Independent Trials and Trial Size

Computational Physics Exercise 1

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In this paper a stochastic method for computing π is introduced and analyzed. The Idea is that the area of the unit circle is given by $\pi \approx 3.1416$ and the area of a square with a width of 2 is 4. Using this it is possible to calculate π by generating P two-dimensional points (from a uniform distribution) inside the square and counting how many of them also are located inside the unit circle.

The points are labeled $\vec{r}_p = (x_p, y_p)$ with $\vec{r}_p \in [-1,1] \times [-1,1]$. By using the Iverson Bracket

$$[X] = \begin{cases} 1 & \text{if X is true} \\ 0 & \text{otherwise} \end{cases}$$

We can write the formula for π_x, where the x stands for the experiment, as

$$\pi_x = \mathrm{E}[4[X^2 + Y^2 \leq 1]] = \frac{4}{P} \sum_{p=1}^{P} [x_p^2 + y_p^2 \leq 1]. \tag{1}$$

The experiment can be repeated X times, so that we get a final answer

$$\pi_\mathrm{f} = \frac{1}{X} \sum_{x=1}^{X} \pi_x \qquad \Delta\pi_\mathrm{f} = \sqrt{\frac{1}{X-1} \sum_{x=1}^{X} (\pi_\mathrm{f} - \pi_x)^2}. \tag{2}$$

1 One big experiment

Let $P = 10\,000$ and $X = 1$. Since only one experiment is done, eq. (2) cannot be used to calculate the uncertainty. Because of that the uncertainty is defined as

$$\Delta\pi_x = 4\sqrt{\mathrm{Var}([X^2 + Y^2 \leq 1])}$$

with $\mathrm{Var}(X)$ being the variance of the random variable X. Letting the experiment run results in a value of

$$\pi_x = 3.1 \pm 1.7, \tag{3}$$

which is considering the uncertainty of approximately 53 % close to the real value.

When doing just one experiment, the distribution of the generated points is of interest. For this the histogram of the radii $R = \sqrt{X^2 + Y^2}$ and the squared radii are plotted in figs. 1 and 2. For the radii we can see a linear rising distribution with a maximum at $r = 1$, after which the distribution drops. This drop is also visible for the squared radii, but in the region $[0, 1]$ a constant distribution is visible. The drop after $r = 1$ is easily explainable with the fact that points insied a 2×2 square we generated and a circle with radius 1 is the biggest circle that fits inside. Whith this in mind the amount of points generated on the circumference of a circle with bigger radius has to get smaller. **TO BE CONTINUED**

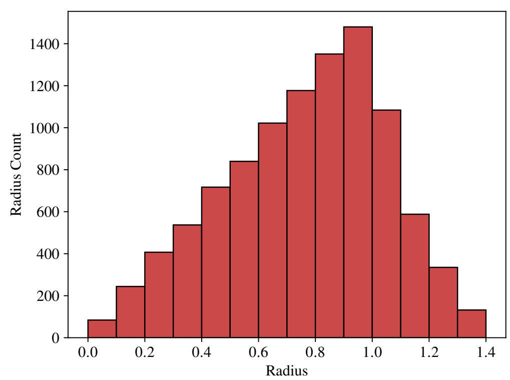

Figure 1: Histogram of radii from all the generated points of one big experiment with $P = 10\,000$.

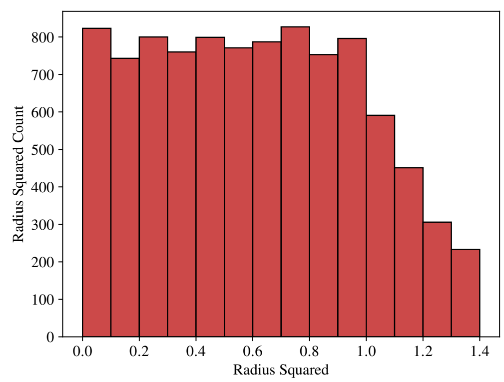

Figure 2: Histogram of the squared radii from all the generated points of one big experiment with $P = 10\,000$.

