Dolines of Dinaric Karst
Case Study of Menišija, Slovenia

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Abstract

Dolines are a frequent karst feature. Their shape, genesis and dynamics are conceptually described by various models. However, to the author's knowledge there was no data about exact shape and size of a larger set of karst dolines that could be used for statistical analysis. We developed and used a numerical method to analyze $60km^2$ of $1m$ grid resolution lidar data of digital relief model of Menišija, a levelled karst surface, former polje near Postojna, Slovenia. We identified 8.700 dolines (about 145 dolines$/m^2$). We then used numerical tools to calculate the average shape of the identified dolines and proposed a function to describe this shape.

Due to the geological history of Menišija and similarity of dolines in the area we propose that they were shaped by the same geomorphological processes, that ultimately lead to a common geomorphologically stable form of doline which is already reached in this area.

Using this hypothesis we propose two possible dynamical models for dolines that would lead to the shape of relief that we observe in Menišija today.

About dinaric karst dolines

Dolines of Dinaric karst are closed, bowl shaped depression with diameter ranging from about 10 to 100 meters. While each doline has its specific history, we can notice similarities in size and shape of dolines found in the same region. This leads us to believe, that no matter the history, they are ultimateley shaped by the same process. This conclusion gives motivation for the presented study.

It is believed that predominant factor of genesis and growth of dolines is chemical dissolution of limestone. The dissolved limestone is washed into the aquifer trough fractures in the bedrock, while the surface is lowered and fractures widened. Lowering causes further concetration of surface water flow that further dissolves limestone. A feedback loop is established that drives the growth of the doline. However the growth is not unbound, as all dolines eventualy reach a stable, adult, form that we most commonly observe in nature. We assume this is caused by creation of efficient drainage system in the doline, that minimizes the dissolution of limestone, locking the dissolution of doline surface in step with adjecent flat surface.

To study a statistically relavant set of dolines we used a high resolution lidar digital relied model provided by (to-do: ime instituta + CITAT).

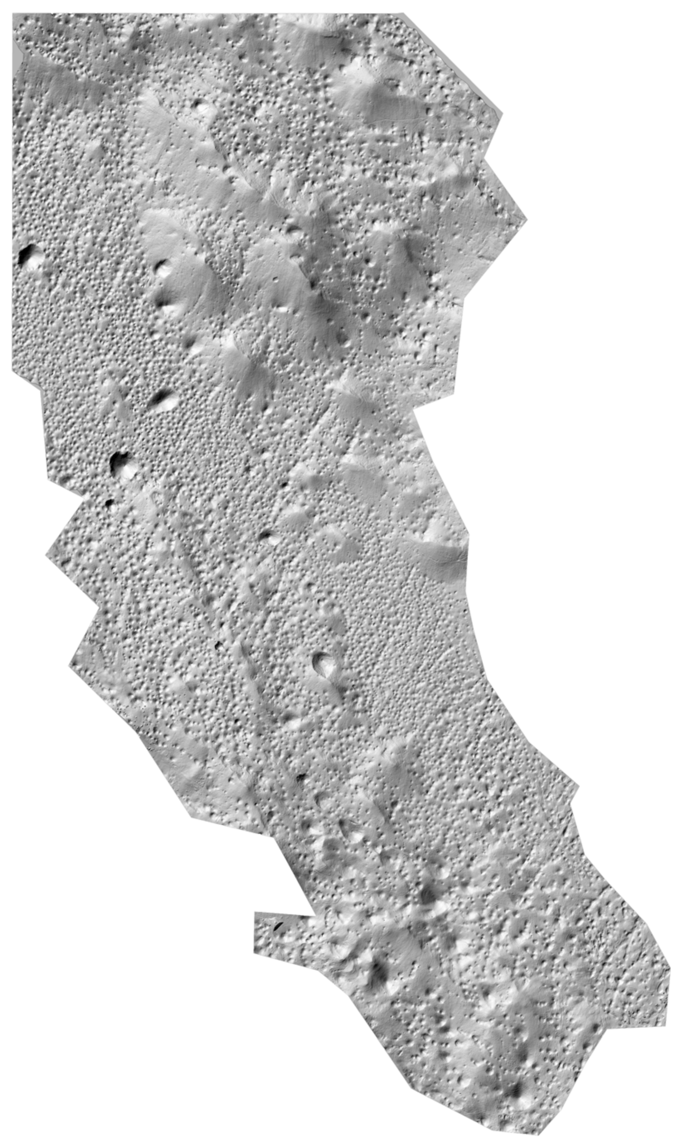

Figure 1: Figure caption

Computer vision method

To isolate dolines from the digital relief model we calculated concavity of each point in the relief by assigning the point a value calculated by subtracting the points height from the average height of points on a concentric ring around it. By varying the rings inner and outer radious we highlighted concave shapes of various sizes. We then segmented the area into concave zones and chose the most fitting candidates for dolines.

