Impact behavior of hollow balls

Article *in* American Journal of Physics · February 2014

DOI: 10.1119/1.4839055

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Abstract

Measurements are presented of the force acting on ping pong and squash balls impacting on a force plate. Both ball types are hollow and have the same diameter, but deform in very different ways. Ping pong balls are relatively stiff and buckle inwards at high impact speeds, while squash balls are softer and tend to squash or flatten. The buckling process generates a large amplitude, high frequency oscillation of the force acting on a ping pong ball. Squash balls are initially very stiff before they soften, with the result that the force on the ball rises to about half its maximum value in the first 20 microseconds. Ping pong balls have a high coefficient of restitution (COR), while squash balls have a low COR. Results for both ball types are interpreted in terms of additional experimental observations.

I. INTRODUCTION

The impact of a ball with another object has been studied for many years, primarily in an effort to characterise the outgoing speed, rebound angle and spin of the ball in terms of parameters such as the coefficient of friction and the coefficient of restitution. Newton himself was the first to undertake detailed measurements of the head-on collision between two balls. He discovered that, for any given pair of balls, the relative speed after the collision is a fixed fraction of the relative speed before the collision. A more difficult task is to describe the behavior of a ball during the impact itself. In most cases of interest, the ball distorts elastically and then recovers its shape, although complete recovery or "restitution" often occurs after the impact is over. An interesting example of the latter effect is provided by the small rubber ball known as an "unhappy" ball.[1] Its coefficient of restitution is very low since the ball takes about 10 ms to recover its shape after it bounces.

One of the earliest ball impact models was provided by Hertz,[2–4] who showed that a solid sphere can be described by a force law of the form $F = kx^{1.5}$ where F is the impact force, x is the compression of the sphere and k is a constant. A solid sphere becomes stiffer the more it is compressed, primarily because the contact area increases with x.[5] A consequence of the force law is that the peak force, F_o, on the sphere increases with the impact speed, v, according to the relation $F_o = Cv^{1.2}$. An additional consequence of ball stiffening is that the impact duration decreases as the impact speed increases, a result that is observed with a wide variety of both solid and hollow balls.

A question of interest is what happens to a hollow ball when it bounces. Several authors have examined the problem theoretically, indicating that the contact region of a hollow ball impacting on a flat surface can either flatten against the surface, or buckle inwards, depending on the stiffness of the wall and the stiffness arising from air compression within the ball.[6–9] Balls with a soft wall, such as a basketball or football, generally require inflation to bounce well, and tend to flatten when they bounce. Other balls, such as ping pong balls, have stiffer walls, do not require inflation, and tend to buckle at high impact speeds.[10–13] Some balls, such as tennis and squash balls, rely both on wall stiffness and compression of the air inside the ball to achieve the desired bounce properties.[14–16] The buckling of a spherical shell has also been widely studied in connection with the engineering strength of large structures such as missiles, space vehicles, aircraft, nuclear reactor vessels, submarines and containment vessels, as well as microspheres such as pharmaceutical capsules, blood cells and pollen grains.[17–19].

The stiffness of a ball can be measured by compressing it in a materials testing machine, but such a measurement tends to underestimate the stiffness of a bouncing ball, for two reasons. One is that the ball is compressed on both sides rather than just the impact side. The other reason is that many balls are viscoelastic and experience stress relaxation when they are compressed slowly. That is, if the compression is held constant then the applied force decreases with time. A better procedure is to impact the ball on a force plate to measure the impact force vs time.[20,21] Double integration of the force over time then yields the displacement of the center of mass. The latter procedure was used in the present experiment to measure the impact behavior of ping pong and squash balls, with surprising results. The results are presented below, together with other measurements designed to assist in the interpretation of the force plate data.

II. EXPERIMENTAL DETAILS

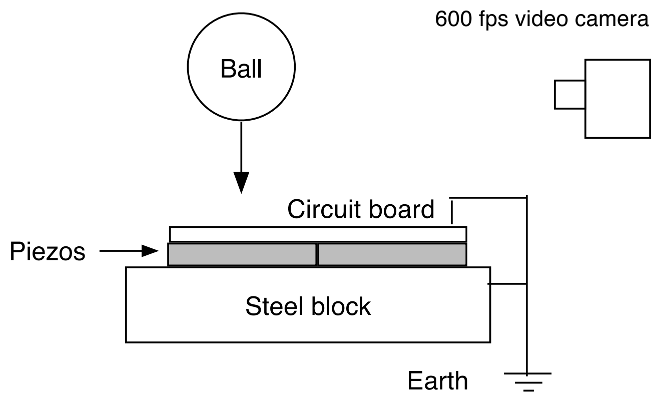

FIG. 1: Experimental arrangement used to measure the impact of a ball falling vertically onto a horizontal force plate. The video camera was used to measure the incident speed, v_1, and the rebound speed, v_2, in order to calculate the COR $= v_2/v_1$.

