Artificial Intelligence Methods TDT4171- Assignment 1

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1 5-card Poker Hands

a)

We want to choose five cards from 52 possible, which can be found as

$$\frac{52!}{(52-5)!} = 52 \times 51 \times 50 \times 49 \times 48 \tag{1}$$

However, we are seeking the amount of distinct hands, and thus have to divide by the different ways the given hands can be ordered. Thus the amount of distinct hands is:

$$\frac{52!}{5!(52-5)!} = {52 \choose 5} = 2598960 \tag{2}$$

b)

As we have found the amount of distinct hands, the probability of drawing a given hand is

$$\frac{\text{desired outcomes}}{\text{possible outcomes}} = \frac{1}{2598960}$$
 (3)

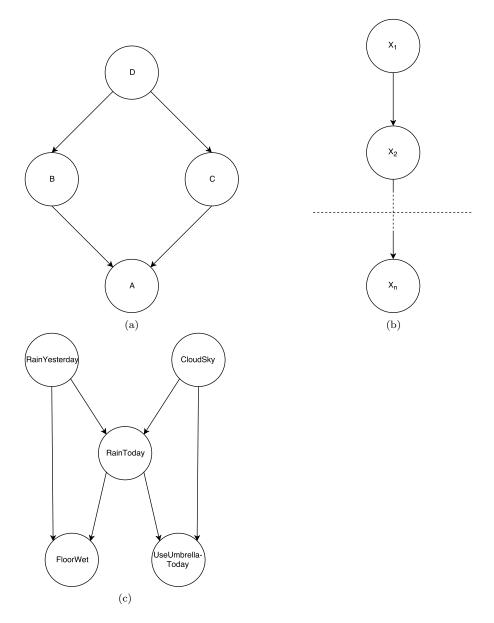
c)

There is only one royal straight flush and the probability of drawing is thus given by equation 3. With a four card hand there are 13 ways of getting four of a kind. Adding a fifth card to the hand will introduce 48 additional hands. Thus:

$$\frac{\text{desired outcomes}}{\text{possible outcomes}} = \frac{5 \times 48}{2598960} = \frac{240}{2598960}$$
 (4)

2 Bayesian Network Construction

a)



The independence structure between the variables can be taken advantage of to minimize the size of the probability tables.

For the problem in figure a) the first variable D is independent of all others. Thus the conditional probability table is given by a single line.

The two next events, B and C, both depend on D with conditional probability tables

$$\begin{array}{c|cccc} \mathbf{B} & \mathbf{D} & \mathbf{C} & \mathbf{D} \\ \hline p(B|D) & \mathbf{T} & & & p(C|D) & \mathbf{T} \\ p(B|\neg D) & \mathbf{F} & & p(C|\neg D) & \mathbf{F} \\ \end{array}$$

Finally, the event A depends on both B and C

| A | В | \mathbf{C} |
|-----------------------|--------------|--------------|
| p(A B,C) | Т | Τ |
| $p(A B, \neg C)$ | \mathbf{T} | \mathbf{F} |
| $p(A \neg B, C)$ | \mathbf{F} | \mathbf{T} |
| $p(A \neg B, \neg C)$ | \mathbf{F} | \mathbf{F} |

Except for the very first, every event in B depends solely on the event before. Thus the conditional probability tables are equal to the ones of B and C in a).

3 Bayesian Network Application

Figures 1 and 2 shows the network with and without supplying evidence.

Figure 3 shows the conditional probability table for Opened by official. It states that the official cannot open neither the door with the prize, nor can he open the door chosen by the player. In the event that the player has chosen the door with the prize, the official opens one of the remaining two doors with

