch 37, Loss: 1767.2689
Epoch 38, Loss: 1685.4140
Epoch 39, Loss: 1604.6907
Epoch 40, Loss: 1566.7514
Epoch 41, Loss: 1527.4735
Epoch 42, Loss: 1455.8788
Epoch 43, Loss: 1404.7006
Epoch 44, Loss: 1342.2911
Epoch 45, Loss: 1307.0476
Epoch 46, Loss: 1279.6443
Epoch 47, Loss: 1235.6010
Epoch 48, Loss: 1256.7415
Epoch 49, Loss: 1250.9291
Epoch 50, Loss: 1246.0749
Epoch 51, Loss: 1204.0565
Epoch 52, Loss: 1156.6501
Epoch 53, Loss: 1107.4404
Epoch 54, Loss: 1080.3976
Epoch 55, Loss: 1055.7768
Epoch 56, Loss: 1040.4172
Epoch 57, Loss: 996.9813
Epoch 58, Loss: 996.1762
Epoch 59, Loss: 990.9617
Epoch 60, Loss: 956.8306
```

```
Epoch 61, Loss: 938.3430
Epoch 62, Loss: 940.9349
Epoch 63, Loss: 942.9159
Epoch 64, Loss: 961.4120
Epoch 65, Loss: 963.7977
Epoch 66, Loss: 981.4424
Epoch 67, Loss: 1003.0753
Epoch 68, Loss: 995.7081
Epoch 69, Loss: 934.1452
Epoch 70, Loss: 829.9614
Epoch 71, Loss: 763.0854
Epoch 72, Loss: 731.9277
Epoch 73, Loss: 695.4336
Epoch 74, Loss: 662.1157
Epoch 75, Loss: 645.2271
Epoch 76, Loss: 623.4058
Epoch 77, Loss: 605.6898
Epoch 78, Loss: 583.0952
Epoch 79, Loss: 567.9419
Epoch 80, Loss: 557.7659
Epoch 81, Loss: 542.4434
Epoch 82, Loss: 537.8825
Epoch 83, Loss: 530.2899
Epoch 84, Loss: 518.9950
Epoch 85, Loss: 517.1479
Epoch 86, Loss: 516.5048
Epoch 87, Loss: 503.3692
Epoch 88, Loss: 508.8968
Epoch 89, Loss: 507.5735
Epoch 90, Loss: 517.7974
Epoch 91, Loss: 517.5896
Epoch 92, Loss: 530.0786
Epoch 93, Loss: 538.0578
Epoch 94, Loss: 541.5439
Epoch 95, Loss: 552.9220
Epoch 96, Loss: 556.2973
Epoch 97, Loss: 550.5939
Epoch 98, Loss: 528.5003
Epoch 99, Loss: 492.7271
Epoch 100, Loss: 453.6254
```



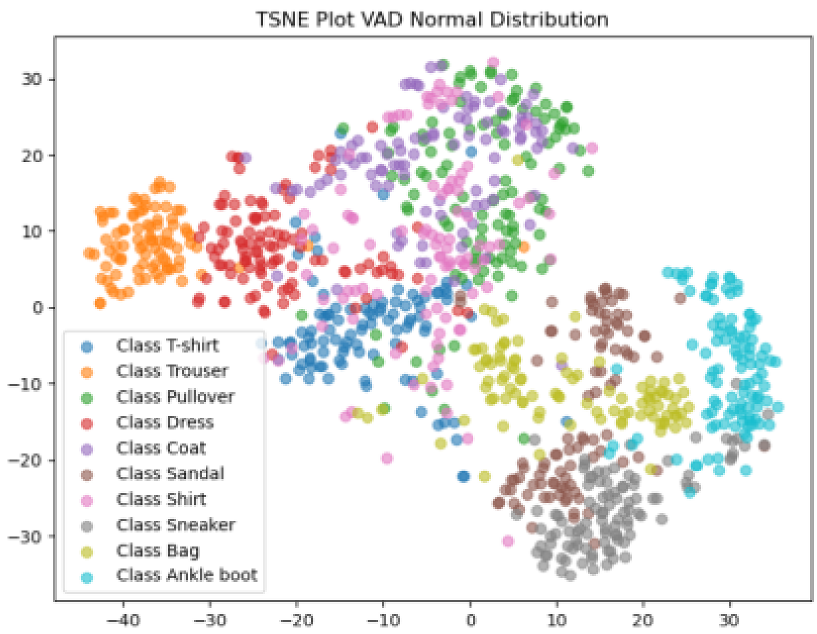

```
Training loss: 453.6254
Testing the VAD model...
Test loss: 0.1279
```

Plotting TSNE plot.

In [9]:
```
file_name = 'tsne_plot_VAD_normal'
image_title = 'TSNE Plot VAD Normal Distribution'
model_normal.plot_tsne(file_name=file_name, plot_title=image_title)
```

```
Generating t-SNE plot...
<Figure size 800x600 with 0 Axes>
```

In [10]:
```
image_path = f'{file_name}.png'
img = mpimg.imread(image_path)
plt.imshow(img)
plt.axis('off')  # Hide axes
plt.show()
```



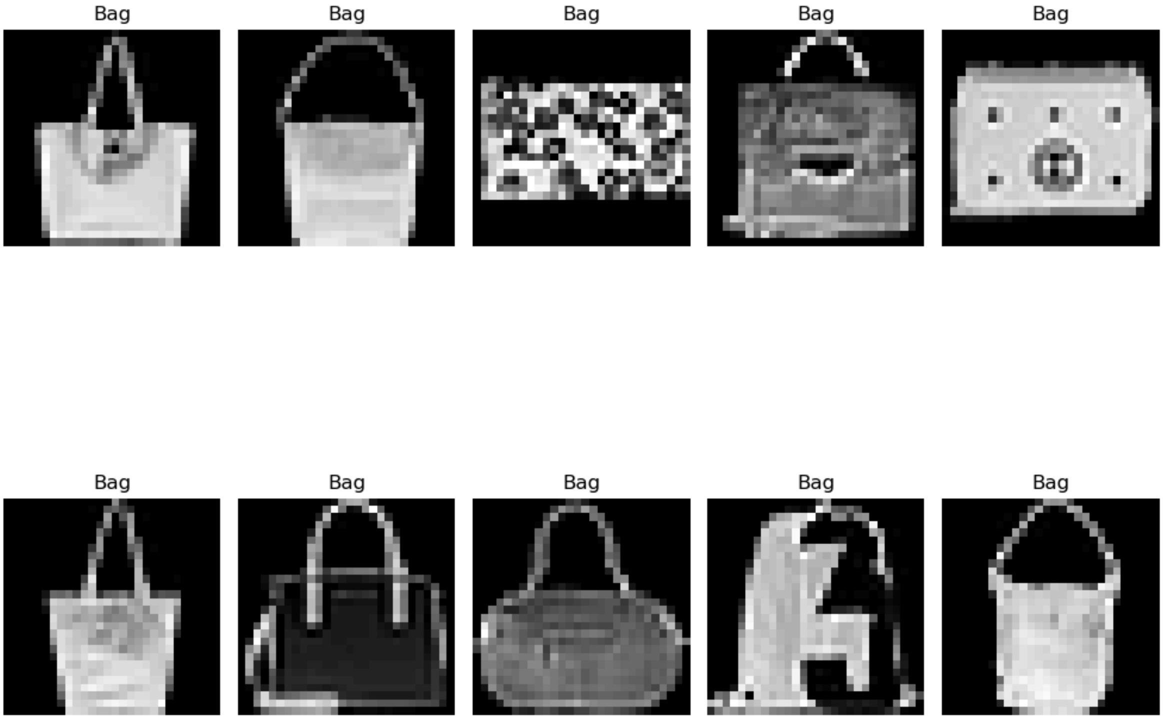

# Results Explanation

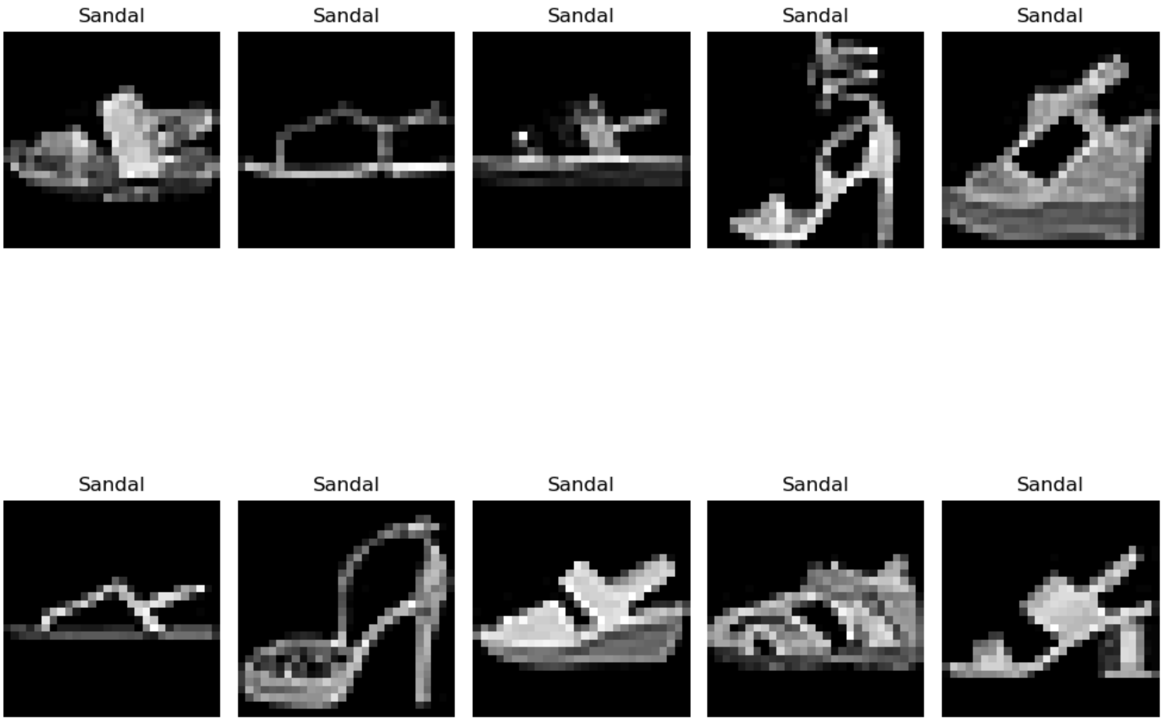
We can see that over all the classes are organized well. From the plot we can observe that the different shoe classes are near each others, but the model learnt to distinguish between the different kind of shoes (sandals, sneakers, ankle boots). The same explanation applies to the dress and trousers classes. Besides that we can observe that the classes coat, pullover and and shirt are not seperated well in the latent space. This due to the fact that these classes have features that are difficult for the model to capture and differentuate between them. Lastly we see that the bag class is very much apart from other classes but has two different clusters. This is probably due to the fact that are two kinds of bags.

### Bags & Sandals Classes - different bags & Sandals

In [11]:
```
BAG_CLASS = 8
SANDALS_CLASS = 5
bag_images = [model_normal.train_ds.X[i] for i in range(len(model_normal.train_ds
bag_images = bag_images[-5:] + bag_images[:5]


