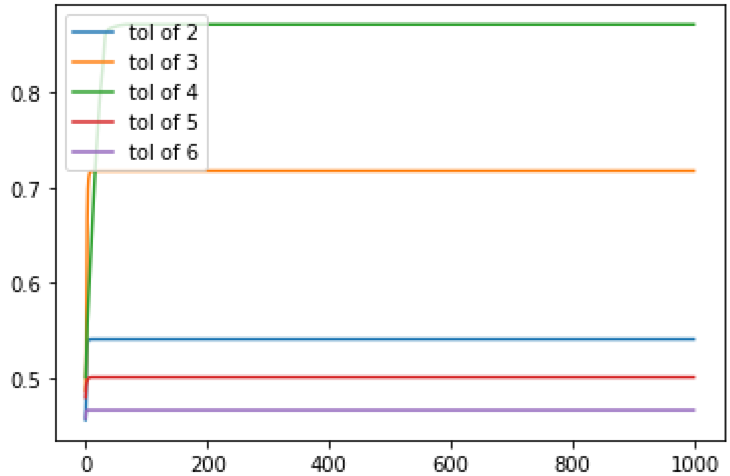
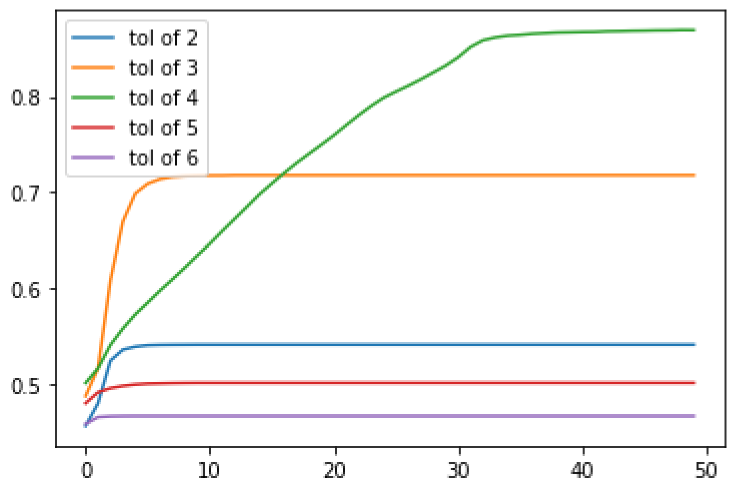
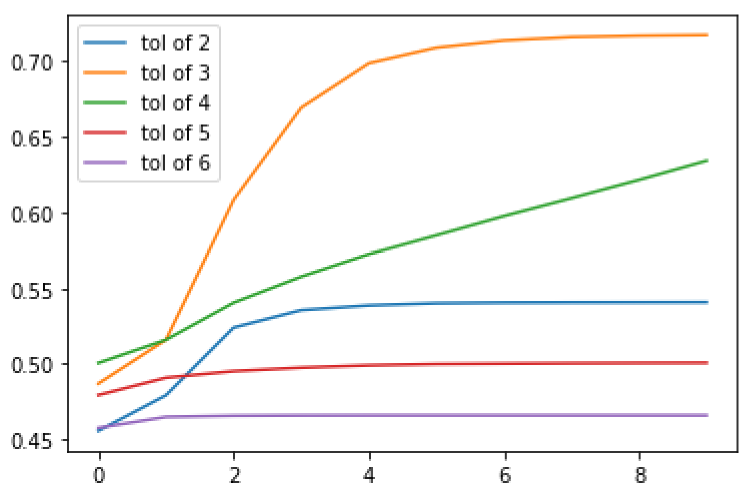
1. Programming Question (40 points)

Here are my results at varying ranges:







From tolerances of 2 to 4 (2, 3, 4) as tolerance increases, so does the proportion of similar neighbors after 1000 iterations. This is to be expected as lower tolerances do not motivate players to find better positions. Further more, as tolerance increases, the rate of convergence decreases. Higher tolerances increase the requirements for satisfaction, increasing the amount of turns needed to satisfy all players.

From tolerances of 4 to 5, however, the trend swaps. As tolerance increases, the proportion of similar neighbors decreases. This is most likely due to the scarcity of valid spaces; higher cause the game to end early since players are no longer able to move.

We can conclude that there exists a specific tolerance (depending on the number of free spaces) to optimize the proportion of similar neighbors and that this proportion may be inversely related to the rate of convergence. If the tolerance is too high the game will end prematurely due to a scarcity of valid spaces. If the tolerance is too low the players will become complacent and stop looking for better positions.

2. Qualitative Questions (60 points)

1. HITS Algorithm Computation

a. Authority = [0.61111111, 0.38888889, 0, 0, 0, 0]
   Hub = [0, 0, 0.23404255, 0.23404255, 0.38297872, 0.14893617]

b. 1 link:
   Auth = [0.57894737, 0.36842105, 0, 0, 0, 0, 0.05263158, 0]
   Hub = [0, 0, 0.22916667, 0.22916667, 0.375, 0.14583333, 0, 0.02083333]
   X Auth = 0.005263158

   3 links:
   Auth = [0.46938776, 0.36734694, 0, 0, 0, 0, 0.16326531, 0]
   [0, 0, 0.14935065, 0.14935065, 0.26623377, 0.11688312, 0, 0.31818182]
   X Auth = 0.16326531

   X has a higher authority score when y points to A and B as well. Intuitively this makes sense; since x only has one source, its authority score is completely determined by the hub score of that source (page Y). In the second scenario, Y is able to increase its hub score since it refers to more pages and thus also increases the authority score of all of the pages it points to (page X).

c. First, since B has a lower authority score and has less nodes pointing to it, B will be the node we will try to surpass. Inorder to increase X's authority score we need to first increase the number of pages pointing to it; this can easily be achieved by having both Y and Z point to it. At this point B will still have a higher score, so the next thing we can try to do is increase the hub scores of the nodes pointing to X. Since A has the highest authority score, having Y and Z point to it will increase both of their hub scores. While having Y and Z point to B will also increase their hub scores, these connections will also increase B's score which we do not want. With the final graph including connections from both X and Y to both A and Z, Z will have the second highest authority score.

