



## Problem no.2 - Airbounce IPT 2022

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## Official Problem Statement

When a Frisbee is thrown in a certain way it can be made to bounce in mid-air. Study the physics of this phenomenon.



## Ideas and Hypotheses

- Normal component of Frisbee velocity will decrease faster because of its shape.
- Frisbee will appear to bounce in mid-air.

- Frisbee in the original video is stable. Angle to the ground is constant.
- Assumptions:
  - Frisbee keeps constant angle to the ground during the whole flight because of gyroscopic stability.
  - Frisbee travels in a straight line. (no Magnus effect...)

## Axis graphs:

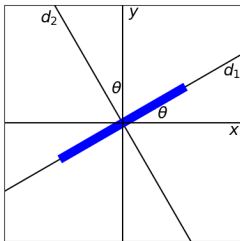


Figure 1:  
Ground coordinate system: N

$\theta$  = angle to the ground

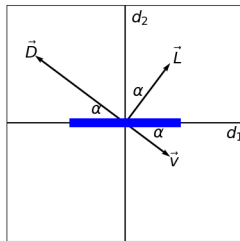


Figure 2:  
Coordinate system of Frisbee: D

$\alpha$  = angle of attack

## Lift and drag force

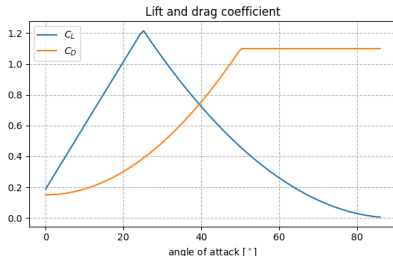
$$L = \frac{1}{2} A \rho C_L v^2 \quad D = \frac{1}{2} A \rho C_D v^2 \quad (1)$$

## Lift and drag coefficient depending on angle of attack [1]

$$C_L = C_{L0} + C_{L\alpha} \alpha \quad C_D = C_{D0} + C_{D\alpha} \alpha^2 \quad (2)$$

## Cutoff

- $C_D$  cutoff; when  $C_D = 1.1$  (drag coefficient of a disc perpendicular to velocity)
- $C_L$  cutoff; at stall angle =  $25^\circ$



**Figure 3:**  $C_{L0} = 0.188$ ,  $C_{L\alpha} = 2.37$ ,  $C_{D0} = 0.15$ ,  $C_{D\alpha} = 1.24$   
M. Hubbard, S. A. Hummel. *Simulation of Frisbee Flight*. (2000). [1]  
(short flights)

# Theoretical Description: Forces

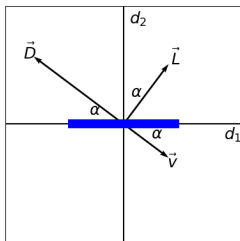


Figure 4: Coordinate system of Frisbee: D

$$K = \frac{A\rho}{2m} \quad \tan \alpha = \frac{-v_2}{v_1}$$
$$v = \sqrt{v_1^2 + v_2^2}$$

$$\mathbf{L} = mKC_L v^2 \begin{pmatrix} \sin \alpha \\ \cos \alpha \end{pmatrix}_D \quad (3)$$

$$\mathbf{L} = mKC_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D$$

$$\mathbf{D} = mKC_D v^2 \begin{pmatrix} -\cos \alpha \\ \sin \alpha \end{pmatrix}_D \quad (4)$$

$$\mathbf{D} = mKC_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D$$

$$\mathbf{F}_g = -mg \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}_D \quad (5)$$



$$m\mathbf{a} = \mathbf{L} + \mathbf{D} + \mathbf{F}_g \quad (6)$$

$$\begin{pmatrix} \dot{v}_1 \\ \dot{v}_2 \end{pmatrix}_D = KC_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D + KC_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D - g \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}_D \quad (7)$$

$$\begin{pmatrix} \dot{d}_1 \\ \dot{d}_2 \end{pmatrix}_D = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}_D \quad (8)$$

Solve for:  $d_1, d_2, v_1, v_2$  and rotate to ground coordinate system N.

$$R = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (9)$$

$$\begin{pmatrix} x \\ y \end{pmatrix}_N = R \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}_D \quad \begin{pmatrix} v_x \\ v_y \end{pmatrix}_N = R \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}_D \quad (10)$$

- Video analysis of a throw.
- Analysed only stable throws.
- Problems:
  - Frisbee is not stable as in the original video.
  - Parallax error. Throw is not perpendicular to the camera.
  - Wind speed was not measured.
  - Rotation of Frisbee not measured, coefficients of lift and drag for rotating Frisbee.

$m[\text{kg}]$	$A[\text{m}^2]$	$\rho[\text{kg}/\text{m}^3]$	$g[\text{m}/\text{s}^2]$
0.175	0.0616	1.23	9.8

Table 1: Frisbee parameters and constants.



Figure 5: Example of a throw.

