



Problem no.2 - Airbounce

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Official Problem Statement

When a Frisbee is thrown in a certain way it can be made to bounce in mid-air. Study the physics of this phenomenon.



Ideas and Hypotheses

- Normal component of Frisbee velocity will decrease faster because of its shape.
- Frisbee will appear to bounce in mid-air.

- Frisbee in the original video is stable. Angle to the ground is constant.
- Assumptions:
 - Frisbee keeps constant angle to the ground during the whole flight because of gyroscopic stability.
 - Frisbee travels in a straight line. (no Magnus effect...)

Axis graphs:

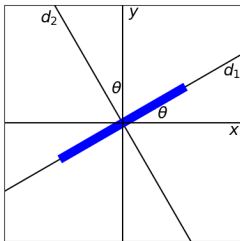


Figure 1:
Ground coordinate system: N

θ = angle to the ground

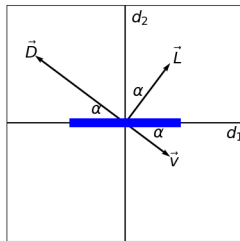


Figure 2:
Coordinate system of Frisbee: D

α = angle of attack

Lift and drag force

$$L = \frac{1}{2} A \rho C_L v^2 \quad D = \frac{1}{2} A \rho C_D v^2 \quad (1)$$

Lift and drag coefficient depending on angle of attack [1]

$$C_L = C_{L0} + C_{L\alpha} \alpha \quad C_D = C_{D0} + C_{D\alpha} \alpha^2 \quad (2)$$

Cutoff

- C_D cutoff; when $C_D = 1.1$ (drag coefficient of a disc perpendicular to velocity)
- C_L cutoff; at stall angle = 25°

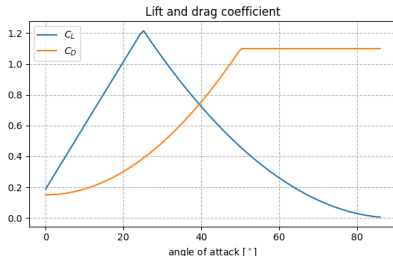


Figure 3: $C_{L0} = 0.188$, $C_{L\alpha} = 2.37$, $C_{D0} = 0.15$, $C_{D\alpha} = 1.24$
M. Hubbard, S. A. Hummel. *Simulation of Frisbee Flight*. (2000). [1]
(short flights)

Theoretical Description: Forces

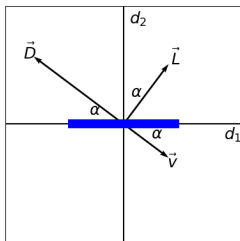


Figure 4: Coordinate system of Frisbee: D

$$K = \frac{A\rho}{2m} \quad \tan \alpha = \frac{-v_2}{v_1}$$
$$v = \sqrt{v_1^2 + v_2^2}$$

$$\mathbf{L} = mKC_L v^2 \begin{pmatrix} \sin \alpha \\ \cos \alpha \end{pmatrix}_D \quad (3)$$

$$\mathbf{L} = mKC_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D$$

$$\mathbf{D} = mKC_D v^2 \begin{pmatrix} -\cos \alpha \\ \sin \alpha \end{pmatrix}_D \quad (4)$$

$$\mathbf{D} = mKC_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D$$

$$\mathbf{F}_g = -mg \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}_D \quad (5)$$

$$m\mathbf{a} = \mathbf{L} + \mathbf{D} + \mathbf{F}_g \quad (6)$$

$$\begin{pmatrix} \dot{v}_1 \\ \dot{v}_2 \end{pmatrix}_D = KC_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D + KC_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D - g \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}_D \quad (7)$$

$$\begin{pmatrix} \dot{d}_1 \\ \dot{d}_2 \end{pmatrix}_D = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}_D \quad (8)$$

Solve for: d_1, d_2, v_1, v_2 and rotate to ground coordinate system N.

$$R = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (9)$$

$$\begin{pmatrix} x \\ y \end{pmatrix}_N = R \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}_D \quad \begin{pmatrix} v_x \\ v_y \end{pmatrix}_N = R \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}_D \quad (10)$$

- Video analysis of a throw.
- Analysed only stable throws.
- Problems:
 - Frisbee is not stable as in the original video.
 - Parallax error. Throw is not perpendicular to the camera.
 - Wind speed was not measured.
 - Rotation of Frisbee not measured, coefficients of lift and drag for rotating Frisbee.

