

ASSIGNMENT 7

We will be investigating the use of Polynomial Regression on the function $y = x^3$. We want to add some randomness to x^3 , so we will add a fraction of a random value selected from a normal distribution with a standard deviation of 1 centered around 0:

$$y = x^3 + 0.5 * \text{np.random.normal}(0,1,1)$$

```
In [9]: from random import randint
import numpy as num
import matplotlib.pyplot as plt
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import PolynomialFeatures
from sklearn import linear_model
from sklearn.metrics import mean_squared_error
import math
from math import sqrt
```

```
In [10]: def func(x):
        return x**3 + 0.5 * num.random.normal(0,1,1)

print(func(10))

[999.44084088]
```

1. Test Points

Make a list of 10 random points between -2 and 2 (using a uniform distribution). Pass this list into the function described above to get a set of x and y coordinates. Display the (x,y) coordinates as a data frame.

```
In [19]: func_test = num.vectorize(func)
num.random.seed(100)
xaxis = num.sort(num.random.uniform(-2,2,10))
yaxis = func_test(xaxis)
print("X coordinates")
print(xaxis)
print()
print("Y coordinates")
print(yaxis)

X coordinates
[-1.98112458 -1.51372352 -1.45317364 -0.88652246 -0.30192964  0.17361977
 0.30037332  0.68299634  1.30341102  1.37910453]

Y coordinates
[-7.64812515 -3.69749732 -2.85110484 -0.98853509  0.38089918  0.34159397
-0.02510465  0.05296668  2.72920563  2.4038915 ]
```

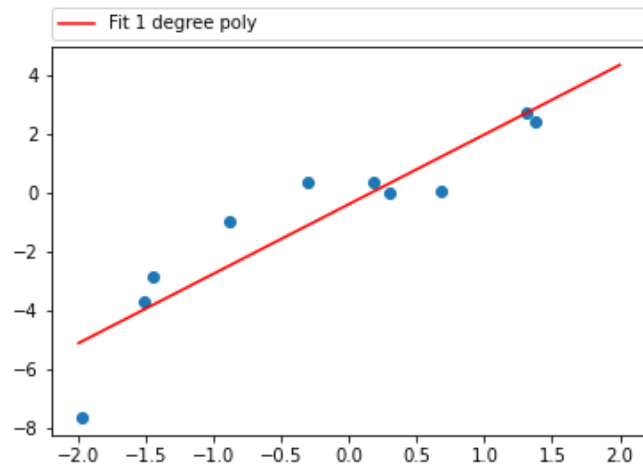
2. Create Graphs

Now, create scatter plots of the (x,y) coordinates and:

a degree 1 (linear) regression model,

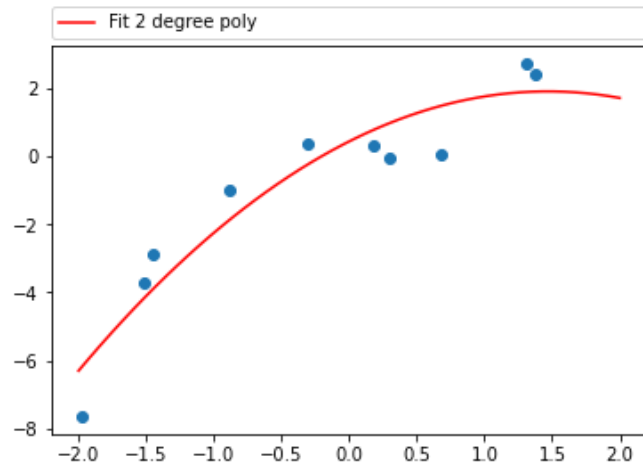
There should be 5 separate graphs for this step each clearly labeled and annotated.

```
In [20]: x2 = num.linspace(-2,2)
m = Pipeline([('poly', PolynomialFeatures(degree=1)),('linear', linear_model.LinearRegression())])
m = m.fit(xaxis[:,np.newaxis], yaxis[:,np.newaxis])
y2 = m.predict(x2[:, num.newaxis])
plt.scatter(xaxis, yaxis)
plt.plot(x2, y2, 'r', label="Fit "+str(1)+ " degree poly")
plt.legend(bbox_to_anchor=(0., 1.02, 1., .102), loc=3,
           ncol=2, mode="expand", borderaxespad=0.)
plt.show()
```