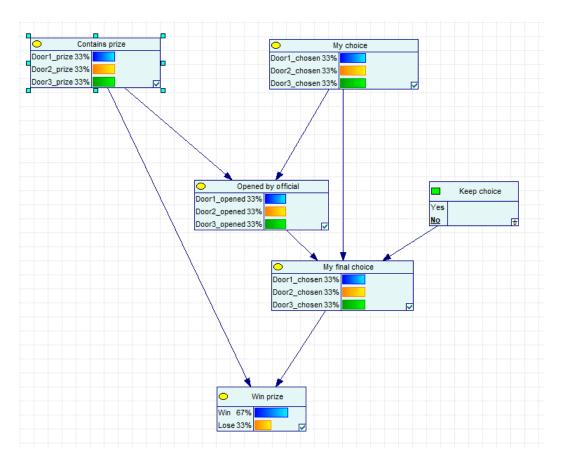


Figure 1: Changing the initial choice yields a better chance at winning

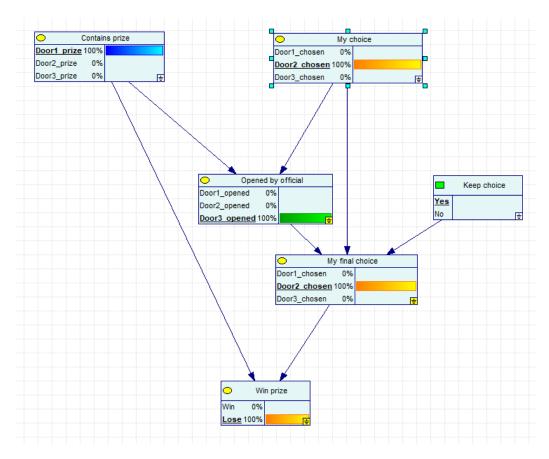


Figure 2: Correct guess with no change

equal probability.

| | Contains prize | | Door1_prize | | | Door2_prize | | □ Door3_prize | | | |
|---|----------------|-----------|-------------|-----------|-----------|-------------|-----------|---------------|-----------|-----------|--|
| | My choice | Door1_cho | Door2_cho | Door3_cho | Door1_cho | Door2_cho | Door3_cho | Door1_cho | Door2_cho | Door3_cho | |
| ſ | ▶ Door1_opened | 0 | 0 | 0 | 0 | 0.5 | 1 | 0 | 1 | 0.5 | |
| ſ | Door2_opened | 0.5 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0.5 | |
| Ī | Door3_opened | 0.5 | 1 | 0 | 1 | 0.5 | 0 | 0 | 0 | 0 | |

Figure 3: Conditional probability table for Opened by official

Figure 4 shows the conditional probability table of deciding to keep or change the initial guess. The event that the official opens the door chosen by the player will never happen and has thus been described by dummy values (0.333333). As one of the three doors have been opened at this point, changing or sticking with the initial choice is a binary action equivalent of deciding between two doors.

| My choice | □ Door1_chosen | | | | | | ☐ Door2_chosen | | | | | □ Door3_chosen | | | | | | |
|--------------------|---------------------|-----------|-----------|------------|-----------|-----------|----------------|-------------|-----------|------------|------------|----------------|-----------|-----------|------------|-----------|-----------|------------|
| Keep choice | Keep choice ☐ Yes ☐ | | | = | No | | = | Yes | | = | No | | = | Yes | | | No | |
| Opened by official | Door1_ope | Door2_ope | Door3_ope | Door1_ope | Door2_ope | Door3_ope | Door1_ope | . Door2_ope | Door3_ope | Door1_ope. | Door2_ope | Door3_ope | Door1_ope | Door2_ope | Door3_ope | Door1_ope | Door2_ope | Door3_ope |
| ▶ Door1_chosen | 0.33333333 |] 1 | 1 | 0.33333333 | 0 | 0 | (| 0.33333333 | 0 | (| 0.33333333 | 1 | 0 | 0 | 0.33333333 | 0 | 1 | 0.33333333 |
| Door2_chosen | 0.33333333 | 0 | 0 | 0.33333333 | 0 | 1 | 1 | 0.33333333 | 1 | (| 0.33333333 | 0 | 0 | 0 | 0.33333333 | 1 | 0 | 0.33333333 |
| Door3_chosen | 0.33333333 | 0 | 0 | 0.33333333 | 1 | 0 | (| 0.33333333 | 0 | | 0.33333333 | 0 | 1 | 1 | 0.33333333 | 0 | 0 | 0.33333333 |

Figure 4: Conditional probability table for Final choice

Figure 5 shows the conditional probability of winning or losing. It simply states that the player wins by choosing the door with the prize behind it while losing otherwise.

| | Contains prize | | Door1_prize | | Ξ | Door2_prize | | □ Door3_prize | | | |
|-----------------|----------------|-----------|-------------|-----------|-----------|-------------|-----------|---------------|-----------|-----------|--|
| My final choice | | Door1_cho | Door2_cho | Door3_cho | Door1_cho | Door2_cho | Door3_cho | Door1_cho | Door2_cho | Door3_cho | |
| D | Win | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | |
| | Lose | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | |

Figure 5: Conditional probability table for Win prize

From the calculations it is evident that changing the initial choice doubles the player's chances of winning.