MECH 498: Introduction to Robotics

Direct (Forward) Manipulator Kinematics

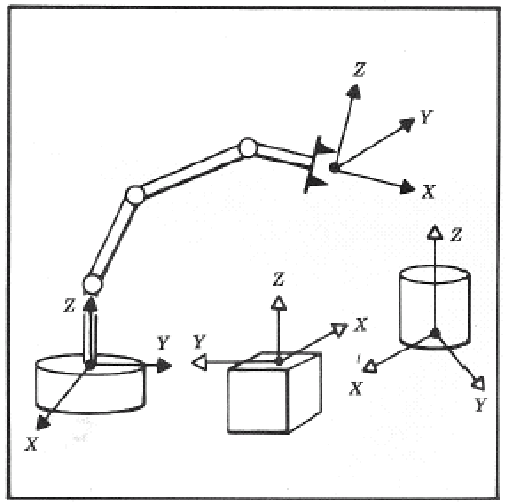
M. O'Malley

Kinematics - Introduction

- ***Kinematics*** - the science of motion which treat motions without regard to the forces that cause them
  - e.g. position, velocity, acceleration, higher derivatives of the position
- ***Kinematics of Manipulators*** - All the geometrical and time based properties of the motion

Central Topic

- **Problem**
  - Given: The manipulator geometrical parameters
  - Specify: The position and orientation of manipulator
- **Solution**
  - Coordinate system or "Frames" are attached to the manipulator and objects in the environment following the Denenvit-Hartenberg notation.



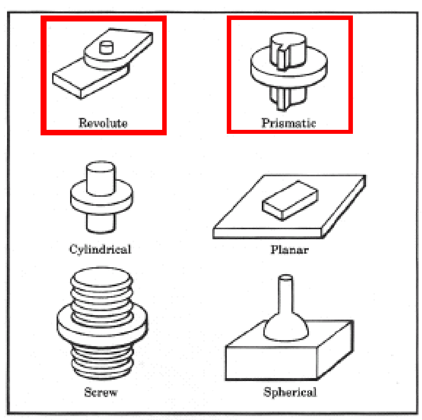
Joint/Link Description

- ***Lower pair*** - The connection between a pair of bodies when the relative motion is characterize by two surfaces sliding over one another.

Mechanical Design Constraints

1 DOF Joint
Revolute Joint
Prismatic Joint

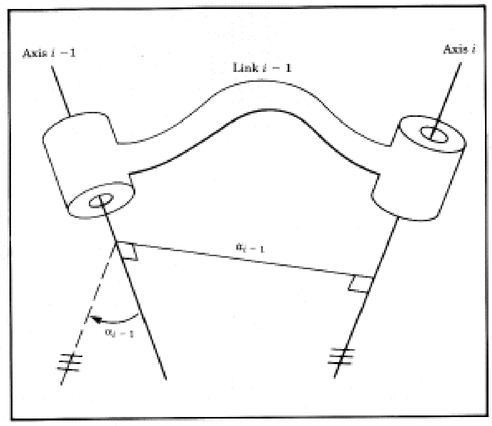
- ***Link*** - A rigid body which defines the relationship between two neighboring joint axes of the manipulator



Link Parameters (Denevit-Hartenberg) – Length & Twist)

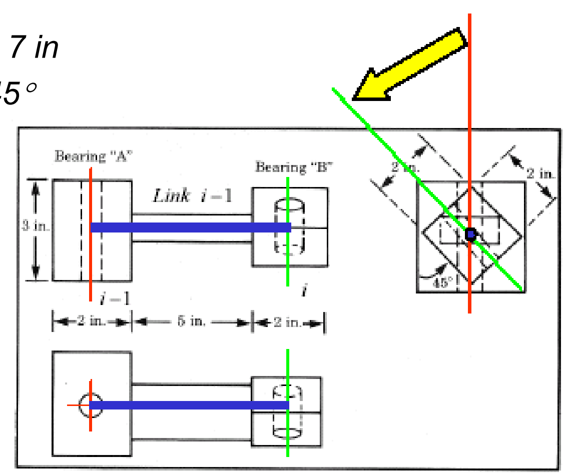
- ***Joint Axis -*** A line in space (or a vector direction) about which link ***i*** rotates relative to link ***i-1***
- ***Link Length –*** a_{i-1}
  - The distance between axis ***i*** and axis ***i-1***

  Notes:
  - Expanding cylinder analogy
  - Distance
    - Parallel axes → ∞
    - Non-Parallel axes → 1
  - Sign → $a_{i-1} \geq 0$
- ***Link Twist –*** α_{i-1}
  - The angle measured from axis ***i-1*** to axis ***i***
- ***Note :*** *Sign* α_{i-1} *by right hand rule*



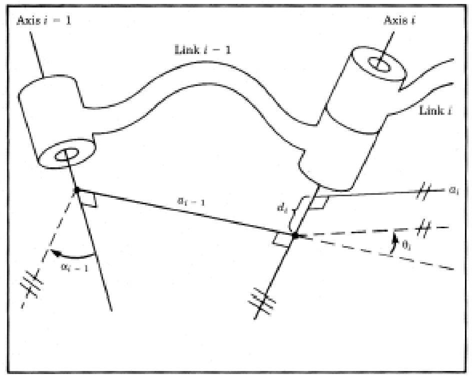
Link Parameters - Example

- Axes
- Link Length → $a_{i-1} = 7\ in$
- Link Twist → $a_{i-1} = 45^{o}$

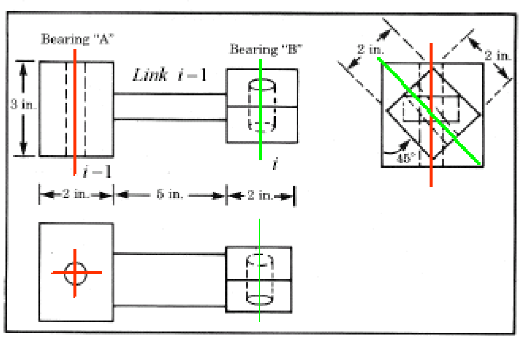


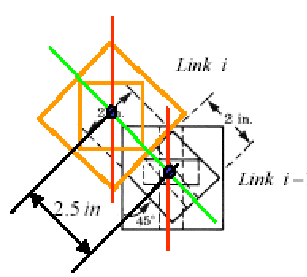
Joint Variables (Denevit-Hartenberg) – Angle & Offset

- **Link Offset – d_i**
  - The signed distance measured along the axis of joint **i** from the point where $\mathbf{a}_{i-1}$ intersects the axis to the point where $\mathbf{a}_i$ intersects the axis
    - The link offset $\mathbf{d}_i$ is variable if joint **i** is prismatic
    - Sign of $\mathbf{d}_i$
- **Joint Angle – θ_i**
  - The signed angle made between an extension of $\mathbf{a}_{i-1}$ and $\mathbf{a}_i$ measured about the the axis of the joint **i**
- **Note:**
  - The joint angle θ_i is variable if the joint **i** is revolute
- Sign - θ_i → Right hand rule



Link Parameters - Example





Link offset $d_i = 2.5in$

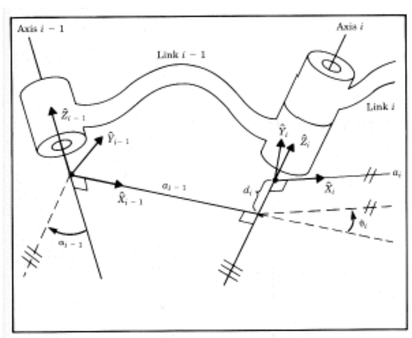
Joint/Link Parameters & Values – First and last links in chain

	$a_1 \rightarrow a_{n-1}$	See Definition
	$a_0 = a_n = 0$	Convention
	$\alpha_1 \rightarrow \alpha_{n-1}$	See Definition
	$\alpha_0 = \alpha_n = 0$	Convention
	$d_2 \rightarrow d_{n-1}$	See Definition
	$\theta_2 \rightarrow \theta_{n-1}$	
Joint 1 - Revolute Joint	$\theta_1 = 0$	Arbitrary
	$d_1 = 0$	Convention
Joint 1 - Prismatic Joint	$\theta_1 = 0$	Convention
	$d_1 = 0$	Arbitrary

