Implementing Axiom Weakening for SROIQ

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Abstract

Axiom weakening is a technique that allows for a fine-grained repair of inconsistent ontologies. Its main advantage is that it repairs ontologies by making axioms less restrictive rather than by deleting them, employing refinement operators. In this paper, we build on previously introduced axiom weakening for $\mathcal{ALC}$, and show how it can be extended to deal with $\mathcal{SROIQ}$, the expressive and decidable description logic underlying OWL 2 DL. We here focus on describing a prototype implementation computing axiom weakening for $\mathcal{SROIQ}$ and discuss a number of performance and evaluation aspects.

Keywords

Description Logic, Knowledge refinement, Protégé

1. Introduction: Weakening for debugging

Example 1.

2. Axiom Weakening for $\mathcal{ALC}$

Formally, an ontology is a set of statements expressed in a suitable logical language and with the purpose of describing a specific domain of interest.

Example 2.

Example 3.

Example 4.

3. Extending Weakening to $\mathcal{SROIQ}$

We now give a brief description of the DL $\mathcal{SROIQ}$; for full details see [1, 2]. The syntax of $\mathcal{SROIQ}$ is based on a vocabulary of three disjoint sets N_C, N_R, N_I of respectively *concept names*, *role names*, and *individual names*. The set of $\mathcal{SROIQ}$ *concepts* and *roles* is generated by the following grammar.

$$R, S ::= U \mid E \mid r \mid r^- \ ,$$

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$$C ::= \bot \mid \top \mid A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \forall R.C \mid \exists R.C \mid$$
$$\geq n\ S.C \mid \leq n\ S.C \mid \exists S.\mathit{Self} \mid \{i\}\ ,$$

where $A \in N_C$ is a concept name, $r \in N_R$ is a role name, $i \in N_I$ is an individual name and $n \in \mathbb{N}_0$ is a non-negative integer. U and E are respectively the universal role and empty role. S is a *simple role* (see below) in the RBox $\mathcal{R}$. In the following, $\mathcal{L}(N_C, N_R, N_I)$ and $\mathcal{L}(N_R) = N_R \cup \{U, E\} \cup \{r^- \mid r \in N_R\}$ denote respectively the set of concepts and roles that can be built over N_C, N_R, and N_I in $\mathcal{SROIQ}$.

A *TBox* $\mathcal{T}$ is a finite set of concept inclusions (GCIs) of the form $C \sqsubseteq D$ where C and D are concepts. The TBox is used to stores terminological knowledge concerning the relationship between concepts. A *ABox* $\mathcal{A}$ is a finite set of statements of the form $R(a)$, $\neg R(a)$, $a = b$, and $a \neq b$ where R is a role and a and b are individual names. The ABox expresses knowledge regarding individuals in the domain. A *RBox* $\mathcal{R}$ is a finite set of role inclusions (RIAs) of the form $R_1 \circ \cdots \circ R_n \sqsubseteq R$, and disjoint role axioms $\mathit{disjoint}(S_1, S_2)$ where R, $R_1, \ldots, R_n$, S_1, and S_2 are roles. S_1 and S_2 are simple (see next) in the RBox $\mathcal{R}$. The special case of $n = 1$ is a simple role inclusion, while we call the cases where $n > 1$ complex role inclusions. The RBox represents knowledge about the relationship between roles.

The set of *non-simple* roles in $\mathcal{R}$ is the smallest set such that: U and E are non-simple; any role R that appears on the right-hand side of a complex RIA $R_1 \circ \cdots \circ R_n \sqsubseteq R$ where $n > 1$ is non-simple; any role R that appears on the right-hand side of a simple RIA $S \sqsubseteq R$ where S is non-simple, is also non-simple; and a role r is non-simple if and only if r^- is non-simple. All other roles are *simple*.

For convenience, let us define the function $\mathit{inv}(R)$ such that $\mathit{inv}(r) = r^-$ and $\mathit{inv}(r^-) = r$ for all role names $r \in N_R$. A RBox $\mathcal{R}$ is *regular* if there exists a pre-order $\preceq$, i.e., a transitive and reflexive relation, over the set of roles such that $R \preceq S \iff \mathit{inv}(R) \preceq \mathit{inv}(S)$, $R \preceq S \iff \mathit{inv}(R) \preceq S$, and all RIAs in $\mathcal{R}$ are of the forms: $\mathit{inv}(R) \sqsubseteq R$, $R \circ R \sqsubseteq R$, $S \sqsubseteq R$, $R \circ S_1 \circ \cdots \circ S_n \sqsubseteq R$, $S_1 \circ \cdots \circ S_n \circ R \sqsubseteq R$, or $S_1 \circ \cdots \circ S_n \sqsubseteq R$, where $r \in N_R$ is a role name and R, S, $S_1, \cdots, S_n$ are roles such that $S \preceq R$, $S_i \preceq R$, and $R \not\preceq S_i$ for $i = 1, \ldots, n$.

A $\mathcal{SROIQ}$ ontology $\mathcal{O} = \mathcal{T} \cup \mathcal{A} \cup \mathcal{R}$ consists of a TBox $\mathcal{T}$, an ABox $\mathcal{A}$, and a RBox $\mathcal{R}$, where $\mathcal{R}$ is regular.

The semantics of $\mathcal{SROIQ}$ are defined using *interpretations* $I = \langle \Delta^I, \cdot^I \rangle$ where Δ^I is a non-empty *domain* and $\cdot^I$ is a function associating to each individual name a an element of the domain $a^I \in \Delta^I$, to each concept C a subset of the domain $C^I \subseteq \Delta^I$, and to each role R a binary relation on the domain $R^I \subseteq \Delta^I \times \Delta^I$; see [1, 2] for further details. An interpretation I is a *model* for $\mathcal{O}$ if it satisfies all the axioms in $\mathcal{O}$.

Given two concepts C and D we say that C is *subsumed* by D (or D *subsumes* C) with respect to the ontology $\mathcal{O}$, written $C \sqsubseteq_{\mathcal{O}} D$, if $C^I \subseteq D^I$ in every model I of $\mathcal{O}$. Further C is *strictly subsumed by* D, written $C \sqsubset_{\mathcal{O}} D$, if $C \sqsubseteq_{\mathcal{O}} D$ but not $D \sqsubseteq_{\mathcal{O}} C$. Analogously, given two roles R and S, R is subsumed by S with respect to $\mathcal{O}$ ($R \sqsubseteq_{\mathcal{O}} S$) if $R^I \sqsubseteq S^I$ in all models I of $\mathcal{O}$. Again, $R \sqsubset_{\mathcal{O}} S$ holds if $R \sqsubseteq_{\mathcal{O}} S$ but not $D \sqsubseteq_{\mathcal{O}} C$

The main difficulties that arise when weakening axioms in $\mathcal{SROIQ}$ ontologies, and especially when weakening RIAs, are related to ensuring that the constraints on the use of non-simple roles and the regularity of the role hierarchy are maintained. Not every weaker axiom can be inserted into a valid $\mathcal{SROIQ}$ ontology without causing a violation of these restrictions.

Example 5. *Take the ontology $\mathcal{O} = \{r \circ s \circ r \sqsubseteq t, r \sqsubseteq s, \top \sqsubseteq \forall t.\bot, \exists s.Self \sqsubseteq \top\}$. Since t is empty in every model of this ontology, the axiom $r \sqsubseteq s$ could be weakened to $t \sqsubseteq s$ if we ignore the additional constraints. This would result in an ontology where s is non-simple, which is not allowed since s is used as part of a self constraint. Additionally, using this weakening would also cause a non-regular RBox, because for any pre-order $\preceq$, $t \not\preceq s$ must hold for the complex RIA and $t \preceq s$ must hold for the new axiom. Yet, this is a contradiction.*

To prevent these kinds of issues, we restrict how concepts are refined, and RIA can be weakened. To achieve this, we must ensure that only simple roles are used when weakening disjoint role axioms or refining cardinality and self constraints. Further, it must be guaranteed that all roles that are currently used in such context remain simple when adding the weakened axioms to the ontology. Finally, the addition of a weakened axiom must maintain the regularity of the role hierarchy. We discuss now the restrictions we applied in order to satisfy these requirements.

Firstly, the covers and refinement operators for roles operate only on roles that are simple. A similar restriction has already been applied in the refinement operator suggested in [3]. Restricting the refinement to simple roles guarantees that the new axioms created by weakening will not contain non-simple roles in axioms or concepts where they are not allowed. An important detail to not here is that the roles over which the covers operate must be simple in all ontologies that the weaker axioms are used in. It is therefore not generally sufficient to use the roles that are simple in the reference ontology, since the reference ontology may not contain all RBox axioms, and therefore contain simple roles that are not simple in the full ontology. For this reason we give to the upward and downward cover as an argument not only the reference ontology $\mathcal{O}^{\text{ref}}$, but also the full ontology $\mathcal{O}^{\text{full}}$. Both $\mathcal{O}^{\text{ref}}$ and $\mathcal{O}^{\text{full}}$ share the same vocabulary N_I, N_C, and N_R. In the context of repairing inconsistent ontologies, $\mathcal{O}^{\text{full}}$ can be chosen to be the inconsistent ontology that we want to repair.

Then, to ensure further that by adding weakened axioms we do not cause a constraint violation in existing axioms and concepts, we choose the allowed weakening for RIAs such that all roles that are simple in $\mathcal{O}^{\text{full}}$, are also simple after adding to it a weakening of one of its axioms. We observe that for complex RIAs $S_1 \circ \cdots \circ S_n \sqsubseteq R$ we should not refine the role R. Since all roles returned by our refinement operator are simple in $\mathcal{O}^{\text{full}}$, such a replacement would make a role with was simple in $\mathcal{O}^{\text{full}}$ non-simple. A similar argument can be made for refining R in a simple RIA $S \sqsubseteq R$ where the role S is non-simple in $\mathcal{O}^{\text{full}}$. So the only way to refine the right-hand side during the weakening of a RIA is when it is a simple RIA and additionally the left-hand side of the axiom is simple in $\mathcal{O}^{\text{full}}$.

When it comes to refining the left-hand side of RIAs, we do not need any special restrictions. The main significant observation is that all roles that are returned by the refinement will be simple. This means that in a simple RIA $R \sqsubseteq S$, even if S is simple, replacing R with another simple role will not cause S to become non-simple. For a complex RIA $S_1 \circ \cdots \circ S_n \sqsubseteq R$ on the other hand, the role R must already have been non-simple in $\mathcal{O}^{\text{full}}$, and replacing any S_i with a refinement has no effect on which roles are simple.

A more interesting question is whether such a weakening may still cause a non-regular role hierarchy. The important insight is that simple roles are always allowed on the left-hand side of a RIA. While this is more directly evident in some alternative definitions of regularity (e.g., [4]) it is not so apparent form the one presented in this paper. Intuitively, the constraint given above for regularity disallows dependency cycles that contain complex RIAs. Simple roles can not be part of such a cycle, since the cycle must contain at least one complex RIA to be a violation of the constraint, and all roles that depend in this sense on a complex RIA must be non-simple. A more formal justification for this fact is given in the proof for lemma 1. Since all refinements of the left-hand side of RIAs are performed using simple roles, these can not lead to a non-regular RBox. Further, refinements of the right-hand side of RIAs are only performed on simple RIAs $S \sqsubseteq R$ where S is a simple role. Since S is simple in this case, all refinements of R are allowed, potentially also if the refinement yielded a non-simple role.

Definition 1. *Let $\mathcal{O}$ be a $\mathcal{SROIQ}$ ontology. The set of* subconcepts *of $\mathcal{O}$ is given by*

$$\mathsf{sub}(\mathcal{O}) = \{\top, \bot\} \cup \bigcup_{C(a)\in\mathcal{O}} \mathsf{sub}(C) \cup \bigcup_{C\sqsubseteq D\in\mathcal{O}} (\mathsf{sub}(C) \cup \mathsf{sub}(D)) \ ,$$

where $\mathsf{sub}(C)$ is the set of subconcepts *in C such that*

$$\begin{aligned}
\mathsf{sub}(A) &= \{A\} \quad , A \in N_C \cup \{\top, \bot\} \ , & \mathsf{sub}(\neg C) &= \{\neg C\} \cup \mathsf{sub}(C) \ , \\
\mathsf{sub}(C \sqcup D) &= \{C \sqcup D\} \cup \mathsf{sub}(C) \cup \mathsf{sub}(D) \ , & \mathsf{sub}(\forall R.C) &= \{\forall R.C\} \cup \mathsf{sub}(C) \ , \\
\mathsf{sub}(C \sqcap D) &= \{C \sqcap D\} \cup \mathsf{sub}(C) \cup \mathsf{sub}(D) \ , & \mathsf{sub}(\exists R.C) &= \{\exists R.C\} \cup \mathsf{sub}(C) \ , \\
\mathsf{sub}(\geq n\ R.C) &= \{\geq n\ R.C\} \cup \mathsf{sub}(C) \ , & \mathsf{sub}(\leq n\ R.C) &= \{\leq n\ R.C\} \cup \mathsf{sub}(C) \ , \\
\mathsf{sub}(\exists R.\mathit{Self}) &= \{\exists R.\mathit{Self}\} \ , & \mathsf{sub}(\{i\}) &= \{\{i\}\} \ .
\end{aligned}$$

We will define now the upward and downward cover sets for concepts and roles. Intuitively, for a given concept the upward cover is the set of the most specific generalizations from the set of subconcepts or roles, while the downward cover set contains the most general specializations from the same set of subconcepts and roles. We define the upward and downward cover additionally also for non-negative integers, as they will be useful in the refinement of cardinality constraints.

Definition 2. *Let $\mathcal{O}^{\mathrm{ref}}$ and $\mathcal{O}^{\mathrm{full}}$ be two $\mathcal{SROIQ}$ ontologies that share the same vocabulary N_C, N_R, and N_I. The* upward cover *and* downward cover *for a concept C are given by*

$$\begin{aligned}
\mathsf{UpCover}_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(C) = \{&D \in \mathsf{sub}(\mathcal{O}^{\mathrm{full}}) \mid C \sqsubseteq_{\mathcal{O}^{\mathrm{ref}}} D \ \mathit{and} \\
&\nexists D' \in \mathsf{sub}(\mathcal{O}^{\mathrm{full}}) \ \mathit{with} \ C \sqsubset_{\mathcal{O}^{\mathrm{ref}}} D' \sqsubset_{\mathcal{O}^{\mathrm{ref}}} D\} \ , \\
\mathsf{DownCover}_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(C) = \{&D \in \mathsf{sub}(\mathcal{O}^{\mathrm{full}}) \mid D \sqsubseteq_{\mathcal{O}^{\mathrm{ref}}} C \ \mathit{and} \\
&\nexists D' \in \mathsf{sub}(\mathcal{O}^{\mathrm{full}}) \ \mathit{with} \ D \sqsubset_{\mathcal{O}^{\mathrm{ref}}} D' \sqsubset_{\mathcal{O}^{\mathrm{ref}}} C\} \ .
\end{aligned}$$

The upward and downward covers for a role R are given by

$$\begin{aligned}
\mathsf{UpCover}_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(R) = \{&S \in \mathcal{L}(N_R) \mid R \sqsubseteq_{\mathcal{O}^{\mathrm{ref}}} S \ \mathit{and} \\
&\nexists S' \in \mathcal{L}(N_R) \ \mathit{with} \ R \sqsubset_{\mathcal{O}^{\mathrm{ref}}} S' \sqsubset_{\mathcal{O}^{\mathrm{ref}}} S \ \mathit{and} \\
&S, S' \ \mathit{are\ simple\ in} \ \mathcal{O}^{\mathrm{full}}\} \ , \\
\mathsf{DownCover}_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(R) = \{&S \in \mathcal{L}(N_R) \mid S \sqsubseteq_{\mathcal{O}^{\mathrm{ref}}} R \ \mathit{and}
\end{aligned}$$

$$\nexists S' \in \mathcal{L}(N_R) \text{ with } S \sqsubset_{\mathcal{O}^{\text{ref}}} S' \sqsubset_{\mathcal{O}^{\text{ref}}} R \text{ and}$$
$$S, S' \text{ are simple in } \mathcal{O}^{\text{full}}\} \ .$$

The upward and downward covers for a non-negative integer n are given by

$$\mathsf{UpCover}_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(n) = \{n, n+1\} \ ,$$
$$\mathsf{DownCover}_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(R) = \begin{cases} \{n\} & \text{if } n = 0 \\ \{n, n-1\} & \text{if } n > 0 \end{cases} \ .$$

Since they operate only over the subconcepts of $\mathcal{O}^{\text{full}}$, on their own, the upward and downward covers of concepts are missing some interesting refinements.