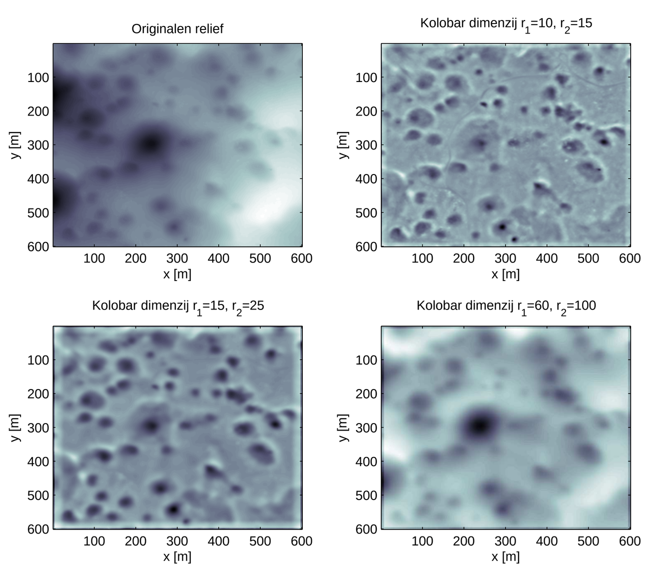

Figure 2: dsadas

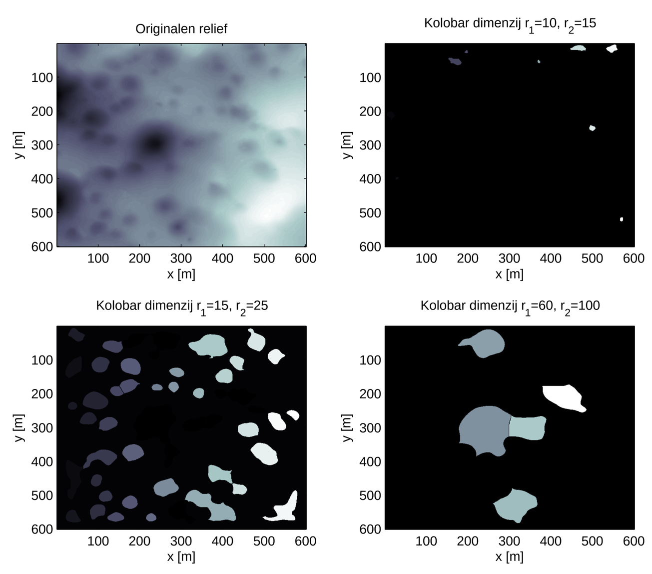

Figure 3: saldk

Results

We then calculated the effective radiouses of found dolines, by counting the pixels of each doline and calculating the radious of a circle with equal surface (see Equation 1).

$$\sum pixels = surface = \pi \cdot r^2 \tag{1}$$

Overlay of the identified dolines and histogram of effective radiouses can be seen on Figure 4.

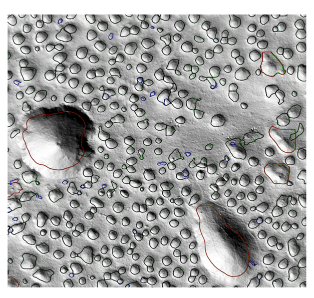

Figure 4: Part of 8600 dolines found in the Menišija and two collapse dolines.

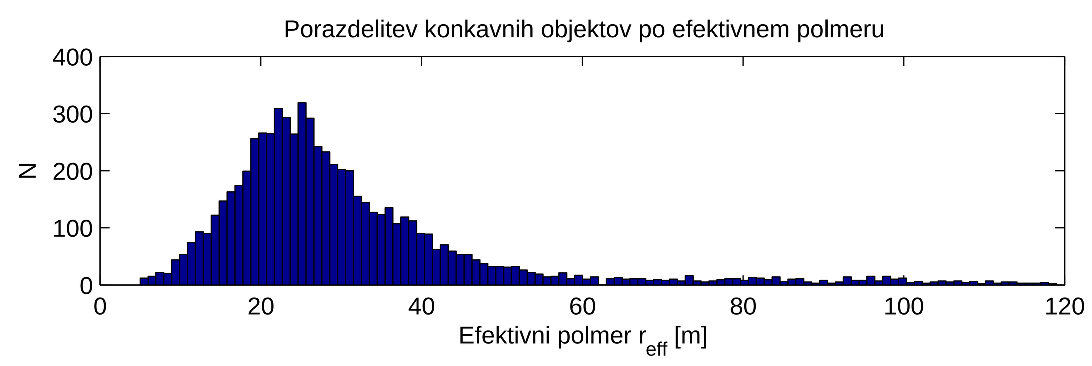

Figure 5: Final result of numeric simulation of Kardar-Parisi-Zhang dynamics, starting from a flat surface after 10^5 steps.

