

Vrtače dinarskega krasa

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Kraške vrtače

So zaprte koncentrične depresije



Jurišče, Slovenija

Kraške vrtače

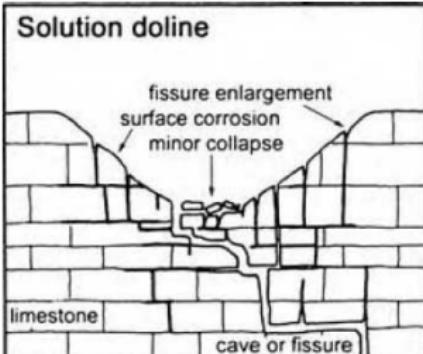
Najdemo jih na starih kraških poljih in planotah



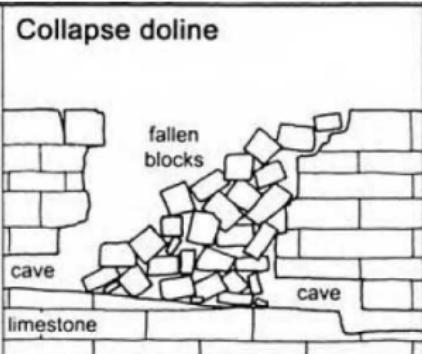
Kapljuh, BiH

Kraške vrtače

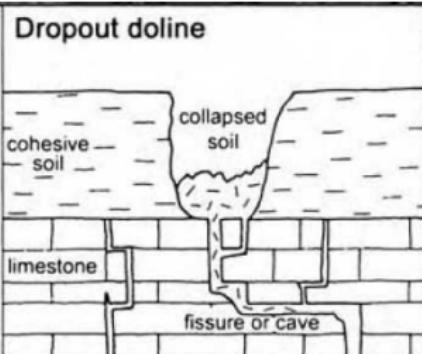
Več predlaganih modelov nastanka



The diagram illustrates a cross-section of a buried doline. At the top, a layer of 'soil' is shown above a horizontal line. An arrow points from the text 'possible compaction depression' to a point on this line. Below the soil layer, the bedrock consists of horizontal limestone layers. A large, irregularly shaped depression has developed in the bedrock, labeled 'caves and fissures'. The base of this depression is at a lower level than the surrounding limestone.



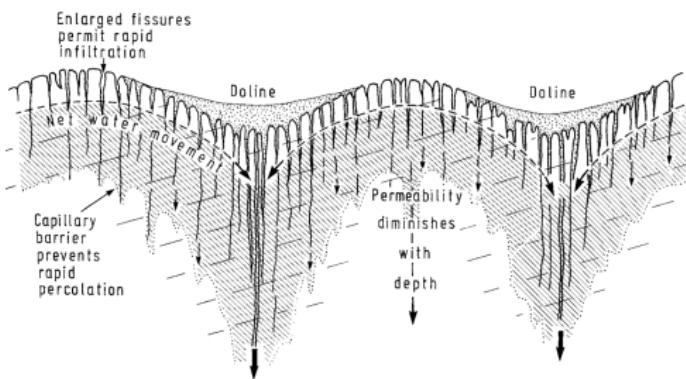
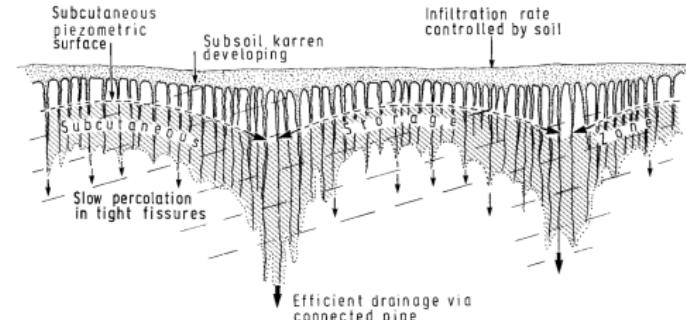
The diagram illustrates the formation of a caprock doline. It shows a cross-section of rock layers. At the top, a layer labeled 'caprock' rests on a bedrock layer. A large, irregularly shaped depression has developed in the caprock, labeled 'caprock doline'. Within this depression, numerous small, rectangular blocks representing talus or collapse material are shown falling or scattered. Two specific locations within the depression are labeled 'cave'. The bedrock layer below the caprock is also labeled 'cave' at its base. The overall process depicted is the gradual collapse of the caprock into the underlying limestone bedrock, creating a depression.



