



## Problem no.2 - Airbounce

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## Official problem statement

When a Frisbee is thrown in a certain way it can be made to bounce in mid-air. Study the physics of this phenomenon.



- Normal component of Frisbee velocity will decrease faster because of its shape.
- Frisbee will appear to bounce in mid-air.

- Frisbee in the original video is stable. Angle to the ground is constant.
- Assumption:  
Frisbee keeps constant angle to the ground during the whole flight because of gyroscopic stability.

## Axis graphs:

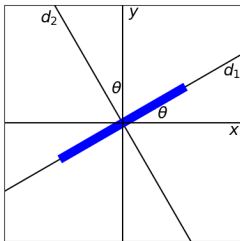


Figure 1:  
Ground coordinate system: N

$\theta$  = angle to the ground

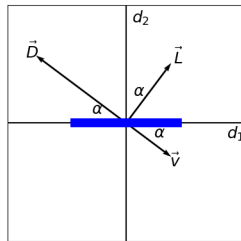


Figure 2:  
Coordinate system of Frisbee: D

$\alpha$  = angle of attack

## Lift and drag force

$$L = \frac{1}{2} A \rho C_L v^2 \quad D = \frac{1}{2} A \rho C_D v^2 \quad (1)$$

## Lift and drag coefficient depending on angle of attack [1]

$$C_L = C_{L0} + C_{L\alpha} \alpha \quad C_D = C_{D0} + C_{D\alpha} \alpha^2 \quad (2)$$

## Cutoff

- $C_D$  cutoff; when  $C_D = 1.1$  (drag coefficient of a disc perpendicular to velocity)
- $C_L$  cutoff; at stall angle =  $25^\circ$

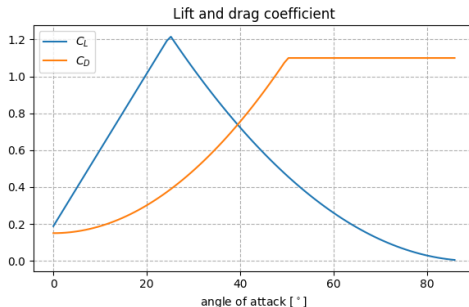


Figure 3:  $C_{L0} = 0.188$ ,  $C_{L\alpha} = 2.37$ ,  $C_{D0} = 0.15$ ,  $C_{D\alpha} = 1.24$  [1]

# Theoretical description: Forces

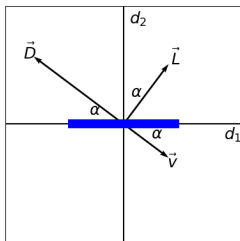


Figure 4: Coordinate system of Frisbee: D

$$K = \frac{A\rho}{2m} \quad \tan \alpha = \frac{-v_2}{v_1}$$
$$v = \sqrt{v_1^2 + v_2^2}$$

$$\mathbf{L} = mKC_L v^2 \begin{pmatrix} \sin \alpha \\ \cos \alpha \end{pmatrix}_D \quad (3)$$

$$\mathbf{L} = mKC_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D$$

$$\mathbf{D} = mKC_D v^2 \begin{pmatrix} -\cos \alpha \\ \sin \alpha \end{pmatrix}_D \quad (4)$$

$$\mathbf{D} = mKC_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D$$

$$\mathbf{F}_g = -mg \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}_D \quad (5)$$



$$m\mathbf{a} = \mathbf{L} + \mathbf{D} + \mathbf{F}_g \quad (6)$$

$$\begin{pmatrix} \dot{v}_1 \\ \dot{v}_2 \end{pmatrix}_D = KC_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D + KC_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D - g \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}_D \quad (7)$$

$$\begin{pmatrix} \dot{d}_1 \\ \dot{d}_2 \end{pmatrix}_D = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}_D \quad (8)$$

Solve for:  $d_1, d_2, v_1, v_2$  and rotate to ground coordinate system N.

$$R = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (9)$$

$$\begin{pmatrix} x \\ y \end{pmatrix}_N = R \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}_D \quad \begin{pmatrix} v_x \\ v_y \end{pmatrix}_N = R \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}_D \quad (10)$$

- Pri mojem metu ni stabilen.
- Wobbling frisbee.

- [1] M. Hubbard, S. A. Hummel. *Simulation of Frisbee Flight*. (2000). [https://www.researchgate.net/publication/253842372\\_Simulation\\_of\\_Frisbee\\_Flight](https://www.researchgate.net/publication/253842372_Simulation_of_Frisbee_Flight)