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Problem no.13 - Chaotic Magnetic Pendulum

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Official Problem Statement

Consider a pendulum consisting of a magnetic bob attached to a string. If the pendulum is allowed to swing over a structure of permanent magnets, it will display complex motion. Study the pendulum dynamics and its dependence on the number of permanent magnets and their arrangement.

Hypotheses

For certain initial parameters pendulum will display chaotic motion.

Theory

- For continuous dynamical systems, the Poincaré–Bendixson theorem shows that chaos only exist in 3 dimensions described by non-linear equations.
- Dynamical chaos can exist in systems starting with effective 3/2 (1/2 for no explicit time dependence) degrees of freedom.
- Integrals of motion (conserved) restrict the solution and make dynamical chaos less likely:

$$N_F = \frac{1}{2}(N - N_{int})$$

Theory

- Chaotic pendulum:

$$N = 4, \quad N_{int} = 1 \Rightarrow N_F = 3/2$$

In the presence of dissipation, a mechanical system relaxes down to one of its local energy minima. Generally, dissipation tends to make chaotic motion regular.

We approximate magnets by magnetic dipoles.
Coordinate system origin at the top of the string.
Force between two dipoles:

$$\mathbf{F_m}(\mathbf{r}', \mathbf{m}_1, \mathbf{m}_2) = \frac{3\mu_0}{4\pi r'^5}\Big[(\mathbf{m}_1 \cdot \mathbf{r}')\mathbf{m}_2 + (\mathbf{m}_2 \cdot \mathbf{r}')\mathbf{m}_1 + \\ + (\mathbf{m}_1 \cdot \mathbf{m}_2)\mathbf{r}' - \frac{5(\mathbf{m}_1 \cdot \mathbf{r}')(\mathbf{m}_2 \cdot \mathbf{r}')}{r'^2}\mathbf{r}'\Big] \tag{1}$$

$$\mathbf{r} = \left(x, y, -l - h + \sqrt{l^2 - x^2 - y^2}\right) \tag{2}$$

All forces:

$$\mathbf{F} = \mathbf{F_g} + \mathbf{F_{mi}} - \mathbf{F_v} \tag{3}$$

$$\mathbf{F} = \mathbf{F_g} + \mathbf{F_{mi}} - \langle \mathbf{F_g} + \mathbf{F_{mi}}, \frac{\mathbf{r}}{r}\rangle \frac{\mathbf{r}}{r} \tag{4}$$

Dimensionless:

$$\{x, y, z\} = L\{\chi, \gamma, \zeta\}, \qquad \boldsymbol{r} = L\boldsymbol{\rho}, \qquad t = T\tau \tag{5}$$

$$\|\mathbf{m_1}\| = \|\mathbf{m_2}\| \tag{6}$$

$$L = \sqrt[4]{\frac{\mu_0 \|\mathbf{m_1}\|^2}{mg}}, \qquad T = \sqrt{\frac{L}{g}} \tag{7}$$

$$\begin{aligned} \mathcal{F}_{\mathbf{mi}}(\boldsymbol{\rho}, \boldsymbol{\rho}_i') = \frac{3}{4\pi\rho_i'^5} \Bigg[& (-\frac{\boldsymbol{\rho}}{\rho} \cdot \boldsymbol{\rho}_i')\hat{\mathbf{e}}_z + (\hat{\mathbf{e}}_{\mathbf{z}} \cdot \boldsymbol{\rho}_i') - \frac{\boldsymbol{\rho}}{\rho} + \\ & + (-\frac{\boldsymbol{\rho}}{\rho} \cdot \hat{\mathbf{e}}_z)\boldsymbol{\rho}_i' - \frac{5(-\frac{\boldsymbol{\rho}}{\rho} \cdot \boldsymbol{\rho}_i')(\hat{\mathbf{e}}_z \cdot \boldsymbol{\rho}_i')}{\rho_i'^2} \boldsymbol{\rho}_i' \Bigg] \end{aligned} \tag{8}$$

Dimensionless:

$$\boldsymbol{\rho} = \left(\chi, \gamma, -\lambda - \eta + \sqrt{\lambda^2 - \chi^2 - \gamma^2}\right) \tag{9}$$

$$\mathcal{F} = \mathcal{F}_{\mathbf{g}} + \mathcal{F}_{\mathbf{mi}} - \mathcal{F}_{\mathbf{v}} \tag{10}$$

$$\mathcal{F} = \mathcal{F}_{\mathbf{g}} + \mathcal{F}_{\mathbf{mi}} - \langle \mathcal{F}_{\mathbf{g}} + \mathcal{F}_{\mathbf{mi}}\,, \frac{\boldsymbol{\rho}}{\rho} \rangle \frac{\boldsymbol{\rho}}{\rho} \tag{11}$$

Projection on x, y plane.

$$\begin{pmatrix} \ddot{\chi} \\ \ddot{\gamma} \end{pmatrix} = \begin{pmatrix} \langle \mathcal{F},\, \hat{e}_x \rangle \\ \langle \mathcal{F},\, \hat{e}_y \rangle \end{pmatrix} \tag{12}$$

We solve for χ, γ.

- We used different magnetic configurations.
- We filmed throws at different initial parameters.