Impact force measurements were made with the apparatus shown in Fig. 1. The balls were dropped or projected by hand at speeds up to about 10 m/s to impact normally on a force plate containing four piezoelectric ceramic plates, each of dimensions 50 mm x 50 mm x 4 mm. The plates were mounted on a heavy block of stainless steel of dimensions 140 mm x 140 mm x 20 mm and electrically connected to the steel block with zinc paste. A fine wire lead was connected to the upper corner of each plate to measure the induced voltage. The upper surfaces of the piezo plates were also glued to a circuit board so that the copper side could be used to shield the plates. The squash ball in particular charged to a high voltage when it bounced.

Only one of the four piezo plates was used in this experiment. A 0.1 μF capacitor was connected in parallel with the plate and its output was measured with a 10 MΩ voltage probe. Impacts of duration up to about 20 ms could therefore be measured reliably, given the 1.0 s time constant for discharge of the capacitor. The output voltage, directly proportional to the impact force, was calibrated from measurements of the incident and rebound speed of balls of known mass, by equating the impulse to the change in momentum.

Of particular concern in this experiment was the frequency response of the force plate. The low frequency response was limited by the 1.0 s time constant. The high frequency response was tested by dropping small steel balls onto the force plate. A vibration response was detected at 40 kHz, corresponding to a transverse oscillation of the ceramic plate. However, that vibration was excited only when the impact duration was less than about 50 μs. It was not excited by ping pong ball impacts, although it was weakly excited in a transient manner by the initial rapid increase in the impact force on the squash ball.

In order to interpret the impact force results, additional experiments were undertaken using the force plate, as described in the relevant sections below. In addition, quasi-static measurements of ball stiffness were made by compressing each ball slowly in a small materials testing machine (Hounsfield Model H1K-S) capable of measuring forces up to 1 kN, at compression rates up to 1000 mm/min, and with a resolution in compression of $\pm$0.001 mm. In this experiment, the balls were compressed at 20 mm/min in the materials testing machine, and at speeds up to 500,000 mm/min in the impact tests.

Table 1. Properties of the three balls tested.

Type	Mass (g)	Diameter (mm)	Wall thickness (mm)
40 mm ping pong	2.49	39.6	0.30
55 mm ping pong	6.16	54.6	0.39
Yellow dot squash	24.2	39.8	4.0

Three balls were tested, with properties as shown in Table 1. The balls were tested at room temperature. Squash balls are known to bounce with a higher COR at higher temperatures,[16] but no attempt was made to warm the squash ball prior to testing. Standard ping pong (table tennis) balls are nominally 40 mm in diameter, but 44 mm and 55 mm diameter balls are also available for beginners. Squash balls are graded with a colored dot, representing the COR of the ball.