Affixing Frames to Links – Intermediate Links in the Chain

- ***Origin of Frame {i}*** –
  - The origin of frame ***{i}*** is located where the $\boldsymbol{a_i}$ perpendicular intersects the joint ***i*** axis
- ***Z Axis -***
  - The $\hat{Z}_i$ axis of frame ***{i}*** is coincident with the joint axis ***I***
- ***X Axis -***
  - The $\hat{X}_i$ axis points along the distance $\boldsymbol{a_i}$ in the direction from joint ***i*** to joint ***i+1***

  Note:
  - For $\boldsymbol{a_i} = 0$, $\hat{X}_i$ is normal to the plane of $\hat{Z}_i$ and $\hat{Z}_{i+1}$
  - The link twist angle α_i is measured in a right hand sense about $\hat{X}_i$
- ***Y Axis-*** $\hat{Y}_i$
  - The $\hat{Y}_i$ axis completes frame ***{i}*** following the right hand rule



Affixing Frames to Links – First & Last Links in the Chain

- ***Frame {0}*** - The frame attached to the base of the robot or link 0 called frame {0} This frame does not move and for the problem of arm kinematics can be considered as the ***reference frame.***
- ***Frame {0} coincides with Frame {1}*** - $\begin{cases} \alpha_0 = 0 \\ a_0 = 0 \end{cases}$

$$\text{Joint 1 - Revolute Joint} \begin{cases} \theta_1 = 0 & \text{Arbitrary} \\ d_1 = 0 & \text{Convention} \end{cases}$$

$$\text{Joint 1 - Prismatic Joint} \begin{cases} \theta_1 = 0 & \text{Convention} \\ d_1 = 0 & \text{Arbitrary} \end{cases}$$

Link Frame Attachment Procedure - Summary

1. Identify the joint axes and imagine (or draw) infinite lines along them. For step 2 through step 5 below, consider two of these neighboring lines (at axes ***i*** and ***i+1***)
2. Identify the common perpendicular between them, or point of intersection. At the point of intersection, or at the point where the common perpendicular meets the ***i*** th axis, assign the link frame origin.
3. Assign the $\hat{Z}_i$ axis pointing along the ***i*** th joint axis.
4. Assign the $\hat{X}_i$ axis pointing along the common perpendicular, or if the axes intersect, assign $\hat{X}_i$ to be normal to the plane containing the two axes
5. Assign the $\hat{Y}_i$ axis to the complete a right hand coordinate system.
6. Assign {0} to match {1} when the first joint veritable is zero. For {N}, choose an origin location and $\hat{X}_N$ direction freely, but generally so as to cause as many linkage parameters as possible to be zero

DH Parameters - Summary

- If the link frame have been attached to the links according to our convention, the following definitions of the DH parameters are valid:

a_i - The distance from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured along $\hat{X}_i$

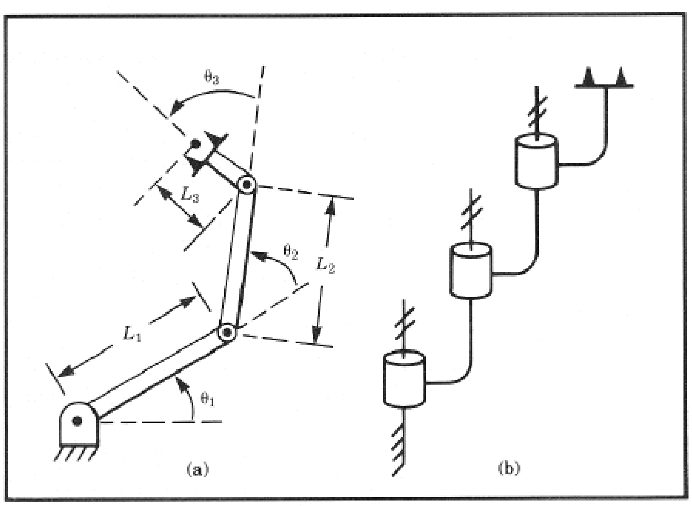
α_i - The angle between $\hat{Z}_i$ and $\hat{Z}_{i+1}$ measured about $\hat{X}_i$

d_i - The distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured along $\hat{Z}_i$

θ_i - The angle between $\hat{X}_{i-1}$ and $\hat{X}_i$ measured about $\hat{Z}_i$

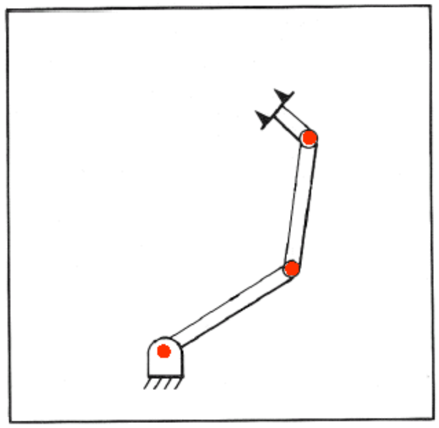
- ***Note:***
  - $\boldsymbol{a_i \geq 0}$, and α_i , $\boldsymbol{d_i}$, and θ_i are signed quantities

DH Parameters - RRR (3R) - Example



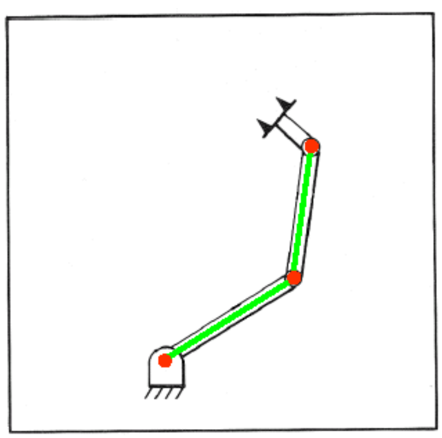
DH Parameters - RRR (3R) - Example

- Identify the joint axes



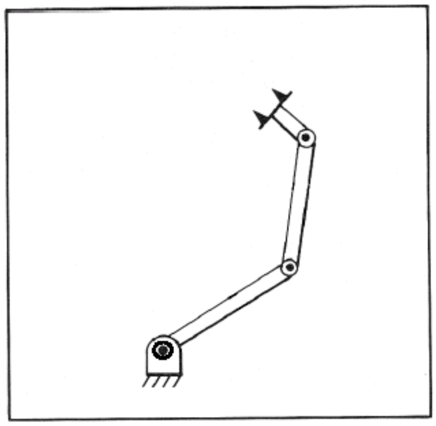
DH Parameters - RRR (3R) - Example

- Identify the common perpendicular between joint axes



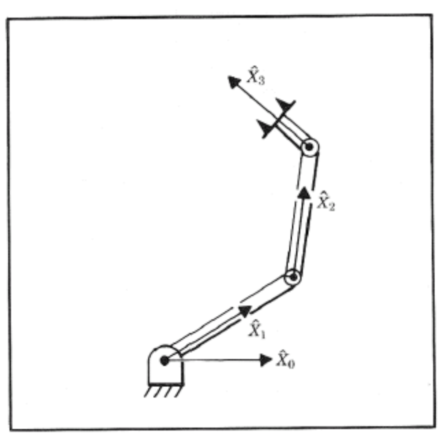
DH Parameters - RRR (3R) - Example

- Assign the $\hat{Z}_i$ axis pointing along the ***i*** th joint axis.



