

- 1) Probability to end at M after N stops
- 2) Probability to stay positive at all times
- 3) Find the probability to spend all the time near 0 (in $[0, \pm k]$), for a given k

1) $\frac{C(N, k)}{2^N}$

Homework

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a) $P(\min(x_1, x_2)) < \frac{1}{3} \Rightarrow$

$\Rightarrow P(x_1 < \frac{1}{3} \text{ or } x_2 < \frac{1}{3}) =$

$= 1 - (P(x_1 > \frac{1}{3}) \text{ and } P(x_2 > \frac{1}{3})) =$

$= 1 - (P(x > \frac{1}{3}) \text{ and } P(x > \frac{1}{3})) =$

$= 1 - \left(\frac{608}{1000} \cdot \frac{608}{1001} \right) = \boxed{0.63107}$

b) $P(\text{exactly } 2) =$

$= 1 - \left(C(4, 0) \left(\frac{1}{3}\right)^4 + C(4, 1) \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right) + C(4, 2) \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2 + C(4, 3) \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^3 + C(4, 4) \left(\frac{2}{3}\right)^4 \right)$

$= 1 - \frac{57}{81} = \frac{24}{81} = \boxed{\frac{8}{27}}$

3)

A - disease

B - test positive

$P(A) = 0.1\% = \frac{1}{1000}$

$P(B|A) = \frac{9}{10} = \frac{P(A \cdot B)}{P(A)}$

$P(B|\bar{A}) = \frac{1}{100} = \frac{P(\bar{A} \cdot B)}{P(\bar{A})} = \frac{1}{1000}$

$P(A \cdot B) = \frac{9}{10000}$

$P(A|B) = \frac{P(A \cdot B)}{P(B)}$

(N3)

$$P(A) = \frac{1}{1000}$$

$$P(B|A) = 90\% = \frac{9}{10}$$

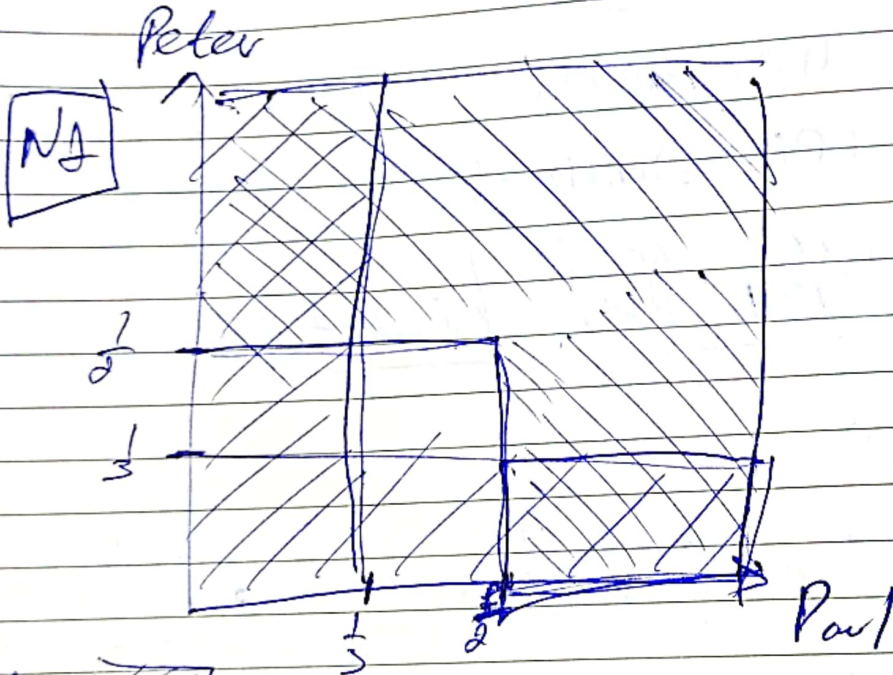
$$P(B|\bar{A}) = 1\% = \frac{1}{100}$$

find $P(A|B)$

Solution

$$P(\text{disease}) = \frac{1}{1000}$$

$$P(\text{positive} | \text{disease}) = \frac{9}{10}$$



$$A = \text{shaded region with } \swarrow \text{ lines}$$

$$B = \text{shaded region with } \searrow \text{ lines}$$

$$P(A \cap B) = 2 \left(\frac{1}{3} \cdot \frac{1}{2} \right) = \frac{1}{3}$$

$$P(A) = 1 - P(\bar{A}) = \frac{4}{9} = \frac{5}{9}$$

$$P(B) = 1 - P(\bar{B}) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$(II) P(A \cap B) = P(A) \cdot P(B) \Rightarrow A \text{ \& B indep.}$$

$$\frac{5}{9} \neq \frac{5}{9} \cdot \frac{3}{4} = \frac{5}{12} \Rightarrow A \text{ \& B dependent}$$

