Problem Set

MA17Q4-D

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[1] Perfectly competitive factor market

Let the production function $Y = F(K, AL)$ have constant returns to scale (CRS), i.e., $F(cK, cAL) = cF(K, AL)$ for any $c > 0$. The profit, measured in Unit/Year, can be written as

$$F(K, AL) - (r + \delta)K - wL,$$

where r and w are real rental and real wage, respectively.

a) Derive the first-order conditions for firms' profit maximization problem.

b) Euler's homogeneous function theorem states that CRS function $f(x, y)$ satisfies $f(x, y) = x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ (Details shown in homework). Use this theorem to show

$$Y = (r + \delta)K + wL,$$

which means that the firms earn zero (economic) profit.

[2] Growth Rates

Assume that Y (output), K (capital), A (technology) and L (labor force) grow at constant annual rate of g_Y, g_K, g_A, g_L, respectively. Compute the growth rate of ...

a) Effective labor, AL

b) Per-capita output, Y/L.

c) Per-capita capital, $(K/L)^\alpha$.

d) Output per unit of effective labor, $Y/(AL)$.

[3] Kaldor's fact

Recall Kaldor's stylized facts.

1. Output per worker Y/L grows at a sustained rate.
2. Capital per worker K/L grows at a sustained rate.
3. Rental rate $r + \delta$ (gross) is constant.
4. Capital output ratio K/Y is constant.
5. Capital share $(r + \delta)K/Y$ and labor share wL/Y are constant, where w is the wage rate.
6. Among the fast growing countries of the world, there is an appreciable variation in the rate of growth.

Verify that two of the above facts are redundant by proving the following statements.

a) Facts 1 and 4 imply Fact 2.

b) Facts 3 and 4 imply Fact 5.

Answer sheet. Please write your name and id number.