$$\binom{n}{\frac{n+m}{2}} 2^{-n}$$

Temirlan Zholanov   Harbour Space

. Prog, that $P(S_n=m) = \binom{n}{\frac{n+m}{2}} 2^{-n}$

Since we assume that m is reachable, we should have some unique k & l, where k = num of paths (1) & l = num of paths (-1). Since their order doesn't matter, we may find $\binom{k+l}{l}$ or $\binom{k+l}{k}$ paths for a given reachable m, Note that $k+l=m$.

thus $N(n,m) = \binom{k+l}{n} = \binom{t}{\frac{t+m}{2}} = \binom{n}{\frac{n+m}{2}} = \binom{n}{\frac{n-m}{2}}$

since $\boxed{n - \frac{n+m}{2} = \frac{n-m}{2}}$

As we have 2^{-n} path's,

$$P(S_n=m) = N(n,m)\cdot 2^{-n} = \frac{\binom{n}{\frac{n-m}{2}}}{2^n}$$

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