III. PING PONG BALL RESULTS

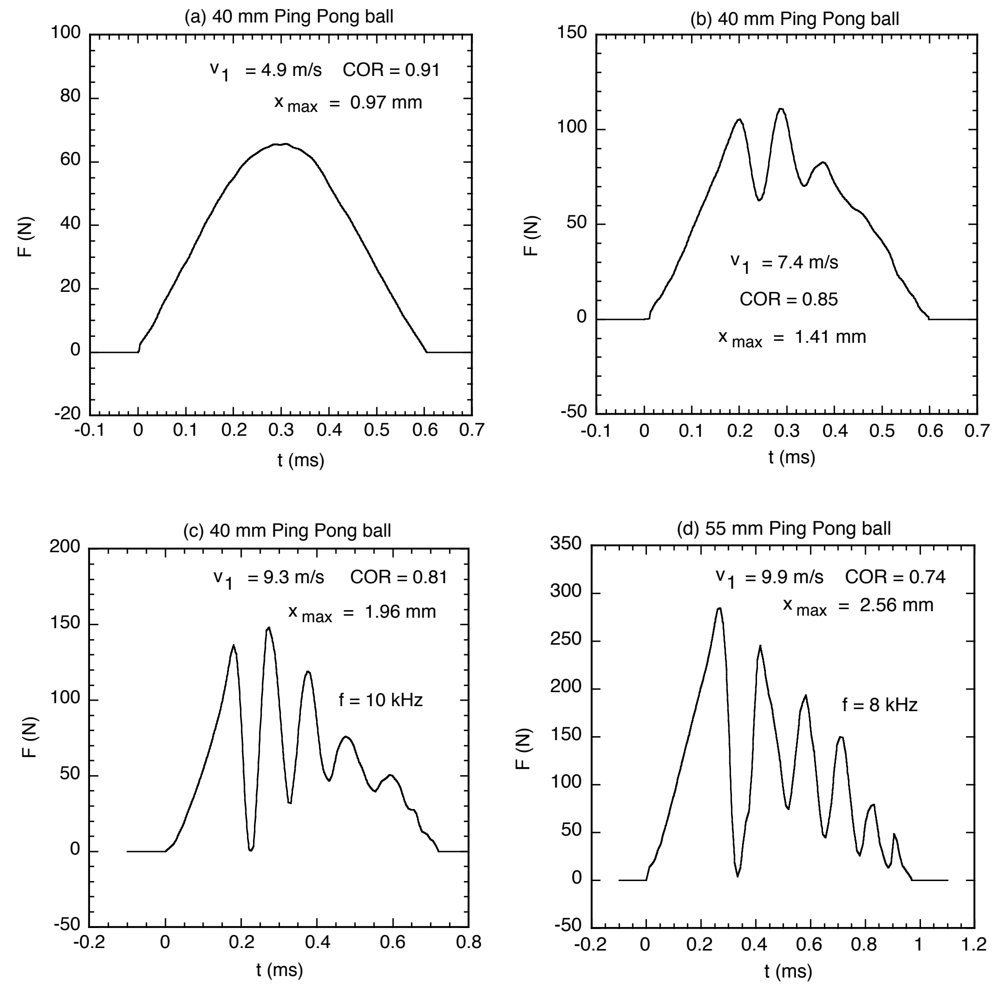

FIG. 2: Impact force results for the ping pong balls. v_1 is the incident speed and x_{max} is the maximum displacement of the center of mass during the impact.

Impact force results for the two ping pong balls are shown in Fig. 2. At impact speeds less than about 5 m/s, the impact force increased smoothly to a maximum and then decreased smoothly to zero, in the same manner as that observed for many other balls. A typical result is shown in Fig. 2(a). The COR at low impact speeds is relatively high, being greater than 0.9 for both balls, comparable to that observed with a superball. At impact speeds between about 5 m/s and 7 m/s, small oscillations in the impact force are observed at a time when the force is near its maximum value, as indicated by the result shown in Fig. 2(b). At impact speeds greater than about 8 m/s, the oscillations in the impact force are larger in amplitude, and the impact duration increased, as indicated in Figs. 2(c) and 2(d). The oscillation frequency was about 10 kHz for the 40 mm ball and about 8 kHz for the 55 mm ball. The oscillations resulted in an audible "crack" sound when the ball bounced, suggesting that the ball may have broken. In fact, there was no visible change to the balls after they bounced. The only outward change in the behaviour of each ball was a decrease in the COR as the impact speed increased, as indicated in Fig. 3. The relatively large scatter in the COR data can be attributed to the fact that the ball sometimes impacted near or on the seam.

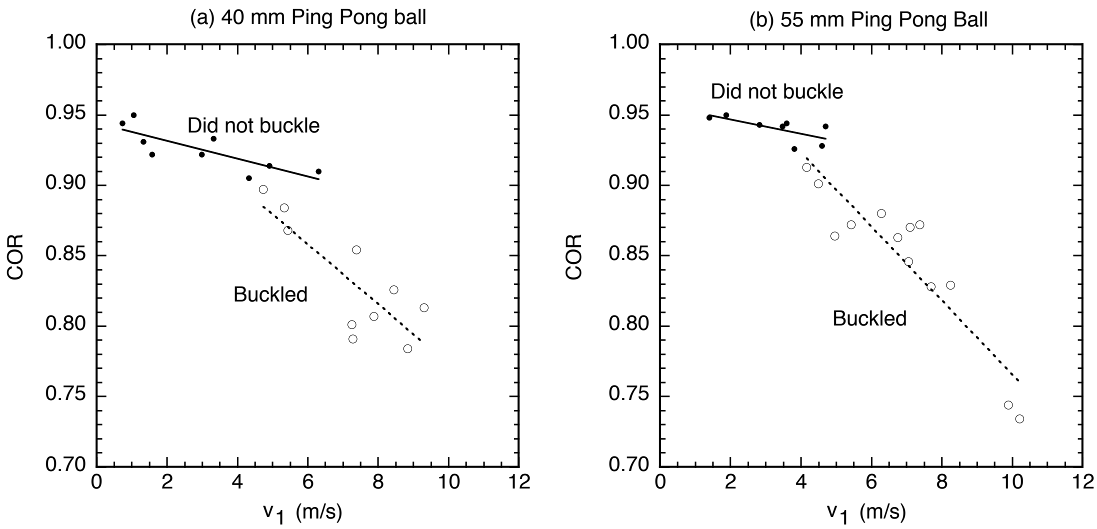

FIG. 3: Measured COR for the two ping pong balls vs the impact speed v_1. Balls that buckled were identified by the presence of a high frequency oscillation in the impact force.