2 Rearranging Calculations

The number of calculations is now split into $P = 100$ and $X = 100$. In this case we get a value of

$$\pi_f = 3.15 \pm 0.16. \tag{4}$$

This value has a much smaller error (5.1 %) compared to section 1. A histogram for the distribution of π_x is shown in fig. 4. With $P = 1$ and $X = 10\,000$ we get a value of

$$\pi_f = 3.2 \pm 1.7, \tag{5}$$

which is identical to the calculation from section 1.

3 some

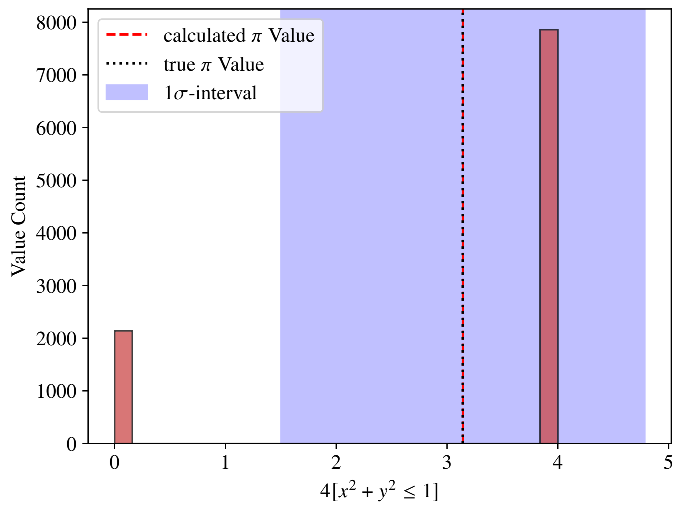

Figure 3: Histogram of the indicator variable $[X^2 + Y^2 \leq 1]$ for an experiment with $P = 10\,000$.

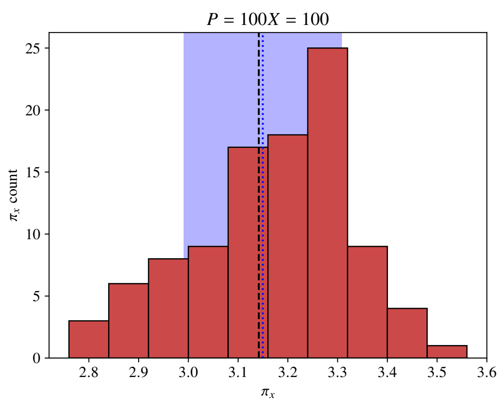

Figure 4: Histogram of calculated π values with $P = 100$ and $X = 100$.

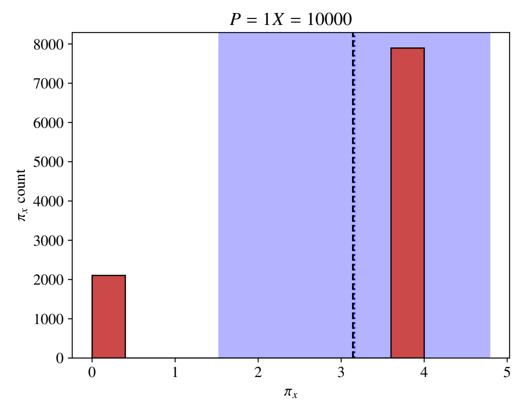

Figure 5: Histogram of calculated π values with $P = 1$ and $X = 10\,000$.

Table 1: Calcuated π values for different P and X.

P X	10	100	1000	10000
10	3.160 ± 0.488	3.208 ± 0.156	3.156 ± 0.046	3.144 ± 0.018
100	3.100 ± 0.608	3.138 ± 0.161	3.139 ± 0.055	3.142 ± 0.016
1000	3.135 ± 0.519	3.143 ± 0.160	3.142 ± 0.052	3.141 ± 0.016
10000	3.134 ± 0.518	3.140 ± 0.164	3.141 ± 0.052	3.142 ± 0.016