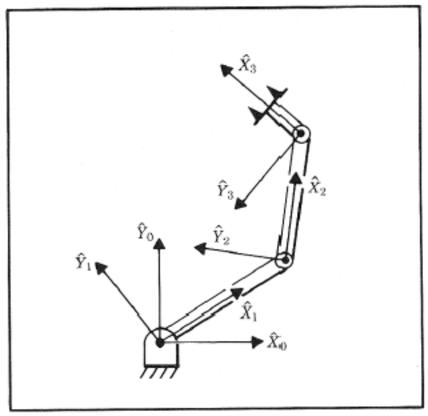
DH Parameters - RRR (3R) - Example

- Assign the $\hat{X}_i$ axis pointing along the common perpendicular



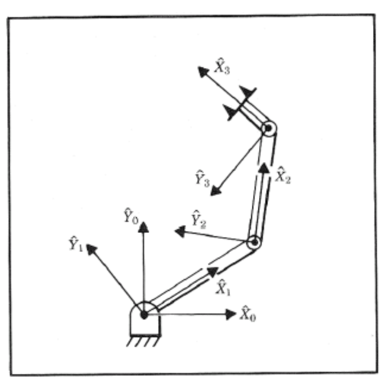
DH Parameters - RRR (3R) - Example

- Assign the $\hat{Y}_i$ axis to the complete a right hand coordinate system



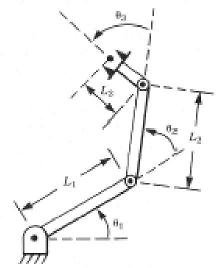
DH Parameters - RRR (3R) - Example

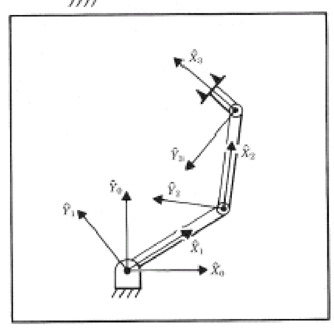
- Assign {0} to match {1} when the first joint variable is zero. For {N} choose an origin location and $\hat{X}_N$ direction freely, but generally so as to cause as many linkage parameters as possible to be zero



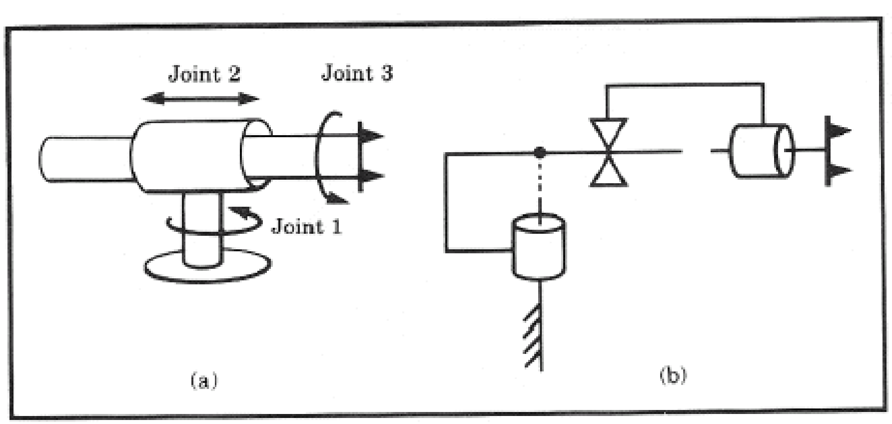
DH Parameters - RRR (3R) - Example

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3



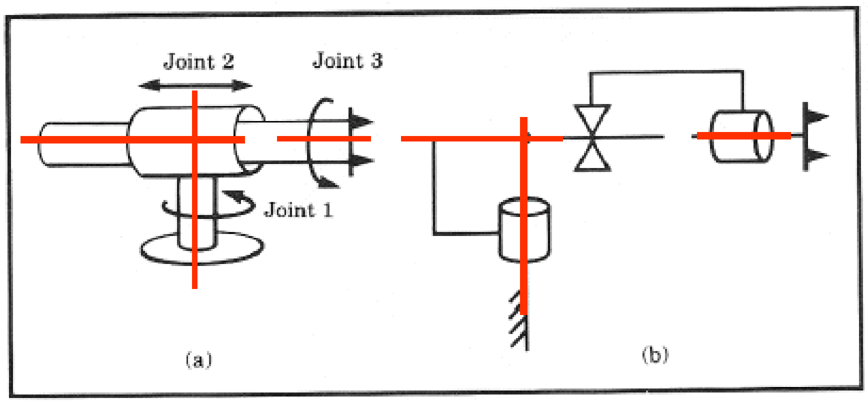


DH Parameters - RPR – Example



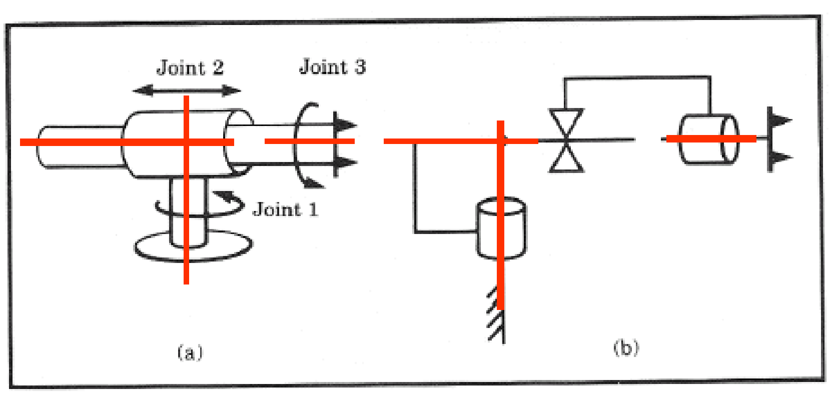
DH Parameters - RPR – Example

- Identify the joint axes



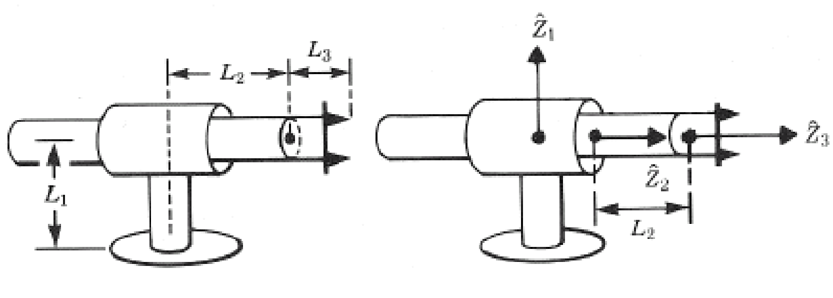
DH Parameters - RPR – Example

- Identify the common perpendicular between axis - NONE



DH Parameters - RPR – Example

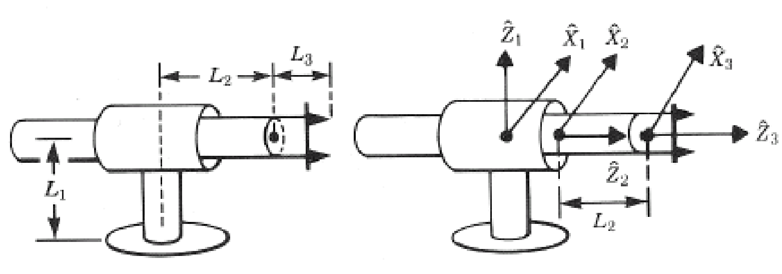
- Assign the $\hat{z}_i$ axis pointing along the ***i*** th joint axis.



(a) (b)

DH Parameters - RPR – Example

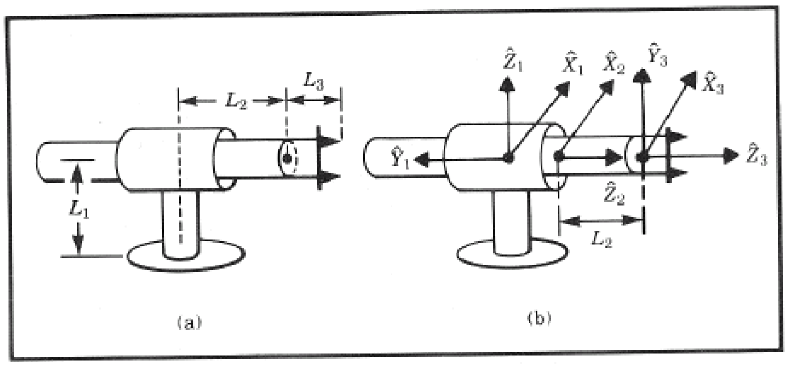
- If the $\hat{Z}_i$ axes intersect, assign $\hat{X}_i$ to be normal to the plane containing the two axes



(a) (b)

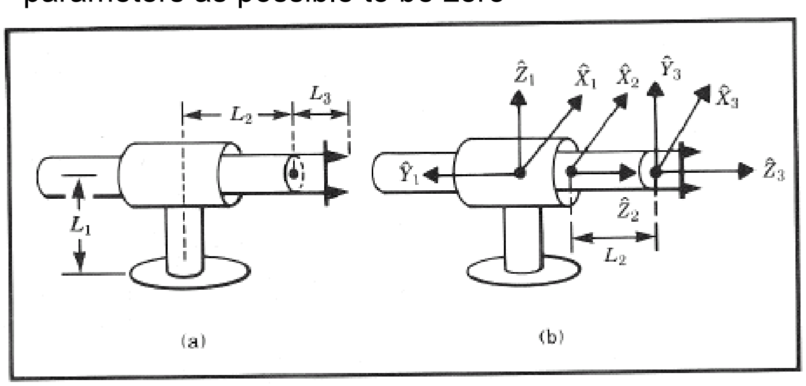
DH Parameters - RPR – Example

- Assign the $\hat{Y}_i$ axis to the complete a right hand coordinate system



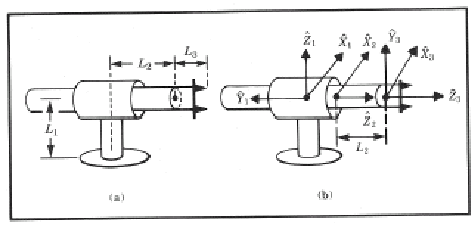
DH Parameters - RPR – Example

- Assign {0} to match {1} when the first joint variable is zero. For {N} choose an origin location and $\hat{X}_N$ direction freely, but generally so as to cause as many linkage parameters as possible to be zero



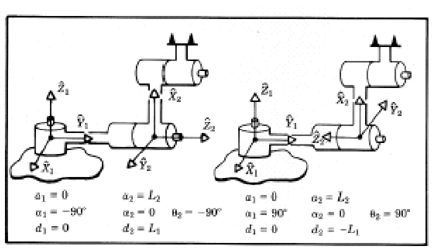
DH Parameters - RPR – Example

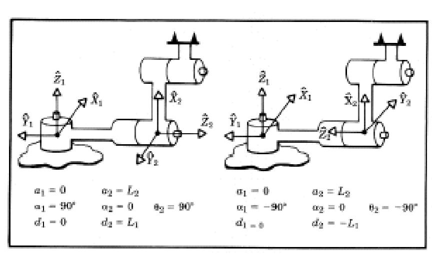
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3



DH Parameters - RRR (3R) - Example

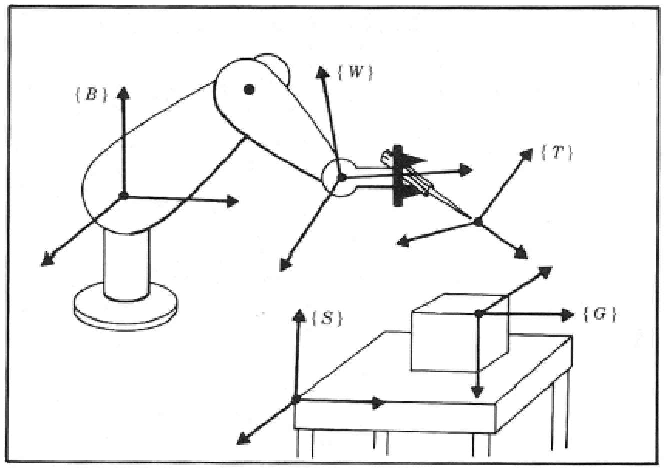
- Orthogonal Axes (Intersection) & Parallel Axes - Non Uniqueness of DH parameters
- When $\hat{Z}_i$ and $\hat{Z}_{i+1}$ intersect there are two choices for $\hat{X}_i$
- There are four more possibilities corresponding to the four configurations but with $\hat{Z}_i$ pointing downward





Central Topic

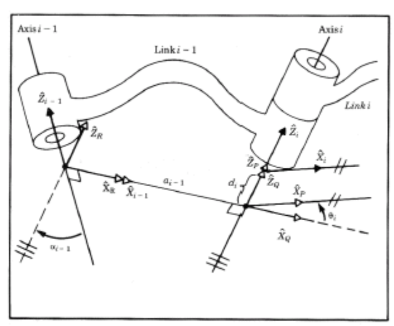
- Where is the tool?



Derivation of Link Homogeneous Transformation

- ***Problem:*** Determine the transformation which defines frame ***{i}*** relative to the frame ***{i+1}***

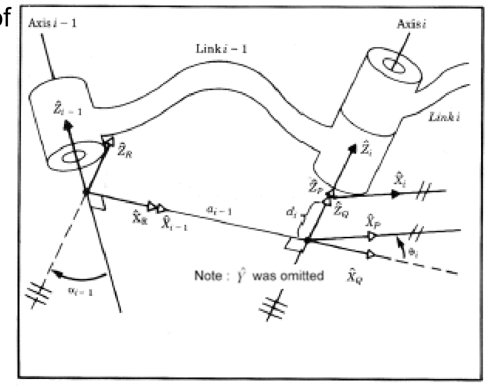
$${}^{i-1}_{\;\;i}T$$



- ***Note:*** For any given link of a robot, ${}^{i-1}_{\;\;i}T$ will be a function of only one variable out of $\boldsymbol{a_i}$, α_i , $\boldsymbol{d_i}$, $\theta_{i.}$ The other three parameters are fixed by mechanical design.
  - Revolute Joint → θ_i
  - Prismatic Joint → $\boldsymbol{d_i}$

Derivation Homogeneous Transformation

- ***Solution:***
  - The problem is further broken into 4 sub problems such that each of the transformations will be a function of one link parameter only
- Define three intermediate frames:
  - *{**P**}*, *{**Q**}*, and *{**R**}*
  - Frame *{**R**}* is different from *{**i+1**}* only by a rotation of α_{i-1}
  - Frame *{**Q**}* is different from *{**R**}* only by a translation a_{i-1}
  - Frame *{**P**}* is different from *{**Q**}* only by a rotation θ_i
  - Frame *{**i**}* is different from *{**P**}* only by a translation d_i



Derivation of link Homogeneous Transformation

Solution: A vector defined in frame {*i*} is expressed in {*i-1*} as follows

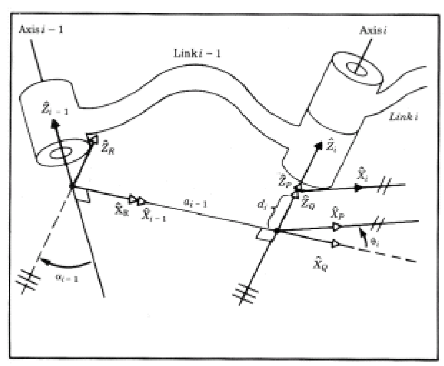
$$^{i-1}P = {}^{i-1}_{R}T\,{}^{R}_{Q}T\,{}^{Q}_{P}T\,{}^{P}_{i}T\,{}^{i}P$$

$$^{i-1}P = {}^{i-1}_{i}T\,{}^{i}P$$

The transformation from frame {*i-1*} to frame {*i*} is defined as follows

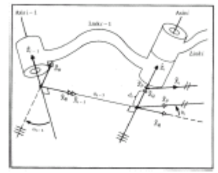
$$^{i-1}_{i}T = {}^{i-1}_{R}T\,{}^{R}_{Q}T\,{}^{Q}_{P}T\,{}^{P}_{i}T$$

$$^{i-1}_{i}T = R_X(\alpha_{i-1})D_X(a_{i-1})R_Z(\theta_i)D_Z(d_i)$$



Note : $\hat{Y}$ was omitted

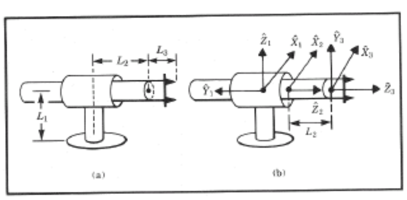
$$ {}^{i-1}_{i}T = R_X(\alpha_{i-1})D_X(a_{i-1})R_Z(\theta_i)D_Z(d_i) $$



$$ {}^{i-1}_{i}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_{i-1} & -s\alpha_{i-1} & 0 \\ 0 & s\alpha_{i-1} & c\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \mathbf{0} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \boldsymbol{d_i} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