Large oscillations in the impact force acting on a ball have not previously been observed as far as the author is aware. In the case of a ping pong ball, it is clear that the oscillations are associated with buckling of the ball. The oscillations were too rapid to be observed with the video camera used in this experiment, nor have they been observed by others using higher speed cameras. Hubbard and Strong[6] recorded the bounce of a ping pong ball impacting on a glass plate at 20 m/s, filming at 9000 frames/s, showing how the ball buckles and then recovers its normal spherical shape as the ball bounces off the plate. Similar results for a ping pong ball were obtained by Zhang et al.,[13] filming at 20,000 frames/s. The latter authors show no evidence of rapid, large scale oscillations during the buckling process, the inference being that the oscillations observed in the present experiment involve relatively small amplitude vibrations of the buckled section of the ball. If the displacement amplitude of a small mass m vibrating at frequency ω is say y_0, then the amplitude of the force acting on that mass is $F_0 = m\omega^2 y_0$. If $F_0 = 50\,\mathrm{N}$ and $f = 10\,\mathrm{kHz}$ for the 40 mm ball, and if we assume that the buckled mass $m \approx 0.2\,\mathrm{g}$, then $y_0 \approx 0.06\,\mathrm{mm}$. In that case, the amplitude of the oscillation would be too small to be observed using a video camera.

Evidence concerning the buckling of a ping pong ball can be obtained simply by squeezing the ball by hand. A light force results in a small, local indentation that recovers when the force is released. A larger force results in a larger, recoverable indentation. If the force is sufficiently large then the ball buckles permanently, although the buckle can often be removed by pressing on the sides of the ball or by floating the ball in hot water. A more scientific test is to compress the ball between two flat plates in a materials testing machine. Results for the 55 mm ball are shown in Fig. 4. Ping pong balls have a relatively thick, internal seam joining the two halves. In Fig. 4, the seam was horizontal and the ball was compressed vertically (ie across the softest diameter).

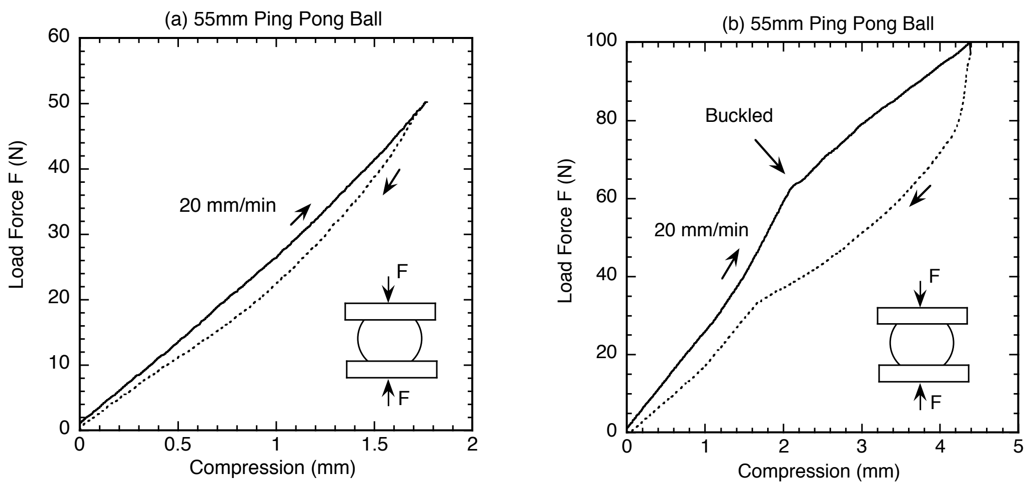

FIG. 4: Load force vs compression for the 55 mm ping pong ball measured in the materials testing machine at a maximum load force of (a) 50 N and (b) 100 N.

At a load force up to about 60 N, the ball flattened in the contact region, without buckling. Most of the stored elastic energy was recovered on unloading, as evidenced by the small area hysteresis curve in Fig. 4(a). The latter result is consistent with the high COR observed in a low speed impact. At a load force above about 60 N, the ball buckled inwards, as indicated in Fig. 4(b), with a significant loss of elastic energy during the unloading stage. The latter result is consistent with the decrease in COR observed at high ball speeds, as indicated in Fig. 3. Similar results were obtained with the 40 mm ball, with buckling occurring at a load force above about 40 N for that ball. The buckling force for a quasi-static compression was significantly lower than that observed in the bounce tests. As shown in Fig. 2, the 40 mm ball bucked at a force greater than 100 N, while the buckling force for the 55 mm ball was about 280 N at the higher impact speeds.