The diagram illustrates the process of suffusion doline formation. It shows a cross-section of a limestone bed (represented by horizontal grey blocks) situated above a layer of non-cohesive soil (represented by a textured grey area). A vertical fissure or cave has developed in the limestone. An arrow labeled "soil washing into fissure" points from the soil layer down into the fissure, indicating how water percolating through the soil erodes the limestone, gradually enlarging the fissure into a doline.

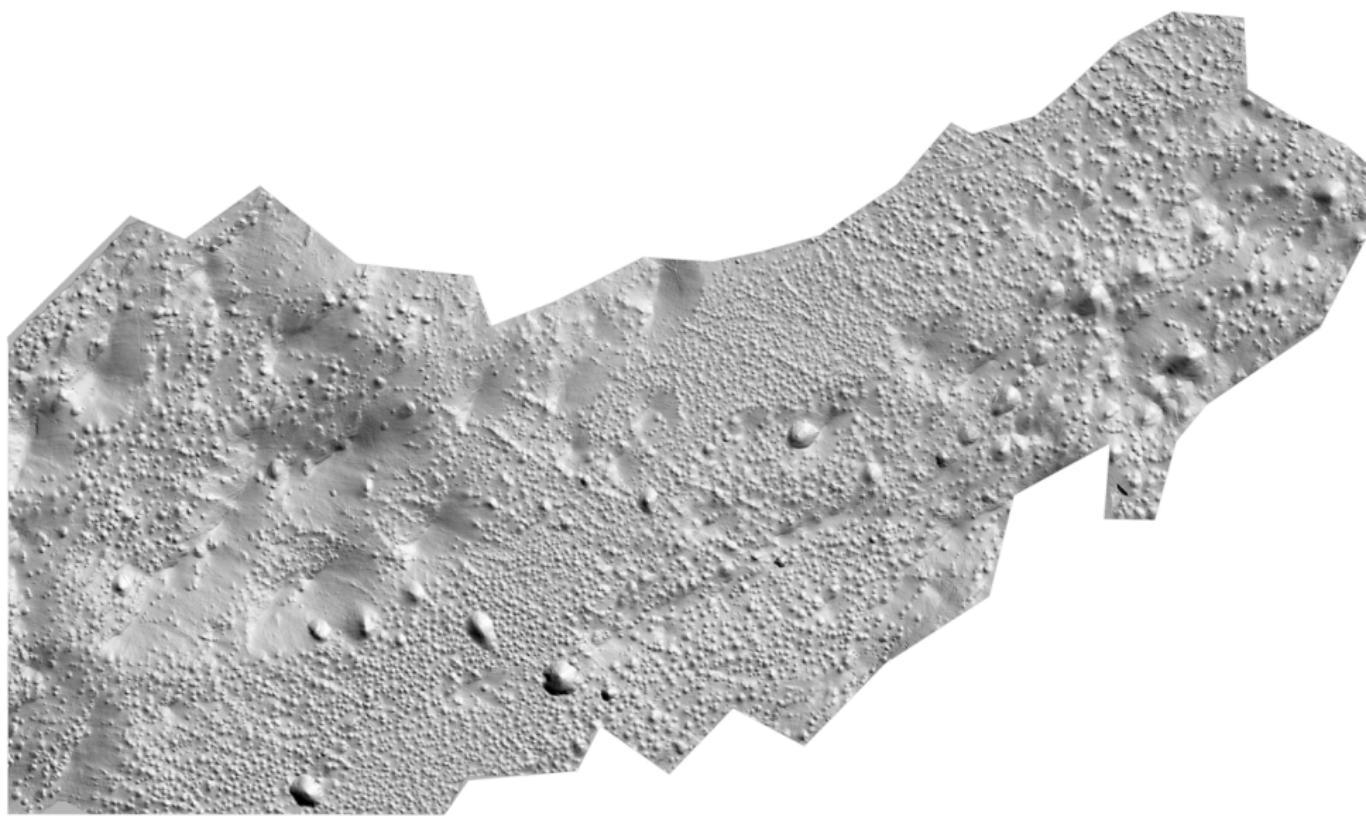
Kraške vrtače

Ni podrobnejših študij procesov, ki jih oblikujejo



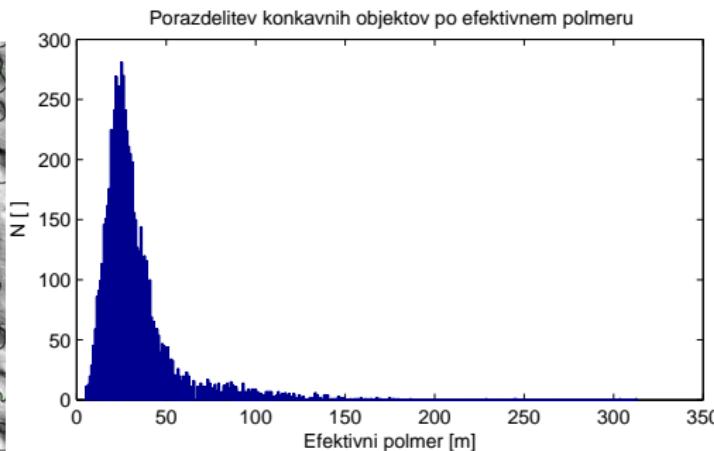
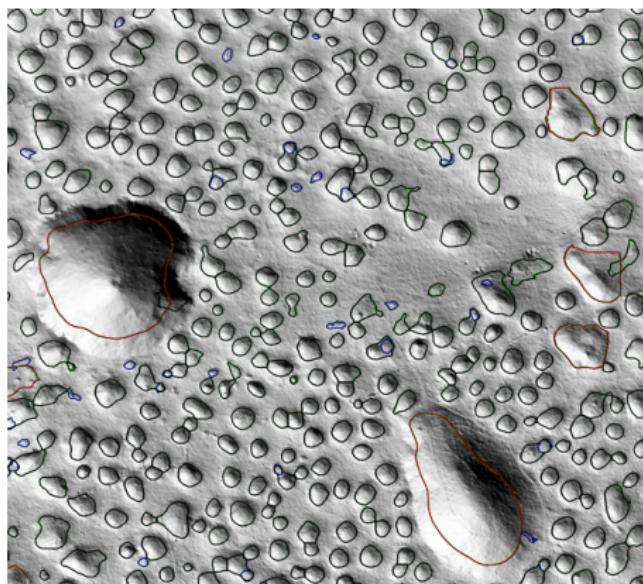
LiDAR