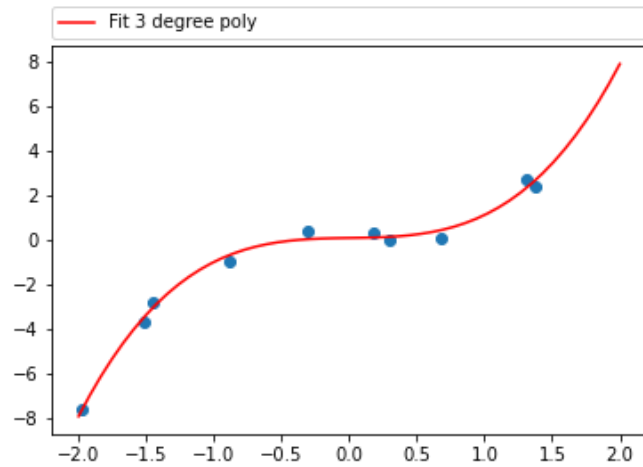
degree 2 (quadratic) regression model,

```
In [21]: x3 = Pipeline([('poly', PolynomialFeatures(degree=2)),('linear', linear_model.LinearRegression())])
x3 = x3.fit(xaxis[:, num.newaxis], yaxis[:, num.newaxis])
y3 = x3.predict(x2[:, num.newaxis])
plt.scatter(xaxis, yaxis)
plt.plot(x2, y3, 'r', label="Fit "+str(2)+ " degree poly")
plt.legend(bbox_to_anchor=(0., 1.02, 1., .102), loc=3,
           ncol=2, mode="expand", borderaxespad=0.)
plt.show()
```



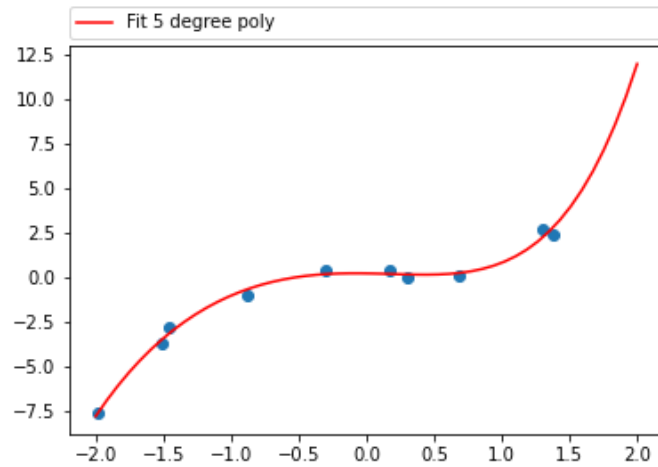
degree 3 (cubic) regression model,

```
In [22]: x4 = Pipeline([('poly', PolynomialFeatures(degree=3)), ('linear', linear_model.LinearRegression())])
x4 = x4.fit(xaxis[:, num.newaxis], yaxis[:, num.newaxis])
y4 = x4.predict(x2[:, num.newaxis])
plt.scatter(xaxis, yaxis)
plt.plot(x2, y4, 'r', label="Fit "+str(3)+ " degree poly")
plt.legend(bbox_to_anchor=(0., 1.02, 1., .102), loc=3,
           ncol=2, mode="expand", borderaxespad=0.)
plt.show()
```



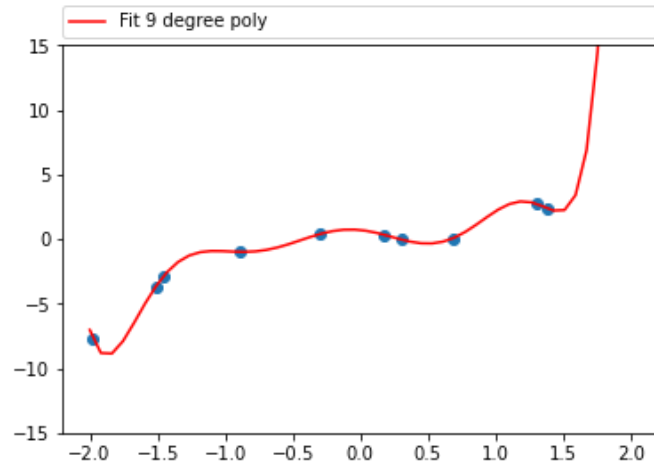
degree 5, and

```
In [23]: x5 = Pipeline([('poly', PolynomialFeatures(degree=5)),('linear', linear_model.LinearRegression())])
x5 =x5.fit(xaxis[:,num.newaxis], yaxis[:,num.newaxis])
y5 = x5.predict(x2[:, num.newaxis])
plt.scatter(xaxis, yaxis)
plt.plot(x2, y5, 'r', label="Fit "+str(5)+ " degree poly")
plt.legend(bbox_to_anchor=(0., 1.02, 1., .102), loc=3,
           ncol=2, mode="expand", borderaxespad=0.)
plt.show()
```



