Problem no.2 - Airbounce

IPT 2022

Team Slovenia
Presenter: Rok Grgič Meško

University of Ljubljana
Faculty of Mathematics and Physics

Official Problem Statement

When a Frisbee is thrown in a certain way it can be made to bounce in mid-air. Study the physics of this phenomenon.

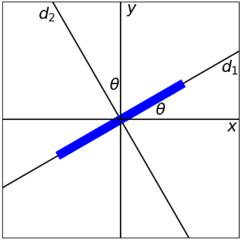
Ideas and Hypotheses

- Normal component of Frisbee velocity will decrease faster because of its shape.
- Frisbee will appear to bounce in mid-air.

Theoretical Description

- Frisbee in the original video is stable. Angle to the ground is constant.
- Assumptions:
  - Frisbee keeps constant angle to the ground during the whole flight because of gyroscopic stability.
  - Frisbee travels in a straight line. (no Magnus effect...)

Axis graphs:



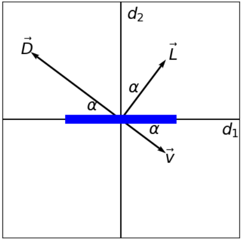


Figure 1:
Ground coordinate system: N

θ = angle to the ground

Figure 2:
Coordinate system of Frisbee: D

α = angle of attack

Theoretical Description

Lift and drag force

$$L = \frac{1}{2} A \rho C_L v^2 \qquad D = \frac{1}{2} A \rho C_D v^2 \tag{1}$$

Lift and drag coefficient depending on angle of attack [1]

$$C_L = C_{L0} + C_{L\alpha}\alpha \qquad C_D = C_{D0} + C_{D\alpha}\alpha^2 \tag{2}$$

Cutoff

- C_D cutoff; when $C_D = 1.1$ (drag coefficient of a disc perpendicular to velocity)
- C_L cutoff; at stall angle $= 25^\circ$

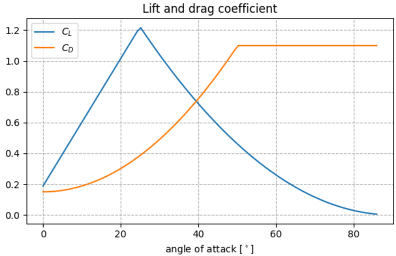


Figure 3: $C_{L0} = 0.188, C_{L\alpha} = 2.37, C_{D0} = 0.15, C_{D\alpha} = 1.24$
M. Hubbard, S. A. Hummel. *Simulation of Frisbee Flight*. (2000). [1]
(short flights)

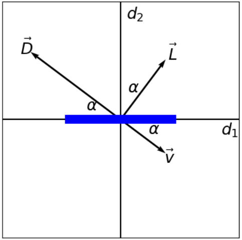


Figure 4: Coordinate system of Frisbee: D

$$K = \frac{A\rho}{2m} \qquad \tan\alpha = \frac{-v_2}{v_1}$$

$$v = \sqrt{v_1^2 + v_2^2}$$

$$\boldsymbol{L} = mKC_L v^2 \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix}_D \qquad \boldsymbol{L} = mKC_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D \tag{3}$$

$$\boldsymbol{D} = mKC_D v^2 \begin{pmatrix} -\cos\alpha \\ \sin\alpha \end{pmatrix}_D \qquad \boldsymbol{D} = mKC_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D \tag{4}$$

$$\boldsymbol{F_g} = -mg \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}_D \tag{5}$$

$$m\boldsymbol{a} = \boldsymbol{L} + \boldsymbol{D} + \boldsymbol{F_g} \tag{6}$$

$$\begin{pmatrix} \dot{v}_1 \\ \dot{v}_2 \end{pmatrix}_D = KC_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D + KC_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D - g \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}_D \tag{7}$$

$$\begin{pmatrix} \dot{d}_1 \\ \dot{d}_2 \end{pmatrix}_D = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}_D \tag{8}$$

Solve for: d_1, d_2, v_1, v_2 and rotate to ground coordinate system N.

