1) $\begin{vmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 1 & 0 & 1 & 1 \\ 1 & 3 & 3 & 0 \end{vmatrix} \xrightarrow{R_1 \leftrightarrow R_4} \begin{vmatrix} 1 & 3 & 3 & 0 \\ 1 & 2 & 4 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{vmatrix} \to \begin{vmatrix} 1 & 3 & 3 & 0 \\ 0 & 1 & -1 & -2 \\ 0 & 3 & 2 & -1 \\ 0 & 1 & 2 & 1 \end{vmatrix} \to \begin{vmatrix} 1 & 3 & 3 & 0 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -3 & -3 \end{vmatrix} \to$

$\to \begin{matrix} 1 & 3 & 3 & 0 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$ pivots in colums 1,2,3 $\Rightarrow$ 3 indep vectors in $\beta(v_1, v_2, v_3, v_4) \Rightarrow$

$\Rightarrow \dim = 3$

2) pivot ~~cols~~ 1,2,3 $\Rightarrow \beta = [v_1, v_2, v_3]$

✓ v_1 & v_2 indep since in v_1 $a_{11} = 0$, in v_2 $a_{12} = 1$

✓ v_1 & v_2 & v_4 check

$\begin{matrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 0 & 1 \\ 1 & 3 & 0 \end{matrix} \to$ independent

✗ v_2 & v_3 & v_4 dependent since $v_3 = v_2 + v_4$

✓ v_1 v_3 v_4 - indep

$\begin{matrix} 0 & 2 & 1 \\ 1 & 4 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & 0 \end{matrix}$ - independent since to get 0 in v_4 at row 4 we should have $-3v_1 + v_3 = \begin{matrix} 2 \\ 1 \\ -2 \\ 0 \end{matrix} \neq v_4$ and is not a scale of v_4

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3) since all e_1, e_2, e_3, e_4 ~~independent in span~~ & e_1, e_2, e_3 are independent, we check if for i in $\{e_1, e_2, e_3, e_4\}$ $[e_i\, u_1, u_2]$ - independent

$\cancel{e_1 \begin{bmatrix} 1 & 3 & 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 & 0 & 1 \end{bmatrix} = 0 \to \text{reducible}}$ as

$\cancel{\left[\begin{matrix} 1 & 0 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 & 4 & 1 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 4 \end{matrix}\middle|\begin{matrix} 0 \\ \vdots \\ \vdots \\ \vdots \end{matrix}\right]}$ since

check for each e_i

1) e_1 $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 4 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ - independent (obs...)

e_2 $\begin{bmatrix} 0 & 1 & 3 \\ 1 & 4 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ - independent by obs

e_3 $\begin{bmatrix} 0 & 1 & 3 \\ 0 & 4 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ - independent by obs

e_4 $\begin{bmatrix} 0 & 1 & 3 \\ 0 & 4 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ - independent by obs

Distance

$d(a, x_1) = \sqrt{-1^2 + -2^2 + -2^2} = \boxed{\sqrt{1+4+4} = 3}$ (x₁)

$d(a, x_2) = \sqrt{0 \quad 0 \quad 5^2} = 5$

$d(a, x_3) = \sqrt{8^2 \quad -3^2 \quad -6^2} =$ — too big ...

$d(a, x_4) = \sqrt{0 \quad +3^2 + 4^2} = 5$

$\cos\omega = \frac{dot}{norm} = ($

$\cos\omega_1 = \cos(\angle a\, x_1) = \frac{d(a, x_1)}{\|a\|_2 \|x_1\|_2} = \frac{39}{\sqrt{26} \cdot \sqrt{61}}$ arccos $=$ ~~0.334~~ π 0.203π

$\cos\omega_2 = \cos(\angle a, x_2) = \frac{d(a, x_2)}{\|a\|_2 \|x_2\|_2} = \frac{40}{\sqrt{26} \cdot \sqrt{81}} = 0.331\pi$

$\cos\omega_3 = \cos(a, x_3) = \frac{d(a, x_3)}{\|a\|_2 \|x_3\|_2} = \frac{47}{\sqrt{26} \cdot \sqrt{117}} = 0.5507\pi$

$\cos\omega_4 = \cos(a, x_4) = \frac{d(a, x_4)}{\|a\|_2 \|x_4\|} = \frac{13}{\sqrt{26}\,\sqrt{25}} = \boxed{1.03\pi \rightarrow \text{largest}}$

$\boxed{x_4 \text{ largest } \omega}$

Det

- $\boxed{\text{Det}} \left[\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix} \begin{matrix} 0 & 1 \\ 3 & 4 \\ 6 & 7 \end{matrix} \right] = 0 + 30 + 42 - 48 - 0 - 24 =$

$= 0$

- eigenvals

$[[2,$

$\begin{matrix} 2 & 7 \\ 7 & 2 \end{matrix}$

char form: $\det \begin{bmatrix} 2-\lambda & 7 \\ 7 & 2-\lambda \end{bmatrix} = 0 \Rightarrow$

$\Rightarrow (2-\lambda)^2 - 49 = 0$

$4 - 2\lambda + \lambda^2 - 49 = 0$

$\cancel{\lambda^2} + \lambda^2 - 4\lambda - 45 = 0$

$(\lambda + 5)(\lambda - 9) = 0 \quad \lambda_1 = -5$

$\lambda_2 = 9$

- eigen vectors

$\lambda_1 = -5$

$\begin{bmatrix} 7 & 7 \\ 7 & 7 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \quad x_1 + x_2 = 0 \Rightarrow x_1 = -x_2$

smallest possible $\boxed{x_1 = 0, x_2 = 0}$

or $\left[\begin{matrix} x_1 = 1 \\ x_2 = -1 \end{matrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right]$

$\lambda_2 = 9$

$\begin{bmatrix} -7 & 7 \\ 7 & -7 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \to \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \quad x_1 - x_2 = 0$

$x_1 = x_2$

$\begin{matrix} x_1 = 1 \\ x_2 = 1 \end{matrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

- Inverse

$A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} \qquad A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$

$\det(A) \neq 0 \Rightarrow A^{-1}$ exist

$\det A = 2 \cdot 5 - 3 \cdot 3 = 1 \qquad A^{-1} = 1 \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$