DH Parameters – RPR – Example



$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

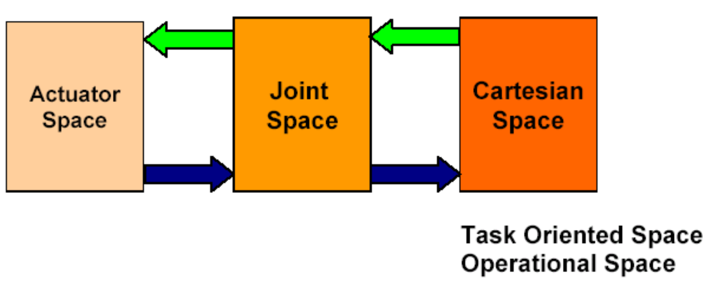
Concatenating Link Transformation

- Define link frames
- Define DH parameters of each link
- Compute the individual link transformation matrix
- Relates frame *{ **N** }* to frame *{ **0** }*

$$ {}^{0}_{N}T = {}^{0}_{1}T\, {}^{1}_{2}T\, {}^{2}_{3}T \ldots {}^{N-1}_{N}T $$

- The transformation ${}^{0}_{N}T$ will be a function of all *n* joint variables.
- If the robot's joint position sensors are measured, the Cartesian position and orientation of the last link may be computed by ${}^{0}_{N}T$

Actuator Space – Joint Space – Cartesian Space



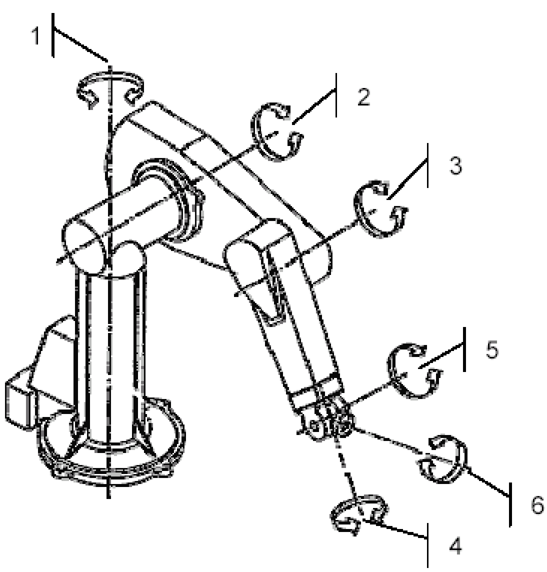
PUMA Family

PUMA 200

PUMA 500

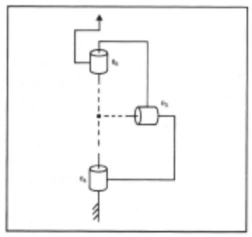
PUMA 700

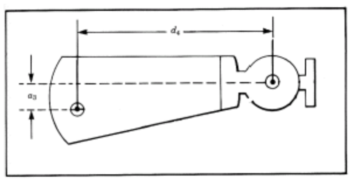
PUMA 560 – 6R

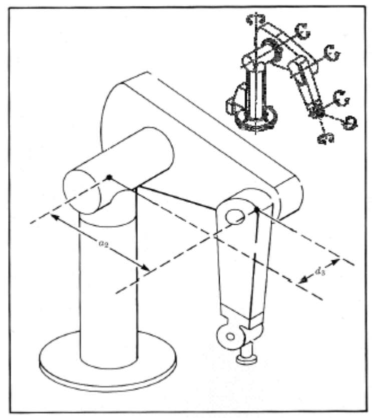


Kinematics of an Industrial Robot – PUMA 560

- The robot position in which all joint angles are equal to zero

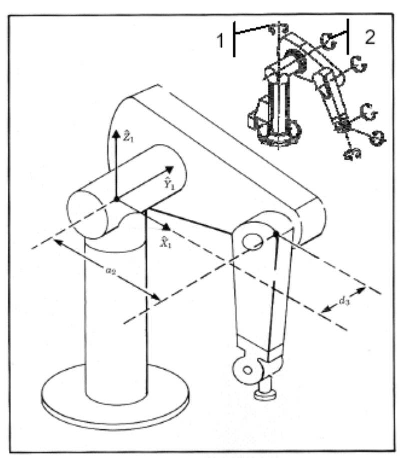






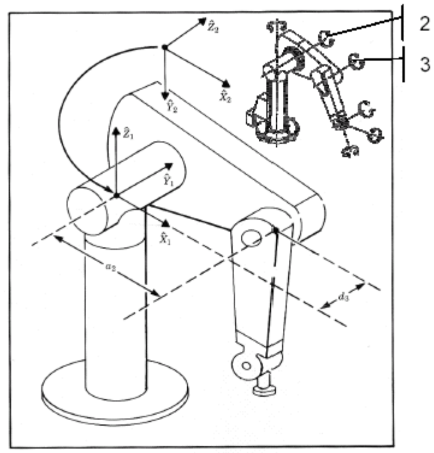
PUMA 560 – Frame Assignments – {0} and {1}

- Assign {0} to match {1} when the first joint variable is zero. Frame {0} is coincident with Frame {1}
- Assign the $\hat{Z}_1$ axis pointing along the *1st* joint axis.
- Assign the $\hat{X}_1$ axis pointing along the common perpendicular, or if the axes intersect, assign $\hat{X}_1$ to be normal to the plane containing the two axes
- Assign the $\hat{Y}_1$ axis to the complete a right hand coordinate system.



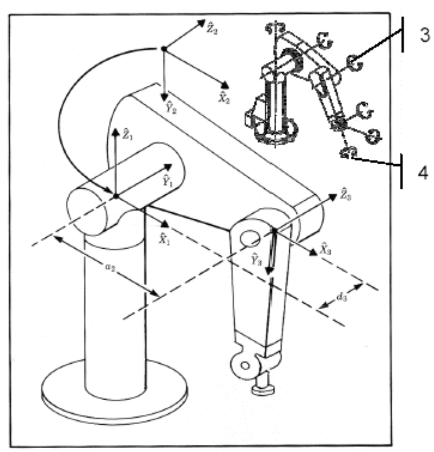
PUMA 560 – Frame Assignments – {2}

- Assign the $\hat{Z}_2$ axis pointing along the *2nd* joint axis.
- Assign the $\hat{X}_2$ axis pointing along the common perpendicular
- Assign the $\hat{Y}_2$ axis to the complete a right hand coordinate system.



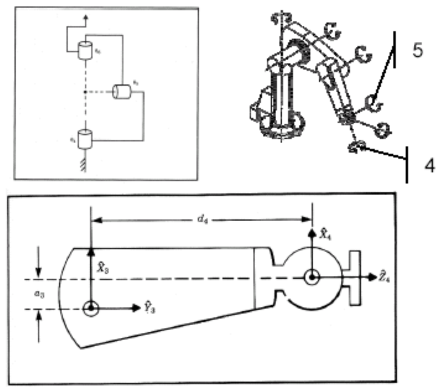
PUMA 560 – Frame Assignments – {3}

- Assign the $\hat{Z}_3$ axis pointing along the *3rd* joint axis
- Assign the $\hat{X}_3$ axis pointing along the common perpendicular
- Assign the $\hat{Y}_3$ axis to the complete a right hand coordinate system.



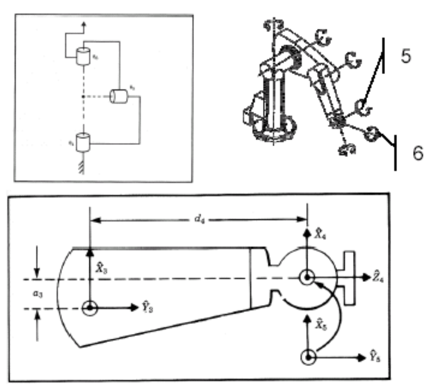
PUMA 560 – Frame Assignments – {4}

- Assign the $\hat{Z}_4$ axis pointing along the *4th* joint axis.
- Assign the $\hat{X}_4$ axis pointing along the common perpendicular if the axes intersect, assign X_4 to be normal to the plane containing the two axes
- Assign the $\hat{Y}_4$ axis to the complete a right hand coordinate system.