Adjacency matrix:

	A	B	C	D	E	F	X	Y	Z
A	0	0	0	0	0	0	0	0	0
B	0	0	0	0	0	0	0	0	0
C	1	0	0	0	0	0	0	0	0
D	1	0	0	0	0	0	0	0	0
E	1	1	0	0	0	0	0	0	0
F	0	1	0	0	0	0	0	0	0
X	0	0	0	0	0	0	0	0	0
Y	1	0	0	0	0	0	1	0	0
Z	1	0	0	0	0	0	1	0	0

Authority scores:
A = 0.57407407
X = 0.25925926
B = 0.16666667
Rest = 0

2. Pagerank

a. The equilibrium alternates between two sets. Since all of the corners/sides will experience the same change, we can caculate one and apply it to all of the similar nodes. Given this, the following equation can be used to caculate the value of each node:

Corner = Side*⅔
Middle = Side*4/3
Side = Corner + Middle/4

Following these rules, the iterations look like:

Corner	Middle	Side
1	1	1

2/3	4/3	5/4
5/6	5/3	1
2/3	4/3	5/4

Since we've repeated a set, any further iterations will result in a repeat of what weve seen

b. With the scaled model, we can adjust our equations to be:

Let s = scaling factor
And n = number of nodes
Corner = s * (Side*⅔) + (1-s)*1/n
Middle = s*(Side*4/3) + (1-s)*1/n
Side = s*(Corner + Middle/4) + (1-s)*1/n

Since we know n equals 9, we can rewrite the last part as (1-s)/9 and rewrite our previous iterations as:

Corner	Middle	Side
1	1	1
s*(⅔) + (1-s)/9	s*(4/3) + (1-s)/9	s*(5/4) + (1-s)/9
s*(⅚) + (1-s)/9	s*(5/3) + (1-s)/9	s + (1-s)/9
s*(⅔) + (1-s)/9	s*(4/3) + (1-s)/9	s*(5/4) + (1-s)/9

3. Game theory

a.

	B:0	B:1	B:2	B:3
A:0	(1.5,1.5)	(0,2)	(0,1)	(0,0)
A:1	(2,0)	(1,1)	(0,1)	(0,0)
A:2	(1,0)	(1,0)	(.5,.5)	(0,0)
A:3	(0,0)	(0,0)	(0,0)	(0,0)

b. There are no strictly dominated strategies since all strategies will result in 0 when any player bids \$3. Bidding \$0 or \$3, however, is weakly dominated by bidding \$1 since bidding \$1 will always result in an equal or greater payoff.

c. The pure-strategy Nash equilibrium is for both players to bid \$2. In this scenario, deviating would only result in a decreased payoff. Any strategies not on the diagonal will not be equilibria since there is an incentive for the lower player to match the other player's bid in order to gain a chance at winning. On all the other diagonals, there is also an alternative strategy that will either match or increase one's utility. Furthermore \$2 is the max bid both players should be willing to bid since bidding \$3 will result in a payoff of 0 for both parties.

4. Game theory

a.

	Firm2: A	Firm2: B	Firm2: None
Firm1: A	(-10, -10)	(10, 10)	(15, 0)
Firm1: B	(10, 10)	(5, 5)	(30, 0)
Firm1: None	(0, 15)	(0, 30)	(0,0)

b. This is correct. Since entering with b will always produce a positive gain no matter what firm 2 picks and not entering will produce nothing, it is at least better to always enter with b than to not enter at all.

c. The pure strategy Nash equilibria are for Firm 1 to produce A while Firm 2 produces B and vice versa. Not entering is a strictly dominated strategy so it can be removed as an option. From here if both firms produce the same product, there is an incentive to diversify in order to increase utility.

5. Game theory

a. ABC:
There are no pure strictly dominated strategies but b is strictly dominated by the mixed strategy a/c
There are also no pure weak

XYZ:
Y is weakly dominated by Z and is strictly dominated by X

b. 1.

	X	Y	Z
A	3,2	9,1	1,1
B	2,6	10,5	0,5
C	1,1	11,0	2,3

2.

	X	Z
A	3,2	1,1
B	2,6	0,5
C	1,1	2,3

3.

	X	Z
A	3,2	1,1
C	1,1	2,3

c. A/X and C/Z are equalibria. I started by eliminating strictly dominated strategies since there are no scenarios in which a player will be incentivized to choose them. After removing the strictly dominated strategies, I was left with a 2x2 matrix. From here, I marked each player's optimal strategy in every scenario and found two in which both players' optimal strategies matched. Observing the graph, given A/X or C/Z, neither player has an incentive to deviate as it will reduce their score signifying equilibrium.