Iterative Correction of Sensor Degradation and a Bayesian Multi-Sensor Data Fusion Method applied to TSI Data from Pmo6-V Radiometers

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MOTIVATION

Observation and measurement of Sun's activity provide key and irreplaceable insights into our understanding of the Universe and with greater knowledge of the Universe we understand ourselves and Life itself better. Total Solar Irradiance (TSI) is one of these measurements and describes the amount of incoming energy from the Sun to Earth.

TSI is measured by two instruments A and B mounted on a spacecraft orbiting around the Sun. Each measurement of TSI incurs some damage to the instrument, i.e. the instrument degrades as it is exposed to Sun's irradiance. Question to be answered is: *Is it possible to recover the ground-truth TSI from two differently degraded instruments A and B?*

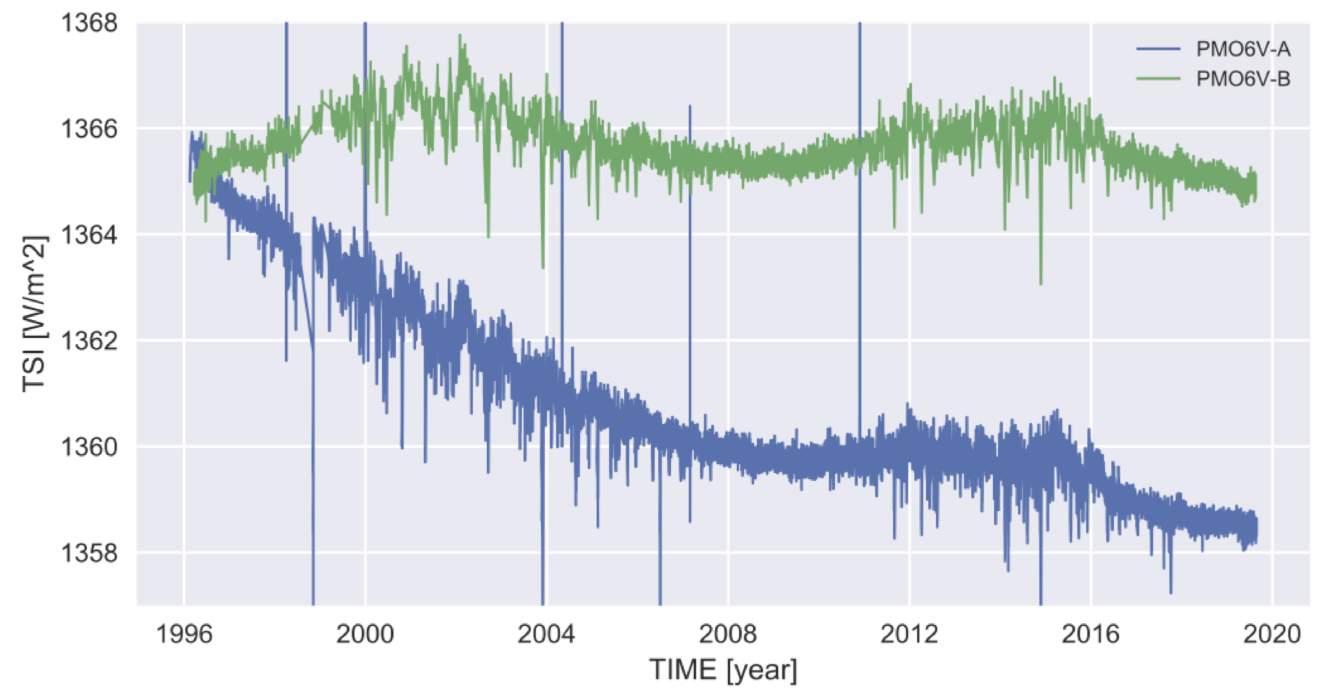


Figure 1: Initial TSI data from PMO6V-A and PMO6V-B radiometers on SOHO spacecraft.