Example 6. *Let $N_C = \{A, B, C\}$, $N_R = \{r, s\}$, and $\mathcal{O} = \{A \sqsubseteq B, r \sqsubseteq s\}$. $\mathsf{sub}(\mathcal{O}) = \{\top, \bot, A, B\}$. The upward cover of $C \sqcup A$ is equal to $\mathsf{UpCover}_{\mathcal{O},\mathcal{O}}(C \sqcup A) = \{\top\}$. The potentiality refinement to $C \sqcup B$ will be missed even by iterated application of the upward cover because $C \sqcup B \notin \mathsf{sub}(\mathcal{O})$. Similarly, $\mathsf{UpCover}_{\mathcal{O},\mathcal{O}}(\forall r.A) = \{\top\}$, even if $\forall r.B$ and $\forall s.A$ are reasonable generalizations.*

To also capture these omissions, we define generalization and specialization operators that exploit the recursive structure of the concept being refined to generate more complex refinements. For convenience, we define these operators also for roles.

Definition 3. *Let $\uparrow$ and $\downarrow$ be two functions with domain $\mathcal{L}(N_C, N_R, N_I) \cup \mathcal{L}(N_C) \cup \mathbb{N}_0$. They map every concept to a finite subset of $\mathcal{L}(N_C, N_R, N_I)$, every role to a subset of $\mathcal{L}(N_C)$, and every non-negative integer to a finite subset of $\mathbb{N}_0$. The* abstract refinement operator *is defined recursively by induction on the structure of concepts as follows.*

$$\begin{aligned}
\zeta_{\uparrow,\downarrow}(A) &= \uparrow(A) \quad , A \in N_C \cup \{\top, \bot\} \ , \\
\zeta_{\uparrow,\downarrow}(\neg C) &= \uparrow(\neg C) \cup \{\neg C' \mid C' \in \zeta_{\downarrow,\uparrow}(C)\} \ , \\
\zeta_{\uparrow,\downarrow}(C \sqcap D) &= \uparrow(C \sqcap D) \cup \{C' \sqcap D \mid C' \in \zeta_{\uparrow,\downarrow}(C)\} \cup \{C \sqcap D' \mid D' \in \zeta_{\uparrow,\downarrow}(D)\} \ , \\
\zeta_{\uparrow,\downarrow}(C \sqcup D) &= \uparrow(C \sqcup D) \cup \{C' \sqcup D \mid C' \in \zeta_{\uparrow,\downarrow}(C)\} \cup \{C \sqcup D' \mid D' \in \zeta_{\uparrow,\downarrow}(D)\} \ , \\
\zeta_{\uparrow,\downarrow}(\forall R.C) &= \uparrow(\forall R.C) \cup \{\forall R'.C \mid R' \in \downarrow(R)\} \cup \{\forall R.C' \mid C' \in \zeta_{\uparrow,\downarrow}(C)\} \ , \\
\zeta_{\uparrow,\downarrow}(\exists R.C) &= \uparrow(\exists R.C) \cup \{\exists R'.C \mid R' \in \uparrow(R)\} \cup \{\exists R.C' \mid C' \in \zeta_{\uparrow,\downarrow}(C)\} \ , \\
&\mathcal{SROIQ} \textit{ concepts:} \\
\zeta_{\uparrow,\downarrow}(\{i\}) &= \uparrow(\{i\}) \ , \\
\zeta_{\uparrow,\downarrow}(\exists R.\mathit{Self}) &= \uparrow(\exists R.\mathit{Self}) \cup \{\exists R'.\mathit{Self} \mid R' \in \uparrow(R)\} \ , \\
\zeta_{\uparrow,\downarrow}(\geq n\ R.C) &= \uparrow(\geq n\ R.C) \cup \{\geq n\ R'.C \mid R' \in \uparrow(R)\} \\
&\quad \cup \{\geq n\ R.C' \mid C' \in \zeta_{\uparrow,\downarrow}(C)\} \cup \{\geq n'R.C \mid n' \in \downarrow(C)\} \ , \\
\zeta_{\uparrow,\downarrow}(\leq n\ R.C) &= \uparrow(\leq n\ R.C) \cup \{\leq n\ R'.C \mid R' \in \downarrow(R)\} \\
&\quad \cup \{\leq n\ R.C' \mid C' \in \zeta_{\downarrow,\uparrow}(C)\} \cup \{\leq n'R.C \mid n' \in \uparrow(C)\} \ , \\
&\mathcal{SROIQ} \textit{ roles:} \\
\zeta_{\uparrow,\downarrow}(R) &= \uparrow(R) \ .
\end{aligned}$$

From the abstract refinement operator $\zeta_{\uparrow,\downarrow}$, two concrete refinement operators, the generalization operator *and* specialization operator *are respectively defined as*

$$\gamma_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}} = \zeta_{\mathsf{UpCover}_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}},\mathsf{DownCover}_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}} \quad \textit{and}$$
$$\rho_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}} = \zeta_{\mathsf{DownCover}_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}},\mathsf{UpCover}_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}} \quad .$$

Revisiting the case in example 6 we observe that $\gamma_{\mathcal{O},\mathcal{O}}(C \sqcup A) = \{\top, \top \sqcup A, C \sqcup A, C \sqcup B\}$ does contain $C \sqcup B$ as a possible refinement. Similarly, $\gamma_{\mathcal{O},\mathcal{O}}(\forall r.A) = \{\top, \forall r.A, \forall s.A, \forall r.B\}$ contains $\forall r.B$.

We define now the *axiom weakening operator* using these generalization and specialization operators.

Definition 4. *Given an axiom ϕ, the set of* weakenings *with respect to the reference ontology $\mathcal{O}^{\mathrm{ref}}$ and full ontology $\mathcal{O}^{\mathrm{full}}$, written $g_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(\phi)$ is defined such that*

$$\begin{aligned}
g_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(C \sqsubseteq D) &= \{C' \sqsubseteq D \mid C' \in \rho_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(C)\} \cup \{C \sqsubseteq D' \mid D' \in \gamma_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(D)\} \ , \\
g_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(C(a)) &= \{C'(a) \mid C' \in \gamma_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(C)\} \ , \\
g_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(R(a,b)) &= \{R'(a,b) \mid R' \in \gamma_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(R)\} \cup \{R(a,b), \bot \sqsubseteq \top\} \ , \\
g_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(\neg R(a,b)) &= \{\neg R'(a,b) \mid R' \in \rho_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(R)\} \cup \{\neg R(a,b), \bot \sqsubseteq \top\} \ , \\
g_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(a = b) &= \{a = b, \bot \sqsubseteq \top\} \ , \quad g_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(a \neq b) = \{a \neq b, \bot \sqsubseteq \top\} \ , \\
g_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(S_1 \circ \cdots \circ S_n \sqsubseteq R) &= \{S_1 \circ \cdots \circ S'_i \circ \cdots \circ S_n \sqsubseteq R \mid S'_i \in \rho_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}}(S_i) \textit{ for } i = 1, \ldots, n\} \\
&\quad \cup \{S_1 \sqsubseteq R' \mid R' \in \gamma_{\mathcal{O}^{\mathrm{ref}},\mathcal{O}^{\mathrm{full}}} \textit{ and } n = 1 \textit{ and } S_1 \textit{ is simple in } \mathcal{O}^{\mathrm{full}}\} \\
&\quad \cup \{S_1 \circ \cdots \circ S_n \sqsubseteq R, \bot \sqsubseteq \top\} \ .
\end{aligned}$$

Lemma 1.

Proof. (*Sketch*) □

4. Implementing Axiom Weakening for $\mathcal{SROIQ}$

5. Weakening makes you strong: evaluation aspects

6. Outlook

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