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}_N = R \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}_D \qquad \begin{pmatrix} v_x \\ v_y \end{pmatrix}_N = R \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}_D \tag{10}$$

Experiment

- Video analysis of a throw.
- Analysed only stable throws.
- Problems:
  - Frisbee is not stable as in the original video.
  - Parallax error. Throw is not perpendicular to the camera.
  - Wind speed was not measured.
  - Rotation of Frisbee not measured, coefficients of lift and drag for rotating Frisbee.

m[kg]	A[m^2]	ρ[kg/m^3]	g[m/s^2]
0.175	0.0616	1.23	9.8

Table 1: Frisbee parameters and constants.

Figure 5: Example of a throw.

Parallax Error Correction

dx is measured, dx' is correct

$$k = \frac{\text{final frisbee size}}{\text{initial frisbee size}} \qquad l = \text{lenght of a throw} \tag{11}$$

$$dx = \left(\frac{k-1}{l}x + 1\right) dx' \tag{12}$$

$$x' = \int_0^x \left(\frac{k-1}{l}x + 1\right)^{-1} dx \tag{13}$$

$$x' = \frac{l}{k-1}\left[\ln((k-1)x + l) - \ln l\right] \tag{14}$$

Results, example of a throw:

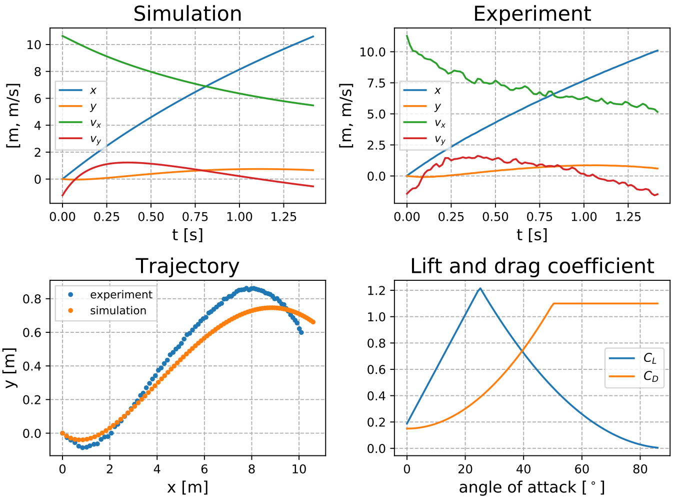


Figure 6: $\theta = 19°$, $v_x(t=0) = 10.64\,\frac{\mathrm{m}}{\mathrm{s}}$, $v_y(t=0) = -1.21\,\frac{\mathrm{m}}{\mathrm{s}}$
C_L, C_D: article [1]

Fitting C_L and C_D

Angle to the ground (θ) in not constant, we used effective (average) angle θ_{ef}.
Parameters in finding C_L and C_D:
θ_{ef}, C_{L0}, $C_{L\alpha}$, C_{D0}, $C_{D\alpha}$

Minimization of s over k experiments:

$$s_k = \sum_i w_i \left\| (x_i, y_i)_{sim} - (x_i, y_i)_{exp} \right\|^2 \tag{15}$$

$$s = \sum_k s_k \tag{16}$$

$$w_i = \frac{1}{\Delta x_i^2} \tag{17}$$

```
scipy.optimize.minimize(method='TNC')
```

Fitting C_L and C_D

Weights

$$x_i = \int_0^{t_i} \left(\int_0^{t_i} a \, dt \right) dt \tag{18}$$

$$\Delta x_i = \frac{\partial x_i}{\partial a} \Delta a + \Delta x_0 \tag{19}$$

$$w_i = \left(\frac{\Delta a}{2} \, t_i^2 + \Delta x_0 \right)^{-2} \tag{20}$$

$$\Delta a \approx 0.1 \frac{\mathrm{m}}{\mathrm{s}^2} \qquad \Delta x_0 \approx 1\mathrm{cm}$$