Published high speed video images[6,13] indicate that buckling of a ping pong ball involves a sudden inversion of the initially spherical cap on the contact surface so that it protrudes into the ball. Any overshoot past the equilibrium position could then result in vibration of the cap. The vibration frequency of a shallow cap was measured by cutting a 29 mm diameter, 7 mm tall cap from a 40 mm ball and attaching it to the force plate with double-sided adhesive tape. Vibrations were induced by dropping a 6 mm diameter, 0.9 g steel ball onto the top of the cap. The result is shown in Fig. 5(a). Figure 5(b) shows a result where the steel ball was dropped onto the 50 mm ping pong ball when it was taped to the force plate. The latter impact was too low in energy to buckle the ball, but it provided a useful measure of the fundamental vibration frequency of the whole ball.

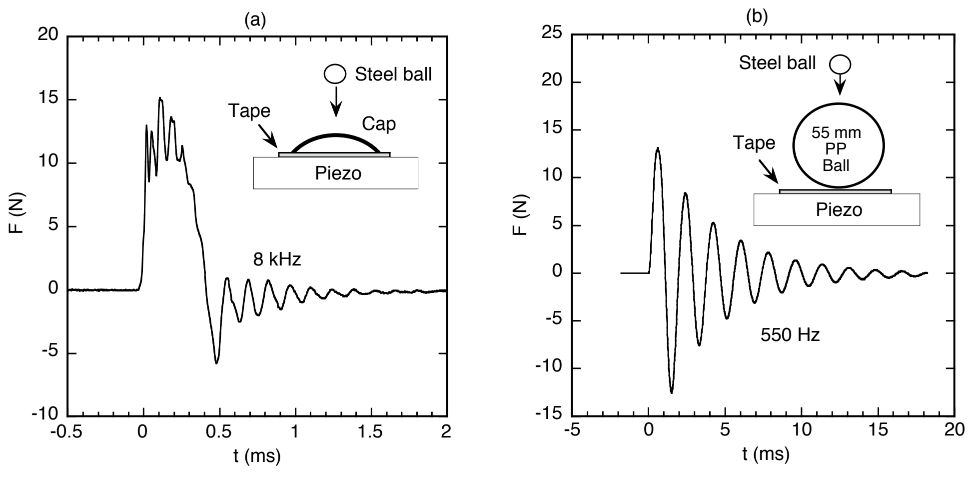

FIG. 5: Results of dropping a 0.9 g steel ball onto (a) a 29 mm diameter spherical cap cut from a 40 mm ping pong ball and (b) the 55 mm ping pong ball, both taped to the force plate. In both cases, the steel ball bounced upwards at about $t = 0.5$ ms.

The vibration frequency of the cap, after the steel ball bounced off the cap, was about 8 kHz, slightly lower than the vibration frequency of the 40 mm ball shown in Fig. 2. Nevertheless, the observed frequency was sufficiently close to indicate that the oscillations seen in Fig. 2 are indeed due to vibrations of the cap. A smaller cap than the one tested would vibrate at a higher frequency. Much larger spherical caps, such as those used as cymbals and gongs, vibrate at much lower fundamental vibration frequencies.

The result shown in Fig. 5(b) is interesting since it shows that the whole ball vibrates at a much lower frequency than any small, isolated part. The observed frequency corresponds to the vibration mode involved in the bounce itself. Only the first half cycle of that vibration is seen when the ball is allowed to bounce off the force plate, as in Fig. 2(a). If the ball remains attached to the force plate, then many vibration cycles can be observed if the ball is lightly damped or if it has a high COR. Only two vibration cycles were observed with the squash ball since internal damping is much larger in a low COR ball. The corresponding result for the squash ball is shown in Fig. 6 for comparison.

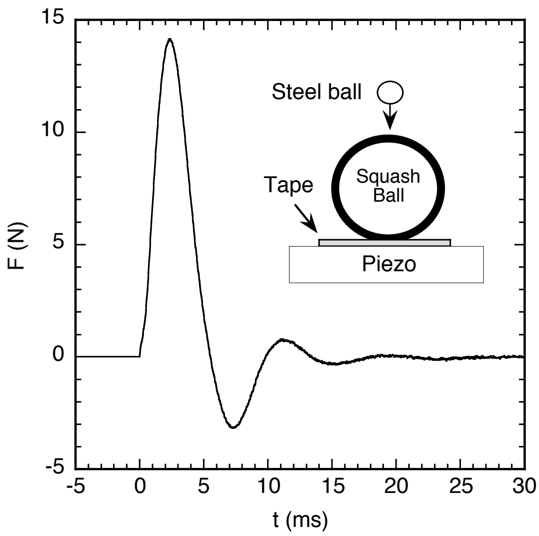

FIG. 6: Result of dropping a 0.9 g steel ball onto the squash ball. The squash ball was taped to the force plate, as shown in the inset. The steel ball bounced off the squash ball at about 0.5 ms, generating a heavily damped vibration of the squash ball.