The diameter of identified dolines appears to be normally distributed with a maximum at $22m$.

Average doline

We then proceded by averaging all the identified dolines, to remove historical and directional bias. The result can be seen in Figure 6.

To approximate the shape of the average doline we proposed an upside down Gaussian function (2).

$$f(r) = -A \cdot e^{\frac{r^2}{\sigma^2}} + C \tag{2}$$

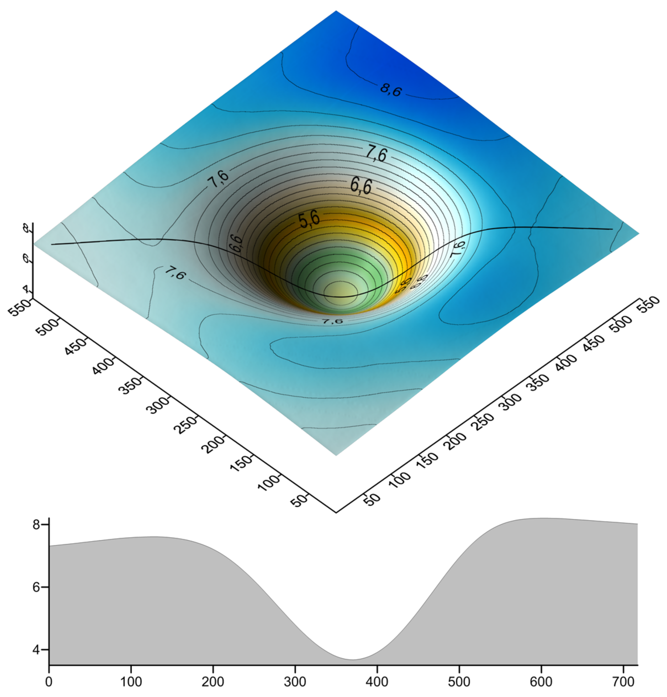

Figure 6: Figure caption

Results and comments

To get a better idea about how similar the dolines of Menišija are to one another we now fit the proposed function 2 to identified dolines and plot the optimal fitting parameters on figure 7. The fitting parameters are the depth A and width σ. From the area of concavity we already calculated effective radious r_{eff} according to equation 1.

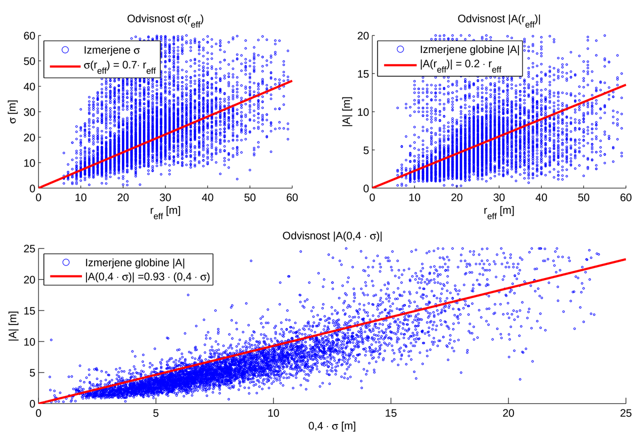

Figure 7: Figure caption

The results suggest:

- There is a linear correlation betweenm σ and r_{eff}, indicating proposed function 2 is good enough for describing the area of dolines.
- There is linear correlation between the depth A and $reff$ as well as A and σ, meaning the wider the doline is, the deeper it will be.

Proposed dynamic model

From geological background of Menišija we know it was a levelled karst polje up until 3 million years ago. Around that time a tectonic shift diverted the water that was levelling Menišija to the nearby Cerkniško polje. The dolines appeared in the time period from then to present. Judging by our findings about depth and size distributions, the dolines appear to have reached a stable shape and their depths and sizes stopped to develop. This fact motivated us to study partial differential equations that take initial conditions of a flat surface and ultimately produce doline like shapes, that are stable and do not grow or diminish in time. We propose two models for this dynamic.