Fitting C_L and D_D

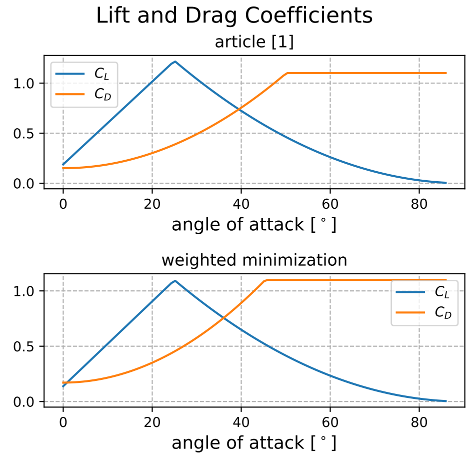


Figure 7: $C_{L0} = 0.188, C_{L\alpha} = 2.37, C_{D0} = 0.15, C_{D\alpha} = 1.24$ [1]
$C_{L0} = 0.138, C_{L\alpha} = 2.20, C_{D0} = 0.171, C_{D\alpha} = 1.47$ [minimization]

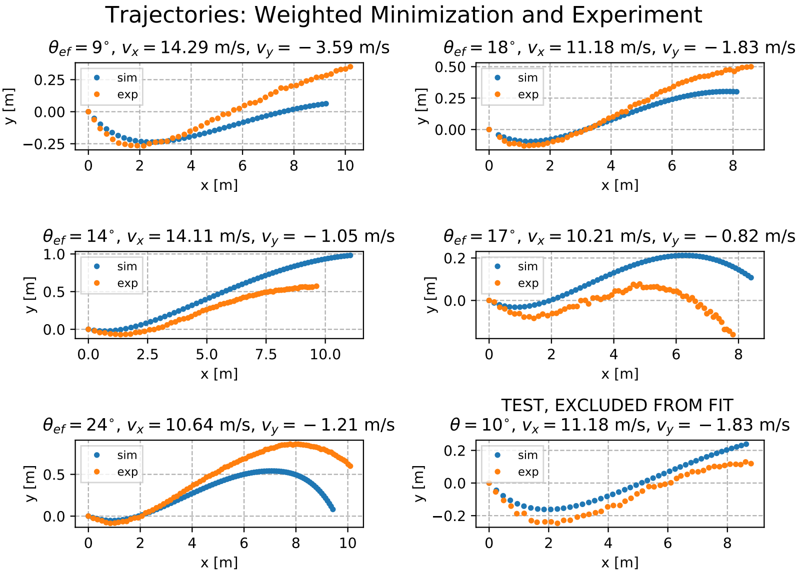


Figure 8: All experiments.

Minimization: Weighted vs Unweighted

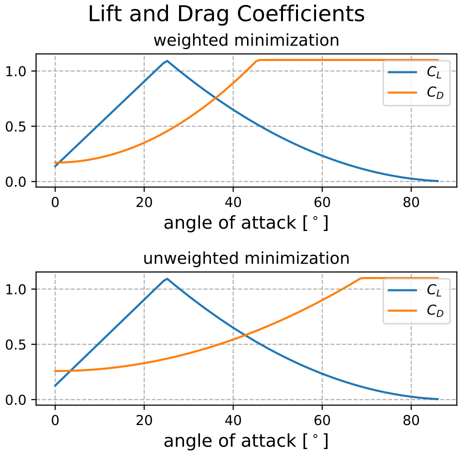


Figure 9: $C_{L0} = 0.138, C_{L\alpha} = 2.20, C_{D0} = 0.171, C_{D\alpha} = 1.47$ (weighted)
$C_{L0} = 0.127, C_{L\alpha} = 2.23, C_{D0} = 0.258, C_{D\alpha} = 0.585$ (unweighted)