$dx$  is measured,  $dx'$  is correct

$$k = \frac{\text{final frisbee size}}{\text{initial frisbee size}} \quad l = \text{lenght of a throw} \quad (11)$$

$$dx = \left( \frac{k-1}{l} x + 1 \right) dx' \quad (12)$$

$$x' = \int_0^x \left( \frac{k-1}{l} x + 1 \right)^{-1} dx \quad (13)$$

$$x' = \frac{l}{k-1} [\ln((k-1)x + l) - \ln l] \quad (14)$$

# Results, example of a throw:

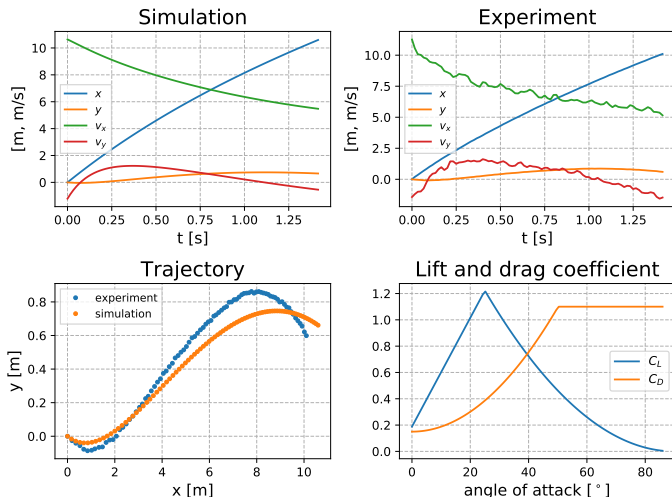


Figure 6:  $\theta = 19^\circ$ ,  $v_x(t=0) = 10.64 \frac{\text{m}}{\text{s}}$ ,  $v_y(t=0) = -1.21 \frac{\text{m}}{\text{s}}$   
 $C_L, C_D$ : article [1]

# Fitting $C_L$ and $C_D$

Angle to the ground ( $\theta$ ) is not constant, we used effective (average) angle  $\theta_{ef}$ .

Parameters in finding  $C_L$  and  $C_D$ :

$$\theta_{ef}, C_{L0}, C_{L\alpha}, C_{D0}, C_{D\alpha}$$

Minimization of  $s$  over  $k$  experiments:

$$s_k = \sum_i w_i \|(x_i, y_i)_{sim} - (x_i, y_i)_{exp}\|^2 \quad (15)$$

$$s = \sum_k s_k \quad (16)$$

$$w_i = \frac{1}{\Delta x_i^2} \quad (17)$$

```
scipy.optimize.minimize(method='TNC')
```

## Weights

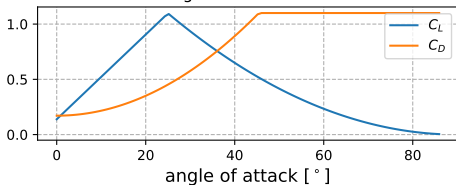
$$x_i = \int_0^{t_i} \left( \int_0^{t_i} a \, dt \right) dt \quad (18)$$

$$\Delta x_i = \frac{\partial x_i}{\partial a} \Delta a + \Delta x_0 \quad (19)$$

$$w_i = \left( \frac{\Delta a}{2} t_i^2 + \Delta x_0 \right)^{-2} \quad (20)$$

$$\Delta a \approx 0.1 \frac{\text{m}}{\text{s}^2} \quad \Delta x_0 \approx 1 \text{cm}$$

article [1]



## 2. Airbounce



## Trajectories: Weighted Minimization and Experiment

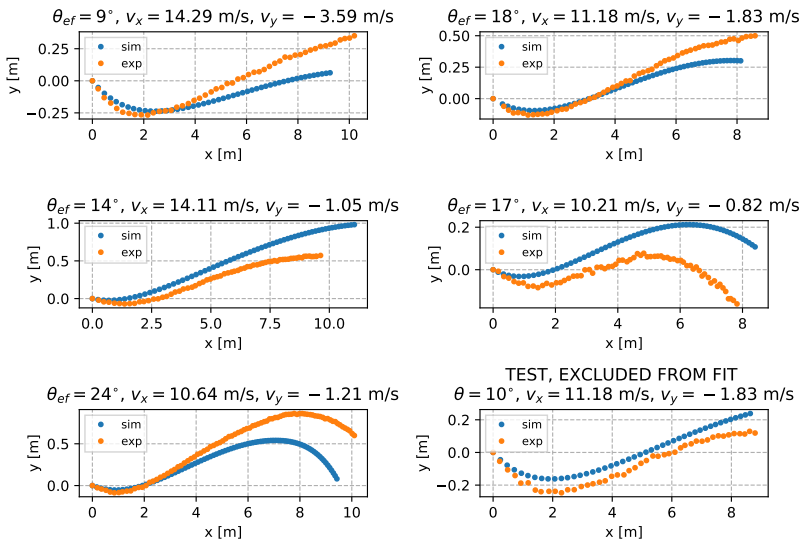


Figure 8: All experiments.

