Temirlan Zholaman Final Exam

(2)

$$F(X) = \begin{cases} 0, & x < -2 \\ \frac{1}{5}, & -2 \le x < 1 \\ \frac{x^2}{4}, & 1 \le x < 2 \\ 1, & x \ge 2 \end{cases}$$ , the pdf of $F(x)$ is,

pdf, $f(x) = \begin{cases} \frac{1}{2}x & \text{if } x \in [1;2] \\ 0 & \text{otherwise} \end{cases}$

$$E[x] = \int_{-\infty}^{\infty} x f(x) = \int_1^2 x \frac{1}{2} x = \left[\frac{x^3}{6}\right]_1^2 = \frac{8}{6} - \frac{1}{6} = \boxed{\frac{7}{6}}$$

Answer: $\boxed{E(x] = \frac{7}{6}}$

(6) find $T_7$ at $x_0 = 0$, for $f(x) = \sqrt{1+\cos x}$

$$T_7 = \sum_{n=0}^{7} \frac{f^n(x_0)}{n!}(x-x_0)^n$$

$f(0) = \sqrt{\cos(0)+1} = \sqrt{2}$

$f'(0) = 0$

$f''(0) = -\frac{\sqrt{2}}{4}$

$f'''(0) = 0$ (due to cos derivatives)

$f^{4}(0) = \frac{\sqrt{2}}{16}$

$f^{5}(0)$ $0$

$f^{6}(0) = -\frac{\sqrt{2}}{64}$ $\Rightarrow$

$f^{7}(0) = 0$

$$\Rightarrow T_7 = \frac{\sqrt{2}}{0!}x^0 + \frac{0}{1!}x^1 + \frac{-\frac{\sqrt{2}}{4}}{2!}x^2 + \frac{0}{3!}x^3 +$$

$$+ \frac{\frac{\sqrt{2}}{16}}{4!}x^4 + \frac{0}{5!}x^5 + \frac{-\frac{\sqrt{2}}{64}}{6!}x^6 + \frac{0}{7!}x^7 =$$

$$\boxed{= \sqrt{2} + \frac{\sqrt{2}x^2}{8} + \frac{\sqrt{2}}{384}x^4 + \frac{\sqrt{2}x^6}{46080}}$$

7)

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 9 \\ 3 & 2 & 1 & 0 \\ 9 & 6 & 5 & 4 \end{bmatrix} \xrightarrow{R_4 \leftrightarrow R_1} \begin{bmatrix} 9 & 6 & 5 & 4 \\ 4 & 5 & 6 & 9 \\ 3 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix} \rightarrow$$

$$\Rightarrow \begin{matrix} R_2 = 4R_1 - 9R_2 \\ R_3 = R_1 - 3R_3 \end{matrix} \rightarrow \begin{bmatrix} 9 & 6 & 5 & 4 \\ 0 & -1 & -34 & -65 \\ 0 & 0 & 2 & 4 \\ 0 & 1 & 2 & 3 \end{bmatrix} \rightarrow R_4 = R_3 + R_4$$

$$\rightarrow \begin{bmatrix} 9 & 6 & 5 & 4 \\ 0 & -1 & -34 & -65 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & -32 & -62 \end{bmatrix} \quad R_4 = 16R_3 + R_4$$

$$\rightarrow \begin{bmatrix} 9 & 6 & 5 & 4 \\ 0 & -1 & -34 & -65 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix} \Rightarrow$$

Rank = 4
since 4 pivot rows

$$E[\xi^n] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \xi^n e^{\left(-\frac{\xi^2}{2}\right)}$$

$$= \frac{1}{\sqrt{2\pi}} \left[ (n-1) \xi^{n-2} \xi e^{-\frac{\xi^2}{2}} \right]_{-\infty}^{\infty} - \frac{1}{\sqrt{2\pi}} \int_{-\infty} -(n-1) \xi^{n-2} e^{-\frac{\xi^2}{2}} d\xi$$

$$\Rightarrow \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (n-1) \xi^{n-2} \cdot e^{-\frac{\xi^2}{2}} d\xi$$

$$\Rightarrow \frac{1}{\sqrt{2}\sqrt{\pi}} \int_{-\infty}^{\infty} (n-1) e^{-\frac{\xi^2}{2}} d\xi =$$

$$= (n-1)$$

3) 1) $p(B|A) + p(B|\bar{A}) = 1$

$$= \frac{P(A|B) \cdot P(B)}{P(A)} = 1 - p(B|\bar{A})$$

$p(\bar{B}|A) = p(\bar{B}|\bar{A})$

which is clearly not true

and there are counter examples

Let B = rain

let A = sunny day

~~p(rain | sunny day)~~ $\neq$ p(not rain | not sunny day)

2) $P(B|A) + P(\bar{B}|\bar{A}) = 1$

$P(B|A) = 1 - P(\bar{B}|\bar{A})$

$P(B|A) = P(B|\bar{A})$

which is clearly not true

example Let $B$ = rain

$A$ = sunny day

$P(\text{rain}|\text{sunny day}) \neq P(\text{rain}|\text{not sunny day})$

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$\boxed{\text{Def'n}}$ Vector space

set of vectors is vector space iff

1)
- $\forall\, u, v \in V,\ u + v \in V$
- $\forall\, c \in \mathbb{R}, \forall v \in V,\ c \cdot v \in V$

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2) Limit of a function

$\lim_{x \to x_0} f(x) = L$ iff

$\forall\, \epsilon > 0,\ \exists\, \delta > 0$ s.t. $\forall x$

$0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \epsilon$

3) Linear transform

A matrix $A$ is a linear transform $T$, iff

$T: V \to W$, $\forall u_1, u_2 \in V$, $a, b \in \mathbb{R}^n$

$$T(au_1 + bu_2) = aT(u_1) + bT(u_2)$$

4) CDF

The CDF of r.v. $X$ is defined as

$$F_X(x) = P(X \leq x) \ \forall x \in \mathbb{R}$$