$m[\text{kg}]$	$A[\text{m}^2]$	$\rho[\text{kg}/\text{m}^3]$	$g[\text{m}/\text{s}^2]$
0.175	0.0616	1.23	9.8

Table 1: Frisbee parameters and constants.



Figure 5: Example of a throw.

dx is measured, dx' is correct

$$k = \frac{\text{final frisbee size}}{\text{initial frisbee size}} \quad l = \text{lenght of a throw} \quad (11)$$

$$dx = \left(\frac{k-1}{l} x + 1 \right) dx' \quad (12)$$

$$x' = \int_0^x \left(\frac{k-1}{l} x + 1 \right)^{-1} dx \quad (13)$$

$$x' = \frac{l}{k-1} [\ln((k-1)x + l) - \ln l] \quad (14)$$

Results, example of a throw:

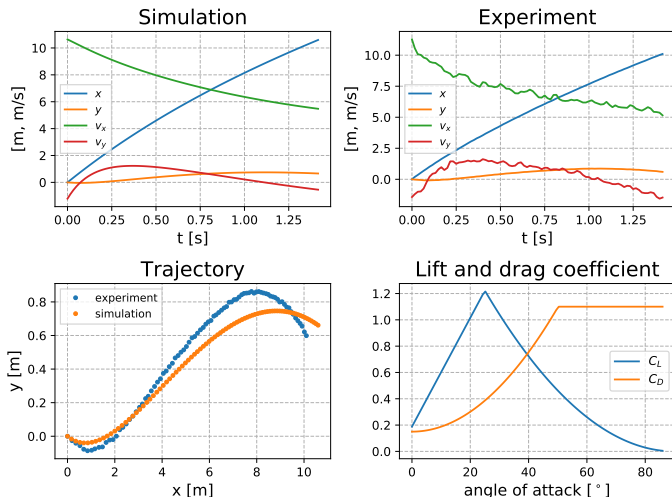


Figure 6: $\theta = 19^\circ$, $v_x(t=0) = 10.64 \frac{\text{m}}{\text{s}}$, $v_y(t=0) = -1.21 \frac{\text{m}}{\text{s}}$
 C_L, C_D : article [1]

Fitting C_L and C_D

Angle to the ground (θ) is not constant, we used effective (average) angle θ_{ef} .

Parameters in finding C_L and C_D :

$$\theta_{ef}, C_{L0}, C_{L\alpha}, C_{D0}, C_{D\alpha}$$

Minimization of s over k experiments:

$$s_k = \sum_i w_i \|(x_i, y_i)_{sim} - (x_i, y_i)_{exp}\|^2 \quad (15)$$

$$s = \sum_k s_k \quad (16)$$

$$w_i = \frac{1}{\Delta x_i^2} \quad (17)$$

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scipy.optimize.minimize(method='TNC')
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Weights

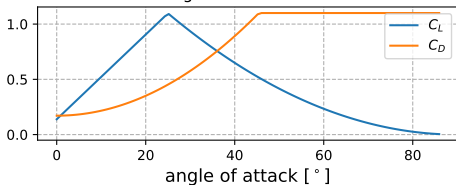
$$x_i = \int_0^{t_i} \left(\int_0^{t_i} a \, dt \right) dt \quad (18)$$

$$\Delta x_i = \frac{\partial x_i}{\partial a} \Delta a + \Delta x_0 \quad (19)$$

$$w_i = \left(\frac{\Delta a}{2} t_i^2 + \Delta x_0 \right)^{-2} \quad (20)$$

$$\Delta a \approx 0.1 \frac{\text{m}}{\text{s}^2} \quad \Delta x_0 \approx 1 \text{cm}$$

article [1]