Posnetek območja Menišije, ločljivost $1m^2$



Računalniški vid

Identificiramo konkavnosti v reliefu in jih označimo

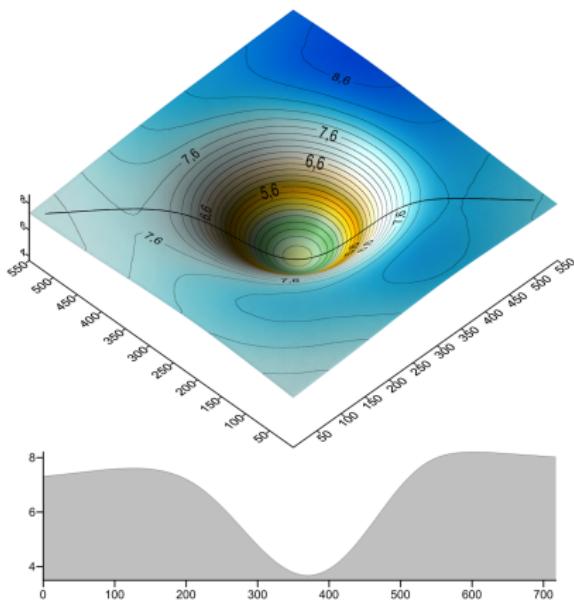


$$A = \sum \text{pixli}$$

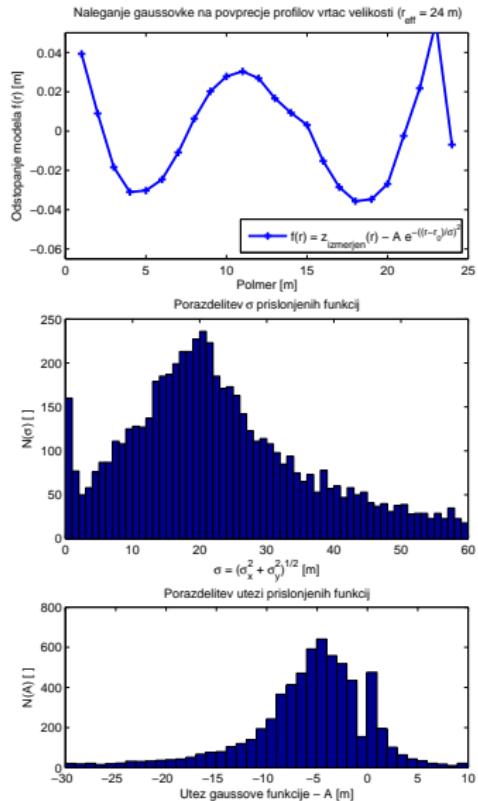
$$r_{\text{eff}} = \sqrt{\frac{A}{\pi}}$$