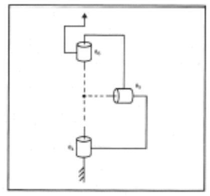
PUMA 560 – Frame Assignments – {5}

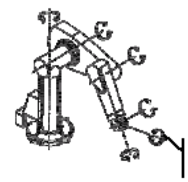
- Assign the $\hat{Z}_5$ axis pointing along the *5th* joint axis.
- Assign the $\hat{X}_5$ axis pointing along the common perpendicular if the axes intersect, assign X_5 to be normal to the plane containing the two axes
- Assign the $\hat{Y}_5$ axis to the complete a right hand coordinate system.

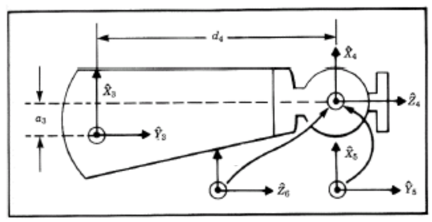


PUMA 560 – Frame Assignments – {6}

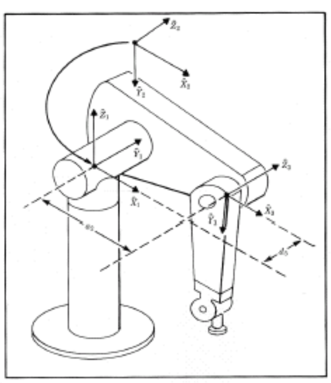
- Assign the $\hat{Z}_6$ axis pointing along the *6th* joint axis.
- For frame {N} ({6}) choose an origin location and $\hat{X}_6$ direction freely, but generally so as to cause as many linkage parameters as possible to be zero
- Assign the $\hat{Y}_6$ axis to the complete a right hand coordinate system.

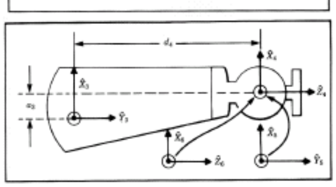






PUMA 560 – DH Parameters





i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6

PUMA 560 – Link Transformations

$$
{}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6

$$
{}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}_{4}T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{4}_{5}T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{5}_{6}T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_5 & -c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

PUMA 560 – Kinematic Equations

- The kinematics equations of PUMA 560 specify how to compute the position & orientation of frame {6} (tool) relative to frame {0} (base) of the robot. These are the basic equations for all kinematic analysis of this manipulator.

$$
{}^{0}_{6}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T\,{}^{5}_{6}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$r_{11} = c_1\left[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6\right] + s_1(s_4c_5c_6 + c_4s_6),$$
$$r_{21} = s_1\left[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6\right] - c_1(s_4c_5c_6 + c_4s_6),$$
$$r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6,$$

$$r_{12} = c_1\left[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6\right] + s_1(c_4c_6 - s_4c_5s_6),$$
$$r_{22} = s_1\left[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6\right] - c_1(c_4c_6 - s_4c_5s_6),$$
$$r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6,$$

$$r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5,$$
$$r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5,$$
$$r_{33} = s_{23}c_4s_5 - c_{23}c_5,$$

$$p_x = c_1\left[a_2c_2 + a_3c_{23} - d_4s_{23}\right] - d_3s_1,$$
$$p_y = s_1\left[a_2c_2 + a_3c_{23} - d_4s_{23}\right] + d_3c_1,$$
$$p_z = -a_3s_{23} - a_2s_2 - d_4c_{23}.$$

Notations:

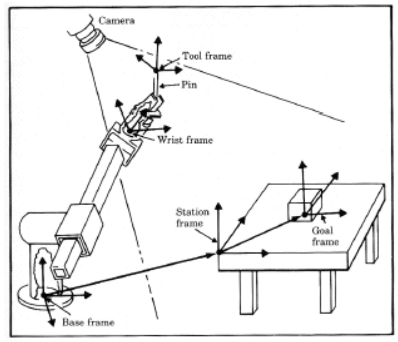
$\cos(\theta_1) = c\theta_1 = c_1$

$\cos(\theta_1 + \theta_2) = c_{12} = c_1c_2 - s_1s_2$

$\sin(\theta_1 + \theta_2) = s_{12} = c_1s_2 + s_1c_2$

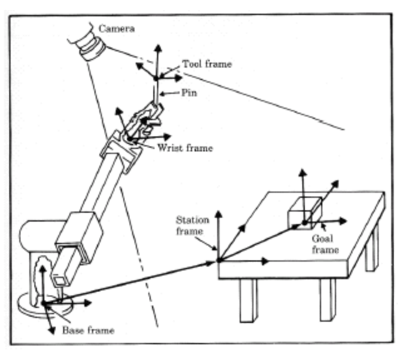
Frame with Standard Names

- ***Base Frame* {B}** –
  - **{B}** is located at the base of the manipulator affixed to the nonmoving part of the robot (another name for frame *{0})*
- ***Station Frame* {S}** –
  - **{S}** is located in a task relevant location (e.g. at the corner of the table upon the which the robot is to work). From the user perspective **{S}** is the universe frame (task frame or world frame) and all action of the robot are made relative to it. The station frame **{S}** is always specify with respect to the base frame **{B},** i.e. ${}^{B}_{S}T$



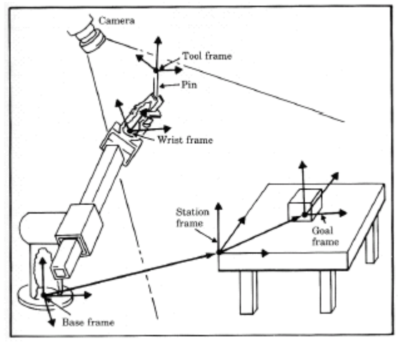
Frame with Standard Names

- ***Wrist Frame* {W}** –
  - **{W}** is affixed to the last link of the manipulator – the wrist (another name for frame {N}). The wrist frame **{W}** is defined relative to the base frame i.e. ${}^{B}_{W}T = {}^{0}_{N}T$
- ***Tool Frame* {T}** –
  - **{T}** is affixed to the end of any tool the robot happens to be holding. When the hand is empty, **{T}** is located with its origin between the fingertips of the robot. The tool frame **{T}** is always specified with respect to the wrist frame **{W}** i.e. ${}^{W}_{T}T$



Frame with Standard Names

- ***Goal Frame* {G}** –
  - **{G}** is describing the location to which the robot is about to move the tool. At the end of the robot motion the tool frame **{T}** is about to coincide with the goal frame **{G}.** The goal frame is always specified with respect to the station frame **{S}** i.e. ${}^{S}_{G}T$

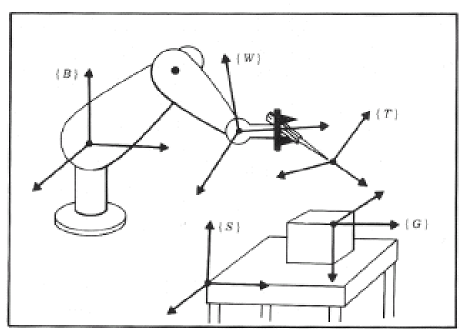


Where is the tool?