sandals_images = [model_normal.train_ds.X[i] for i in range(len(model_normal.trai
sandals_images = sandals_images[-5:] + sandals_images[:5]

# Plot 10 bag images
plt.figure(figsize=(10, 10))
for i, img in enumerate(bag_images):
    plt.subplot(2, 5, i + 1)
    plt.imshow(img.numpy(), cmap="gray")
    plt.axis("off")
    plt.title("Bag")
plt.tight_layout()
plt.show()
```

```python
# Plot 10 sandals images
plt.figure(figsize=(10, 10))
for i, img in enumerate(sandals_images):
    plt.subplot(2, 5, i + 1)
    plt.imshow(img.numpy(), cmap="gray")
    plt.axis("off")
    plt.title("Sandal")
plt.tight_layout()
plt.show()
```



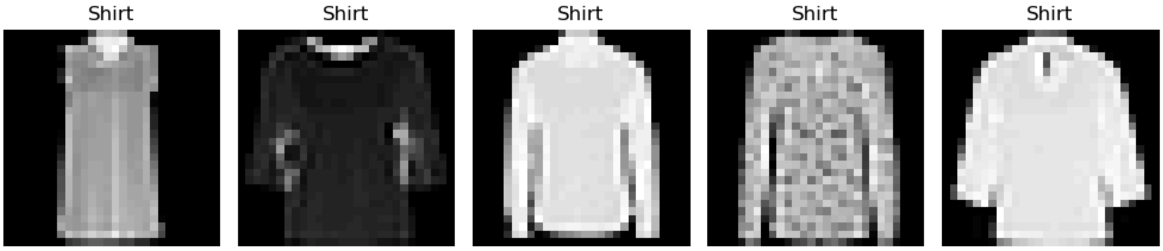

We can see that there are two different types of bags - bags with a strap and amore enveloped type of bag. In addition there are two different types of sandals - flat sandals and heeled sandals.

### Shirt, Pullover & Coat Classes

In [12]:
```
PULLOVER_CLASS = 2
COAT_CLASS = 4
SHIRT_CLASS = 6

pullover_images = [model_normal.train_ds.X[i] for i in range(len(model_normal.tra
pullover_images = pullover_images[-5:] + pullover_images[:5]

coat_images = [model_normal.train_ds.X[i] for i in range(len(model_normal.train_d
coat_images = coat_images[-5:] + coat_images[:5]

shirt_images = [model_normal.train_ds.X[i] for i in range(len(model_normal.train_
shirt_images = shirt_images[-5:] + shirt_images[:5]
```

In [13]:
```
# Plot 10 pullover images
plt.figure(figsize=(10, 10))
for i, img in enumerate(pullover_images):
    plt.subplot(2, 5, i + 1)
    plt.imshow(img.numpy(), cmap="gray")
    plt.axis("off")
    plt.title("Pullover")
plt.tight_layout()
plt.show()

# Plot 10 coat images
plt.figure(figsize=(10, 10))
for i, img in enumerate(coat_images):
    plt.subplot(2, 5, i + 1)
    plt.imshow(img.numpy(), cmap="gray")
    plt.axis("off")
    plt.title("Coat")
plt.tight_layout()
plt.show()

# Plot 10 shirt images
plt.figure(figsize=(10, 10))
for i, img in enumerate(shirt_images):
    plt.subplot(2, 5, i + 1)
    plt.imshow(img.numpy(), cmap="gray")
    plt.axis("off")
    plt.title("Shirt")
plt.tight_layout()
plt.show()
```


Pullover
Pullover
Pullover
Pullover
Pullover
Pullover
Pullover
Pullover
Pullover
Pullover
Coat
Coat
Coat
Coat
Coat
Coat
Coat
Coat
Coat
Coat

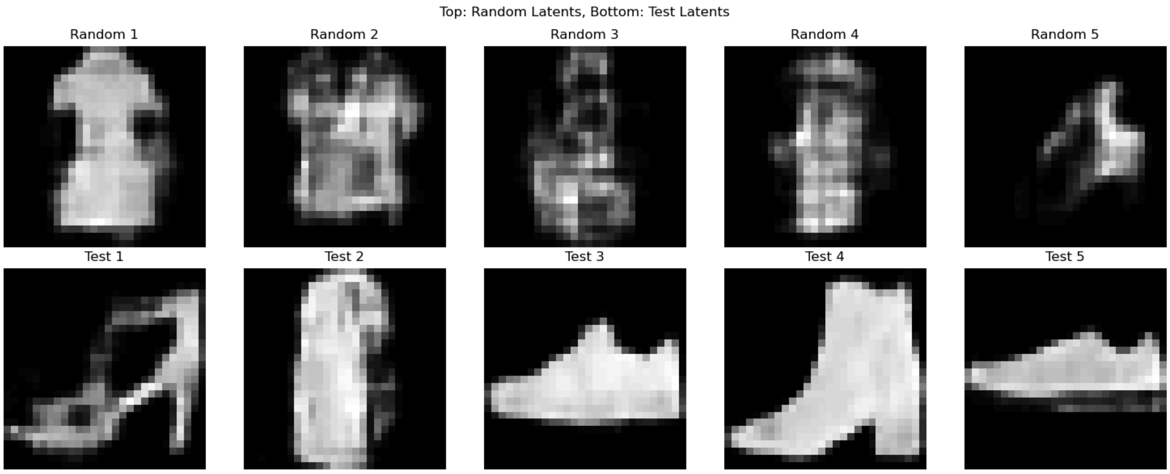




As we can see from the images above, it is very hard to tell the difference between the three classes.

## Testing Samples

In [98]:
```
random_latents = torch.randn(5, model_normal.latent_dim).to(model_normal.device)
random_decoded = model_normal.decode(random_latents)  # Decode the sampled latent

