Homework Temirlan Zholaman

N4) $\bar{A} \cup \bar{B} \cup \bar{C} = (\bar{A} \cup \bar{B}) \cup (\bar{C})$

- since $A \cdot B \cdot C = \emptyset$, we can say that
  (i) A, B, C are mutually exclusive
  (ii) there is an mutually exclusive set from two others (it could be either A, B or C).
- ~~if~~ suppose $A \cap B = \emptyset \Rightarrow \bar{A} \cup \bar{B} = U$, since
  $(A \cap B)' = \emptyset^c$
  $A^c \cup B^c = U$ (U is a complement of $\emptyset$)
- thus we may assume, that $\bar{A} \cup \bar{B} \cup \bar{C} = \emptyset^c = U$
  since $(\bar{A} \cup \bar{B} \cup \bar{C})^c = A \cap B \cap C = \emptyset \Rightarrow \emptyset^c = \bar{A} \cup \bar{B} \cup \bar{C} = U = \Omega$
- so for $P(\bar{A} \cup \bar{B} \cup \bar{C} | A) = \dfrac{P((\bar{A} \cup \bar{B} \cup \bar{C}) \cdot A)}{P(A)} = \dfrac{P(\Omega \cdot A)}{P(A)} =$
  $= \dfrac{P(A)}{P(A)} = 1$ (since $\Omega \cap A = A$)

$\therefore$ thus $\boxed{P(\bar{A} \cup \bar{B} \cup \bar{C} | A) = 1}$

N6) ~~$P(\bar{A}) = \frac{3}{4} \Rightarrow P(A) = \frac{1}{4}$ $P(B|A) = \frac{1}{3}$ find $P(A\bar{B})$~~

~~Sol'n~~

- ~~$P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{1}{3} \Rightarrow \dfrac{P(A \cap B)}{\frac{1}{4}} = \dfrac{1}{3} \Rightarrow P(A \cap B) = \dfrac{1}{12}$~~
- ~~$P(A \cap B) = P(A) + P(B) - P(A) \cdot P(B)$, let $P(B) = x \Rightarrow$~~
  ~~$\Rightarrow \frac{1}{4} + x - \frac{1}{4} x = \frac{1}{12}$~~
  ~~$\Rightarrow \frac{1}{4} + \frac{3}{4} x = \frac{1}{12}$~~

- $P(\bar{A}) = \frac{3}{4}$
  $\Rightarrow P(A) = \frac{1}{4}$
- $P(B|A) = \frac{1}{3}$
  $\Rightarrow P(\bar{B}|A) = \frac{2}{3}$
  ~~by~~ law of total probability

~~$P(A\bar{B}) = 1 - P(A\bar{B})' =$~~
~~$= 1 - P(\bar{A} \cup$~~

$P(\bar{B}|A) = \dfrac{P(A \cdot \bar{B})}{P(A)} \Rightarrow$

$\Rightarrow P(A\bar{B}) = P(\bar{B}|A) \cdot P(A)$
$= \frac{2}{3} \cdot \frac{1}{4} = \boxed{\frac{1}{6}}$