ARS Explanation

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Github account for the final version of the project

Final version of the project resides in RJ's Github account. Github user name: rokjunlee

The very first main task that our `ars()` needs to be able to accomplish is to determine if the input function is logarithmically concave or not. The function should not move on to the next task if the input function is not even logarithmically concave. To verify log-concavity, we first created an auxiliary function that simply puts `log()` around the input function. Then the second derivative at numerious points in between the maximum and minimum (if they are given) were computed numerically. In order to numerically evaluate second derivative of the log-function, the following formula was used:

$$f''(x) = lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x+h)}{h^2}$$

, which is also known as the second symmetric derivative [https://en.wikipedia.org/wiki/Second_derivative].

After second derivatives were evaluated at several points in the domain, the sign of the largest evaluated value was checked. If the sign is negative, then we accept that the input function is logarithmically concave since all the other evaluated values must also be negative. Otherwise, we stop the function from moving on to the next task.

Next step is to transform the input function f into g which is equal to $cf(x)$ for some constant c. We created function so that c equals $\frac{1}{\int_D f(x)}$ to make sure that function $g(x)$ becomes a sensible density function that integrates to one over its support.

Specific Contributions of each team member

Although we divided the task as below we all contributed to all the tasks to some extent by checking for errors and making codes faster/better.

Malvika Rajeev

vectorizing computation of Zjs, computing the upper linear hull, computing the cdf, sampling from the exponential density.

Yihuan Song

The basic structure of the ars function (for loop and while loop) The functions of h(x), h'(x), uk(x), lk(x), sk(x), and finding zj's The squeezing and rejection tests

RJ Lee

Function that checks for log-concavity of the input function by evaluating second derivative numerically Function that can join tangent segments Finding tangent lines Simple linear models to find endpoints of each tangent lines Function that chooses the starting point: X_1 and X_k (k=2) g_func that transforms f into g as described in the paper.