Povprečimo konkavnosti

Dobimo 'povprečno vrtačo', prilegamo gaussovko na vse najdene konkavnosti

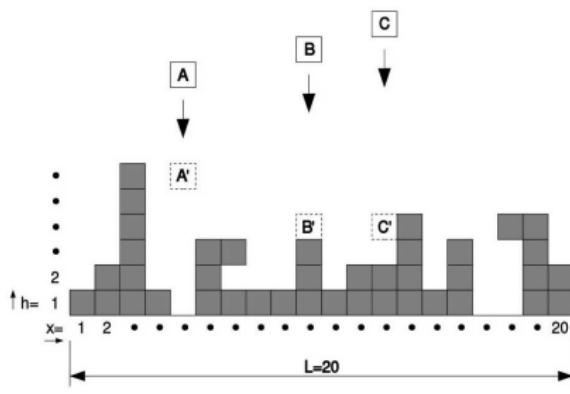


$$f(r) = A \cdot e^{-\frac{(r-r_0)^2}{\sigma^2}} + C \quad (1)$$



Rast vmesnikov

Širina vmesnika, priraščanje



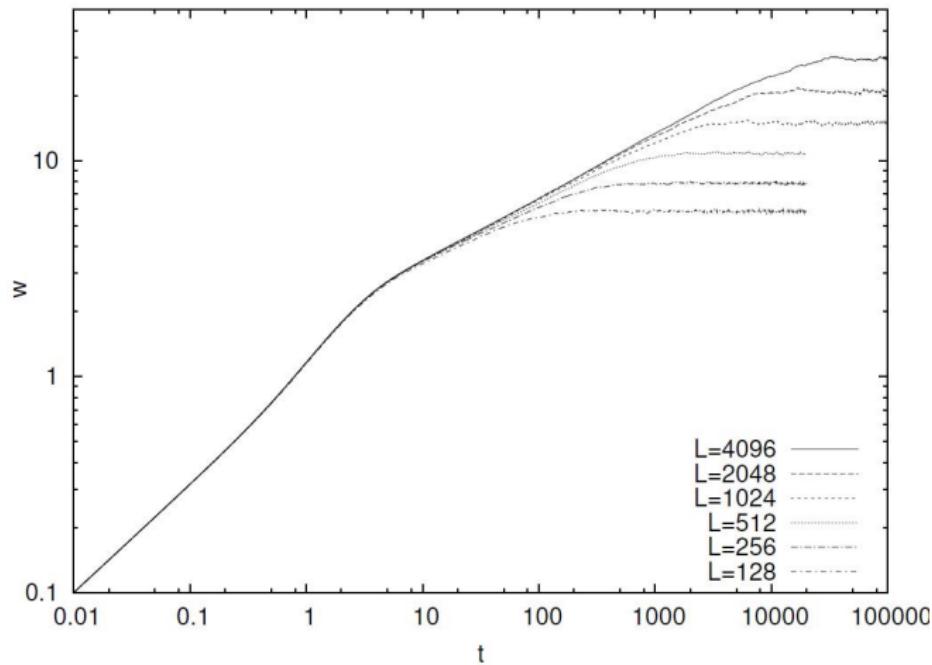
$$\bar{h} = \frac{1}{L} \sum_{i=1}^L h(i, t) \quad (2)$$

$$w(L, t) = \sqrt{\frac{1}{L} \sum_{i=1}^L (h(i, t) - \bar{h}(t))^2} \quad (3)$$

$$\bar{h}(t) \sim t \quad (4)$$

Rast vmesnikov

Povprečna širina vmesnika v času, eksponent hrapavosti

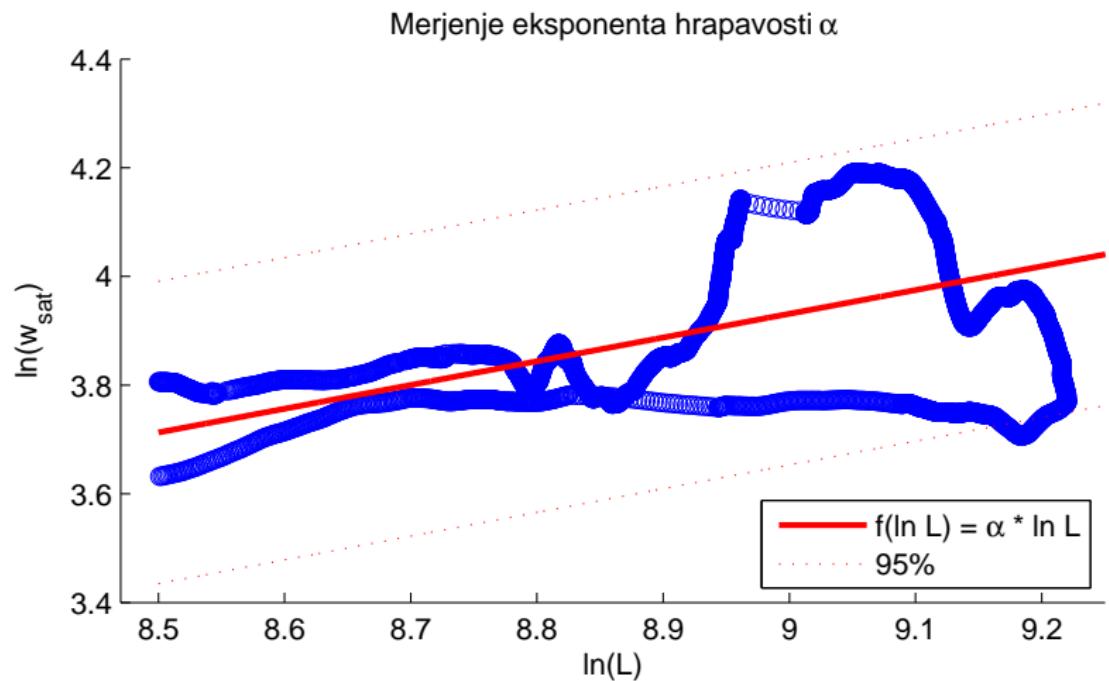


$$w(L, t) = \sqrt{\left\langle \frac{1}{L} \sum_{i=1}^L (h(i, t) - \bar{h}(t))^2 \right\rangle} \quad (5)$$

