

Measurements of the Hall effect

Magnus F. Ivarsen (1) and Róbert K. Lárusson (2)

University of Iceland. 1) mfi2@hi.is 2) rkl@hi.is

April 30, 2014

Abstract:

The Hall effect

The basic principle in the Hall effect is the Lorentz force \mathbf{F} on a charged particle (with charge q) moving with velocity \mathbf{v} in a magnetic field \mathbf{B} ,

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}, \quad (1)$$

in a semiconductor material. As an introduction to the effect, consider a P-type semiconductor slab depicted in fig REF, with a current of holes going through it in the y -direction. Then, the velocity of the charged particles (holes) is

$$v = \frac{j}{qp}, \quad (2)$$

where j is the current density and p is the concentration of holes in the semiconductor. Furthermore, at time $t = 0$ a magnetic field in z -direction is turned on. Then, according to the Lorentz force law, a force in the x -direction amounting to

$$F = -qvB = jB/p$$

will change the trajectory of the holes to a parabolic course into the side of the semiconductor bar. The accumulation of holes causes an electric field in the x -direction to appear, which will oppose the magnetic field:

$$E = jB/qp.$$

In the fullness of time, the current will continue undirected in the y -direction, with a measurable voltage V_H between the sides perpendicular to the current flow. The voltage can be deduced from the electric field, and amounts to

$$\frac{IB}{qp d}, \quad (3)$$

where d is the thickness of the semiconductor bar. Hall himself showed empirically that this can be written as

$$V_H = R_H \frac{IB}{d}, \quad (4)$$

where R_H is called the Hall coefficient, which according to the theoretical value for V_H [eq. (3)] has the value

$$R_H = \frac{1}{qp}. \quad (5)$$

A plot of V_H for a cryostat-cooled graphene sample as a function of temperature is exhibited in figure ?? . To

generalize, repetition of the argument with a N-doped semiconductor with electron concentration n leads to a Hall coefficient having the value

$$R_H = -\frac{1}{qn}. \quad (6)$$

It is then prudent to define the Hall mobility as

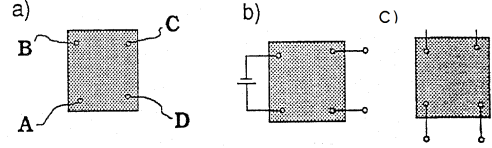


Figure 1: title

$$\sigma_{n,p} = \frac{\mu_{n,p}}{R_H}, \quad (7)$$

for the different doping respectively, given that $\sigma_n = q\mu_n n$ and vice versa for p . In figure ?? a plot of $\ln \mu_p = \ln[\sigma_p/pe]$ versus $\ln T$ exhibiting a dependency

$$\mu_p \propto T^\alpha \Leftrightarrow \ln \mu_p \propto \alpha \ln T,$$

for the linear parts of the graph in figure ??, where initially $\alpha = 213$ and then $\alpha = -8.15$.

Ionization energy

Doped semiconductors, the concentrations of occupied donors, n_D and p_A with doping concentrations N_D and N_A are

$$n_D = \frac{N_D}{1 + \frac{1}{g_D} \exp\left(\frac{E_D - E_F}{kT}\right)}, \quad (8)$$

and

$$p_A = \frac{N_D}{1 + \frac{1}{g_A} \exp\left(\frac{E_F - E_A}{kT}\right)}, \quad (9)$$

where E_F is the Fermi energy level, kT the thermal energy, $g_{A,D}$ doping degeneracy levels, while E_D and E_A are the donor and acceptor doping energy level respectively. Generally, the concentration of holes and electrons in the valence and conduction bands can be expressed as

$$n = 2 \int_{E_C}^{\infty} \frac{N_C(E)}{1 + \exp\left(\frac{E_F - E}{kT}\right)} dE, \quad (10)$$

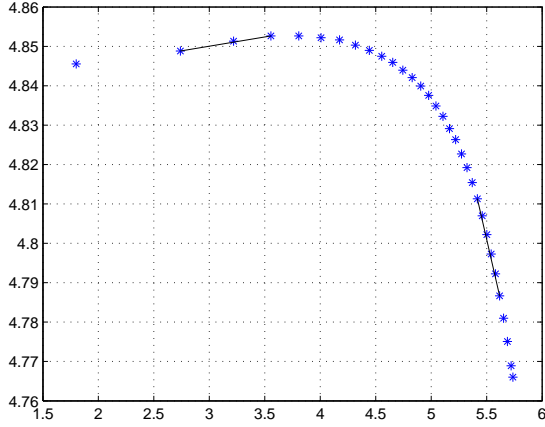


Figure 2: A plot of the natural logarithm of T as a function of the natural logarithm of carrier mobility μ , with two linear portions marked with straight lines from which α was calculated.

