Problem Set

MA17Q4-O

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OLG: Government purchases in the OLG model

Consider the following optimization problem.

$$\max_{c_t^Y, c_{t+1}^O, s_t} \frac{\left(c_t^Y\right)^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \frac{\left(c_{t+1}^O\right)^{1-\theta}}{1-\theta}$$

subject to

$$c_t^Y + s_t + G_t = w_t,$$
$$c_{t+1}^O = (1 + r_{t+1})s_t,$$

where G_t is the lump-sum tax. Assume

$$\theta = 1, \quad \text{and} \quad F(K, AL) = K^\alpha (AL)^{1-\alpha}.$$

1. Under the assumption that $A_{t+1} = (1+g)A_t$ and $L_{t+1} = (1+n)L_t$, derive the dynamic equation

$$\hat{k}_{t+1} = \frac{1}{(1+g)(1+n)(2+\rho)} \left[(1-\alpha)\hat{k}_t^\alpha - \hat{G}_t \right],$$

where $\hat{G} = G/A$ and $\hat{k} := K/(AL)$.

2. Suppose that the economy is on the balanced growth path (i.e., $\hat{k}_{t+1} = \hat{k}_t$) with $\hat{G}_t = 0$. At the beginning of period $t = 0$, the government puts a permanent tax increase in force, effective immediately. Describe what would happen after that.

3. Suppose that the economy is on the balanced growth path (i.e., $\hat{k}_{t+1} = \hat{k}_t$) with $\hat{G}_t = 0$. At the beginning of period $t = 0$, the government announces a permanent tax increase as of $t = 2$. Describe what would happen after this announcement.

Ricardian Equivalence

Consider the budget constraint for consumers

$$C(t) + \dot{S}(t) = r(t)S(t) + W(t) - T(t) \quad \text{with} \quad \lim_{t \to \infty} e^{-\int_0^t r(s)ds} S(t) \geq 0, \tag{1}$$

where C, S, r, W, T are consumption, savings, real interest rate, wage and tax, respectively, and that for the government

$$\dot{D}(t) = r(t)D(t) + G(t) - T(t) \quad \text{with} \quad \lim_{t \to \infty} e^{-\int_0^t r(s)ds} D(s) \leq 0, \tag{2}$$

where $G(t)$ and $D(t)$ are government purchases and govenment debt.

1. Use (1) to derive the lifetime budget constraint for the consumer.

2. Use (2) to derive the "lifetime" budget constraint for the govenment.

3. Show that taxes and debt financing have an equivalent effect on the consumer's total utility $\int_0^\infty e^{-\rho t} u(C(t))dt$. That is, the government's plan about how and when to raise tax in order to finance the govenment purchases $G(t)$ does not affect the utility.