Temirlan Zholaman [Harbour Space]

Problem 1.2

$$P(\xi < C) = p_1,\quad P(\eta < C) = p_2$$

1) since $\xi, \eta : \Omega \to \mathbb{R}$ independent, =>
=> $P(\xi < C \cap \eta < C) = P(\xi < C) \cdot P(\eta < C) = p_1 \cdot p_2$

$$P(\max(\xi,\eta) < C) = P(\xi < C \;\boxed{\text{and}}\; \eta < C) =$$
$$= P(\xi < C) \cdot P(\eta < C) = p_1 \cdot p_2$$

2) $P(\min(\xi,\eta) < C) = P(\xi < C \text{ or } \eta < C) = P((\xi < C) \cup (\eta < C)) =$
$= P(\xi < C) + P(\eta < C) - P(\xi < C \text{ and } \eta < C) =$
$= p_1 + p_2 - p_1 \cdot p_2$

3) $P(\max(\xi,\eta) \geq C) = P(\xi \geq C \text{ or } \eta \geq C) =$
$= P((\xi \geq C) \cup (\eta \geq C)) = (1-p_1) + (1-p_2) - (1-p_1)(1-p_2)$

4) $P(\min(\xi,\eta) \geq C) = P(\xi \geq C \text{ and } \eta \geq C) =$
$= P(\xi \geq C \cap \eta \geq C) = P(\xi \geq C) \cdot P(\eta \geq C) =$
$= (1-p_1)(1-p_2)$

Problem 1.3

1) $P(\alpha = \beta) = \frac{1}{32} + \frac{1}{32} + \frac{1}{6} + \frac{1}{32} + \frac{1}{32} =$
$= \frac{4}{32} + \frac{1}{6} = \frac{1}{8} + \frac{1}{6} = \frac{14}{48} = \boxed{\frac{7}{24}}$

2) $P(\alpha > \beta) = P(\alpha, \beta) = P(-1, -2) +$
$+ P(0, -1) + P(0, -2) +$
$+ P(1, 0) + P(1, -1) + P(1, -2)$
$+ P(2, 1) + P(2, 0) + P(2, -1) + P(2, -2)$
$= \frac{1}{32} + \frac{1}{24} + \frac{1}{24} + \frac{1}{32} + \frac{1}{32} + \frac{1}{24} + \frac{1}{32} + \frac{1}{32} + \frac{1}{24} + \frac{1}{32}$
$= \frac{6}{32} + \frac{4}{24} = \frac{3}{16} + \frac{1}{6} = \frac{9}{48} + \frac{8}{48} = \boxed{\frac{17}{48}}$

3) $P(\alpha \le \beta) = P(\alpha, \beta) = P(-2,-2) + P(-2,-1) + P(-2,0) + P(-2,1) + P(-2,2)$

$+ P(-1,-1) + P(-1,0) + P(-1,1) + P(-1,2) +$

$+ P(0,0) + P(0,1) + P(0,2)$

$+ P(1,1) + P(1,2)$

$+ P(2,2)$

$= \frac{1}{32} + \frac{1}{32} + \frac{1}{24} + \frac{1}{32} + \frac{1}{32} + \frac{1}{32} + \frac{1}{24} + \frac{1}{32} + \frac{1}{32} + \frac{1}{6} + \frac{1}{24} + \frac{1}{24} +$

$+ \frac{1}{32} + \frac{1}{32} + \frac{1}{32} = \frac{10}{32} + \frac{4}{24} + \frac{1}{6} = \frac{5}{16} + \frac{1}{6} + \frac{1}{6} =$

$= \frac{5}{16} + \frac{1}{3} = \frac{15+16}{48} =$

$= \boxed{\frac{31}{48}}$

4) $P(|\alpha| > |\beta|) = P(\alpha, \beta) = P(-2,-1) + P(-2,0) + P(-2,1)$

$+ P(2,-1) + P(-2,0) + P(2,1)$

$+ P(1,0) + P(-1,0)$

$= \underbrace{\frac{1}{32} + \frac{1}{32} + \frac{1}{24}}\ + \underbrace{\frac{1}{32} + \frac{1}{32} + \frac{1}{24}} + \frac{1}{24} + \frac{1}{24}$

$= \frac{4}{64} + \frac{4}{24} = \frac{1}{16} + \frac{1}{6} = \frac{3+8}{48} = \frac{11}{48} =$

$\boxed{= \frac{11}{48}}$

5) $P(\max(\alpha,\beta) > 0) = P(\alpha,\beta) = 1 - P(\alpha, \beta \le 0)$ both

$= 1 - (P(0,0) + P(0,-1) + P(0,-2) +$

$+ P(-1,-1) + P(-1,-2)$

$+ P(-2, -1$

$= 1 - \begin{pmatrix} P(0,0) + P(0,-1) + P(0,-2) \\ P(-1,0) + P(-1,-1) + P(-1,-2) \\ P(-2,0) + P(-2,-1) + P(-2,-2) \end{pmatrix} =$

$= 1 - \left(\frac{1}{6} + \frac{1}{24} + \frac{1}{24} + \frac{1}{24} + \frac{1}{32} + \frac{1}{32} + \frac{1}{24} + \frac{1}{32} + \frac{1}{32} \right) =$

$= 1 - \left(\frac{1}{6} + \frac{4}{24} + \frac{4}{32} \right) = 1 - \left(\frac{1}{6} + \frac{1}{6} + \frac{1}{8} \right) = 1 - \left(\frac{11}{24} \right) =$

$= \frac{13}{24}$

Problem 1.1

1.) $1_A \cdot 1_B = 1_{AB}$

Define

$$1_A = \begin{cases} 1 & \text{if } \varepsilon \in A \\ 0 & \text{if } \varepsilon \notin A \end{cases}$$

$$1_B = \begin{cases} 1 & \text{if } \varepsilon \in B \\ 0 & \text{if } \varepsilon \notin B \end{cases}$$

statement #1 is true, since

$$1_{A \cdot B} = \begin{cases} 1 & \text{if } \varepsilon \in A \cdot B \\ 0 & \text{if } \varepsilon \notin A \cdot B \end{cases}$$

& so given $\varepsilon \in A,\ \varepsilon \notin B = 1_A \cdot 1_B = 0 = 1_{AB}$

$\varepsilon \in A,\ \varepsilon \in B = 1_A \cdot 1_B = 1 = 1_{AB}$

$\varepsilon \notin A,\ \varepsilon \notin B = 1_A \cdot 1_B = 0 = 0 = 1_{AB}$

$\varepsilon \notin A,\ \varepsilon \in B = 1_A \cdot 1_B = 0 = 0 = 1_{AB}$

2) True, since

$$\boxed{\begin{array}{l} 1_A \cdot 1_A = 0 \text{ if } 1_A = 0 \\ 1_A \cdot 1_A = 1 \text{ if } 1_A = 1 \end{array}}$$

3) $1_A + 1_B = 1_{A+B}$ false

since $1_A + 1_B = 2$, when

$\varepsilon \in A \cdot B \neq \boxed{1_{A+B}}$

4) $1_A \cdot (1 - 1_B) = 1_{A \setminus B}$ True

since $\boxed{\text{case 1}}$ $\varepsilon \in A,\ \varepsilon \in B \Rightarrow$

$1_A \cdot (1 - 1_B) = 0 = 1_{A \setminus B}$

$\boxed{\text{case 2}}$ $\varepsilon \in A,\ \varepsilon \notin B \Rightarrow$

$\Rightarrow 1_A \cdot (1 - 1_B) = 1 = 1_{A \setminus B}$

$\boxed{\text{case 3}}$ $\xi \notin A$, $\xi \in B \Rightarrow$

$\Rightarrow 1_A(1 - 1_B) = 0 = 1_{A \setminus B}$

$\boxed{\text{case 4}}$ $\xi \notin A$, $\xi \notin B \Rightarrow$

$\Rightarrow 1_A(1 - 1_B) = 0 = 1_{A \setminus B} \Rightarrow$

$\Rightarrow \boxed{\text{valid}}$

Conclusion:

$$\boxed{\begin{array}{l} 1)\ \text{valid} \\ 2)\ \text{valid} \\ 3)\ \text{invalid} \\ 4)\ \text{valid} \end{array}}$$