Simple Sampling Experiments

Computational Physics Exercise 2

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1 Sequences of Fair Coin Flips

Suppose we flip a fair coin p times and record f_1 through f_p. If the coin lands heads on the j-th flip, $f_j = 1$ and if it lands tails, $f_j = 0$. Each sequence of p flips will be interpreted as a binary number with fixed precision of p digits,

$$n_p(\vec{f}) = 0.f_1 f_2 \dots f_p = \sum_{j=1}^{p} \frac{f_j}{2^j}. \tag{1}$$

Since f_k can only be 1 or 0, it is easy to see that

$$\min\{n_p(\vec{f})\} = 0$$

and

$$\max\{n_p(\vec{f})\} = \sum_{j=1}^{p} \frac{1}{2^j}.$$

For $p = 16$ we get

$$\max\{n_{16}(\vec{f})\} = \frac{65\,535}{65\,536}.$$

For the special case where $p \to \infty$ we can rewrite the sequence,

$$\max\{n_\infty(\vec{f})\} = \sum_{j=1}^{\infty} \frac{1}{2^j} = \sum_{j=0}^{\infty} \frac{1}{2^j} - 1 = g\left(\frac{1}{2}\right) - 1 = 1$$

where $g(x)$ is the geometric series with the known limit $1/(1-x)$ for $|x| < 1$.

Now we look at the distribution of n_p. For this we generate sequences of $p = 32$ and plot the corresponding histograms for different amounts S of sequences. This is shown in fig. 1. One can see that for higher number of sequences the distribution gets closer to a uniform distribution.

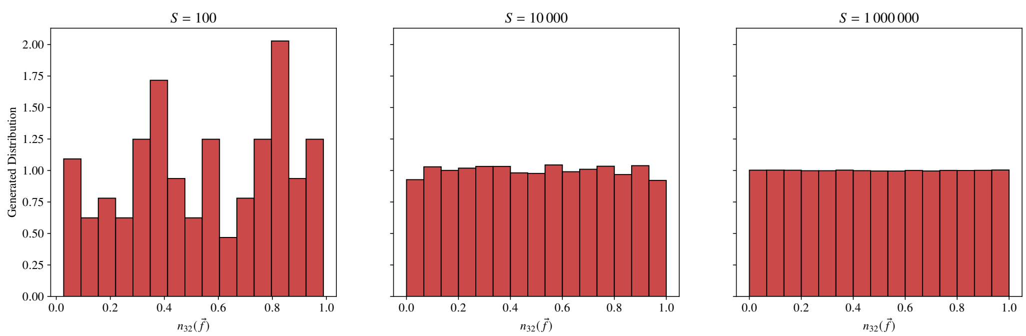

Figure 1: Histograms of $n_{32}(\vec{f})$ for different number sequences.