Problem Set

MA17Q4-M

mail@**kenjisato**.jp

2018/1/25

Discrete-time Ramsey model

To make the model simple, assume that there is no population growth ($n = 0$) or techinical growth ($g = 0$). Assume further that $\delta = 1$. We omit hat ($\hat{}$) for simplicity.

The capital accumulation equation is

$$k_t = f(k_{t-1}) - c_t,$$

where k_t is the capital per capita for period t, c_t consumption per capita. The intensive form production function, f, assumes standard properties. The representative consumer solves

$$\max \sum_{t=1}^{\infty} \beta^{t-1} u(c_t) \quad \text{subject to } k_t = f(k_{t-1}) - c_t,$$

where $k_0 > 0$ is given and $\beta = \frac{1}{1+\rho}$ is the discount factor. We assume that $u(c) = \ln c$ and that $f(k) = k^\alpha$.

1. Derive the Euler condition:
$$\frac{c_{t+1}}{c_t} = \alpha\beta k_t^{\alpha-1}.$$

2. The dynamics of this economy is determined by the system of difference equations:
$$\begin{aligned} k_t &= k_{t-1}^{\alpha} - c_t \\ \frac{c_{t+1}}{c_t} &= \alpha\beta k_t^{\alpha-1}. \end{aligned}$$
Under our simplifying assumption, we can derive the Solow assumption of constant saving rate. That is
$$c_t = (1-s)k_{t-1}^{\alpha}.$$
This result gives a microeconomic foundation for the Solow model.
    a) Find an appropriate s such that the above consumption function satisfies the Euler equation.
    b) Show that the sequence $(c_t, k_t)_{t\geq 0}$ satisfies the terminal condition (known as Transversality Condition)
    $$\lim_{t\to\infty} \beta^t u'(c_t) k_t = 0.$$