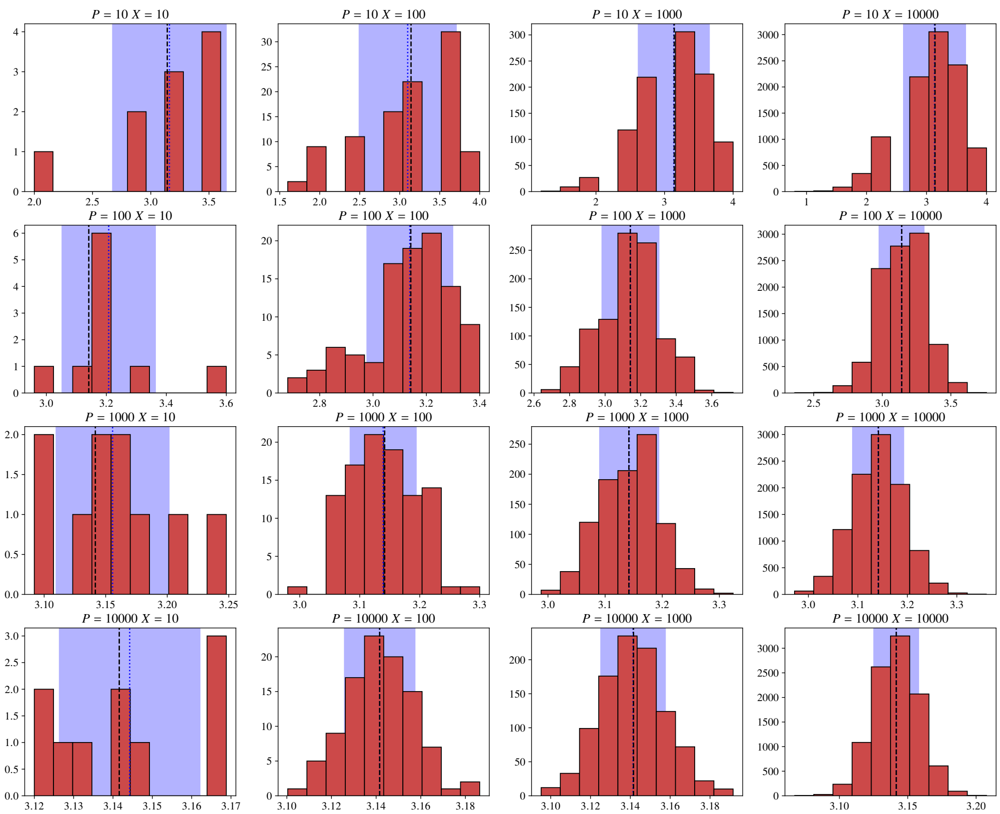

Figure 6:

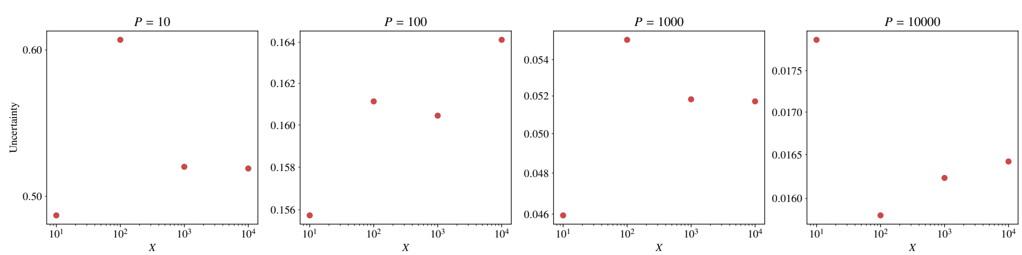

Figure 7:

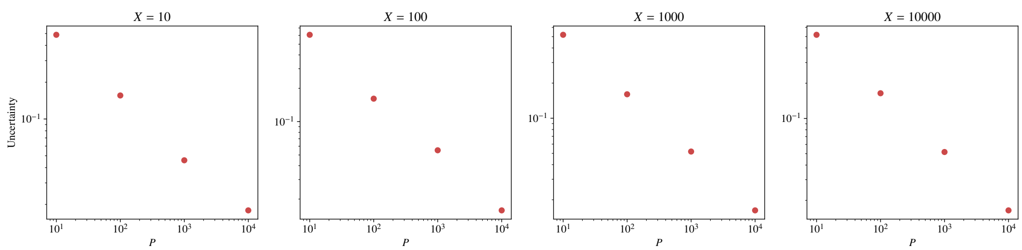

Figure 8: