Independent Trials and Trial Size

Computational Physics Exercise 1

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In this paper a stochastic method for computing π is introduced and analyzed. The Idea is that the area of the unit circle is given by π and the area of a square with a width of 2 is 4. Using this it is possible to calculate π by generating P two-dimensional points (from a uniform distribution) inside the square and counting how many of them also are located inside the unit circle.

The points are labeled $\vec{r}_p = (x_p, y_p)$ with $\vec{r}_p \in [-1, 1] \times [-1, 1]$. By using the Iverson Bracket

$$[P] = \begin{cases} 1 & \text{if P is true} \\ 0 & \text{otherwise} \end{cases}$$

We can write the formula for π_x, where the x stands for the experiment, as

$$\pi_X = \frac{4}{P} \sum_{p=1}^{P} [x_p^2 + y_p^2 \leq 1]. \tag{1}$$

The experiment can be repeated X times, so that we get a final answer

$$\pi_\mathrm{f} = \frac{1}{X} \sum_{x=1}^{X} \pi_x \qquad \Delta\pi_\mathrm{f} = \sqrt{\frac{1}{X-1} \sum_{x=1}^{X} (\pi_\mathrm{f} - \pi_x)^2}. \tag{2}$$