Implementing Axiom Weakening for SROIQ

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Abstract

Axiom weakening is a technique that allows for a fine-grained repair of inconsistent ontologies. Its main advantage is that it repairs ontologies by making axioms less restrictive rather than by deleting them, employing refinement operators. In this paper, we build on previously introduced axiom weakening for $\mathcal{ALC}$, and show how it can be extended to deal with $\mathcal{SROIQ}$, the expressive and decidable description logic underlying OWL 2 DL. We here focus on describing a prototype implementation computing axiom weakening for $\mathcal{SROIQ}$ and discuss a number of performance and evaluation aspects.

Keywords

Description Logic, Knowledge refinement, Protégé

1. Introduction: Weakening for debugging

2. Extending Weakening to $\mathcal{SROIQ}$

We now give a brief description of the DL $\mathcal{SROIQ}$; for full details, see [1, 2]. The syntax of $\mathcal{SROIQ}$ is based on a vocabulary of three disjoint sets N_C, N_R, N_I of respectively *concept names*, *role names*, and *individual names*. The set of $\mathcal{SROIQ}$ *concepts* and *roles* is generated by the following grammar.

$$\begin{aligned} R, S &::= U \mid E \mid r \mid r^- \ , \\ C &::= \bot \mid \top \mid A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \forall R.C \mid \exists R.C \mid \\ &\quad\ \geq n\ S.C \mid \leq n\ S.C \mid \exists S.\mathit{Self} \mid \{i\} \ , \end{aligned}$$

where $A \in N_C$ is a concept name, $r \in N_R$ is a role name, $i \in N_I$ is an individual name and $n \in \mathbb{N}_0$ is a non-negative integer. U and E are respectively the universal role and existential role. S is a *simple role* (see below) in the RBox $\mathcal{R}$. In the following, $\mathcal{L}(N_C, N_R, N_I)$ and $\mathcal{L}(N_R) = N_R \cup \{U, E\} \cup \{r^- \mid r \in N_R\}$ denote respectively the set of concepts and roles that can be built over N_C, N_R, and N_I in $\mathcal{SROIQ}$.

A *TBox* $\mathcal{T}$ is a finite set of concept inclusions (GCIs) of the form $C \sqsubseteq D$ where C and D are concepts. The TBox is used to stores terminological knowledge concerning the relationship between concepts. A *ABox* $\mathcal{A}$ is a finite set of statements of the form $R(a)$, $\neg R(a)$, $a = b$, and $a \neq b$ where R is a role and a and b are individual names. The ABox expresses knowledge

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regarding individuals in the domain. A *RBox* $\mathcal{R}$ is a finite set of role inclusions (RIAs) of the form $R_1 \circ \cdots \circ R_n \sqsubseteq R$, and disjoint role axioms $disjoint(S_1, S_2)$ where R, $R_1, \ldots, R_n$, S_1, and S_2 are roles. S_1 and S_2 are simple (see next) in the RBox $\mathcal{R}$. The special case of $n = 1$ is a simple role inclusion, while we call the cases where $n > 1$ complex role inclusions. The RBox represents knowledge about the relationship between roles.

The set of *non-simple* roles in $\mathcal{R}$ is the smallest set such that: U and E are non-simple; any role R that appears as the super role of a complex RIA $R_1 \circ \cdots \circ R_n \sqsubseteq R$ where $n > 1$ is non-simple; any role R that appears on the right-hand side of a simple RIA $S \sqsubseteq R$ where S is non-simple, is also non-simple; and a role r is non-simple if and only if r^- is non-simple. All other roles are *simple*.