Logistic growth

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = D\frac{\partial^2}{\partial \mathbf{x}^2}h(\mathbf{x},t) + a \cdot h(\mathbf{x},t) \cdot (1 - \frac{h(\mathbf{x},t)}{K}) \tag{3}$$

Logistic growth equation produces a dynamics, where doline grows slowly at first, but as it's size increases so does it's growth rate. But, as the doline approaches the carrying capacity (see the second term in (3)), the growth diminishes and stops. Carrying capacity is defined by the variables of the environment. Given border conditions:

$$\begin{aligned} h(\mathbf{x},0) &= -e^{-\mathbf{x}^2}, \mathbf{x} \in D, \\ h(\mathbf{x},t) &= 0, \mathbf{x} \in \partial D, \\ \mathbf{n} \cdot \nabla h(\mathbf{x},t) &= 0, \mathbf{x} \in \partial D, \end{aligned} \tag{4}$$

we get dynamics as seen on Figure 8.

Kardar-Parisi-Zhang

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = D\nabla^2 h(\mathbf{x},t) + F(h(\mathbf{x},t)) \tag{5}$$

Kardar-Parisi-Zhang equation gives us a non-deterministic dynamic model for growth of dolines. Starting from a flat surface this model will produce a Figure 9 like surface. The size and depth of dolines in this model depend on the diffusion constant D and stochastic term $F(h(\mathbf{x},t))$ as seen in Equation 5.

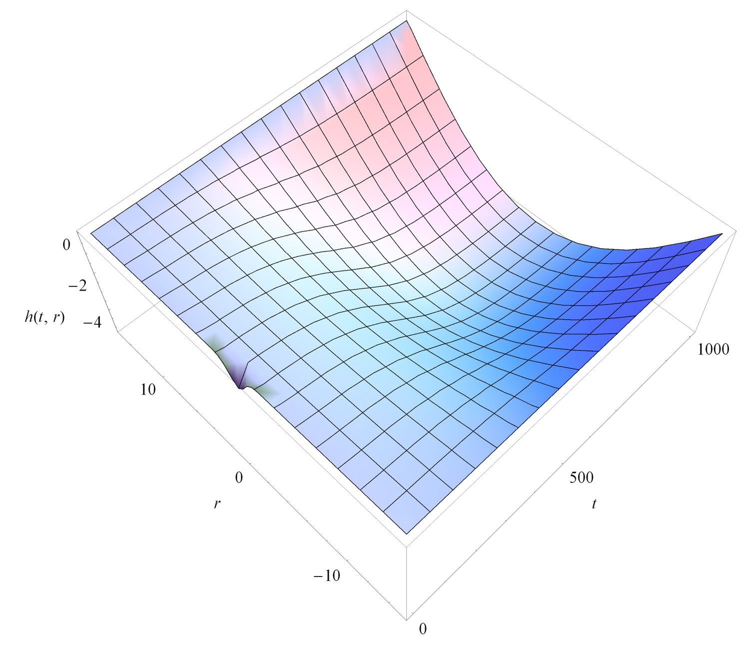

Figure 8: Time evolution of logistic diffusion growth given by Equation 3. We set $D = 1$, $a = 50$, $K = -10$.

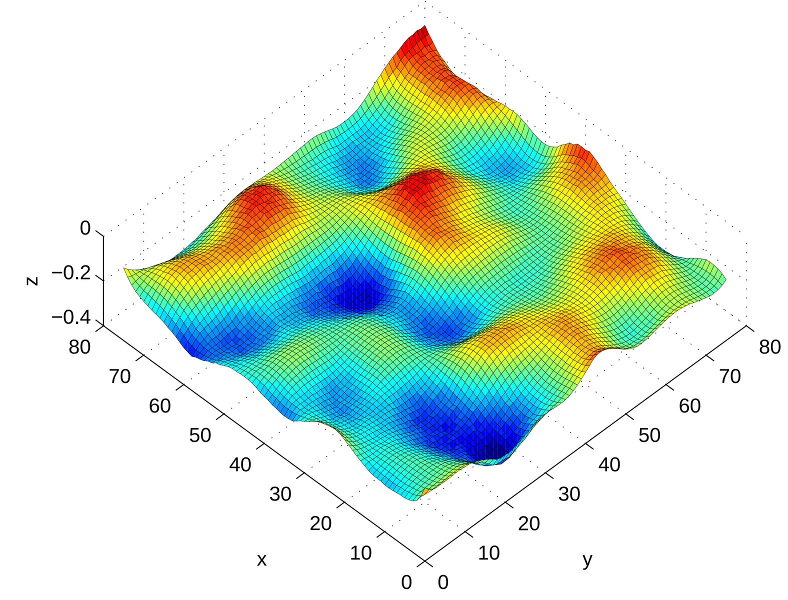

Figure 9: Final result of numeric simulation of Kardar-Parisi-Zhang dynamics, starting from a flat surface after 10^5 steps.

References

[1] A. B. Jones and J. M. Smith. Article Title. *Journal title*, 13(52):123–456, March 2013.

[2] J. M. Smith and A. B. Jones. *Book Title*. Publisher, 7th edition, 2012.