## Problem Set

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## [1] Growth rates

The below table shows nominal GDP of the USA.

Year	GDP in bil. 2011 US\$	Symbol
2011	15,591	$Y_1$
2012	15,978	$Y_2$
2013	16,274	<i>Y</i> <sub>3</sub>
2014	16,705	$Y_4$

Table 1: GDP of the USA

For notational simplicity, let  $Y_1$ ,  $Y_2$ ,  $Y_3$ ,  $Y_4$  denote the GDP for years 2011, 2012, 2013 and 2014, respectively. The net annual growth rate between 2011 and 2012 is defined by

$$g_{2,1} = \frac{Y_2 - Y_1}{Y_1} = \frac{Y_2}{Y_1} - 1.$$

 $g_{3,2}$  and  $g_{4,3}$  are defined similarly.

- (a) Calculate the annual growth rates  $g_{2,1}$ ,  $g_{3,2}$  and  $g_{4,3}$ .
- (b) Compute compound annual growth rate between 2011 and 2014 defined by

$$g_{4,1} = \left(\frac{Y_4}{Y_1}\right)^{\frac{1}{3}} - 1.$$

Note: Since  $\frac{Y_4}{Y_1} = \frac{Y_4}{Y_3} \frac{Y_2}{Y_2} \frac{Y_2}{Y_1}$  holds, the compound annual growth rate is the geometric average of annual growth rates over multiple years.

(c) Observe that

$$\frac{1}{3}\left(\ln Y_4 - \ln Y_1\right)$$

gives a nice approximation to the compound annual growth rate.

[2] Effective interest rate.

Assume that a bank offers an annual, nominal interest rate of 6% **compounded monthly** and that you make a deposit of one thousand dollars (\$1,000) at the bank today.

- (a) How much do you expect to have in the bank account in one year from now?

  Assume that there is no other engagement with the bank before and after that deposit.
- (b) How much will you have after 2 years, 3 years, and t years?
- (c) Compute the annual effective rate of interest.
- (d) How do the above results change if the interest is compounded daily?

Answer sheet. Please write your name and id number.