Computer Lab in Nonlinear Dynamics

May 13, 2015

1 Introduction

1.1 Programming issues

- Simplest C++ programs, `for` loop, output operator `cout`.
- Compilation and execution of the program.
- Writing results into a data file with
  `./myprogram.x >> my_results.dat`
- Plotting the results.

1.2 Examples

Logistic map

$$x_{n+1} = r x_n (1 - x_n) \ ,$$

- Writing `for` loop that iterates over n, and each step calculates new x and output it to the file then plotting $x(n)$.
- For the bifurcation diagram, two `for` loops is needed with taking into account transient transition. r and x have to be putted out. Then plot $x(r)$, note that it has to be multiple x for the same r.
- Creating Cobweb plot. Two separate `for` loops is needed, that will be put in two different functions in the next section

2 Introduction: functions and arrays

2.1 Programming issues

- Functions. Argument passing: pass-by-value and pass-by-reference.
- Numerical Recipes Library: arrays.
- Writing results into a data file with

```
ofstream fout1("name.dat",ios::out);
fout1.setf(ios::scientific); fout1.precision(12);
```

2.2 Examples

Ikeda map:

$$E_{n+1} = A + B E_n \exp\left(\mathrm{i}|E_n|^2\right)$$

for $B = 0.2,\ A = 3,\ 4.5,\ 6.$

3 ODE integration

3.1 Programming issues

- Euler and Runge-Kutta methods.
- Numerical Recipes rk integrator
- `atan2(y,x)` function

3.2 Examples

- Van der Pol equation
$$\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$$
for different μ.
- Forced van der Pol equation,
$$\ddot{x} - \mu(1 - x^2)\dot{x} + x = \varepsilon \cos(\nu t) \ ,$$
frequency and phase locking.
- Example of a strange attractor: the Lorenz system.
$$\begin{aligned}\dot{x} &= 10(y - x)\\ \dot{y} &= 28x - y - xz\\ \dot{z} &= -\frac{8}{3}z + xy\end{aligned}$$
- Lorenz map $z_{n+1}^{max} = f(z_n^{max})$; detection of maxima via parabolic fit.
- The Rössler system:
$$\begin{aligned}\dot{x} &= -y - z\\ \dot{y} &= x + 0.15y\\ \dot{z} &= 0.4 + z(x - 8.5)\end{aligned}$$
First return map via Hénon trick
- The Rössler system:
$$\begin{aligned}\dot{x} &= -y - z\\ \dot{y} &= x + \mu y\\ \dot{z} &= 0.4 + z(x - 8.5)\end{aligned}$$
Bifurcation diagram for $0 \leq \mu \leq 0.2$.

3.3 Delay differential equations

- Predictor-Corrector technique
- Example: Mackey-Glass equation
$$\dot{x} = \beta \frac{x_\tau}{1 + x_\tau^n} - \gamma x$$
with $x_\tau = x(t - \tau)$. Parameters: $\gamma = 1$, $\beta = 2$, $\tau = 2$. Transition to chaos with variation of n. ($n = 7$ periodic, $n = 7.75$ period 2, $n = 8.79$ period 4, $n = 9.65$ chaos).

3.4 Ensembles

- The Kuramoto-Sakaguchi model, identical oscillators:

$$\dot{\varphi}_k = \omega + \varepsilon R \sin(\Theta - \varphi_k + \beta) \;, \quad k = 1, \ldots, N \;, \quad Re^{i\Theta} = N^{-1} \sum_1^N e^{i\varphi_k}$$

  Optimization of the code for speed (how to compute sine and cosine functions only once, how to use function `syncos`).
  Simple test: (i) take initial conditions (almost) uniformly distributed and $|\beta| < \pi/2$ and plot $R(t)$; (ii) take initial conditions (almost) identical and $|\beta| > \pi/2$ and plot $R(t)$.

- The nonlinear Kuramoto-Sakaguchi model:

$$\dot{\varphi}_k = \omega + \varepsilon R \sin(\Theta - \varphi_k + \beta(\varepsilon, R)) \;, \quad k = 1, \ldots, N \;, \quad Re^{i\Theta} = N^{-1} \sum_1^N e^{i\varphi_k}$$

  with $\beta = \beta_0 + \varepsilon^2 R^2$ and e.g. $\beta_0 = \pi/4$. Plot plot time average of the order parameter vs coupling strength ε. Next, plot also time averaged frequencies of an oscillator and of the mean field vs coupling strength ε. Animation for some $\varepsilon > \varepsilon_{cr}$.

- The Kuramoto-Sakaguchi model with the Lorentzian frequency distribution; plot time average of the order parameter vs coupling strength ε.

- The van Vreeswijk model.

3.5 Animation with gnuplot

- The nonlinear Kuramoto-Sakaguchi model.

- Globally coupled Hindmarsh-Rose neurons.