test_indices = torch.randint(0, len(model_normal.test_ds), (5,))
test_parameters = model_normal.test_parameters[test_indices]

test_mu = test_parameters[:, :model_normal.latent_dim]
test_log_var = test_parameters[:, model_normal.latent_dim:]

test_latents = model_normal._reparameterize(test_mu, test_log_var)
test_decoded = model_normal.decode(test_latents)

fig, axes = plt.subplots(2, 5, figsize=(15, 6))

for i, (random_img, test_img) in enumerate(zip(random_decoded, test_decoded)):
    axes[0, i].imshow(random_img.cpu().detach().numpy(), cmap='gray')
    axes[0, i].axis('off')
    axes[0, i].set_title(f"Random {i + 1}")

    axes[1, i].imshow(test_img.cpu().detach().numpy(), cmap='gray')
    axes[1, i].axis('off')
    axes[1, i].set_title(f"Test {i + 1}")

plt.suptitle("Top: Random Latents, Bottom: Test Latents")
plt.tight_layout()
plt.show()
```




## Explanation

The images reconstructed from test latents are generally better because:

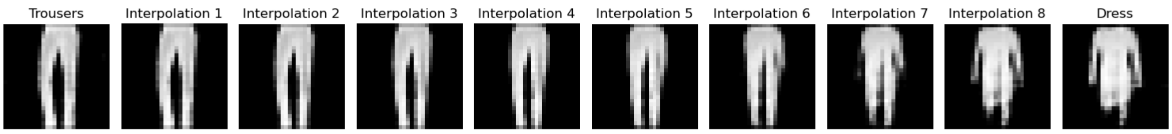
1. Test latent vectors are optimized during inference to minimize the reconstruction loss,ensuring that the latent representations capture meaningful features of the input data. This means they're found in regions of the latent space that the decoder has learned to map to realistic images during training.
2. While we train the VAD to push the latent distributions towards N(0, I), the actual learned distribution might not perfectly match this target distribution. There might be specific regions within the N(0, I) space that the model learns to associate with realistic images, while other regions might map to less realistic outputs. Random latents sampled from N(0, I) are not specific to the data distribution and may lie in regions of latent space that are less meaningful or poorly supported by the decoder. This highlights how the latent space learned by the model is data-dependent and structured to capture patterns in the training data.

## Interpolating Samples

It was unclear from the task how many interpolated samples we need to present - that task stated five, but also refered to an example with 10 images with 8 interpolations (original images in the start and the end).

In [99]:
```
TROUSERS_CLASS = 1
DRESS_CLASS = 3

trousers_images = [i for i in range(len(model_normal.test_ds)) if model_normal.te
dress_images = [i for i in range(len(model_normal.test_ds)) if model_normal.test_

trousers_index = trousers_images[0]
dress_index = dress_images[0]

trousers_params = model_normal.test_parameters[trousers_index]
dress_params = model_normal.test_parameters[dress_index]

trousers_mu = trousers_params[:model_normal.latent_dim]
trousers_log_var = trousers_params[model_normal.latent_dim:]
```

```
dress_mu = dress_params[:model_normal.latent_dim]
dress_log_var = dress_params[model_normal.latent_dim:]

latent_trousers = model_normal._reparameterize(trousers_mu, trousers_log_var)
latent_dress = model_normal._reparameterize(dress_mu, dress_log_var)




interpolation_coeffs = torch.linspace(0.1, 0.9, steps=9)

# Step 2: Interpolate between latent_bag and latent_sandal
interpolated_latents = [
    (1 - t) * latent_trousers + t * latent_dress for t in interpolation_coeffs
]

# Step 3: Decode the original and interpolated latent vectors
decoded_images = [model_normal.decode(latent.unsqueeze(0)).squeeze(0) for latent

# Step 4: Plot the images
fig, axes = plt.subplots(1, len(decoded_images)-1, figsize=(15, 3))
titles = ['Trousers', 'Interpolation 1', 'Interpolation 2', 'Interpolation 3', 'I

for ax, image, title in zip(axes, decoded_images, titles):
    ax.imshow(image.cpu().detach().numpy(), cmap='gray')
    ax.axis('off')
    ax.set_title(title)