degree 9.

```
In [24]: x6 = Pipeline([('poly', PolynomialFeatures(degree=9)),('linear', linear_model.LinearRegression())])
x6 = x6.fit(xaxis[:, num.newaxis], yaxis[:, num.newaxis])
y6 = x6.predict(x2[:, num.newaxis])
plt.scatter(xaxis, yaxis)
plt.plot(x2, y6, 'r', label="Fit "+str(9)+ " degree poly")
plt.legend(bbox_to_anchor=(0., 1.02, 1., .102), loc=3,
           ncol=2, mode="expand", borderaxespad=0.)
plt.ylim(-15,15)
plt.show()
```

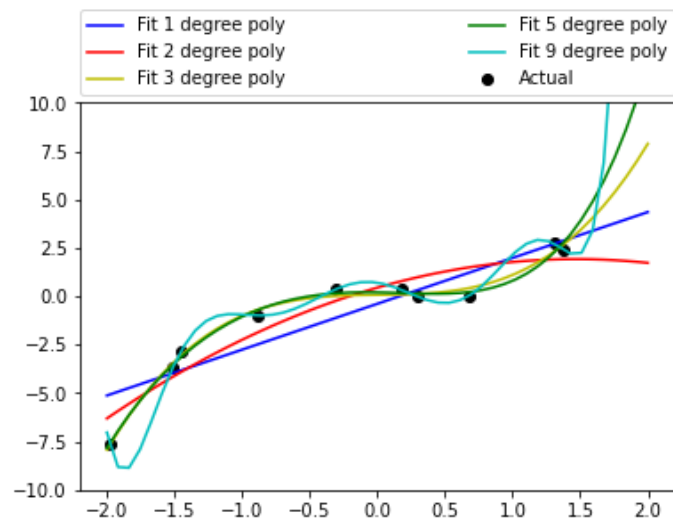


3.Combine Graphs

Next, create a single graph that contains all 5 regression models from the last step as well as the (x,y) coordinates. Make sure there is a useful legend as well.

```
In [25]: plot_config=['b', 'r', 'y', 'g', 'c']
plt.scatter(xaxis, yaxis, color = 'black', marker = 'o', label="Actual")
d_degree = [1,2,3,5,9]
i = 0
for degree in d_degree:
    model = Pipeline([('poly', PolynomialFeatures(degree=degree)),('linear', linear_model.LinearRegression())])
    model=model.fit(xaxis[:,num.newaxis], yaxis[:,num.newaxis])
    predict_sk=model.predict(x2[:,np.newaxis])
    plt.plot(x2, predict_sk, plot_config[i], label="Fit "+str(degree)+ " degree poly")
    plt.legend(bbox_to_anchor=(0., 1.02, 1., .102), loc=3,
               ncol=2, mode="expand", borderaxespad=0.)
    plt.ylim(-10,10)
    i = i+1

plt.show()
```



4. Test the Models

We want to see which model was the best predictor for our function. Which one do you think would be the best? To figure this out, create a set of 100 x coordinates randomly generated from a uniform distribution between -2 and 2. Then generate the corresponding y coordinates by passing in the x's to the $x^3 + (\text{degree of randomness})$ function from earlier. Display the first few (x,y) coordinates to make sure they are what you expect.


```
In [26]: X_test = num.random.uniform(-2,2,100)
y_test = func_test(X_test)
print("X values")
print(X_test)
print()
print("Y values")
print(y_test)
```