IV. SQUASH BALL RESULTS

Representative low and high speed impacts of the squash ball on the force plate are shown in Fig. 7. The force waveforms are quite different from those for a ping pong ball, having a longer impact duration due to the larger mass and the lower ball stiffness. The most interesting result is that the force rises to about half its maximum value in less than 100 μs, even at low impact speeds. The rise time is only 20 μs at impact speeds around 9 m/s. The ball is therefore very stiff initially and then rapidly softens.

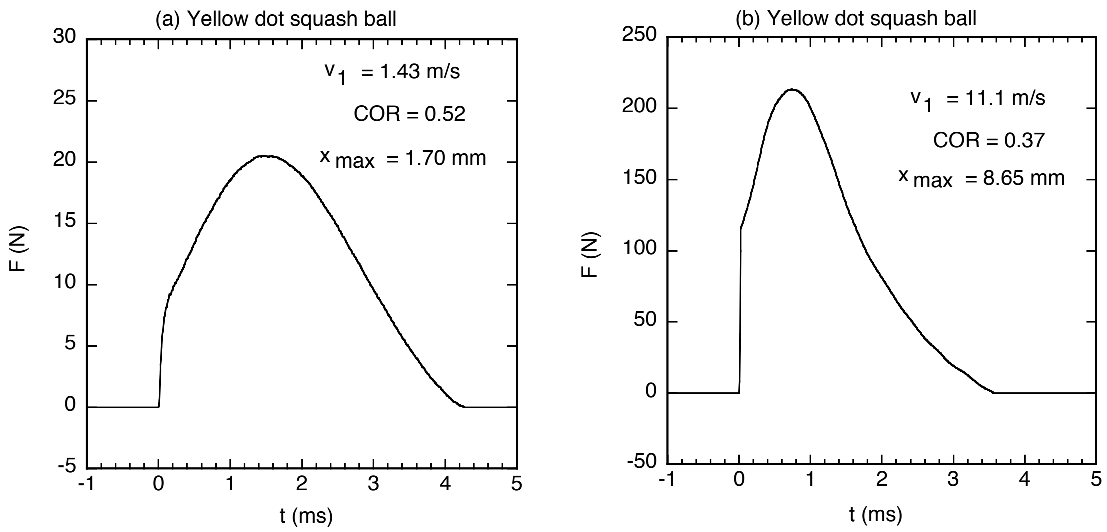

FIG. 7: Impact force results for the squash ball incident at low and high speeds on the force plate.

The stiffness of the high speed ball can be estimated directly from the result in Fig. 7(b). During the first 20 μs, the speed of the ball remains essentially constant at 11.1 m/s, so the center of mass advances by a distance $x = 0.22$ m while the impact force rises to $F = 115$ N. If we assume that the contact region also compresses by about 0.22 mm, then the ball stiffness is about $F/x = 5 \times 10^5$ N/m. At maximum ball compression, where $F = 215$ N, the center of mass advanced 8.65 mm, giving an average stiffness over the first 0.7 ms of about $F/x = 2.4 \times 10^4$ N/m, 21 times smaller than the initial stiffness. The dynamic stiffness $(\Delta F/\Delta x)$ from $F = 115$ N to $F = 215$ N is about half that value or about 1.2×10^4 N/m.

The ball stiffness estimates obtained from Fig. 7(b) can be compared with those measured from a slow compression of the squash ball in the materials testing machine, shown in Fig. 8. Figure 8(a) shows the result for the whole ball and Fig. 8(b) shows the result of compressing a 22 mm diameter, 7 mm tall spherical cap cut from a nominally identical squash ball. From Fig. 8(a), the average stiffness up to $F = 100$ N is 4350 N/m. Since the ball squashes mainly on the contact side when it bounces, rather than on both sides, the relevant stiffness of the ball can be taken as about 8700 N/mm, which is comparable to the dynamic stiffness found from the impact data.