plt.tight_layout()
plt.show()
```




## Using Different Distributions

### Laplace Distibution

We decided to use the laplace distribution. The laplace distribution is known for producing sparse representations due to its sharper peak around the mean. Sparse representations allow the latent space to focus on the most important features of the data. Moreover, the laplace distribution has a sharp peak around the mean, and the probability for extreme values decreases more slowly than it does for the normal distribution. This is why the distribution's extreme values are still relatively more likely. In data that contains rare but significant features, the laplace distribution can represent such variability. This can help distinguish between similar classes. In addition, it can potentially improve the model’s ability to generate realistic and varied samples.

#### Modifying The Reparametrization Trick

The trick transforms a sample from $\text{Uniform}(0, 1)$ into a sample from $\text{Laplace}(\mu, b)$.

#### 1. Laplace Distribution Properties

- **PDF**:

$$f(x) = \frac{1}{2b} \exp\left(-\frac{|x-\mu|}{b}\right)$$

- **CDF**:

$$F(x) = \begin{cases} \frac{1}{2} \exp\left(\frac{x-\mu}{b}\right) & \text{if } x \leq \mu, \\ 1 - \frac{1}{2} \exp\left(-\frac{x-\mu}{b}\right) & \text{if } x > \mu. \end{cases}$$

**2. Using Cases** The Laplace CDF is piecewise-defined:

- For $x \leq \mu$, the CDF grows exponentially toward ( 0.5 ).
- For $x > \mu$, the CDF grows exponentially from ( 0.5 ) to ( 1 ).

This requires splitting the inverse CDF into two cases:

- $u \leq 0.5$: Corresponds to the left half of the Laplace distribution.
- $u > 0.5$: Corresponds to the right half.

**Deriving the Inverse CDF** To reparameterize, we solve ( u = F(x) ) for ( x ):

- For $u \leq 0.5$:

$$u = \frac{1}{2} \exp\left(\frac{x-\mu}{b}\right) \implies x = \mu + b \log(2u).$$

- For $u > 0.5$:

$$u = 1 - \frac{1}{2} \exp\left(-\frac{x-\mu}{b}\right) \implies x = \mu - b \log(2 - 2u).$$

**4. Final Reparameterization Formula** To sample $z \sim \text{Laplace}(\mu, b)$:

$$z = \mu + b \cdot F^{-1}(u),$$

where $F^{-1}(u)$ is the inverse CDF:

- If $u \leq 0.5$:

$$z = \mu + b \cdot \log(2u).$$

- If $u > 0.5$:

$$z = \mu - b \cdot \log(2 - 2u).$$

**KL Divergence**

The KL divergence between a posterior Laplace distribution $q(z)$ and a prior Laplace distribution $p(z)$ is:

- Posterior: $q(z) \sim \text{Laplace}(0, 1)$

$$q(z) = \frac{1}{2} \exp(-|z|)$$

- Prior: $p(z) \sim \text{Laplace}(\mu_p, b_p)$

$$p(z) = \frac{1}{2b_p} \exp\left(-\frac{|z-\mu_p|}{b_p}\right)$$

The formula for KL divergence is:

$$D_{KL}(q(z)\,||\,p(z)) = \mathbb{E}_{z\sim q(z)}\left[\log\frac{q(z)}{p(z)}\right]$$

**Step 1: Substitute the formulas for $q(z)$ and $p(z)$**

$$\log\frac{q(z)}{p(z)} = \log\left(\frac{\frac{1}{2}\exp(-|z|)}{\frac{1}{2b_p}\exp\left(-\frac{|z-\mu_p|}{b_p}\right)}\right)$$

Simplifying:

$$= \log\left(\frac{b_p}{1}\right) - \left(|z| - \frac{|z-\mu_p|}{b_p}\right)$$

**Step 2: Compute the expectation**

$$D_{KL}(q(z)\,||\,p(z)) = \mathbb{E}_{z\sim q(z)}\left[\log(b_p) - |z| + \frac{|z-\mu_p|}{b_p}\right]$$

Breaking it into terms:

1. $\mathbb{E}_{z\sim q(z)}\left[\log(b_p)\right] = \log(b_p)$
2. $\mathbb{E}_{z\sim q(z)}\left[|z|\right] = 1$ (since $b_q = 1$)
3. $\mathbb{E}_{z\sim q(z)}\left[|z-\mu_p|\right] = 1 + |\mu_p|$

**Final Expression:**

$$D_{KL}(q(z)\,||\,p(z)) = \log(b_p) - 1 + \frac{1+|\mu_p|}{b_p}$$

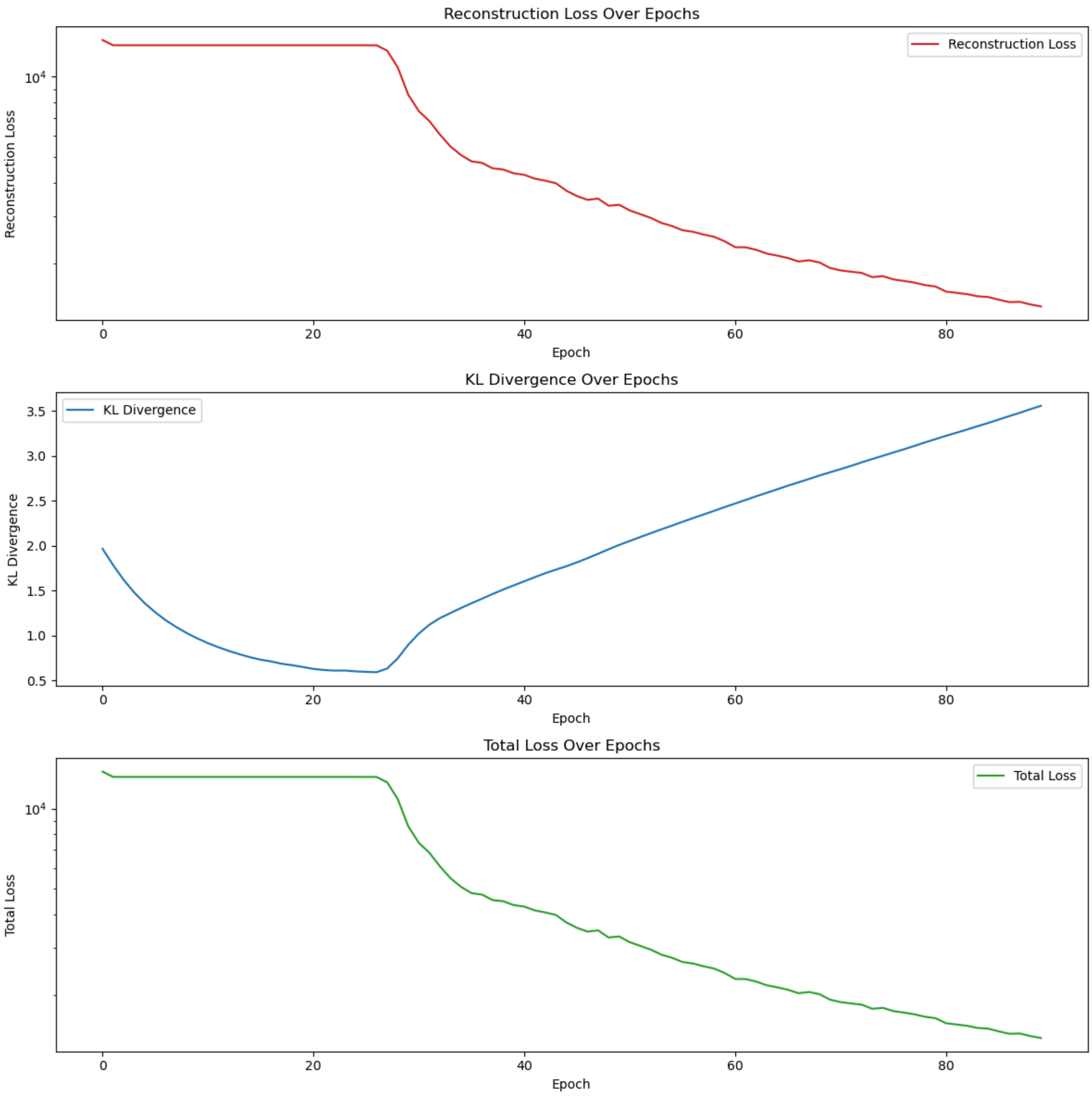
## Training VAD with Laplace Distribution

In [2]:

```
epochs = 90
beta = 1
learning_rate = 0.02
latent_dim = 60
model_laplace = VariationalAutoDecoderLaplace(latent_dim=latent_dim,lr=learning_r
train_loss,_,_ = model_laplace.fit_and_train(num_epochs=epochs, beta=beta)
print(f'Training loss: {train_loss:.4f}')
test_loss = model_laplace.test_vad(num_epochs=epochs, learning_rate=learning_rate
print(f'Test loss: {test_loss:.4f}')
```

```
Training the VAD model...
Epoch 1, Loss: 13743.8640
Epoch 2, Loss: 13134.5948
Epoch 3, Loss: 13134.4305
Epoch 4, Loss: 13134.2817
Epoch 5, Loss: 13134.1601
Epoch 6, Loss: 13134.0676
Epoch 7, Loss: 13133.9484
Epoch 8, Loss: 13133.9030
Epoch 9, Loss: 13133.8365
Epoch 10, Loss: 13133.7779
Epoch 11, Loss: 13133.7101
Epoch 12, Loss: 13133.6739
Epoch 13, Loss: 13133.6288
Epoch 14, Loss: 13133.5950
Epoch 15, Loss: 13133.5382
Epoch 16, Loss: 13133.4032
Epoch 17, Loss: 13133.5184
Epoch 18, Loss: 13133.3806
Epoch 19, Loss: 13133.4402
Epoch 20, Loss: 13133.4518
Epoch 21, Loss: 13133.2715
Epoch 22, Loss: 13132.9129
Epoch 23, Loss: 13133.2447
Epoch 24, Loss: 13133.2500
Epoch 25, Loss: 13132.8489
Epoch 26, Loss: 13131.6227
Epoch 27, Loss: 13125.5274
Epoch 28, Loss: 12529.9809
Epoch 29, Loss: 10844.6006
Epoch 30, Loss: 8561.9750
Epoch 31, Loss: 7436.5455
Epoch 32, Loss: 6826.2465
Epoch 33, Loss: 6075.0935
Epoch 34, Loss: 5482.9710
Epoch 35, Loss: 5090.4682
Epoch 36, Loss: 4821.6385
Epoch 37, Loss: 4759.9085
Epoch 38, Loss: 4540.2810
Epoch 39, Loss: 4496.3453
Epoch 40, Loss: 4350.2697
Epoch 41, Loss: 4293.7078
Epoch 42, Loss: 4154.3269
Epoch 43, Loss: 4082.1231
Epoch 44, Loss: 3993.5322
Epoch 45, Loss: 3742.9174
Epoch 46, Loss: 3574.5239
Epoch 47, Loss: 3461.4798
Epoch 48, Loss: 3498.4614
Epoch 49, Loss: 3288.6298
Epoch 50, Loss: 3317.4967
Epoch 51, Loss: 3158.6140
Epoch 52, Loss: 3059.6849
Epoch 53, Loss: 2961.4515
Epoch 54, Loss: 2836.7270
Epoch 55, Loss: 2763.0870
Epoch 56, Loss: 2663.6425
Epoch 57, Loss: 2628.2404
Epoch 58, Loss: 2565.9812
Epoch 59, Loss: 2516.6151
Epoch 60, Loss: 2422.1553
```

