Temirlan Zholaman Harbour Space HW

Problem 3.31

14) $E(\alpha) = \sum_{i=1}^{n} p_i \alpha_i$

- $P(\alpha=-2) = \frac{1}{8} + \frac{1}{24} = \frac{1}{6}$
- $P(\alpha=-1) = \frac{1}{8} + \frac{1}{24} = \frac{1}{6}$
- $P(\alpha=0) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$
- $P(\alpha=1) = \frac{1}{8} + \frac{1}{24} = \frac{1}{6}$
- $P(\alpha=2) = \frac{1}{8} + \frac{1}{24} = \frac{1}{6}$

$P(\beta=-2) = \frac{1}{6}$

$P(\beta=-1) = \frac{1}{6}$

$P(\beta=0) = \frac{1}{3}$

$P(\beta=1) = \frac{1}{6}$

$P(\beta=2) = \frac{1}{6}$

$$E(\alpha) = \sum_{x \in \{-2,-1,0,1,2\}} P(\alpha = x) \cdot x$$

$$= -2 \cdot \frac{1}{6} + \left(-1 \cdot \frac{1}{6}\right) + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} = \boxed{0} = E[\alpha]$$

$E[\beta] = 0$ too

$E(\alpha^2) = 4 \cdot \frac{1}{6} + 1 \cdot \frac{1}{6} + 0 + 1 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} = \frac{10}{6} = \frac{5}{3}$

$E(\beta^2) = \frac{5}{3}$

$$Var(\alpha) = D(\alpha) = E(\alpha^2) - (E(\alpha))^2 =$$

$$= \frac{5}{3} - 0 = \frac{5}{3}$$

$Var(\beta) = \frac{5}{3}$ too

15) $Cov(\alpha, \beta) = m_{11}(\alpha, \beta) = E((\alpha - E(\alpha))(\beta - E(\beta)))$

$= E(\alpha\beta)$

$E(\alpha\beta) = 4\cdot\frac{1}{32} + 2\cdot\frac{1}{32} + 0 - 2\cdot\frac{1}{32} + 4\cdot\frac{1}{32} = 0$

$+ 2\cdot\frac{1}{32} + 1\cdot\frac{1}{32} + 0 - 1\cdot\frac{1}{32} - 2\cdot\frac{1}{32} = 0$

$+ 0 + 0 + 0 + 0 + 0 + 0 = 0$

$+ 2\cdot\frac{1}{32} - 1\cdot\frac{1}{32} + 0 + 1\cdot\frac{1}{32} + 2\cdot\frac{1}{32} = 0$

$- 4\cdot\frac{1}{32} + 2\cdot\frac{1}{32} + 0 + 2\cdot\frac{1}{32} + 4\cdot\frac{1}{32} = 0$

thus $E(\alpha\beta) = 0 \Rightarrow$

$\Rightarrow Cov(\alpha,\beta) = 0$

$\Rightarrow Correlation(\alpha,\beta) = 0$

4)

5)

Problem 1.5

let ξ & η be indep. $\Rightarrow$

$\Rightarrow P(\xi=a, \eta=b) = P(\xi=a)\cdot P(\eta=b)$

a) $P(\xi > \eta)$

$\Rightarrow \Omega = P(\xi,\eta)_{\xi>\eta} = P(-1,-2) + P(0,-2) + P(1,-2) + P(2,-2)$

$+ P(0,-1) + P(1,-1) + P(2,-1)$

$+ P(1,0) + P(2,0)$

$+ P(2,1)$

$= \frac{1}{64} + \frac{1}{32} + \frac{1}{64} + \frac{1}{32}$ $\quad \frac{3}{64}$

$+ \frac{1}{16} + \frac{1}{16} + \frac{1}{32} + \frac{1}{16}$ $\quad = \frac{7}{32} = \boxed{\frac{27}{64}}$

$+ \frac{1}{32} + \frac{1}{16}$ $\quad = \frac{3}{32}$

$+ \frac{1}{16}$ $\quad = \frac{1}{16}$

$\therefore P(\xi > \eta) = \boxed{\frac{27}{64}}$

4)

$\xi+\eta$	-4	-3	-2	-1	0	1	2	3	4
	$\frac{1}{32}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{11}{64}$	$\frac{6}{32}$	$\frac{11}{64}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{32}$

$\xi-\eta$	-4	-3	-2	-1	0	1	2	3	4
	$\frac{1}{32}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{11}{64}$	$\frac{6}{32}$	$\frac{11}{64}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{32}$

5) $E(\xi+\eta) = E(\xi) + E(\eta) =$

$= 0 + 0 = \boxed{0}$

$E(\xi-\eta) = E(\xi) - E(\eta) = \boxed{0}$

$Var(\xi+\eta) = E((\xi+\eta)^2) - E[\xi+\eta]^2 =$

P(3,2)

$= E((\xi+\eta)^2) =$

$= \left[16\cdot\frac{1}{32} + 9\cdot\frac{3}{32} + 4\cdot\frac{1}{8} + 1\cdot\frac{1}{64}\right]\cdot 2$ (by symmetry)

$= \left[\frac{1}{2} + \frac{27}{32} + \frac{1}{2} + \frac{1}{64}\right]\cdot 2 =$

$= 2 + \frac{1}{32} + \frac{27}{16} = \frac{64}{32} + \frac{1}{32} + \frac{54}{32} =$

$= \frac{119}{32} = \boxed{3\frac{23}{32}}$

$Var(\xi+\eta) = \boxed{3\frac{23}{32}}$

$Var(\xi-\eta) = \boxed{3\frac{23}{32}}$

b) $Cov(\varepsilon+\eta, \varepsilon-\eta) =$

$$= E\left((\varepsilon+\eta - E(\varepsilon+\eta))(\varepsilon-\eta - E[\varepsilon-\eta])\right) =$$

$$= E\left((\varepsilon+\eta)(\varepsilon-\eta)\right) =$$

$$= E(\varepsilon^2 - \eta^2) = E(\varepsilon^2) - E(\eta^2)$$

$$= 3\frac{25}{32} - 3\frac{23}{32} = 0$$

$$\boxed{Cov(\varepsilon+\eta, \varepsilon-\eta) = 0}$$

$$\boxed{Cov(\varepsilon+\eta, \varepsilon-\eta) = 0}$$

Done!