ITERATIVE CORRECTION

Given two time series of measurements, $a[1], \ldots, a[n_a]$ and $b[1], \ldots, b[n_b]$ ($n_b \ll n_a$) and knowing that

$$a(t) = s(t) \cdot d(e_a(t)) + \varepsilon_a(t), \;\; \varepsilon_a(t) \sim \mathcal{N}(0, \sigma_a^2)$$
$$b(t) = s(t) \cdot d(e_b(t)) + \varepsilon_b(t), \;\; \varepsilon_b(t) \sim \mathcal{N}(0, \sigma_b^2)$$

holds, can we retrieve s? Degradation function is denoted by d and exposure functions of instruments A and B by e_a and e_b respectively.

- Correct them iteratively as described in CORRECTBOTH.
- Correction algorithm converges very quickly. Convergence in the noise free case is guaranteed by Theorem 1.

Algorithm 1 CORRECTBOTH(a, b, e_a, e_b)

while not converged **do**
  $r \leftarrow \frac{a}{b}$
  $f(\cdot) \leftarrow$ FITCURVETO(e_a, r) ▷ Learn $f : e_a \mapsto f(e_a)$
  $a \leftarrow \frac{a}{f(e_a)}$; $b \leftarrow \frac{b}{f(e_b)}$ ▷ Correction update
end while
$a_c \leftarrow a$; $b_c \leftarrow b$ ▷ $a_c(t) \approx s(t)$ and $b_c(t) \approx s(t)$
$r_c \leftarrow \frac{a}{b_c}$ ▷ $r_c(e_a) \approx d(e_a)$
$d_c(\cdot) \leftarrow$ FITCURVETO(e_a, r_c) ▷ Get $d(\cdot)$
return a_c, b_c, $d_c(\cdot)$

SMOOTHED MONOTONIC REGRESSION

Degradation is a smooth, monotonically non-increasing function with $d(0) = 1$.

- For fitting use smoothed monotonic regression to enforce constraints.

$$\min_{\boldsymbol{\theta}} \sum_{i=1}^{n} (\theta_i - y_i)^2 + \sum_{i=1}^{n-1} (\theta_{i+1} - \theta_i)^2$$
$$\text{s. t. } \theta_i \geq \theta_{i+1} \text{ for } i \in \{1, \ldots, n-1\} \text{ and } \theta_1 = 1.$$

CONVERGENCE

Theorem 1. *Let $a_0(t) = s(t) \cdot d(e_a(t))$ and $b_0(t) = s(t) \cdot d(e_b(t))$ for $t \geq 0$, where $s(t) > 0$ is the ground-truth signal, $d : \mathbb{R}_{\geq 0} \to [0, 1]$ is a continuous degradation function with $d(0) = 1$ and $e_a(t), e_b(t) : [0, \infty) \to [0, \infty)$ are the continuous exposure function of signal a and b respectively. Let us further assume $e_a(0) = e_b(0) = 0$, $e_b(t) < e_a(t)$ for all $t > 0$ and assume that there exist function $e_a^{-1} : [0, \infty) \to [0, \infty)$. For $n = 0, 1, \ldots$*

$$r_n(t) = \frac{a_n(t)}{b_n(t)}, \quad a_{n+1}(t) = \frac{a_n(t)}{r_n(t)}, \quad b_{n+1}(t) = \frac{b_n(t)}{r_n((e_a^{-1} \circ e_b)(t))},$$

then it holds $\forall\, t \geq 0 : \lim_{n \to \infty} a_n(t) = \lim_{n \to \infty} b_n(t) = s(t)$.