For convenience, let us define the function $inv(R)$ such that $inv(r) = r^-$ and $inv(r^-) = r$ for all role names $r \in N_R$. A RBox $\mathcal{R}$ is *regular* if there exists a pre-order $\preceq$, i.e., a transitive and reflexive relation, over the set of roles such that $R \preceq S \iff inv(R) \preceq inv(S)$, $R \preceq S \iff inv(R) \preceq S$, and all RIAs in $\mathcal{R}$ are of the forms: $inv(R) \sqsubseteq R$, $R \circ R \sqsubseteq R$, $S \sqsubseteq R$, $R \circ S_1 \circ \cdots \circ S_n \sqsubseteq R$, $S_1 \circ \cdots \circ S_n \circ R \sqsubseteq R$, or $S_1 \circ \cdots \circ S_n \sqsubseteq R$, where $r \in N_R$ is a role name, $n > 1$ and $R, S, S_1, \cdots, S_n$ are roles such that $S \preceq R$, $S_i \preceq R$, and $R \not\preceq S_i$ for $i = 1, \ldots, n$.[1]

A $\mathcal{SROIQ}$ ontology $\mathcal{O} = \mathcal{T} \cup \mathcal{A} \cup \mathcal{R}$ consists of a TBox $\mathcal{T}$, an ABox $\mathcal{A}$, and a RBox $\mathcal{R}$, where $\mathcal{R}$ is regular.

The semantics of $\mathcal{SROIQ}$ are defined using *interpretations* $I = \langle \Delta^I, \cdot^I \rangle$ where Δ^I is a non-empty *domain* and $\cdot^I$ is a function associating to each individual name a an element of the domain $a^I \in \Delta^I$, to each concept C a subset of the domain $C^I \subseteq \Delta^I$, and to each role R a binary relation on the domain $R^I \subseteq \Delta^I \times \Delta^I$; see [1, 2] for further details. An interpretation I is a *model* for $\mathcal{O}$ if it satisfies all the axioms in $\mathcal{O}$.

Given two concepts C and D we say that C is *subsumed* by D (or D *subsumes* C) with respect to the ontology $\mathcal{O}$, written $C \sqsubseteq_{\mathcal{O}} D$, if $C^I \subseteq D^I$ in every model I of $\mathcal{O}$. Further, C is *strictly subsumed by* D, written $C \sqsubset_{\mathcal{O}} D$, if $C \sqsubseteq_{\mathcal{O}} D$ but not $D \sqsubseteq_{\mathcal{O}} C$. Analogously, given two roles R and S, R is subsumed by S with respect to $\mathcal{O}$ ($R \sqsubseteq_{\mathcal{O}} S$) if $R^I \sqsubseteq S^I$ in all models I of $\mathcal{O}$. Again, $R \sqsubset_{\mathcal{O}} S$ holds if $R \sqsubseteq_{\mathcal{O}} S$ but not $D \sqsubseteq_{\mathcal{O}} C$

The main difficulties that arise when weakening axioms in $\mathcal{SROIQ}$ ontologies, and especially when weakening RIAs, are related to ensuring that the constraints on the use of non-simple roles and the regularity of the role hierarchy are maintained. Not every weaker axiom can be inserted into a valid $\mathcal{SROIQ}$ ontology without causing a violation of these restrictions.

Example 1. *Take the ontology* $\mathcal{O} = \{r \circ s \circ r \sqsubseteq t, r \sqsubseteq s, \top \sqsubseteq \forall t.\bot, \exists s.\mathit{Self} \sqsubseteq \top\}$. *Since* t *is empty in every model of this ontology, the axiom* $r \sqsubseteq s$ *could be weakened to* $t \sqsubseteq s$ *if we ignore the additional constraints. This would result in an ontology where* s *is non-simple, which is not allowed since* s *is used as part of a self constraint. Additionally, using this weakening would also cause a non-regular RBox, because for any pre-order* $\preceq$, $t \not\preceq s$ *must hold for the complex RIA and* $t \preceq s$ *must hold for the new axiom. Yet, this is a contradiction.*

To prevent these kinds of issues, we restrict how concepts are refined and RIAs weakened. In [4] the refinement of RIAs was not considered at all to avoid these problems. In this paper,

[1]The definitions for simple roles and regularity used here conform to the global restrictions in OWL 2 DL [3].

however, we have extended the axiom weakening operator to handle also RIAs. To achieve this, we must ensure that only simple roles are used when weakening disjoint role axioms or refining cardinality and self constraints. Further, it must be guaranteed that all roles that are currently used in such context remain simple when adding the weakened axioms to the ontology. Finally, the addition of a weakened axiom must maintain the regularity of the role hierarchy. We discuss now the restrictions we applied in order to satisfy these requirements.