```
Epoch 61, Loss: 2301.4221
Epoch 62, Loss: 2298.1621
Epoch 63, Loss: 2249.5881
Epoch 64, Loss: 2177.7917
Epoch 65, Loss: 2139.8567
Epoch 66, Loss: 2096.2815
Epoch 67, Loss: 2034.8020
Epoch 68, Loss: 2056.2908
Epoch 69, Loss: 2017.3422
Epoch 70, Loss: 1924.4603
Epoch 71, Loss: 1884.4903
Epoch 72, Loss: 1863.1423
Epoch 73, Loss: 1843.9804
Epoch 74, Loss: 1778.5161
Epoch 75, Loss: 1793.7593
Epoch 76, Loss: 1744.1486
Epoch 77, Loss: 1721.2694
Epoch 78, Loss: 1696.4536
Epoch 79, Loss: 1660.7898
Epoch 80, Loss: 1640.4129
Epoch 81, Loss: 1570.2993
Epoch 82, Loss: 1553.4363
Epoch 83, Loss: 1536.0896
Epoch 84, Loss: 1508.2995
Epoch 85, Loss: 1498.7517
Epoch 86, Loss: 1464.4112
Epoch 87, Loss: 1434.3290
Epoch 88, Loss: 1437.8208
Epoch 89, Loss: 1405.9732
Epoch 90, Loss: 1382.7998
```

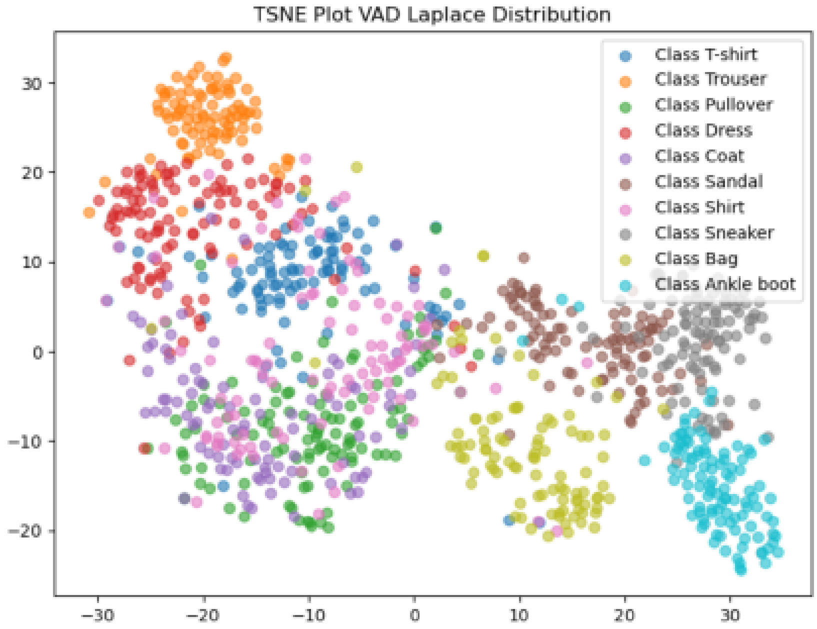



```
Training loss: 1382.7998
Testing the VAD model...
Test loss: 0.1538
```

In [47]:
```
file_name = 'tsne_plot_VAD_laplace'
image_title = 'TSNE Plot VAD Laplace Distribution'
model_laplace.plot_tsne(file_name=file_name, plot_title=image_title)
```

```
Generating t-SNE plot...
<Figure size 800x600 with 0 Axes>
```

In [48]:
```
image_path = f'{file_name}.png'
img = mpimg.imread(image_path)
plt.imshow(img)
plt.axis('off')  # Hide axes
plt.show()
```

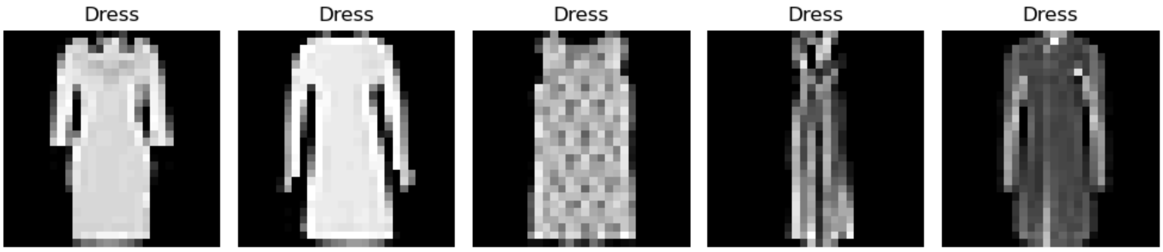



# Results Explanation

We can observe that the dress class is more scattered rather than in the previous plot. This is probably due to characteristics of the laplace distribution - able to represent rare features in the data. So, the model learnt to distinguish well between the different dresses - with and without sleeve (see below). The pullover class is more concetrated than in the previous plot, suggesting that the model learnt its features better.

In [52]:

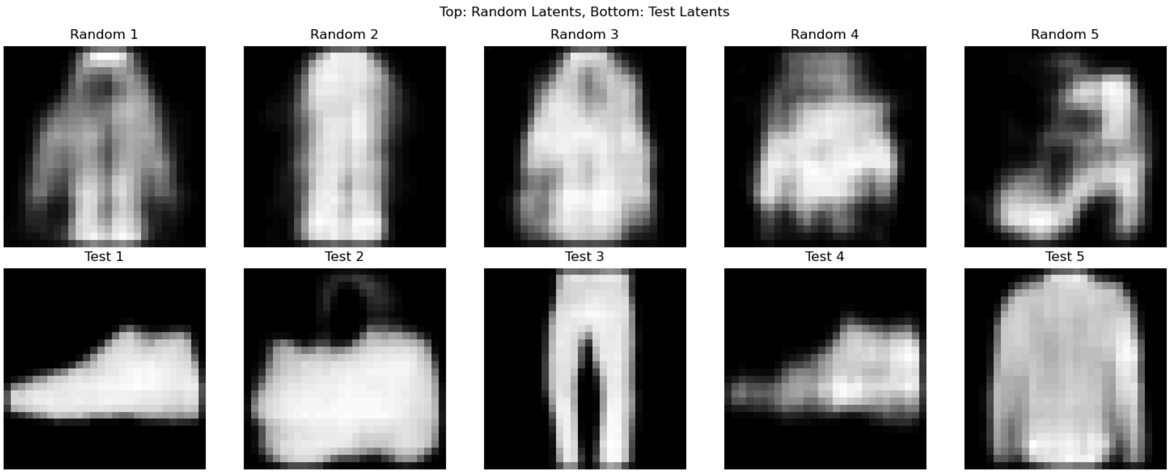
```
DRESS_CLASS = 3

dress_images = [model_laplace.train_ds.X[i] for i in range(len(model_laplace.trai
dress_images = dress_images[-5:] + dress_images[:5]

# Plot 10 dress images
plt.figure(figsize=(10, 10))
for i, img in enumerate(dress_images):
    plt.subplot(2, 5, i + 1)
    plt.imshow(img.numpy(), cmap="gray")
    plt.axis("off")
    plt.title("Dress")
plt.tight_layout()
plt.show()
```


We can see that there are dresses without sleeves and some are with. The model probably learnt this difference in the between the kind of dresses.

## Testing Samples

In [29]:
```
laplace_dist = torch.distributions.Laplace(0,1)
random_latents = laplace_dist.sample((5, model_laplace.latent_dim)).to(model_lapl
random_decoded = model_laplace.decode(random_latents)  # Decode the sampled laten

test_indices = torch.randint(0, len(model_laplace.test_ds), (5,))
test_parameters = model_laplace.test_parameters[test_indices]

test_mu = test_parameters[:, :model_laplace.latent_dim]
test_log_b = test_parameters[:, model_laplace.latent_dim:]

test_latents = model_laplace._reparameterize(test_mu, test_log_b)
test_decoded = model_laplace.decode(test_latents)

fig, axes = plt.subplots(2, 5, figsize=(15, 6))

for i, (random_img, test_img) in enumerate(zip(random_decoded, test_decoded)):
    axes[0, i].imshow(random_img.cpu().detach().numpy(), cmap='gray')
    axes[0, i].axis('off')
    axes[0, i].set_title(f"Random {i + 1}")

    axes[1, i].imshow(test_img.cpu().detach().numpy(), cmap='gray')
    axes[1, i].axis('off')
    axes[1, i].set_title(f"Test {i + 1}")

