Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

   The work begin after 10 min, so the average time increase from 45min to 55min.

   for normal distribution :-

   z = (X-μ)/б

   = (60-55)/8

   = 0.625

   1-pnorm(0.635)=0.2675

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.

   A. More employees at the processing center are older than 44 than between 38 and 44.

   Zscore age 44=(X-mean)/SD=(44-38)/6=1=0.8413=84.13%
   Zscore People above age 44=100-84.13=15.87%=63 ppl
   Zscore for 38=0=50%=200
   People between 38 and 44= 84.13-50=34.13=137(approx.)

   Hence, More employees at the processing center are older than 44 than between 38 and 44. Is **FALSE**

   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**TRUE**
   People age 30=(30-38)/6=-1.33=9.15%=36

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Normal Distribution can be defined by two parameters such as mean(μ),Variance(σ) written as ,$X \sim N(\mu, \sigma^2)$

So, $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are two independent Normal distributions

Addition of those will be,

$X+Y \sim N(\mu_1+ \mu_{2,}\ \sigma^2{}_1+ \sigma^2{}_2)$

Subtraction of those will be,

$X-Y \sim N(\mu_1- \mu_{2,}\ \sigma^2{}_1- \sigma^2{}_2)$

When $Z=\alpha X$,the product of X , $Z \sim N(\alpha\ \mu_{1,}\ \alpha^2\ \sigma^2{}_1)$

Thus, Follwing the property pf product

$2X_1 \sim N(2\ \mu_{,}\ 4\ \sigma^2)$

$X_1 + X_2 \sim N(\mu_{,+}\ \mu_{,,}\ \sigma^2{}_{+}\ \sigma^2) \sim N(2\ \mu, 2\ \sigma^2)$

So the mean of 2X1, $X_1 + X_2$ is same, but Variance of 2X is 2 times higher than the $X_1 + X_2$

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

   $P1 \sim N(5, 3^2)$,$P2 \sim N(7, 4^2)$

   $P \sim N(12, 7^2)$

   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
      =12-1.96X7,12+1.96X7=($1.72M,$25.72M)=(Rs.77.4M,Rs.1157.4M)
   B. Specify the 5th percentile of profit (in Rupees) for the company
      (p-12)/5=-1.644
      P=12-8.22=$3.78M=Rs.170.1M
   C. Which of the two divisions has a larger probability of making a loss in a given year?
      FirstDivision , Loss is when p<0