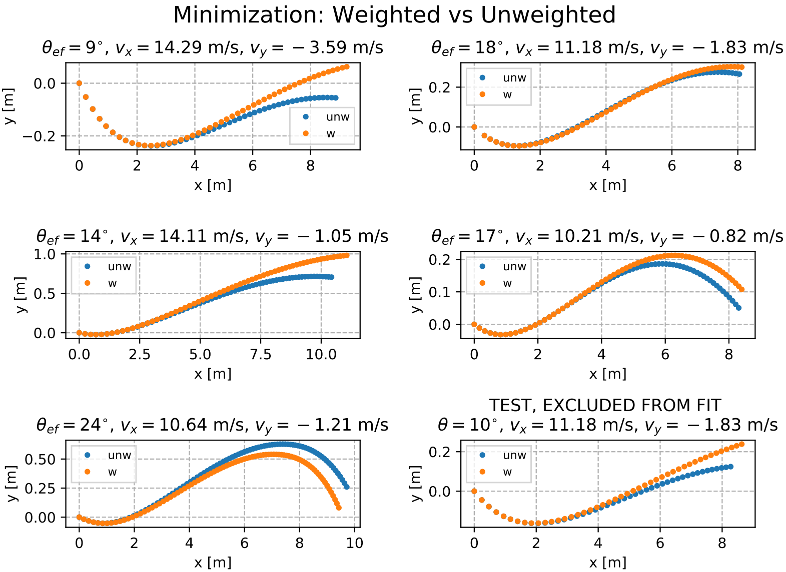


Figure 10: All experiments.

Nondimensionalization

$$t = T_c \tau, \quad d_i = D_c \sigma_i, \quad v_i = \frac{D_c}{T_c}\, \nu_i, \quad \dot{v}_i = \frac{D_c}{T_c^2}\, \nu_i' \tag{21}$$

$$\begin{pmatrix} \dot{v}_1 \\ \dot{v}_2 \end{pmatrix}_D = K C_L v \begin{pmatrix} -v_2 \\ v_1 \end{pmatrix}_D + K C_D v \begin{pmatrix} -v_1 \\ -v_2 \end{pmatrix}_D - g \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}_D \tag{22}$$

$$\Downarrow$$

$$\begin{pmatrix} \nu_1' \\ \nu_2' \end{pmatrix}_D = C_L\, \nu \begin{pmatrix} -\nu_2 \\ \nu_1 \end{pmatrix}_D + C_D\, \nu \begin{pmatrix} -\nu_1 \\ -\nu_2 \end{pmatrix}_D - \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}_D \tag{23}$$

$$D_c = \frac{2m}{A\rho} = 4.6\mathrm{m}, \quad T_c = \sqrt{\frac{2m}{A\rho g}} = 0.68\mathrm{s} \tag{24}$$

Ignore gravity in bounce part

Only α (angle of attack) and v remain as initial parameters.

$$D_c = \frac{2m}{A\rho}, \quad T_c = 1\mathrm{s} \tag{25}$$

$$\begin{pmatrix} \nu_1' \\ \nu_2' \end{pmatrix} = C_L\,\nu \begin{pmatrix} -\nu_2 \\ \nu_1 \end{pmatrix} + C_D\,\nu \begin{pmatrix} -\nu_1 \\ -\nu_2 \end{pmatrix} \tag{26}$$

Changing initial speed only scales in time, trajectory shape is the same.

Phase diagram, no gravity

Conclusion

- Simulation and experiment match.
- Coefficients similar as article: good model
- Experiment improvements:
  - measure wind speed
  - practice more for a stable throw
  - experiment conducted inside
  - measure Frisbee rotation

References

[1] M. Hubbard, S. A. Hummel. *Simulation of Frisbee Flight*. (2000). https://www.researchgate.net/publication/253842372_Simulation_of_Frisbee_Flight

[2] J. Potts, W. J. Crowther. *Disc-wing Aerodynamics*. (2002). https://www.researchgate.net/publication/268559957_FrisbeeTM_Aerodynamics