Harbour Space HW#13 Probability T Zholaman

Problem 2

1) $p(t) = \begin{cases} 0 & \text{otherwise} \\ \frac{1}{b-a} & \text{if } t \in [a,b] \end{cases}$

$F(t) = \int \frac{1}{b-a} dt = \frac{t}{b-a} + c$

we know that $F(t)\Big]_a^b = 1 \Rightarrow \left[\frac{t}{b-a} + c\right]_a^b = 1 \Rightarrow$

$\Rightarrow$ CDF is $\begin{cases} \frac{t-a}{b-a} & \text{if } x \in [a,b] \\ 0 & \text{if } x < a \\ 1 & \text{if } x > b \end{cases}$

2) pdf is $\begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

cdf is $\begin{cases} 1 - \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

3) $pdf = \frac{\theta}{\pi(t^2+\theta^2)}$

$cdf = \frac{1}{2} + \frac{1}{\pi} \arctan\left(\frac{t}{\theta}\right)$

Problem 3

$E[X] = \sum_{i=1}^{6} p_i \cdot x_i = \frac{1}{6}(1+2+3+4+5+6) = \boxed{3.5}$

Problem 4

since X is a Poisson distribution

$P(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$

$E[X]$ by theorem is $\boxed{\lambda}$

Problem 5|

$$MLE(\theta) = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i} =$$

$$\log MLE = \sum_{i=1}^{n} \log p^{x_i}(1-p)^{1-x_i} =$$

$$= \sum_{i=1}^{n} x_i(\log p) + (1-x_i)\log(1-p) =$$

$$= Y\log p + (n-Y)\log(1-p), \quad Y = \sum_{i=1}^{n} x_i$$

to maximize - FOC

$$\frac{d \log MLE}{dp} = Y\frac{1}{p} + (n-Y)\frac{-1}{1-p} = 0$$

$$\hat{p} = \frac{Y}{n} = \frac{\sum_{i=1}^{n} x_i}{n} \quad \boxed{\text{mean}}$$

Problem 6

since discrete values, Assume that and $P(b-) = P(b-1)$

1) $P([a,b]) = F(b) - F(a-)$

since $F(a-) = \sum_{i=1}^{a-1} P(i)$

$F(b) = \sum_{i=1}^{b} P(i)$ $\Rightarrow F(b) - F(a-) =$

$= \sum_{i=1}^{b} P(i) - \sum_{i=1}^{a-1} =$

$= \sum_{a}^{b} P(i)$ as $b \geq a$

$= P([a,b])$ ✓

$\boxed{\text{equal}}$ or $\boxed{\text{True}}$

2) $P((a,b)) = F(b-) - F(a)$

$\sum_{i=1}^{b-1} P(i) - \sum_{i=1}^{a} = \sum_{a-1}^{b-1} P(i) = P((a,b))$ $\boxed{\text{True}}$

3) $P([a,b)) = F(b-) - F(a-)$

$\sum_{i=1}^{b-1} P(i) - \sum_{i=1}^{a-1} P = \sum_{a}^{b-1} P(i) = P([a,b))$ $\rightarrow \boxed{\text{True}}$

4) $P(\{x\}) = F(x) - F(x-)$

$\hookrightarrow P(\{x\}) = \sum_{i=1}^{x} P(i) - \sum_{i=1}^{x-1} P(i) = P(x) = (P = x)$

$\boxed{\text{True}}$, ✓