Firstly, the covers and refinement operators for roles operate only on roles that are simple. A similar restriction has already been applied in the refinement operator suggested in [4]. Restricting the refinement to simple roles guarantees that the new axioms created by weakening will not contain non-simple roles in axioms or concepts where they are not allowed. An important detail that was not considered in [4] is that the roles over which the covers operate must be simple in all ontologies that the weaker axioms are used in. It is therefore not generally sufficient to use the roles that are simple in the reference ontology, since the reference ontology may not contain all RBox axioms, and therefore contain simple roles that are not simple in the full ontology. For this reason, we give to the upward and downward cover as an argument not only the reference ontology $\mathcal{O}^{\text{ref}}$, but also the full ontology $\mathcal{O}^{\text{full}}$. Both $\mathcal{O}^{\text{ref}}$ and $\mathcal{O}^{\text{full}}$ share the same vocabulary N_I, N_C, and N_R. We assume that $\mathcal{O}^{\text{ref}} \subseteq \mathcal{O}^{\text{full}}$. In the context of repairing inconsistent ontologies, $\mathcal{O}^{\text{full}}$ can be chosen to be the inconsistent ontology that we want to repair.

Then, to ensure further that by adding weakened axioms we do not cause a constraint violation in existing axioms and concepts, we choose the allowed weakening for RIAs such that all roles that are simple in $\mathcal{O}^{\text{full}}$, are also simple after adding to it a weakening of one of its axioms. We observe that for complex RIAs $S_1 \circ \cdots \circ S_n \sqsubseteq R$ we should not refine the role R. Since all roles returned by our refinement operator are simple in $\mathcal{O}^{\text{full}}$, such a replacement would make a role with was simple in $\mathcal{O}^{\text{full}}$ non-simple. A similar argument can be made for refining R in a simple RIA $S \sqsubseteq R$ where the role S is non-simple in $\mathcal{O}^{\text{full}}$. So the only way to refine the super role during the weakening of a RIA is when it is a simple RIA and additionally the sub role of the axiom is simple in $\mathcal{O}^{\text{full}}$.

When it comes to refining the left-hand side of RIAs, we do not need any special restrictions. The main significant observation is that all roles that are returned by the refinement will be simple. This means that in a simple RIA $R \sqsubseteq S$, even if S is simple, replacing R with another simple role will not cause S to become non-simple. For a complex RIA $S_1 \circ \cdots \circ S_n \sqsubseteq R$ on the other hand, the role R must already have been non-simple in $\mathcal{O}^{\text{full}}$, and replacing any S_i with a refinement has no effect on which roles are simple.

A more interesting question is whether such a weakening may still cause a non-regular role hierarchy. The important insight is that simple roles are always allowed on the left-hand side of a RIA. While this is more directly evident in some alternative definitions of regularity (e.g., [5]) it is not so apparent from the one presented in this paper. Intuitively, the constraint given above for regularity disallows dependency cycles that contain complex RIAs. Simple roles can not be part of such a cycle, since the cycle must contain at least one complex RIA to be a violation of the constraint, and all roles that depend in this sense on a complex RIA must be non-simple. A more formal justification for this fact is given in the proof for Lemma 4. Since all refinements of the left-hand side of RIAs are performed using simple roles, these can not lead to a non-regular RBox. Further, refinements of the super role of RIAs are only performed on simple RIAs $S \sqsubseteq R$ where S is a simple role. Since S is simple in this case, all refinements of R are allowed, potentially also if the refinement yielded a non-simple role.