# Minimization: Weighted vs Unweighted

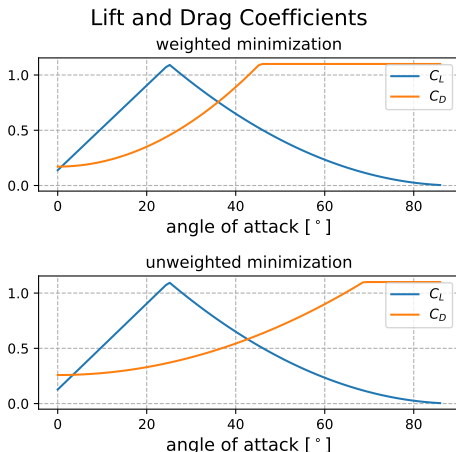


Figure 9:  $C_{L0} = 0.138, C_{L\alpha} = 2.20, C_{D0} = 0.171, C_{D\alpha} = 1.47$  (weighted)  
 $C_{L0} = 0.127, C_{L\alpha} = 2.23, C_{D0} = 0.258, C_{D\alpha} = 0.585$  (unweighted)

## Minimization: Weighted vs Unweighted

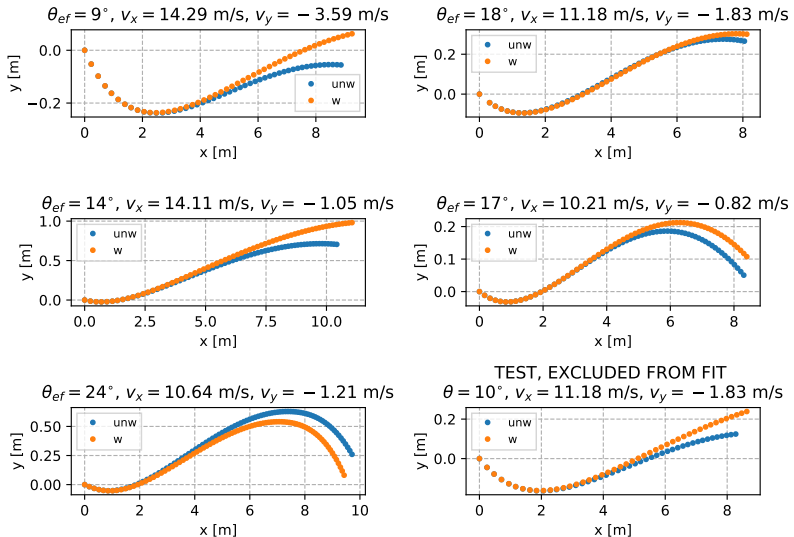


Figure 10: All experiments.

$$t = T_c \tau, \quad d_i = D_c \sigma_i, \quad v_i = \frac{D_c}{T_c} \nu_i, \quad \dot{v}_i = \frac{D_c}{T_c^2} \nu'_i \quad (21)$$

$$\begin{pmatrix} \dot{v}_1 \\ \dot{v}_2 \end{pmatrix}_D = KC_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D + KC_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D - g \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}_D \quad (22)$$

$\Downarrow$

$$\begin{pmatrix} \nu'_1 \\ \nu'_2 \end{pmatrix}_D = C_L \nu \begin{pmatrix} -\nu_2 \\ \nu_1 \end{pmatrix}_D + C_D \nu \begin{pmatrix} -\nu_1 \\ -\nu_2 \end{pmatrix}_D - \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}_D \quad (23)$$

$$D_c = \frac{2m}{A\rho} = 4.6\text{m}, \quad T_c = \sqrt{\frac{2m}{A\rho g}} = 0.68\text{s} \quad (24)$$

Only  $\alpha$  (angle of attack) and  $v$  remain as initial parameters.

$$D_c = \frac{2m}{A\rho}, \quad T_c = 1\text{s} \quad (25)$$

$$\begin{pmatrix} \nu'_1 \\ \nu'_2 \end{pmatrix} = C_L \nu \begin{pmatrix} -\nu_2 \\ \nu_1 \end{pmatrix} + C_D \nu \begin{pmatrix} -\nu_1 \\ -\nu_2 \end{pmatrix} \quad (26)$$

Changing initial speed only scales in time, trajectory shape is the same.

- Simulation and experiment match.
- Coefficients similar as article: good model
- Experiment improvements:
  - measure wind speed
  - practice more for a stable throw
  - experiment conducted inside
  - measure Frisbee rotation

- [1] M. Hubbard, S. A. Hummel. *Simulation of Frisbee Flight*. (2000). [https://www.researchgate.net/publication/253842372\\_Simulation\\_of\\_Frisbee\\_Flight](https://www.researchgate.net/publication/253842372_Simulation_of_Frisbee_Flight)
- [2] J. Potts, W. J. Crowther. *Disc-wing Aerodynamics*. (2002). [https://www.researchgate.net/publication/268559957\\_FrisbeeTM\\_Aerodynamics](https://www.researchgate.net/publication/268559957_FrisbeeTM_Aerodynamics)