$$w_{sat}(L) \sim L^\alpha \quad (6)$$

Rast vmesnikov

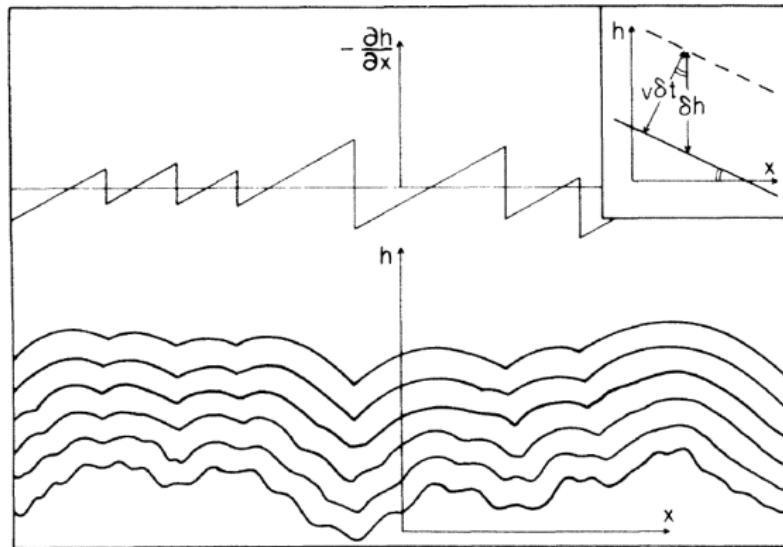
Zasičeni režim, hrapavost Menišije



$$\alpha = \frac{\partial(\ln(w_{sat}))}{\partial(\ln L)} = 0.4368 \pm 0.0005 \quad (7)$$

Kardar-Parisi-Zhang

Stohastična difuzijska enačba, teoretično napovedana hrapavost



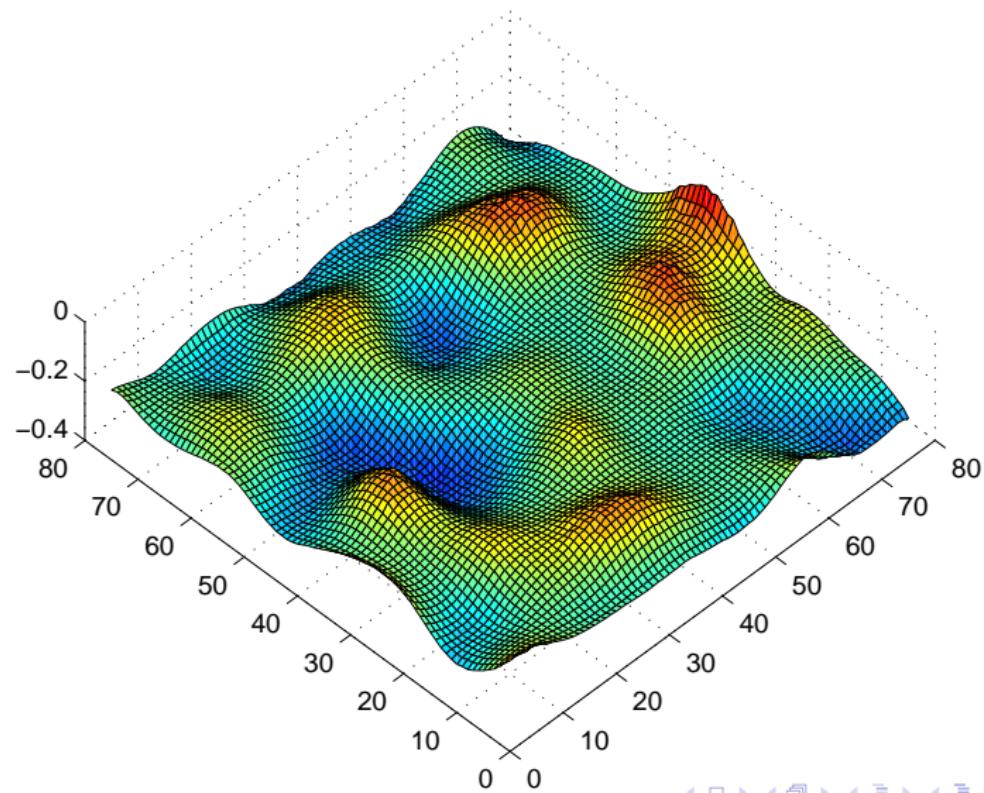
$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta(\mathbf{x}, t) \quad (8)$$