X values

```
[ -1.97724597e+00 -9.90294586e-01  1.18265003e+00 -1.93898012e+00
  3.95373508e-01  4.15218156e-01 -1.57940926e+00 -4.72226220e-01
 -1.85409577e+00  1.56164625e+00  1.92368343e+00 -1.76023204e+00
  1.56218378e+00  3.07605998e-01  9.69918756e-01  5.20735746e-01
  3.27368770e-01 -1.91824347e+00 -1.15989369e+00  1.78739513e-01
  1.07646068e+00 -9.97219083e-01 -8.56417238e-01  1.40958035e+00
  1.90002597e+00  1.53941317e+00 -5.61968624e-01  3.95435784e-01
 -5.80817553e-01 -6.39239139e-01 -1.28767604e+00 -1.04922317e+00
 -1.82055087e+00  2.17257185e-02 -4.94990183e-01  3.71221604e-01
  5.19767502e-01 -1.42959874e+00  1.73536520e+00  1.78551952e+00
  4.09186631e-01 -4.48934879e-01 -5.47247984e-01 -1.18261889e+00
 -8.92939754e-01 -1.01385648e+00 -1.30556799e+00  1.86643878e+00
  1.82805040e+00  3.91894737e-01  9.25203012e-01 -6.38459109e-01
 -1.63177759e+00 -1.46007924e-01  3.47955730e-02 -1.64615931e+00
  1.12140893e-01  1.96863215e+00 -4.19856273e-01 -6.57614233e-01
  1.22180215e+00  1.01739598e+00 -7.47734234e-01  5.36146732e-01
  1.61618301e-01 -8.12824996e-01 -1.55684840e+00 -7.49438808e-01
 -1.72083480e-01  6.35760281e-01 -9.82969929e-01  5.64405035e-01
 -1.19950557e+00  6.30499222e-01  1.11315686e+00  1.11839359e+00
  4.41312613e-01 -7.63998606e-01  7.90939630e-01  1.43847318e+00
  5.01295031e-01  1.92963132e+00  1.90600051e+00 -1.33322348e+00
 -1.90728745e+00 -1.35702181e+00  1.69398730e+00  1.81419940e+00
 -1.15608633e+00 -5.57898997e-01  1.97501047e-01 -9.12676603e-01
 -1.57593516e-01  7.84646259e-01  1.42358670e-03  8.64283962e-01
  1.03823745e-01 -1.99440391e+00 -4.21198853e-01 -3.13321204e-02 ]
```

Y values

```
[ -8.58237194 -1.53929593  0.16746886 -7.2732161 -0.06263946 -0.15350207
 -3.87367557 -0.09419835 -6.21508754  3.43224057  6.47050606 -5.40636291
  3.60052411 -0.56388566  0.7297127 -0.49430584  0.82816968 -6.71178464
 -2.5395075 -0.06169032  0.47705976  0.03167655 -1.32663931  2.25213284
  6.73992487  2.93355699  0.29702779  0.05213529  0.25136061  0.1186364
 -2.88396818 -1.75200051 -5.38591247  0.47614807 -0.72990722 -0.02747621
 -0.6133731 -2.86780403  5.59956688  5.90721724 -0.63900983 -0.41085943
  0.22592313 -1.87305639  0.32541875 -1.21379693 -2.53366549  6.88350621
  6.20537942 -0.11404169  1.94130132 -0.34285988 -4.11178138  0.13188098
 -0.1598734 -5.03469991  0.85322223  7.26838316  0.47283136 -0.39914841
  1.81945752  0.78150206 -0.04153196 -0.65060229  0.97585268 -1.26073891
 -3.70832899  0.05375173 -1.01269022  0.21719838 -0.79925019 -0.66265704
 -1.61466954 -0.09181897  1.31623233  2.39402827  0.34744754 -0.454114
  0.28689219  2.29724458 -0.13124115  7.07690782  7.13538106 -2.91680597
 -6.31977228 -2.6141091  4.50884496  5.67542223 -1.17665294  0.04428684
  0.89570065 -0.50370287  0.58134954  1.52193908 -0.22796101  0.97019515
 -0.08627162 -7.42440244 -0.37471577  0.7880526 ]
```

5. The Results

To find the best model, we compare the root mean square error for each polynomial regression model. The model with the lowest error is the best!

```
In [27]: pred1 = m.predict(X_test[:, np.newaxis])
         = sqrt(mean_squared_error(y_test,y_pred1))
         int('Root mean square error for Degree 1')
         int(r1)

         pred2 = x3.predict(X_test[:, np.newaxis])
         = sqrt(mean_squared_error(y_test,y_pred2))
         int('Root mean square error for Degree 2')
         int(r2)

         pred3 = x4.predict(X_test[:, np.newaxis])
         = sqrt(mean_squared_error(y_test,y_pred3))
         int('Root mean square error for Degree 3')
         int(r3)

         pred5 = x5.predict(X_test[:, np.newaxis])
         = sqrt(mean_squared_error(y_test,y_pred5))
         int('Root mean square error for Degree 5')
         int(r5)

         pred9 = x6.predict(X_test[:, np.newaxis])
         = sqrt(mean_squared_error(y_test,y_pred9))
         int('Root mean square error for Degree 9')
         int(r9)

         "Cubic regression is best because Degree 3:cubic regression has the lowest root mean square error among the others."""
```

```
Root mean square error for Degree 1
1.4902410023308104
Root mean square error for Degree 2
1.8292871801491593
Root mean square error for Degree 3
0.5385454583069078
Root mean square error for Degree 5
1.0665111079926206
Root mean square error for Degree 9
12.374312194587901
```

```
Out[27]: 'Cubic regression is best because Degree 3:cubic regression has the lowest root mean square error among the others.'
```

In []: