

# Implementing Axiom Weakening for SROIQ

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## Abstract

Axiom weakening is a technique that allows for a fine-grained repair of inconsistent ontologies. Its main advantage is that it repairs ontologies by making axioms less restrictive rather than by deleting them, employing refinement operators. In this paper, we build on previously introduced axiom weakening for  $\mathcal{ALC}$ , and show how it can be extended to deal with  $\mathcal{SROIQ}$ , the expressive and decidable description logic underlying OWL 2 DL. We here focus on describing a prototype implementation computing axiom weakening for  $\mathcal{SROIQ}$  and discuss a number of performance and evaluation aspects.

## Keywords

Description Logic, Knowledge refinement, Protégé

## 1. Introduction: Weakening for debugging

Example 1.

## 2. Preliminaries

## 3. Axiom Weakening for $\mathcal{ALC}$

Formally, an ontology is a set of statements expressed in a suitable logical language and with the purpose of describing a specific domain of interest.

Example 2.


Example 3.

Example 4.

## 4. Extending Weakening to $\mathcal{SROIQ}$

We now give a brief description of the DL  $\mathcal{SROIQ}$ ; for full details see [1, 2]. The syntax of  $\mathcal{SROIQ}$  is based on a vocabulary of three disjoint sets  $N_I$ ,  $N_R$ ,  $N_C$  of *individual names*, *role names*, and *concept names*. The set of  $\mathcal{SROIQ}$  *concepts* and *roles* is generated by the following grammar.

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$$\begin{aligned}
R, S &::= U \mid E \mid r \mid r^- , \\
C &::= \perp \mid \top \mid A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \forall R.C \mid \exists R.C \mid \\
&\quad \geq n S.C \mid \leq n S.C \mid \exists S.Self \mid \{i\} ,
\end{aligned}$$

where  $r \in N_R$  is a role name,  $A \in N_C$  is a concept name,  $i \in N_i$  is an individual name and  $n \in \mathbb{N}_0$  is a non-negative integer.  $U$  and  $E$  are respectively the universal role and empty role.  $S$  is a *simple role* (see below) in the RBox  $\mathcal{R}$ .

A TBox  $\mathcal{T}$  is a finite set of concept inclusions (GCIs) of the form  $C \sqsubseteq D$  where  $C$  and  $D$  are concepts. The TBox is used to store terminological knowledge concerning the relationship between concepts. A ABox  $\mathcal{A}$  is a finite set of statements of the form  $R(a)$ ,  $\neg R(a)$ ,  $a = b$ , and  $a \neq b$  where  $R$  is a role and  $a$  and  $b$  are individual names. The ABox expresses knowledge regarding individuals in the domain. A RBox  $\mathcal{R}$  is a finite set of role inclusions (RIAs) of the form  $R_1 \circ \dots \circ R_n \sqsubseteq R$ , and disjoint role axioms  $\text{disjoint}(S_1, S_2)$  where  $R, R_1, \dots, R_n, S_1$ , and  $S_2$  are roles.  $S_1$  and  $S_2$  are simple (see next) in the RBox  $\mathcal{R}$ . The special case of  $n = 1$  is a simple role inclusion, while we call the cases where  $n > 1$  complex role inclusions. The RBox represents knowledge about the relationship between roles.

The set of *non-simple* roles in  $\mathcal{R}$  is the smallest set such that:  $U$  and  $E$  are non-simple; any role  $R$  that appears on the right-hand side of a complex role inclusion  $R_1 \circ \dots \circ R_n \sqsubseteq R$  where  $n > 1$  is non-simple; any role  $R$  that appears on the right-hand side of a simple role inclusion  $S \sqsubseteq R$  where  $S$  is non-simple, is also non-simple; and a role  $r$  is non-simple if and only if  $r^-$  is non-simple. All other roles are *simple*.

For convenience, let us define the function  $\text{inv}(R)$  such that  $\text{inv}(r) = r^-$  and  $\text{inv}(r^-) = r$  for all role names  $r \in N_R$ . A RBox  $\mathcal{R}$  is *regular* if there exists a pre-order  $\preceq$ , i.e., a transitive and reflexive relation, over the set of roles such that  $R \preceq S \iff \text{inv}(R) \preceq \text{inv}(S)$ ,  $R \preceq S \iff \text{inv}(R) \preceq S$ , and all RIAs in  $\mathcal{R}$  are of the forms:  $\text{inv}(R) \sqsubseteq R$ ,  $R \circ R \sqsubseteq R$ ,  $S \sqsubseteq R$ ,  $R \circ S_1 \circ \dots \circ S_n \sqsubseteq R$ ,  $S_1 \circ \dots \circ S_n \circ R \sqsubseteq R$ , or  $S_1 \circ \dots \circ S_n \sqsubseteq R$  where  $r \in N_R$  is a role name and  $R, S, S_1, \dots, S_n$  are roles such that  $S \preceq R$ ,  $S_i \preceq R$ , and  $R \not\preceq S_i$  for  $i = 1, \dots, n$ .

A *SRIOIQ* ontology  $\mathcal{O} = \mathcal{T} \cup \mathcal{A} \cup \mathcal{R}$  consists of a TBox  $\mathcal{T}$ , an ABox  $\mathcal{A}$ , and a RBox  $\mathcal{R}$ , where  $\mathcal{R}$  is regular.

The semantics of *SRIOIQ* are defined using *interpretations*  $I = \langle \Delta^I, \cdot^I \rangle$  where  $\Delta^I$  is a non-empty domain and  $\cdot^I$  is a function associating to each individual name  $a$  an element of the domain  $a^I \in \Delta^I$ , to each concept  $C$  a subset of the domain  $C^I \subseteq \Delta^I$ , and to each role  $R$  a binary relation on the domain  $R^I \subseteq \Delta^I \times \Delta^I$ ; see [1, 2] for further details. An interpretation  $I$  is a *model* for  $\mathcal{O}$  if it satisfies all the axioms in  $\mathcal{O}$ .

Given two concepts  $C$  and  $D$  we say that  $C$  is *subsumed* by  $D$  (or  $D$  *subsumes*  $C$ ) with respect to the ontology  $\mathcal{O}$ , written  $C \sqsubseteq_{\mathcal{O}} D$ , if  $C^I \subseteq D^I$  in every model  $I$  of  $\mathcal{O}$ . Further  $C$  is *strictly subsumed* by  $D$ , written  $C \sqsubset_{\mathcal{O}} D$ , if  $C \sqsubseteq_{\mathcal{O}} D$  but not  $D \sqsubseteq_{\mathcal{O}} C$ . Analogously, given two roles  $R$  and  $S$ ,  $R$  is subsumed by  $S$  with respect to  $\mathcal{O}$  ( $R \sqsubseteq_{\mathcal{O}} S$ ) if  $R^I \subseteq S^I$  in all models  $I$  of  $\mathcal{O}$ . Again,  $R \sqsubset_{\mathcal{O}} S$  holds if  $R \sqsubseteq_{\mathcal{O}} S$  but not  $D \sqsubseteq_{\mathcal{O}} C$ .

**Example 5.**

**Example 6.**

## **5. Implementing Axiom Weakening for *SROIQ***

## **6. Weakening makes you strong: evaluation aspects**

## **7. Outlook**

### **References**

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