$$\langle \eta(\mathbf{x}, t) \eta(\mathbf{x}', t') \rangle = \delta(\mathbf{x} - \mathbf{x}') (t - t') \quad (9)$$

Napove: $\alpha = \frac{1}{2}$

Kardar-Parisi-Zhang

Simulacija



Dinamične enačbe

Primer: Logistična rast

Model

Rast v času je odvisna od višine in razdalje do nosilne kapacitete

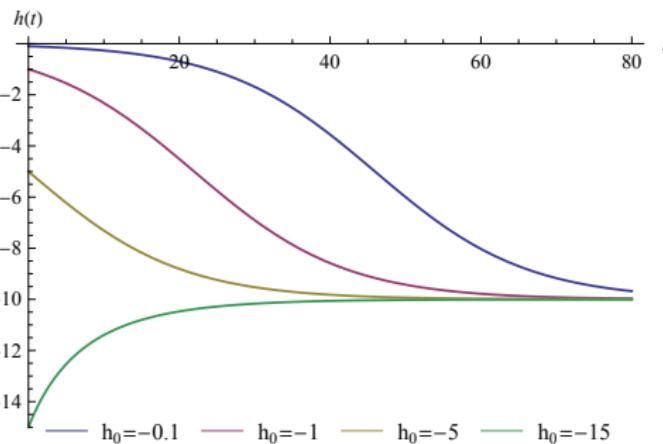
$$\frac{\partial h(t)}{\partial t} = a \cdot \left(1 - \frac{h(t)}{K}\right) h(t) \quad (10)$$