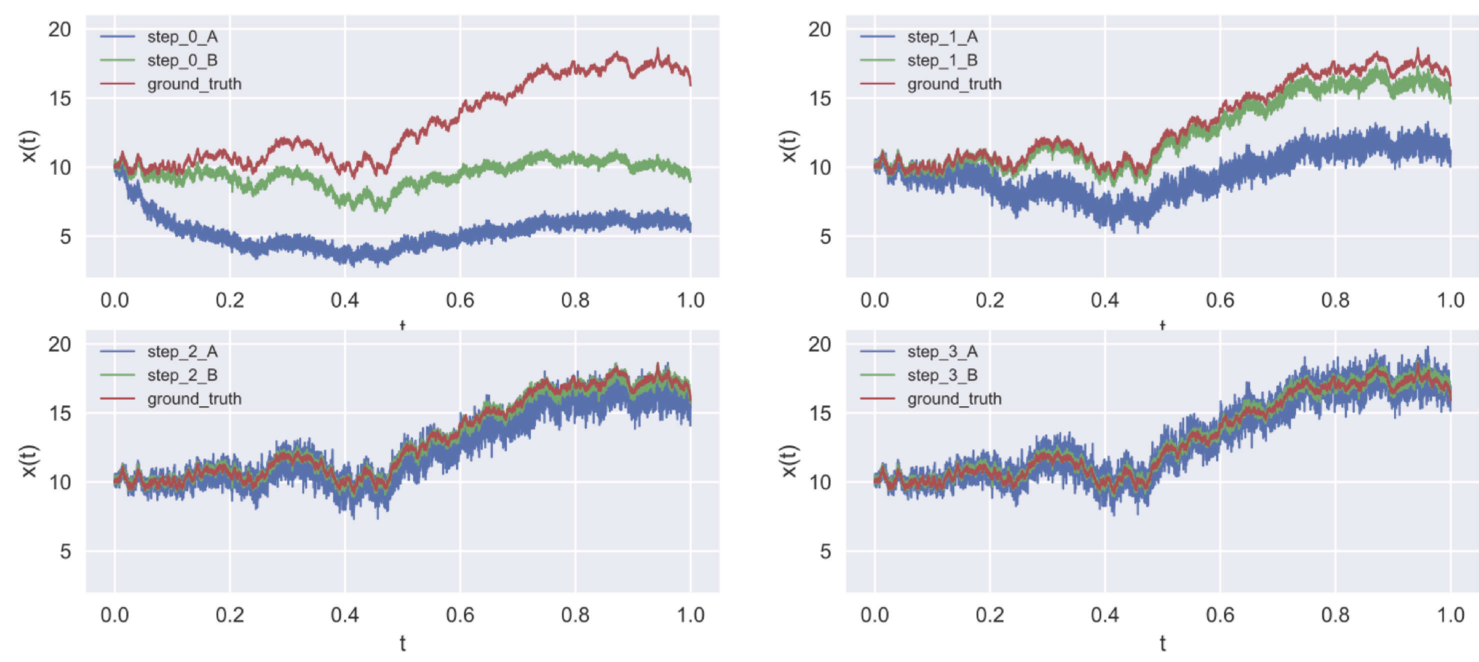


Figure 2: First four correction steps are illustrated on a synthetic Brownian motion ground-truth signal and its two noisy measurement signals.

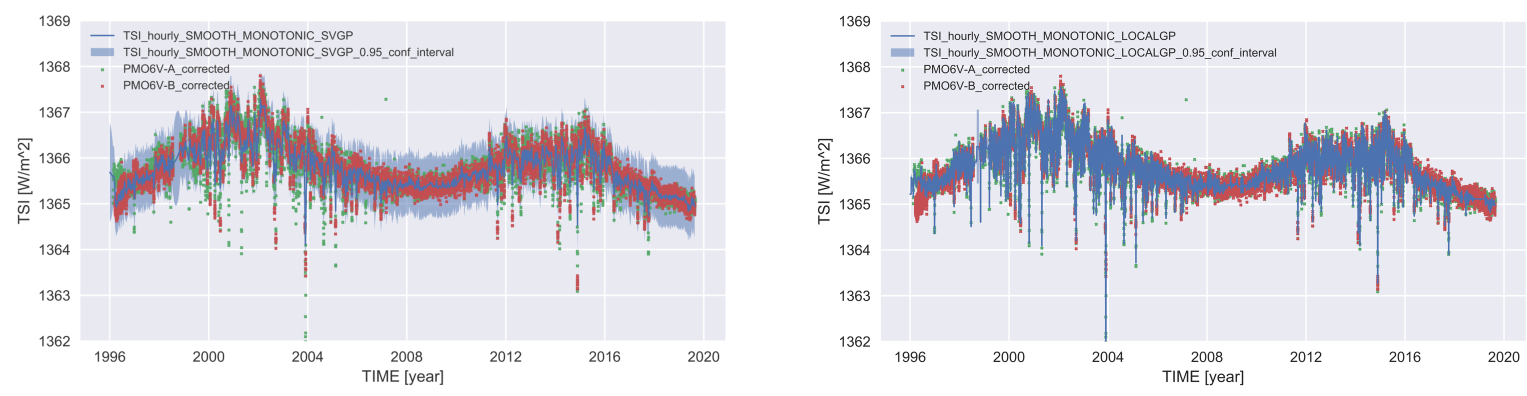
DATA FUSION

After correcting for degradation of a and b, how to combine them to get the best estimate of s? Gaussian processes arise naturally as a data fusion method, enabling incorporation of domain-knowledge prior in s. But what if there are a few millions of data points? And we want to inspect the signal at multiple timescales?