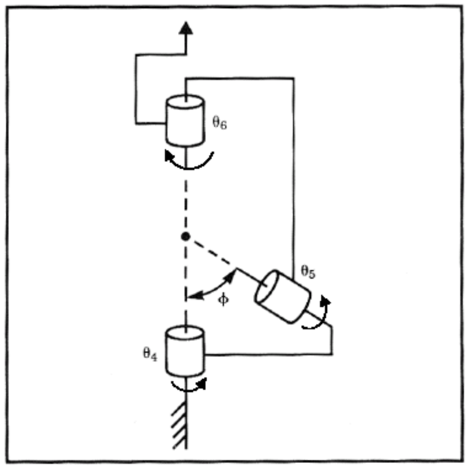
- ***Problem:***
  Calculate the transformation matrix of the the tool frame **{T}** relative to the station frame **{S}** - ${}^{S}_{T}T$

- ***Solution:***
  Cartesian Transformation

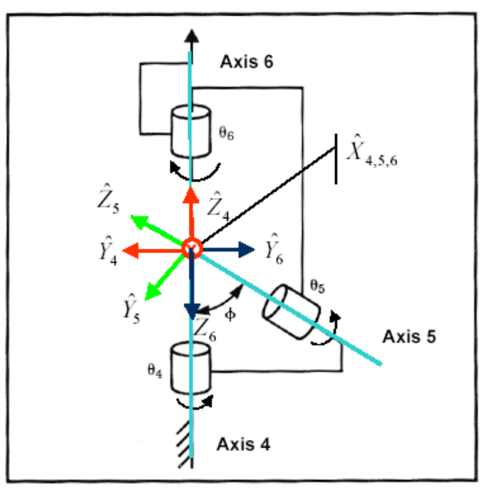
$${}^{S}_{T}T = {}^{B}_{S}T^{-1}\,{}^{B}_{W}T\,{}^{W}_{T}T$$



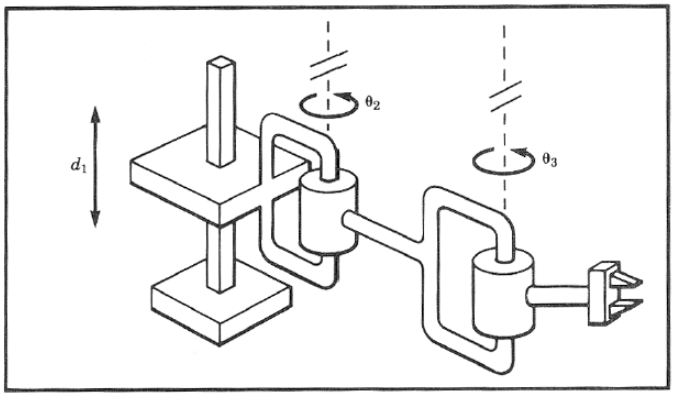
Manipulator Kinematics - Example - 3R - Wrist



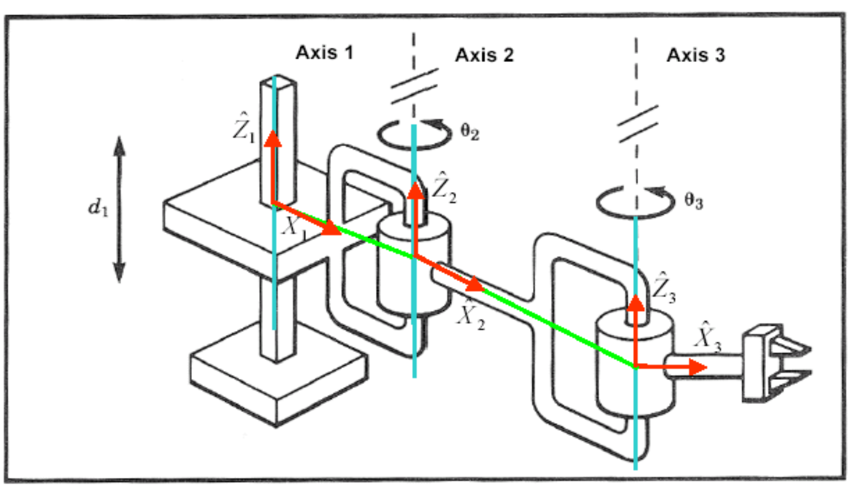
Manipulator Kinematics - Example - 3R - Wrist