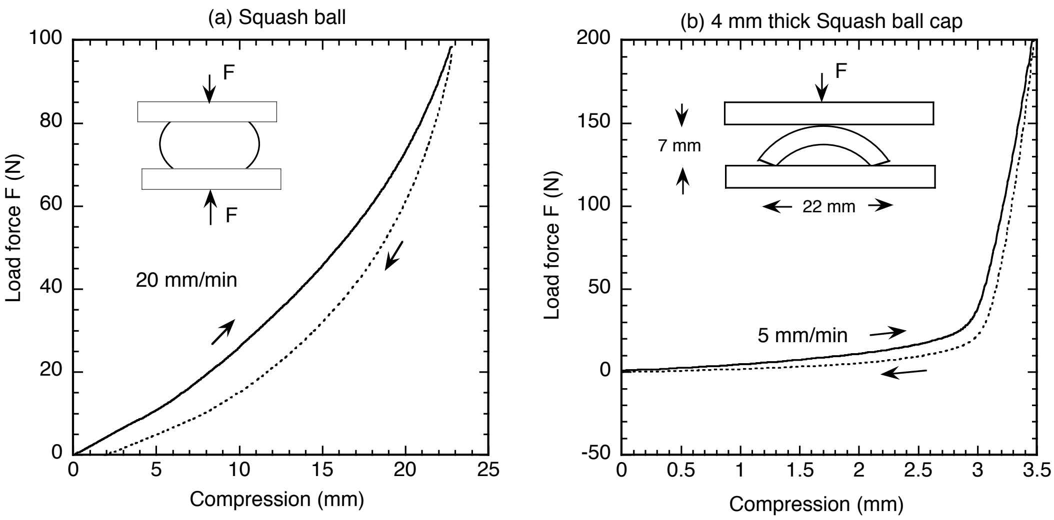

FIG. 8: Slow compression results obtained with the materials testing machine for (a) the squash ball and (b) a 7 mm tall cap cut from a nominally identical squash ball.

On the other hand, the cap by itself was much stiffer than the whole ball. Figure 8(b) indicates that the cap remained soft until it bent into a 4 mm thick, flat rubber disk. Compression of the solid disk by 0.5 mm resulted in a rapid increase in the load force, from about 30 N to 200 N. The average stiffness of the solid disk was about 3.4×10^5 N/m, comparable to the initial stiffness of the ball found from the impact tests.

The implication of these results is that the initial contact of a high speed squash ball results in rapid compression of the rubber wall in a small region surrounding the initial contact point, and that there is negligible bending of the ball during that time. Subsequently, the wall starts to bend, resulting in a rapid decrease in the dynamic stiffness of the ball. In order to test that hypothesis, a simple experiment was devised using a squash ball cut in half. The cut half was attached firmly by friction grip to the bottom end of a solid wood egg having a diameter slightly larger than the internal diameter of the ball. In that way, a small adjustable gap was created between the egg and the ball, as indicated in Fig. 9.

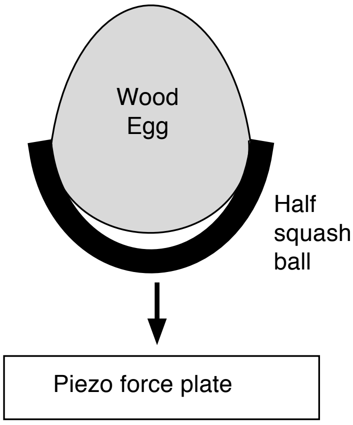

FIG. 9: Arrangement used to test the hypothesis that the rubber in a squash ball compresses as a solid before it starts to bend.

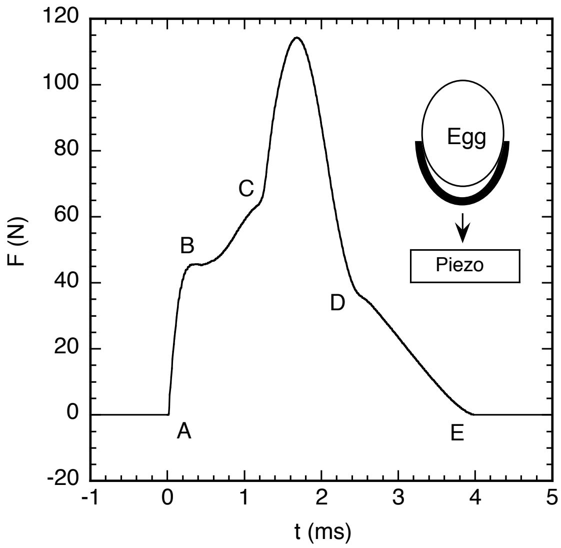

FIG. 10: Result of impacting the egg-in-ball arrangement shown in Fig. 9.

The result of this experiment is shown in Fig. 10. The ball impacted the force plate at point A, resulting in a rapid increase in the impact force. At point B, the ball softened by bending, resulting in closure of the gap between the ball and the egg at point C, and a consequent second rapid increase in the impact force. Given that rubber is much softer than wood, the second rapid increase in the force can be attributed to compression of the rubber against the wood. At point D the rubber separated from the wood, opening up the gap and resulting in a soft bounce off the force plate at point E. The result was repeatable, indicating that there was no slip between the egg and the ball during the bounce. Widening the gap resulted in a longer delay before the gap closed, but there was no change in the initial rapid increase in the impact force and the subsequent bending of the ball. The similarity between the initial rapid increase in F, from A to B, and the subsequent rapid increase at point C, indicates that the rubber compressed without bending during both time intervals.