Rešitev

$$h(t) = \frac{h_0 K e^{at}}{K + h_0(e^{at} - 1)} \quad (11)$$

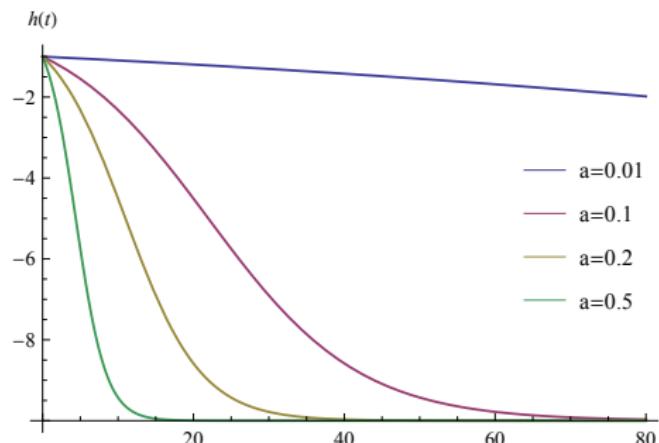
Dinamične enačbe

Primer: Logistična rast



$$K = 10$$

$$a = 0.1$$



$$K = 10$$

$$h_0 = 0.1$$

Difuzijsko dinamične enačbe

Primer: Difuzijska logistična rast

Model

Fisher-Kolmogorov-a enačba, rešitev iščemo numerično

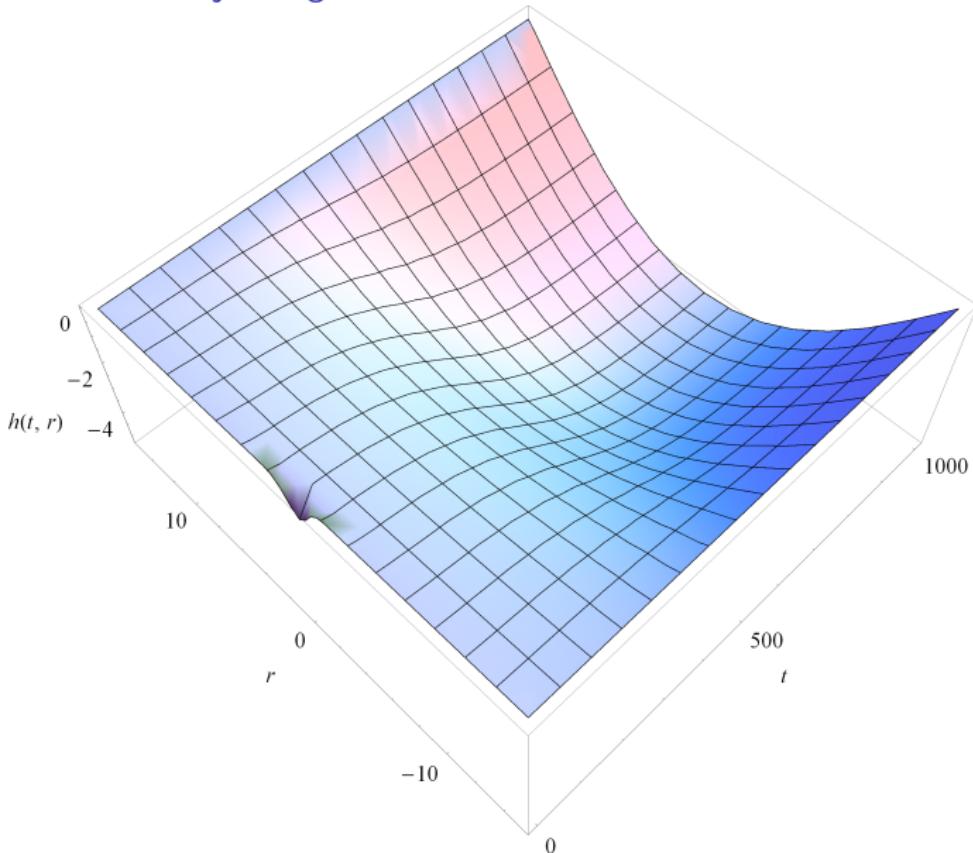
$$\frac{\partial h(t, x)}{\partial t} = D \Delta h(t, x) + a \cdot h(t, x) \cdot \left(1 - \frac{h(t, x)}{K}\right) \quad (12)$$

Robni pogoji

$$\begin{aligned} h(0, x) &= -e^{-x^2}, x \in D \\ h(t, x) &= 0, x \in \partial D \\ \frac{\partial h(t, x)}{\partial n} &= 0, x \in \partial D \end{aligned} \quad (13)$$

Difuzijsko dinamične enačbe

Primer: Difuzijska logistična rast



$$D = 1$$

$$a = \frac{1}{50}$$

$$K = -10$$

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- ▶ Segmentacija in analiza vrtač na digitalnem reliefu je relativno enostavna naloga

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 - ▶ Najmanj napačen se zdi model logistične difuzijske rasti

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- ▶ Segmentacija in analiza vrtač na digitalnem reliefu je relativno enostavna naloga
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- ▶ Izbera fizikalnega modela je zaradi pomanjkanja informacij o dinamiki težka
 - ▶ Najmanj napačen se zdi model logistične difuzijske rasti
- ▶ Natančnejši geološki študij dinamike reliefa bi bil v pomoč