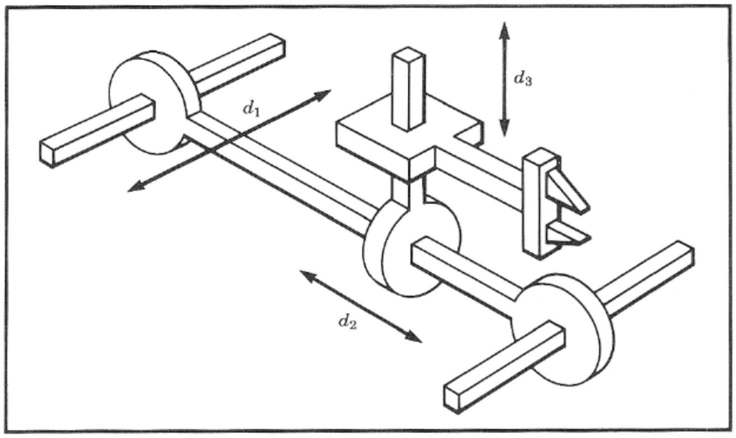
Manipulator Kinematics – Example - PRR



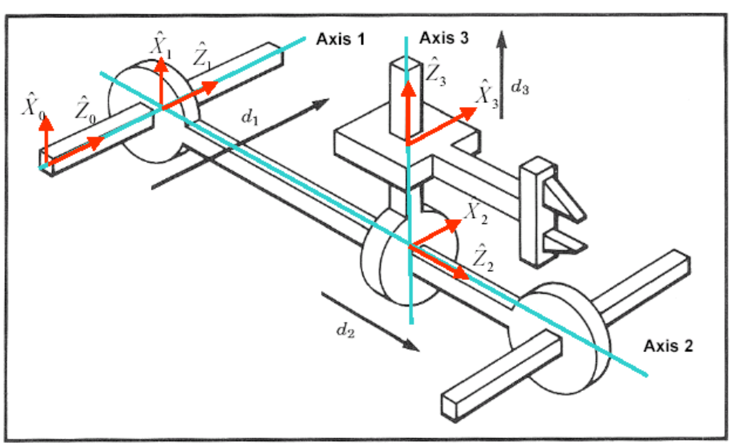
Manipulator Kinematics – Example - PRR



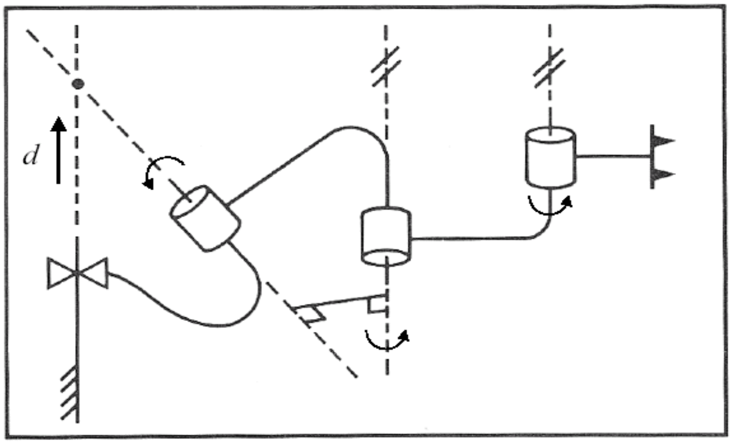
Manipulator Kinematics – Example – 3P



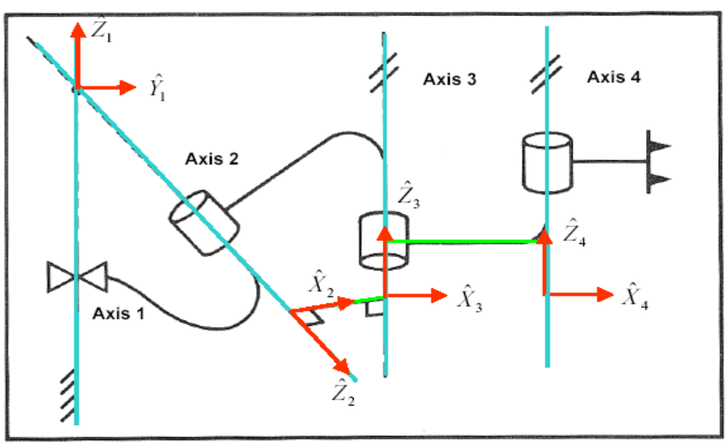
Manipulator Kinematics – Example – 3P



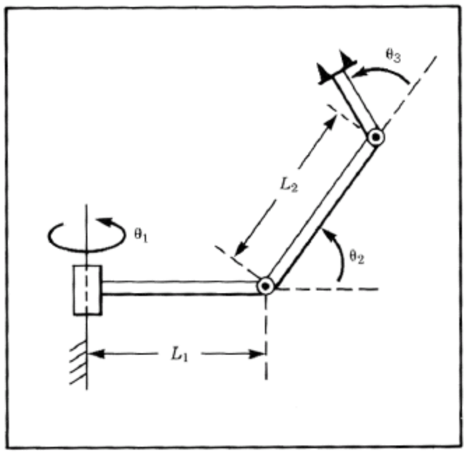
Manipulator Kinematics – Example – PRRR



Manipulator Kinematics – Example – PRRR



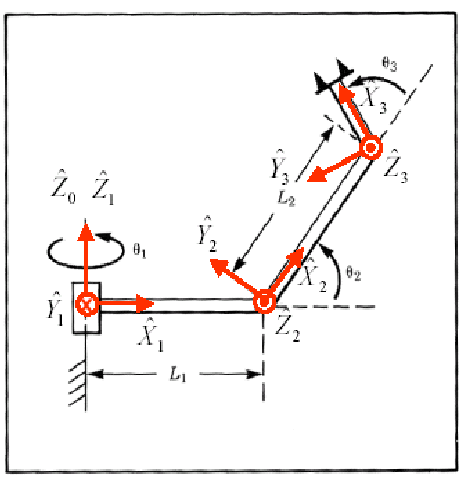
Manipulator Kinematics – Example – 3R



i-1	i	α_{i-1}	a_{i-1}	d_i	θ_i
0	1				
1	2				
2	3				

$^B_W T$

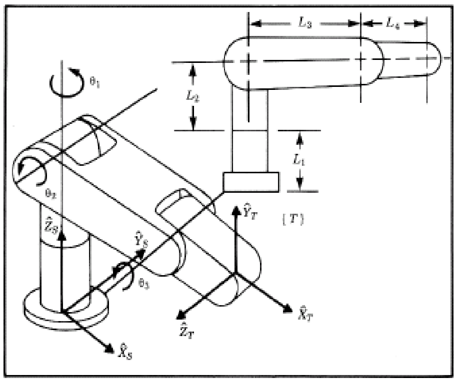
Manipulator Kinematics – Example – 3R



i-1	i	α_{i-1}	a_{i-1}	d_i	θ_i
0	1	0	0	0	θ_1
1	2	90	L_1	0	θ_2
2	3	0	L_2	0	θ_3

$$^B_W T = {}^0_3 T = {}^0_1 T \, {}^1_2 T \, {}^2_3 T$$

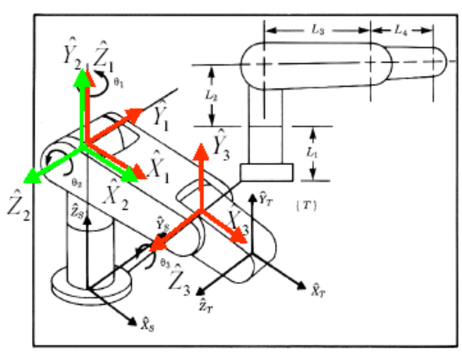
Manipulator Kinematics – Example – 3R



$i-1$	i	α_{i-1}	a_{i-1}	d_i	θ_i
0	1				
1	2				
2	3				
3	4				

${}^B_T T$

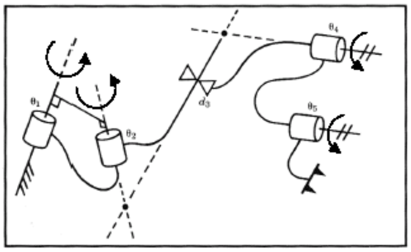
Manipulator Kinematics – Example – 3R



$i-1$	i	α_{i-1}	a_{i-1}	d_i	θ_i
0	1	0	0	$L_1 + L_2$	θ_1
1	2	90	0	0	θ_2
2	3	0	L_3	0	θ_3
3	4	0	L_4	0	0

$${}^S_T T = {}^0_4 T = {}^0_1 T\, {}^1_2 T\, {}^2_3 T\, {}^3_4 T$$

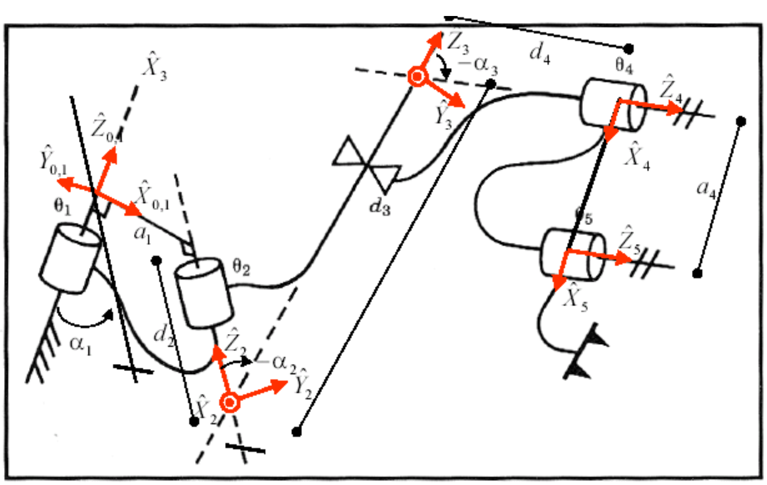
Manipulator Kinematics – Example – RRPRR



i-1	i	α_{i-1}	a_{i-1}	d_i	θ_i
0	1				
1	2				
2	3				
3	4				
4	5				

$^B_W T$

Manipulator Kinematics – Example – RRPRR (2RP2R)



Manipulator Kinematics – Example – RRPRR (2RP2R)

i-1	*i*	α_{i-1}	a_{i-1}	d_i	θ_i
0	**1**	**0**	**0**	**0**	θ_1
1	**2**	α_1	a_1	$-d_1$	θ_2
2	**3**	$-\alpha_2$	**0**	d_2	**0**
3	**4**	$-\alpha_3$	**0**	d_3	θ_4
4	**5**	**0**	a_4	**0**	θ_5

$${}^{B}_{W}T = {}^{0}_{5}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T$$

Link Frame Attachment Procedure - Summary

1. Identify the joint axes and imagine (or draw) infinite lines along them. For step 2 through step 5 below, consider two of these neighboring lines (at axes ***i*** and ***i+1***)
2. Identify the common perpendicular between them, or point of intersection. At the point of intersection, or at the point where the common perpendicular meets the ***i*** th axis, assign the link frame origin.
3. Assign the $\hat{Z}_i$ axis pointing along the ***i*** th joint axis.
4. Assign the $\hat{X}_i$ axis pointing along the common perpendicular, or if the axes intersect, assign $\hat{X}_i$ to be normal to the plane containing the two axes
5. Assign the $\hat{Y}_i$ axis to the complete a right hand coordinate system.
6. Assign {0} to match {1} when the first joint veritable is zero. For {N}, choose an origin location and $\hat{X}_N$ direction freely, but generally so as to cause as many linkage parameters as possible to be zero

DH Parameters - Review

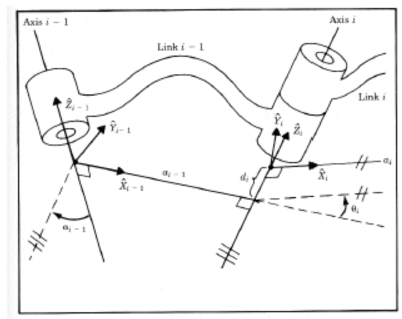
a_i - The distance from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured along $\hat{X}_i$

α_i - The angle between $\hat{Z}_i$ and $\hat{Z}_{i+1}$ measured about $\hat{X}_i$

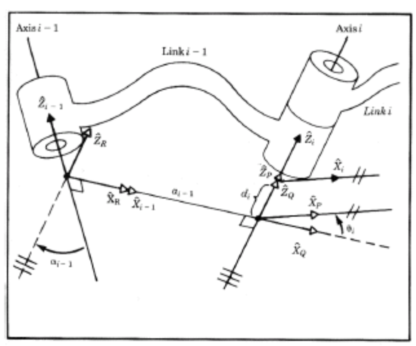
d_i - The distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured along $\hat{Z}_i$

θ_i - The angle between $\hat{X}_{i-1}$ and $\hat{X}_i$ measured about $\hat{Z}_i$

- ***Note:***
  - ***$a_i \geq 0$*** ,and α_i , ***d_i*** , and θ_i are signed quantities



Derivation of Link Homogeneous Transformation



$$
{}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$