The change in slope at point B was further investigated by measuring the impact force on a rubber pencil eraser of mass 16.5 g with dimensions 60 mm x 22 mm x 10 mm. A 10 mm square hole was cut out from one end, in order to simulate a hollow rubber ball at that end. The 10 mm cube of rubber cut from that end was then cut and polished into a 10 mm diameter, 0.80 g solid rubber sphere. Each end of the rectangular eraser was curved, as indicated in Fig. 11, so that it would impact on a curved surface rather than on a flat end or a sharp edge. The effects of impacting each end of the eraser, and the rubber sphere, on the force plate are shown in Fig. 11. The impact speed was the same in each case since the drop height was 20 cm in each case.

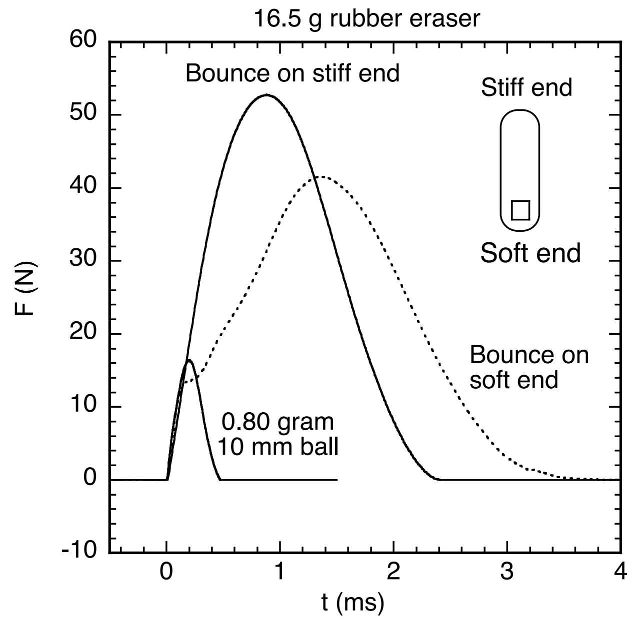

FIG. 11: Results of impacting a rubber eraser and a small rubber ball on the force plate.

The hollow end of the eraser, with the cut-out, bounced in a manner similar to the squash ball. That is, the impact force increased rapidly for the first 100 μs and then more slowly due to bending of the rubber in the vicinity of the cut-out. The initial increase in F is given by $F = kx = kvt$ where k is the initial stiffness and v is the impact speed. The initial increase in F was the same in all three cases, indicating that the rubber compressed in the contact area without bending, and it compressed at the same rate in each case since (a) the impact speed was the same and (b) the curvature of the contact region was the same. Furthermore, the transition from compression to bending at the soft end occurred at about the same time as the compression of the small sphere reached its maximum value. The implication in this case is that the initial contact region at the soft end commenced to bounce upwards while the surrounding region was still falling.

V. DISCUSSION AND CONCLUSIONS

Many experimental and theoretical studies have been undertaken to characterize and understand the behavior of a bouncing ball. It is surprising that there have been no previous measurements of the impact force acting on ping pong and squash balls. The results themselves are also surprising and show that the small spherical cap surrounding the initial contact point plays a major role in both ball types. Buckling of the cap in a ping pong ball occurs only at impact speeds above about 5 m/s, but that is a relatively small speed in the game of table tennis. Playing speeds are typically about 20 to 30 m/s, in which case the ball can be expected to buckle almost every time it is struck. The effect of the soft cushion on a bat was not investigated in this experiment, since the primary focus was to understand the origin of the large amplitude oscillations observed in the impact force.

It was concluded that the oscillations in the impact force on a ping pong ball arise from vibrations of the cap when it buckles. The vibration amplitude is too small to be observed with a high speed video camera, but it has a large effect on the impact force since the vibration frequency is relatively high, typically about 10 kHz. A secondary effect is that the impact duration increased with ball speed. For most ball types, the impact duration decreases as the ball speed increases.

Compression of the spherical cap in a squash ball results in a rapid increase in the impact force, to about half its maximum value in the first 20 μs at impact speeds around 10 m/s. The cap tends to bounce off the impact surface and may even lose contact with the surface at high impact speeds. No direct evidence was obtained for complete separation of the cap and the surface, although such an effect has previously been observed with high speed impacts of a tennis ball.[14] The main effect observed in the present experiment is that the ball softens as a result of bending after the initial rapid rise in the impact force. The subsequent three-dimensional shape of the ball on the contact surface was not measured. Video images suggest that the ball flattens against the surface, although the pressure distribution is not necessarily uniform or constant in time.

ACKNOWLEDEMENTS

I acknowledge helpful comments of Profs Mont Hubbard and Bill Stronge concerning the experiments described in this paper.

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