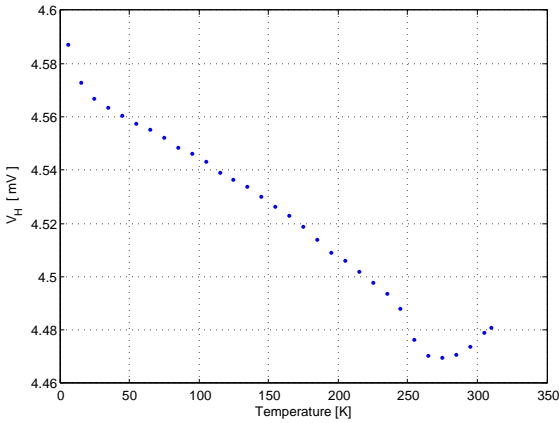


Figure 3: A plot of V_H against T in a cryostat-cooled graphene sample.

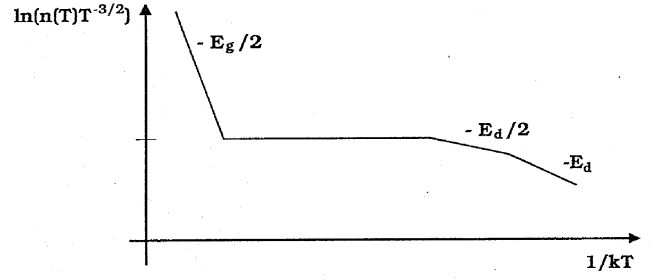


Figure 4: Approximation of $\ln[n(T)T^{-3/2}]$ in a N-doped semiconductor as a function of $(kT)^{-1}$, the graph is approximated by straight lines with slopes $-E_g/2$, $-E_d/2$ and $-E_d$. E_g is the energy gap of the semiconductor and E_d is explained in the text.

and

$$p = 2 \int_{-\infty}^{E_V} \frac{N_V(E)}{1 + \exp\left(\frac{E - E_F}{kT}\right)} dE, \quad (11)$$

where $N_V(E)$ and $N_C(E)$ are valence and conduction band density of states respectively, while E_C and E_V (the conduction and valence band energy level respectively) are related to E_A and E_D through

$$E_d = E_C - E_D \quad \text{and} \\ E_a = E_A - E_V,$$

where E_a and E_d are distances in energy space. Bringing all these concentrations together is the semiconductor neutrality relation

$$n + n_D - p - p_A = N_D - N_A. \quad (12)$$

To solve for n , p , n_D and p_A numerical simulation is in order. However, some very important simplification makes things easier; as figure ?? shows, concentration exhibits different behaviour for certain intervals of reciprocal thermal energy $(kT)^{-1}$, that is, for certain temperatures. As seen in figure 5 shows, this behaviour is rather more opaque in our measurements. We conclude that, perhaps unsurprisingly, graphene does not exhibit the same behaviour as that of gold-doped silicon.

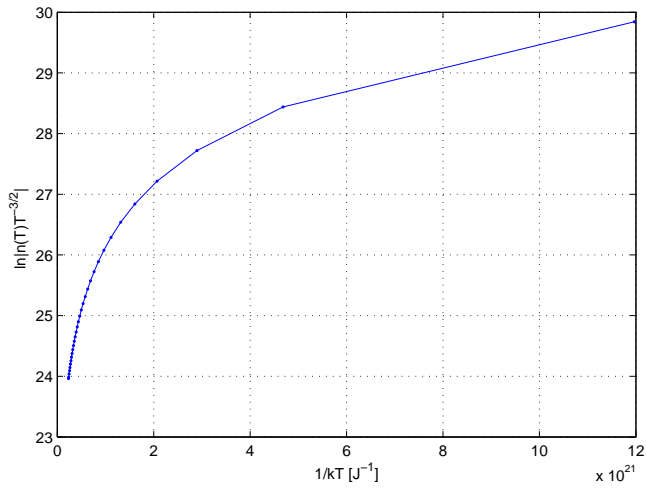


Figure 5: A plot of $\ln[p(T)T^{-3/2}]$ against $(kT)^{-1}$ for a cryostat-cooled graphene sample.