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14th International Physicists' Tournament

9th – 14th of May 2022 Bucaramanga, Colombia

2022.iptnet.info

Universidad Industrial de Santander

Problem no.2 - Airbounce

IPT 2022

Team Slovenia
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Official problem statement

When a Frisbee is thrown in a certain way it can be made to bounce in mid-air. Study the physics of this phenomenon.

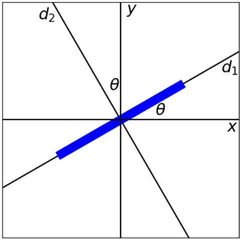
Ideas and hypotheses

- Normal component of Frisbee velocity will decrease faster because of its shape.
- Frisbee will appear to bounce in mid-air.

Theoretical description

- Frisbee in the original video is stable. Angle to the ground is constant.
- Assumptions:
  - Frisbee keeps constant angle to the ground during the whole flight because of gyroscopic stability.
  - Frisbee travels in a straight line. (no Magnus effect...)

Axis graphs:



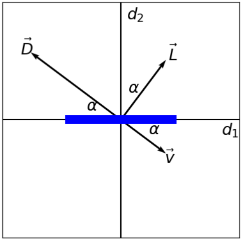


Figure 1:
Ground coordinate system: N

θ = angle to the ground

Figure 2:
Coordinate system of Frisbee: D

α = angle of attack

Theoretical description

Lift and drag force

$$L = \frac{1}{2} A\rho C_L v^2 \qquad D = \frac{1}{2} A\rho C_D v^2 \tag{1}$$

Lift and drag coefficient depending on angle of attack [1]

$$C_L = C_{L0} + C_{L\alpha}\alpha \qquad C_D = C_{D0} + C_{D\alpha}\alpha^2 \tag{2}$$

Cutoff

- C_D cutoff; when $C_D = 1.1$ (drag coefficient of a disc perpendicular to velocity)
- C_L cutoff; at stall angle $= 25°$

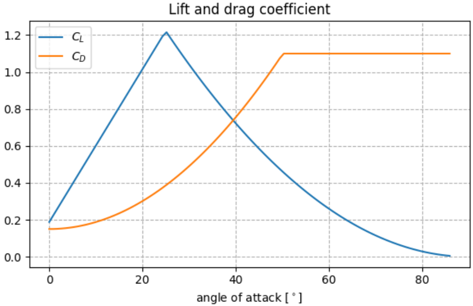


Figure 3: $C_{L0} = 0.188, C_{L\alpha} = 2.37, C_{D0} = 0.15, C_{D\alpha} = 1.24$ [1]

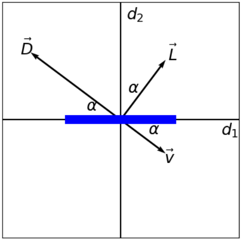


Figure 4: Coordinate system of Frisbee: D

$$K = \frac{A\rho}{2m} \qquad \tan\alpha = \frac{-v_2}{v_1}$$

$$v = \sqrt{v_1^2 + v_2^2}$$

$$\boldsymbol{L} = mKC_L v^2 \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix}_D$$
$$\boldsymbol{L} = mKC_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D \tag{3}$$

$$\boldsymbol{D} = mKC_D v^2 \begin{pmatrix} -\cos\alpha \\ \sin\alpha \end{pmatrix}_D$$
$$\boldsymbol{D} = mKC_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D \tag{4}$$

$$\boldsymbol{F_g} = -mg \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}_D \tag{5}$$

Theoretical description

$$m\boldsymbol{a} = \boldsymbol{L} + \boldsymbol{D} + \boldsymbol{F_g} \tag{6}$$

$$\begin{pmatrix} \dot{v_1} \\ \dot{v_2} \end{pmatrix}_D = KC_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D + KC_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D - g \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}_D \tag{7}$$

$$\begin{pmatrix} \dot{d_1} \\ \dot{d_2} \end{pmatrix}_D = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}_D \tag{8}$$

Solve for: d_1, d_2, v_1, v_2 and rotate to ground coordinate system N.

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}_N = R \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}_D \qquad \begin{pmatrix} v_x \\ v_y \end{pmatrix}_N = R \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}_D \tag{10}$$

- Video analysis of a throw.
- Problems:
  - Frisbee is not stable as in the original video.
  - ?Parallax? error. Throw is not perpendicular to the camera.

Figure 5: Example of a throw.

Parallax error correction

dx is measured, dx' is correct

$$k = \frac{\text{final frisbee size}}{\text{initial frisbee size}} \qquad l = \text{lenght of a throw} \tag{11}$$

$$dx = \mu(x)dx' = \left(\frac{k-1}{l}x + 1\right)dx' \tag{12}$$

$$x = \int_0^x \mu(x)\,dx = \frac{l}{k-1}\left[\ln((k-1)x + l) - \ln l\right] \tag{13}$$

References

[1] M. Hubbard, S. A. Hummel. *Simulation of Frisbee Flight*. (2000). https://www.researchgate.net/publication/253842372_Simulation_of_Frisbee_Flight