2. Airbounce

Trajectories: Weighted Minimization and Experiment

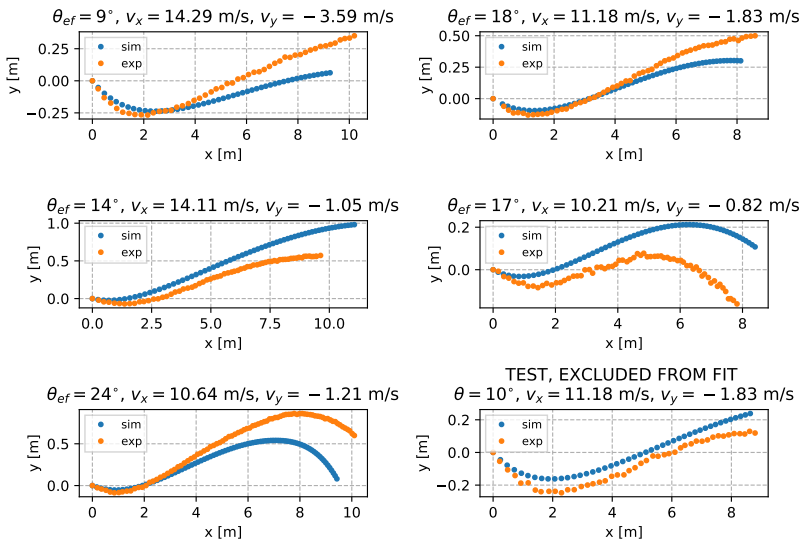


Figure 8: All experiments.

Minimization: Weighted vs Unweighted

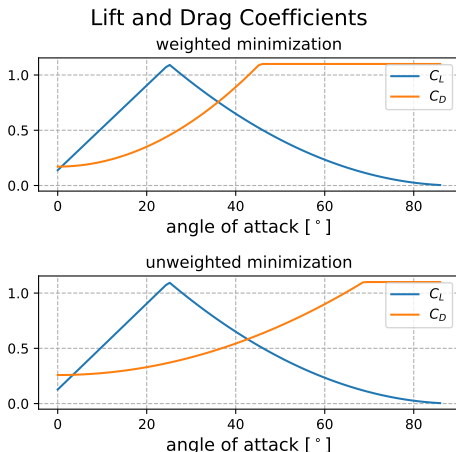


Figure 9: $C_{L0} = 0.138, C_{L\alpha} = 2.20, C_{D0} = 0.171, C_{D\alpha} = 1.47$ (weighted)
 $C_{L0} = 0.127, C_{L\alpha} = 2.23, C_{D0} = 0.258, C_{D\alpha} = 0.585$ (unweighted)

Minimization: Weighted vs Unweighted

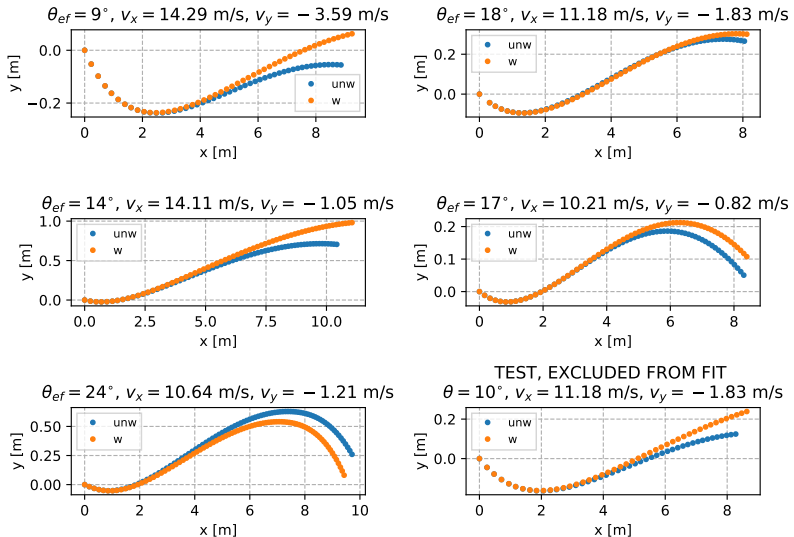


Figure 10: All experiments.