plt.suptitle("Top: Random Latents, Bottom: Test Latents")
plt.tight_layout()
plt.show()
```




### What's Happening in the Images

**Top Row (Random Latents)** These images are generated by sampling random values from a $\text{Laplace}(0, 1)$ distribution and passing them through the model's decoder. Since the model was trained to make its latent space follow this distribution, we expect the outputs to resemble items from the dataset.

The images do look somewhat like pieces of clothing, but they're blurrier and lack detail. This suggests that while the model has learned the general structure of the dataset, it struggles to fully capture fine details when generating from purely random latent points.

**Bottom Row (Test Latents)** These images come from latent vectors directly tied to test data. The model predicts specific parameters (mean and scale) for each test sample, and these parameters are used to generate the latent vectors.

Since these latents are "informed" by real data, the decoder produces clearer and sharper images that closely resemble the original dataset. This shows that the model is effective at reconstructing real examples from the test set.

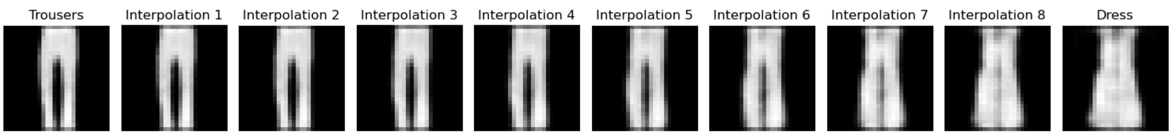
#### What This Comparison Shows

- The random latent images (top row) show how well the model can generate new, unseen examples based only on the prior distribution it was trained on. While the outputs are plausible, the blurriness suggests the model struggles to generate highly detailed images from purely random inputs.
- The test latent images (bottom row) highlight how well the model can recreate data it has seen during training. These images are much clearer and more detailed, indicating that the model has learned to represent the dataset effectively in its latent space.

## Interpolating Samples

In [37]:
```
TROUSERS_CLASS = 1
DRESS_CLASS = 3

trousers_images = [i for i in range(len(model_laplace.test_ds)) if model_laplace.
dress_images = [i for i in range(len(model_laplace.test_ds)) if model_laplace.tes
```

```
trousers_index = trousers_images[0]
dress_index = dress_images[-4]

trousers_params = model_laplace.test_parameters[trousers_index]
dress_params = model_laplace.test_parameters[dress_index]

trousers_mu = trousers_params[:model_laplace.latent_dim]
trousers_log_b = trousers_params[model_laplace.latent_dim:]

dress_mu = dress_params[:model_laplace.latent_dim]
dress_log_b = dress_params[model_laplace.latent_dim:]

latent_trousers = model_laplace._reparameterize(trousers_mu, trousers_log_b)
latent_dress = model_laplace._reparameterize(dress_mu, dress_log_b)




interpolation_coeffs = torch.linspace(0.1, 0.9, steps=9)

interpolated_latents = [
    (1 - t) * latent_trousers + t * latent_dress for t in interpolation_coeffs
]

decoded_images = [model_laplace.decode(latent.unsqueeze(0)).squeeze(0) for latent

fig, axes = plt.subplots(1, len(decoded_images)-1, figsize=(15, 3))
titles = ['Trousers', 'Interpolation 1', 'Interpolation 2', 'Interpolation 3', 'I

for ax, image, title in zip(axes, decoded_images, titles):
    ax.imshow(image.cpu().detach().numpy(), cmap='gray')
    ax.axis('off')
    ax.set_title(title)

plt.tight_layout()
plt.show()
```




## Exponential Distribution

We chose the exponential distribution because it has similar characteristics to the Laplace distribution - Sparse and heavy tail. The probability density peaks near $0$ and decreases rapidly. That means the exponential distribution tends to produce values close to $0$ with usually larger values. As a result, most latent dimensions remain inactive (close to zero), and only a few become active. Moreover, regarding the heavy tail - the exponential decay still allows large values, this helps balance sparsity and the ability to capture significant deviations. In addition, the exponential distribution is strictly defined for positive real numbers. This makes it particularly suited for exploring how non-Gaussian, positive-only latent representations influence model behavior.

**Modifying the Reparameterization Trick**:
The reparameterization trick for the exponential distribution transform a sample from a

uniform distribution $\mathrm{Uniform}(0, 1)$ into a sample from $\mathrm{Exponential}(\lambda)$ using the inverse cumulative distribution function (CDF).

**1. Exponential Distribution Properties**

- **PDF**:

$$f(x; \lambda) = \lambda e^{-\lambda x}, \quad x \geq 0$$

- **CDF**:

$$F(x; \lambda) = 1 - e^{-\lambda x}, \quad x \geq 0$$

**2. Deriving the Inverse CDF and the Final Reparameterization Formula**

The reparameterization involves solving $u = F(x; \lambda)$ for $x$, where $u \sim U(0, 1)$ Substituting the exponential CDF:

$$u = 1 - e^{-\lambda x} \implies e^{-\lambda x} = 1 - u \implies x = -\frac{\ln(1-u)}{\lambda}$$

Here, $u \sim U(0, 1)$, so $1 - u$ is also uniform, ensuring consistent sampling. To avoid numerical issues with $\ln(0)$, $u$ is clamped between $(0 + \epsilon, 1 - \epsilon)$, where $\epsilon$ is a small constant.

To sample $z \sim \mathrm{Exponential}(\lambda)$, we use the formula:

$$z = -\frac{1}{\lambda}\ln(\epsilon), \quad \epsilon \sim U(0, 1)$$

**KL Divergence**

The KL divergence measures the difference between the posterior $q(z) \sim \mathrm{Exponential}(\lambda_q)$ and the prior $p(z) \sim \mathrm{Exponential}(\lambda_p)$ The formula for KL divergence is:

$$D_{KL}(q(z) \,||\, p(z)) = \mathbb{E}_{z \sim q(z)}\left[\log \frac{q(z)}{p(z)}\right]$$

**Step 1: Substitute the PDFs**

The PDFs for the posterior and prior are:

- $q(z) = \lambda_q e^{-\lambda_q z}$
- $p(z) = \lambda_p e^{-\lambda_p z}$

The log ratio is:

$$\log \frac{q(z)}{p(z)} = \log\left(\frac{\lambda_q e^{-\lambda_q z}}{\lambda_p e^{-\lambda_p z}}\right) = \log\left(\frac{\lambda_q}{\lambda_p}\right) + (\lambda_p - \lambda_q)z.$$

**Step 2: Compute the Expectation**

The expectation is taken over $z \sim q(z)$, whose mean is $\mathbb{E}_{z \sim q(z)}[z] = \frac{1}{\lambda_q}$:

$$D_{KL}(q(z)\,||\,p(z)) = \log\left(\frac{\lambda_q}{\lambda_p}\right) + \lambda_p \mathbb{E}[z] - 1 = \log\left(\frac{\lambda_q}{\lambda_p}\right) + \frac{\lambda_p}{\lambda_q} - 1.$$

Breaking this into terms:

- $\mathbb{E}[z]\log\left(\frac{\lambda_q}{\lambda_p}\right) = \log\left(\frac{\lambda_q}{\lambda_p}\right)$
- $(\lambda_p - \lambda_q)\mathbb{E}_{z\sim q(z)}[z] = (\lambda_p - \lambda_q)\frac{1}{\lambda_q}$

**Final Expression:**

$$D_{KL}(q(z)\,||\,p(z)) = \log\left(\frac{\lambda_q}{\lambda_p}\right) + \lambda_p \mathbb{E}[z] - 1 = \log\left(\frac{\lambda_q}{\lambda_p}\right) + \frac{\lambda_p - \lambda_q}{\lambda_q}.$$

This is minimized when $\lambda_q = \lambda_p$, reflecting minimal divergence between the prior and posterior. The divergence increases as the two distributions become more dissimilar.

When $\lambda_p = 1$ we get the expression that we used:

$$D_{KL}(q(z)\,||\,p(z)) = -\log(\lambda_q) + \lambda_q - 1$$

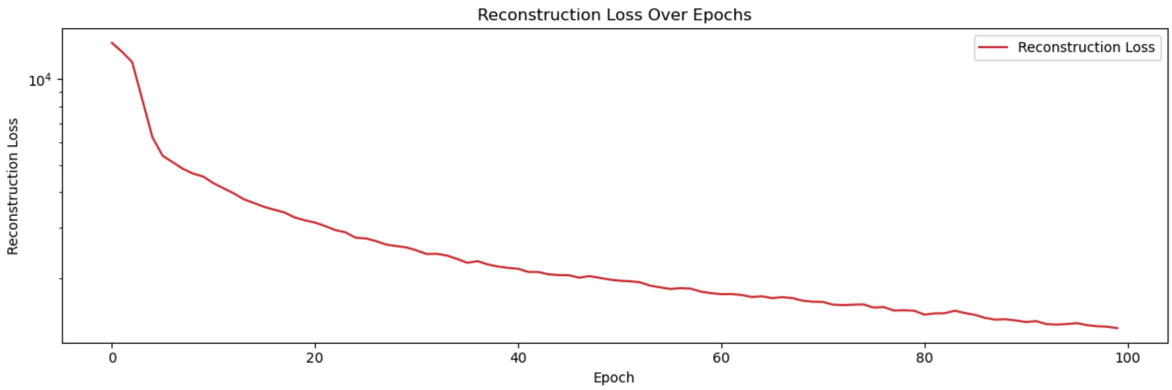
## Training VAD with Exponential Distribution

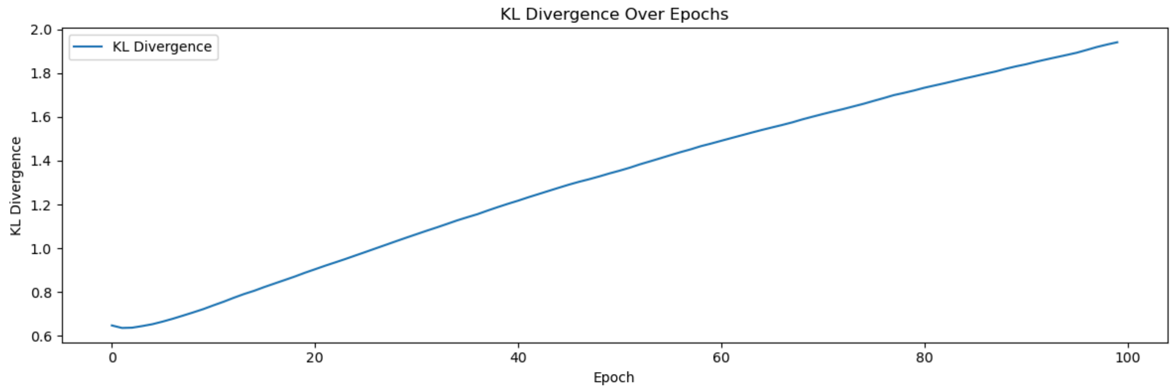
In [2]:

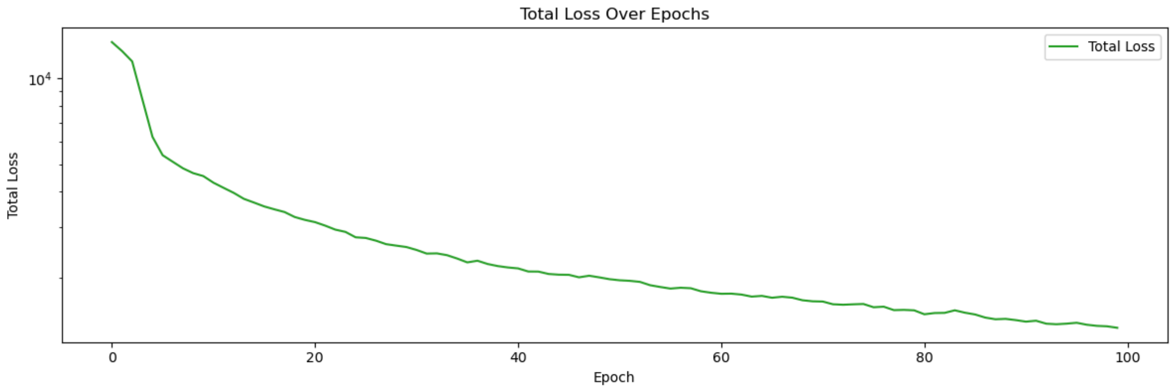
```
epochs = 100
beta = 1
learning_rate = 0.015
latent_dim = 60
model_exp = VariationalAutoDecoderExponential(latent_dim=latent_dim,lr=learning_r
train_loss,_,_ = model_exp.fit_and_train(num_epochs=epochs, beta=beta)
print(f'Training loss: {train_loss:.4f}')
test_loss = model_exp.test_vad(num_epochs=epochs, learning_rate=learning_rate)
print(f'Test loss: {test_loss:.4f}')
```