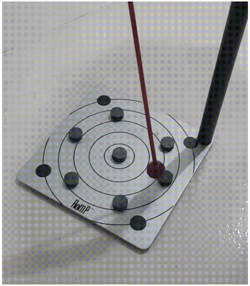

Figure 1: Example of an experiment.

Residual flux density on surface B_r

$$m_1 = \frac{B_r V}{\mu_0} \approx 0.1\mathrm{Am}^2, \quad l = 0.18\mathrm{cm}, \quad h = 0.02\mathrm{cm}, \quad m \approx 3\mathrm{g}$$

Experiment

We found three regimes.

- High kinetic energy:
  pendulum swings sinusoidally in a regular way.
- Medium kinetic energy:
  pendulum swings chaotically.
- Low kinetic energy:
  perturbed sinusoidal swinging in a minimum (spherical pendulum).

Results form simulation

Experiment videos

Animation $\rightarrow$

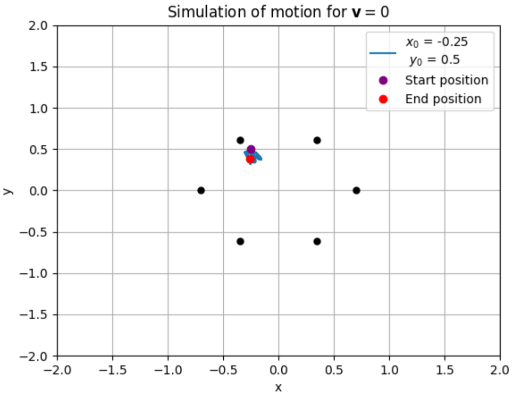

Figure 2: Example of stationary point.

Results form simulation

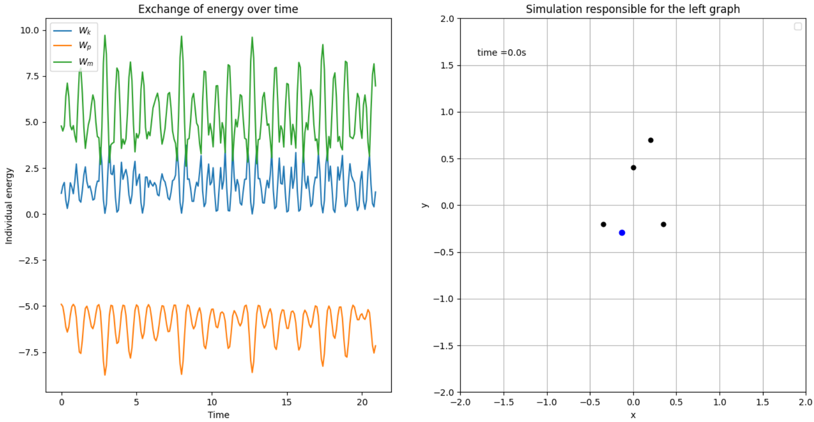

Figure 3: Exchange of energy over time.

Results form simulation

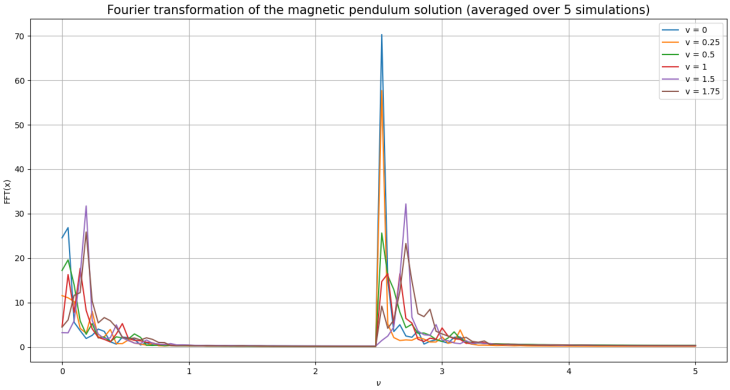

Figure 4: Fourier transformation, eigen frequencies. For higher energies motion is more chaotic. Second peak is redundant, arises form discrete model.

Results form simulation

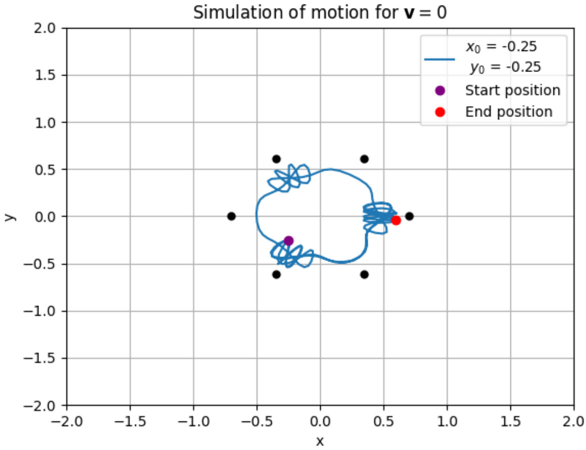

Figure 5: Motion looks like a random walk. Swinging in a minimum and jumping to other minimums.

Conclusion

- We found regular and chaotic regimes of motion in experiment.
- We showed in simulation that for certain initial parameters motion is chaotic.

References

[1] D. Garanin. *Dynamical Chaos*. (2008). https://www.lehman.edu/faculty/dgaranin/Mechanics/Dynamical_Chaos.pdf