$$t = T_c \tau, \quad d_i = D_c \sigma_i, \quad v_i = \frac{D_c}{T_c} \nu_i, \quad \dot{v}_i = \frac{D_c}{T_c^2} \nu'_i \quad (21)$$

$$\begin{pmatrix} \dot{v}_1 \\ \dot{v}_2 \end{pmatrix}_D = KC_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D + KC_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D - g \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}_D \quad (22)$$

\Downarrow

$$\begin{pmatrix} \nu'_1 \\ \nu'_2 \end{pmatrix}_D = C_L \nu \begin{pmatrix} -\nu_2 \\ \nu_1 \end{pmatrix}_D + C_D \nu \begin{pmatrix} -\nu_1 \\ -\nu_2 \end{pmatrix}_D - \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}_D \quad (23)$$

$$D_c = \frac{2m}{A\rho} = 4.6\text{m}, \quad T_c = \sqrt{\frac{2m}{A\rho g}} = 0.68\text{s} \quad (24)$$

Bounce part without gravity

Only α (angle of attack) and v remain as initial parameters.

$$D_c = \frac{2m}{A\rho}, \quad T_c = 1\text{s} \quad (25)$$

$$\begin{pmatrix} \nu'_1 \\ \nu'_2 \end{pmatrix} = C_L \nu \begin{pmatrix} -\nu_2 \\ \nu_1 \end{pmatrix} + C_D \nu \begin{pmatrix} -\nu_1 \\ -\nu_2 \end{pmatrix} \quad (26)$$

Changing initial speed only scales in time, trajectory shape is the same.

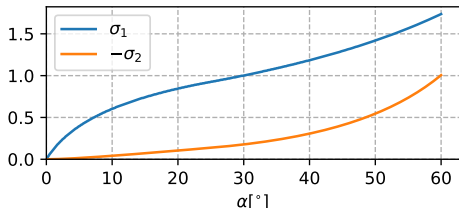


Figure 11: Distance and depth to the bounce minimum depending on angle of attack.

Phase diagram

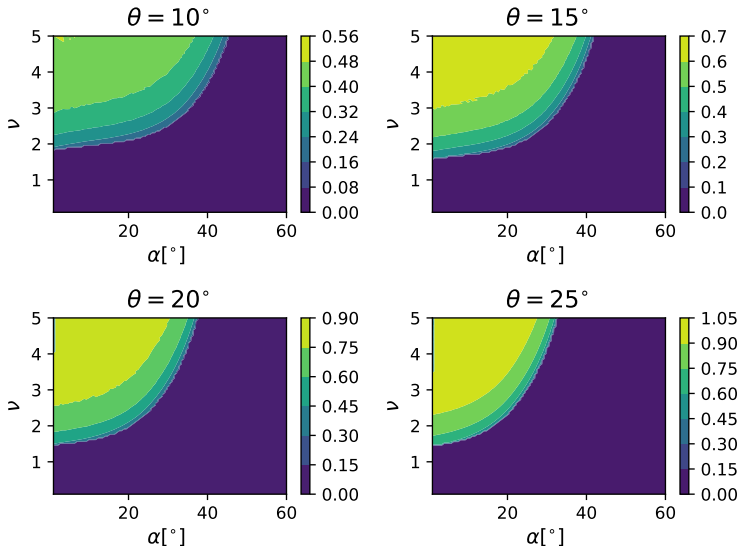


Figure 12: Phase diagram, magnitude is curvature at minimum.

- Simulation and experiment match.
- Coefficients similar as article: good model
- Experiment improvements:
 - measure wind speed
 - practice more for a stable throw
 - experiment conducted inside
 - measure Frisbee rotation

- [1] M. Hubbard, S. A. Hummel. *Simulation of Frisbee Flight*. (2000). https://www.researchgate.net/publication/253842372_Simulation_of_Frisbee_Flight
- [2] J. Potts, W. J. Crowther. *Disc-wing Aerodynamics*. (2002). https://www.researchgate.net/publication/268559957_FrisbeeTM_Aerodynamics