```
Training the VAD model...
Epoch 1, Loss: 13411.9787
Epoch 2, Loss: 12476.5875
Epoch 3, Loss: 11470.8912
Epoch 4, Loss: 8449.1745
Epoch 5, Loss: 6233.2014
Epoch 6, Loss: 5374.5296
Epoch 7, Loss: 5096.2209
Epoch 8, Loss: 4834.8177
Epoch 9, Loss: 4650.3209
Epoch 10, Loss: 4541.3203
Epoch 11, Loss: 4304.9546
Epoch 12, Loss: 4129.5786
Epoch 13, Loss: 3964.6228
Epoch 14, Loss: 3777.5795
Epoch 15, Loss: 3665.2741
Epoch 16, Loss: 3552.9413
Epoch 17, Loss: 3470.6985
Epoch 18, Loss: 3395.8602
Epoch 19, Loss: 3262.2724
Epoch 20, Loss: 3187.2619
Epoch 21, Loss: 3129.8760
Epoch 22, Loss: 3042.2386
Epoch 23, Loss: 2945.3713
Epoch 24, Loss: 2891.9413
Epoch 25, Loss: 2768.2652
Epoch 26, Loss: 2753.3766
Epoch 27, Loss: 2694.1624
Epoch 28, Loss: 2618.8780
Epoch 29, Loss: 2587.8409
Epoch 30, Loss: 2558.6253
Epoch 31, Loss: 2498.9043
Epoch 32, Loss: 2426.4816
Epoch 33, Loss: 2430.5914
Epoch 34, Loss: 2395.5582
Epoch 35, Loss: 2331.2984
Epoch 36, Loss: 2259.8536
Epoch 37, Loss: 2290.4155
Epoch 38, Loss: 2230.7927
Epoch 39, Loss: 2193.3756
Epoch 40, Loss: 2169.0323
Epoch 41, Loss: 2152.5364
Epoch 42, Loss: 2099.1901
Epoch 43, Loss: 2097.7612
Epoch 44, Loss: 2057.9557
Epoch 45, Loss: 2046.0984
Epoch 46, Loss: 2043.7347
Epoch 47, Loss: 2002.9436
Epoch 48, Loss: 2028.8836
Epoch 49, Loss: 2001.8856
Epoch 50, Loss: 1972.9262
Epoch 51, Loss: 1955.2859
Epoch 52, Loss: 1946.9052
Epoch 53, Loss: 1930.2156
Epoch 54, Loss: 1879.0369
Epoch 55, Loss: 1852.2991
Epoch 56, Loss: 1828.5104
Epoch 57, Loss: 1840.9393
Epoch 58, Loss: 1833.6712
Epoch 59, Loss: 1789.5923
Epoch 60, Loss: 1768.2239
```

