**Sums of a random number of random variables**

Let we are given an i.i.d sequence $\{\xi_n\}$ (i.e., a sequence of independent and identically distributed non-negative integer random variables); thus, they all have the same generating function $Q_\xi(z)$. Let, in addition, there is a discrete random variable $\eta$ independent of $\{\xi_n\}$ and it is also non-negative integer r.v, let its generating function is $Q_\eta(z)$. Then we can consider the r.v. $\beta$, which on any outcome $\omega \in \Omega$ takes a value equal to the sum

$$\beta(\omega) = \xi_0(\omega) + \ldots + \xi_{\eta(\omega)}(\omega)$$

the number of summands is $\eta(\omega)$. Then

$$P(\beta = n) = \sum_{k=0}^{\infty} P(\eta = k) P(\xi_0 + \ldots + \xi_k = n)$$

We may decompose $(Q_\xi(z))^k$ and take there the coefficient of $z^n$ in order to find $P(\xi_0 + \ldots + \xi_k = n)$. Therefore

$$Q_\beta(z) = \sum_{k=0}^{\infty} P(\eta = k)(Q_\xi(z))^k = Q_\eta\left(Q_\xi(z)\right)$$

That is the composition of generating functions. For instance if $\xi_k$ are i.i.d indicators with $p$ и $q = 1 - p$, then $Q_\xi(z) = q + pz$ so $Q_\beta(z) = Q_\eta(q + pz)$

1. Let we are given the infinite sequence $\{\xi_i\}$ ,$i = 0, 1 \ldots$ of i.i.d indicators with parameter $p = \frac{1}{2}$ and also r.v. $\beta$ independent from them. Find the generating function for the $\sum_{k=0}^{\beta} \xi_k$ and its expectation if
   (a) $\beta$ is poissonian with $\lambda = 1$
   (b) $\beta$ is binomial with $n = 3$ $p = \frac{1}{2}$.
   (c) $\beta$ is geometric with $p = \frac{1}{2}$.
2. (Evolution model) Each plant has a large number of seeds, however, for individual seed is very unlikely to survive and develop, so it is reasonable to assume that the number of offspring of an individual plant has a Poisson distribution, say, with parameter $\lambda$. Let the distribution of the number of parent plants be a binomial distribution with parameters $n$, $p$, indicate the distribution of the number of offspring in the first generation.