- Use sparse Gaussian processes and exploit the power of a variational approach.
- Or do a local approximation of Gaussian processes, by only considering points that are "close".

What if the instruments are not the same? Possibly, have different measurement noise?

- Design a custom kernel, which simulates additive white noise.

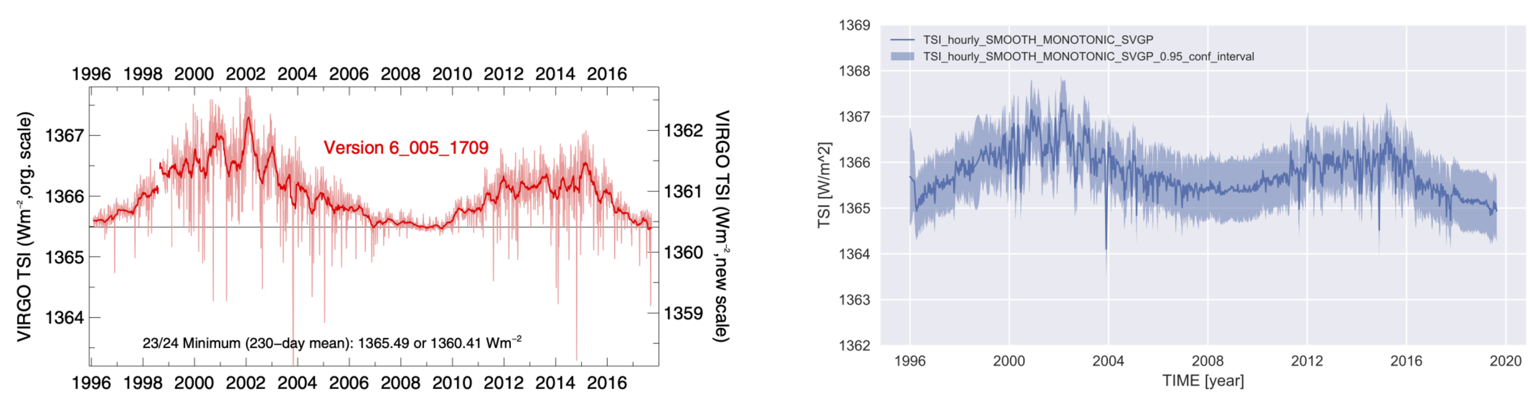


(a) SPARSEVARIATIONALGP (b) LOCALGP

Figure 3: Comparison of data fusion methods. "Sparse GP for merging and smoothing, local GP for merging."

SUMMARY

We introduced a novel degradation correction method that performs iterative correction steps and has a theoretical guarantee for convergence to the ground-truth signal. Smoothed monotonic regression is used for robust fitting and enforcing constrains on degradation function. Next, we propose a general data-driven framework for data fusion from multiple distinct measuring instruments based on sparse and locally applied Gaussian processes. This approach naturally handles data gaps and large number of measurements. Proposed algorithms were applied on level-1 TSI time series of PMO6-V radiometers from SOHO spacecraft to produce a new reliable TSI level-2 composite.



(a) v6.005 (b) Ours

Figure 4: Comparison of final VIRGO TSI between PMOD/WRC Institute's v6.005 and ours with 95 % confidence interval.

ACKNOWLEDGEMENTS

This work is a result of Data Science Lab at ETH Zürich taking part in Autumn 2019. We would like to thank Prof. Dr. Andreas Krause and Prof. Dr. Ce Zhang for their supervision, invaluable insights and suggestions on the modeling methods. Furthermore, we thank Dr. Wolfgang Finsterle for his domain knowledge support and data preparation.