```
Epoch 61, Loss: 1753.4436
Epoch 62, Loss: 1755.2789
Epoch 63, Loss: 1742.0172
Epoch 64, Loss: 1713.8098
Epoch 65, Loss: 1723.8653
Epoch 66, Loss: 1698.2150
Epoch 67, Loss: 1711.9423
Epoch 68, Loss: 1699.0881
Epoch 69, Loss: 1664.0707
Epoch 70, Loss: 1650.0088
Epoch 71, Loss: 1646.9860
Epoch 72, Loss: 1611.0081
Epoch 73, Loss: 1605.0023
Epoch 74, Loss: 1610.3782
Epoch 75, Loss: 1614.7724
Epoch 76, Loss: 1572.6675
Epoch 77, Loss: 1579.8177
Epoch 78, Loss: 1536.5796
Epoch 79, Loss: 1539.4570
Epoch 80, Loss: 1533.8085
Epoch 81, Loss: 1485.9058
Epoch 82, Loss: 1500.7580
Epoch 83, Loss: 1502.8049
Epoch 84, Loss: 1534.2171
Epoch 85, Loss: 1504.8327
Epoch 86, Loss: 1483.3520
Epoch 87, Loss: 1446.6629
Epoch 88, Loss: 1427.1662
Epoch 89, Loss: 1431.6863
Epoch 90, Loss: 1417.4828
Epoch 91, Loss: 1400.0999
Epoch 92, Loss: 1410.3471
Epoch 93, Loss: 1377.0797
Epoch 94, Loss: 1370.8479
Epoch 95, Loss: 1377.3468
Epoch 96, Loss: 1387.2319
Epoch 97, Loss: 1365.2977
Epoch 98, Loss: 1353.0387
Epoch 99, Loss: 1348.0644
Epoch 100, Loss: 1331.9361
```



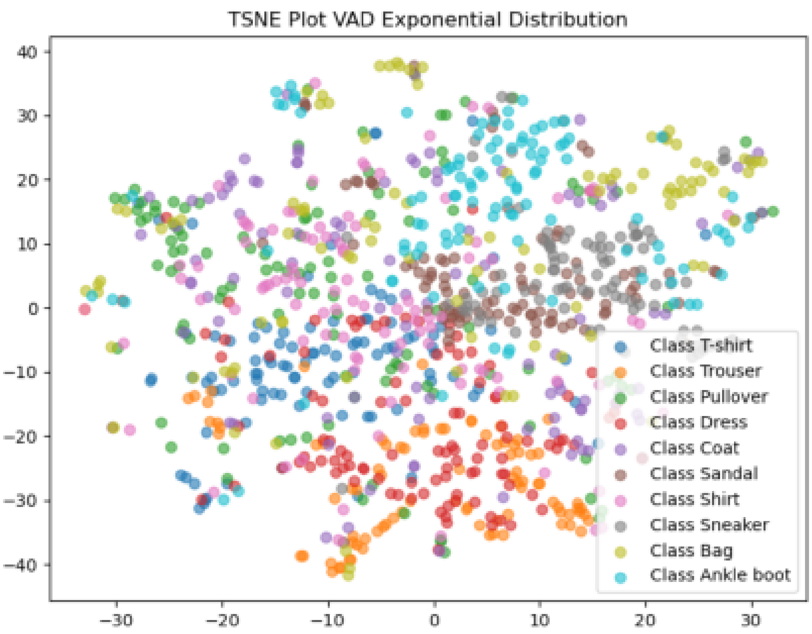




```
Training loss: 1331.9361
Testing the VAD model...
Test loss: 0.1758
```

In [3]:
```
file_name = 'tsne_plot_VAD_exp'
image_title = 'TSNE Plot VAD Exponential Distribution'
model_exp.plot_tsne(file_name=file_name, plot_title=image_title)
```

```
Generating t-SNE plot...
<Figure size 800x600 with 0 Axes>
```

In [4]:
```
image_path = f'{file_name}.png'
img = mpimg.imread(image_path)
plt.imshow(img)
plt.axis('off')  # Hide axes
plt.show()
```




# Results Explanation

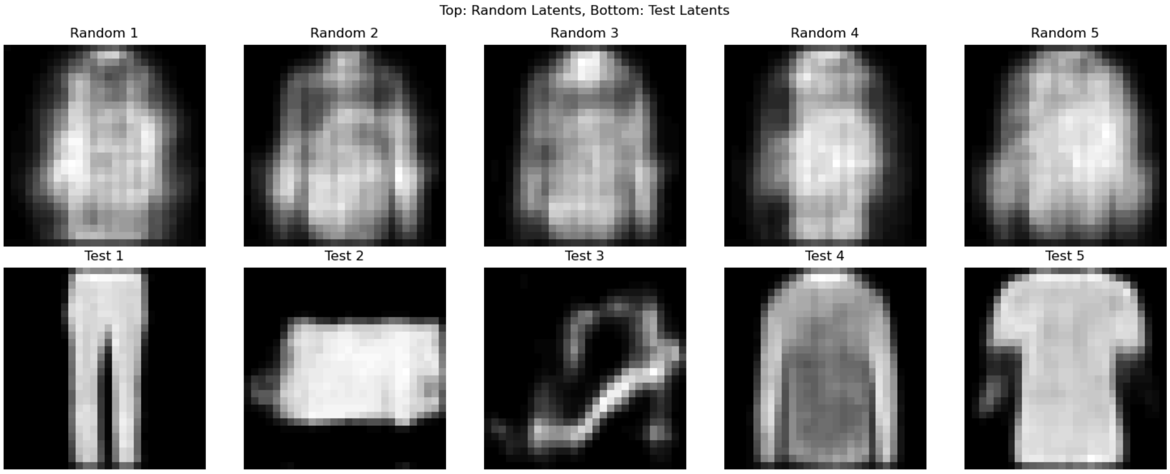
We can see that most of the classes are mixed inside the picture and only a few classes are almost separate so we can think about them as clusters. This happens because the exponential distribution is probably not a good prior decision for this dataset, which means it is difficult for the model to organize the latent space effectively when using this distribution.

## Testing Samples

In [12]:

```
exponential_dist = torch.distributions.exponential.Exponential(1.0)

random_latents = exponential_dist.sample((5, model_exp.latent_dim)).to(model_exp.
random_decoded = model_exp.decode(random_latents)  # Decode the sampled latents

test_indices = torch.randint(0, len(model_exp.test_ds), (5,))
test_log_lambda = model_exp.test_log_lambda[test_indices]


test_latents = model_exp._reparameterize(test_log_lambda)
test_decoded = model_exp.decode(test_latents)

fig, axes = plt.subplots(2, 5, figsize=(15, 6))

for i, (random_img, test_img) in enumerate(zip(random_decoded, test_decoded)):
    axes[0, i].imshow(random_img.cpu().detach().numpy(), cmap='gray')
    axes[0, i].axis('off')
    axes[0, i].set_title(f"Random {i + 1}")

    axes[1, i].imshow(test_img.cpu().detach().numpy(), cmap='gray')
    axes[1, i].axis('off')
    axes[1, i].set_title(f"Test {i + 1}")
```

```
plt.suptitle("Top: Random Latents, Bottom: Test Latents")
plt.tight_layout()
plt.show()
```




## What's Happening in the Images

When decoding latent vectors into images, the quality of the decoded results heavily depends on how well the latent space aligns with the data distribution.

**Exponential Random Samples (Top Row)**: These fail to produce meaningful images because the Exponential distribution is a poor prior for this dataset. It skews the sampling process, leading to latent vectors that fall outside the meaningful regions of the latent space, where the decoder cannot map them to valid outputs.

**Test Set Latents (Bottom Row)**: These are better aligned with the latent space, as they originate from the same distribution as the training data. Decoding these latents results in high-quality images because the decoder was explicitly trained to reconstruct data from these regions of the latent space.

## Interpolating Samples

In [13]:
```
TROUSERS_CLASS = 1
DRESS_CLASS = 3

trousers_images = [i for i in range(len(model_exp.test_ds)) if model_exp.test_ds.
dress_images = [i for i in range(len(model_exp.test_ds)) if model_exp.test_ds.y[i

trousers_index = trousers_images[0]
dress_index = dress_images[-4]

trousers_test_log_lambda = model_exp.test_log_lambda[trousers_index]
dress_test_log_lambda = model_exp.test_log_lambda[dress_index]


latent_trousers = model_exp._reparameterize(trousers_test_log_lambda)
latent_dress = model_exp._reparameterize(dress_test_log_lambda)
```

```python
interpolation_coeffs = torch.linspace(0.1, 0.9, steps=9)

interpolated_latents = [
    (1 - t) * latent_trousers + t * latent_dress for t in interpolation_coeffs
]

decoded_images = [model_exp.decode(latent.unsqueeze(0)).squeeze(0) for latent in

fig, axes = plt.subplots(1, len(decoded_images)-1, figsize=(15, 3))
titles = ['Trousers', 'Interpolation 1', 'Interpolation 2', 'Interpolation 3', 'I

for ax, image, title in zip(axes, decoded_images, titles):
    ax.imshow(image.cpu().detach().numpy(), cmap='gray')
    ax.axis('off')
    ax.set_title(title)

plt.tight_layout()
plt.show()
```