Definition 1. *Let $\mathcal{O}$ be a $\mathcal{SROIQ}$ ontology. The set of* subconcepts *of $\mathcal{O}$ is given by*

$$\mathsf{sub}(\mathcal{O}) = \{\top, \bot\} \cup \bigcup_{C(a)\in\mathcal{O}} \mathsf{sub}(C) \cup \bigcup_{C\sqsubseteq D\in\mathcal{O}} (\mathsf{sub}(C) \cup \mathsf{sub}(D)) \ ,$$

where $\mathsf{sub}(C)$ is the set of subconcepts *in C such that*

$$\begin{aligned}
\mathsf{sub}(A) &= \{A\} \quad , A \in N_C \cup \{\top, \bot\} \ , & \mathsf{sub}(\neg C) &= \{\neg C\} \cup \mathsf{sub}(C) \ , \\
\mathsf{sub}(C \sqcup D) &= \{C \sqcup D\} \cup \mathsf{sub}(C) \cup \mathsf{sub}(D) \ , & \mathsf{sub}(\forall R.C) &= \{\forall R.C\} \cup \mathsf{sub}(C) \ , \\
\mathsf{sub}(C \sqcap D) &= \{C \sqcap D\} \cup \mathsf{sub}(C) \cup \mathsf{sub}(D) \ , & \mathsf{sub}(\exists R.C) &= \{\exists R.C\} \cup \mathsf{sub}(C) \ , \\
\mathsf{sub}(\geq n\ R.C) &= \{\geq n\ R.C\} \cup \mathsf{sub}(C) \ , & \mathsf{sub}(\leq n\ R.C) &= \{\leq n\ R.C\} \cup \mathsf{sub}(C) \ , \\
\mathsf{sub}(\exists R.\mathit{Self}) &= \{\exists R.\mathit{Self}\} \ , & \mathsf{sub}(\{i\}) &= \{\{i\}\} \ .
\end{aligned}$$

We will define now the upward and downward cover sets for concepts and roles. Intuitively, for a given concept the upward cover is the set of the most specific generalizations from the set of subconcepts or roles, while the downward cover set contains the most general specializations from the same set of subconcepts and roles. We define the upward and downward cover additionally also for non-negative integers, as they will be useful in the refinement of cardinality constraints.

Definition 2. *Let $\mathcal{O}^{\text{full}}$ and $\mathcal{O}^{\text{ref}} \subseteq \mathcal{O}^{\text{full}}$ be two $\mathcal{SROIQ}$ ontologies that share the same vocabulary N_C, N_R, and N_I. The* upward cover *and* downward cover *for a concept C are given by*

$$\begin{aligned}
\mathsf{UpCover}_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(C) = \{&D \in \mathsf{sub}(\mathcal{O}^{\text{full}}) \mid C \sqsubseteq_{\mathcal{O}^{\text{ref}}} D \text{ and} \\
&\nexists D' \in \mathsf{sub}(\mathcal{O}^{\text{full}}) \text{ with } C \sqsubset_{\mathcal{O}^{\text{ref}}} D' \sqsubset_{\mathcal{O}^{\text{ref}}} D\} \ , \\
\mathsf{DownCover}_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(C) = \{&D \in \mathsf{sub}(\mathcal{O}^{\text{full}}) \mid D \sqsubseteq_{\mathcal{O}^{\text{ref}}} C \text{ and} \\
&\nexists D' \in \mathsf{sub}(\mathcal{O}^{\text{full}}) \text{ with } D \sqsubset_{\mathcal{O}^{\text{ref}}} D' \sqsubset_{\mathcal{O}^{\text{ref}}} C\} \ .
\end{aligned}$$

The upward and downward covers for a role R are given by

$$\begin{aligned}
\mathsf{UpCover}_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(R) = \{&S \in \mathcal{L}(N_R) \mid R \sqsubseteq_{\mathcal{O}^{\text{ref}}} S \text{ and} \\
&\nexists S' \in \mathcal{L}(N_R) \text{ with } R \sqsubset_{\mathcal{O}^{\text{ref}}} S' \sqsubset_{\mathcal{O}^{\text{ref}}} S \text{ and} \\
&S, S' \text{ are simple in } \mathcal{O}^{\text{full}}\} \ , \\
\mathsf{DownCover}_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(R) = \{&S \in \mathcal{L}(N_R) \mid S \sqsubseteq_{\mathcal{O}^{\text{ref}}} R \text{ and} \\
&\nexists S' \in \mathcal{L}(N_R) \text{ with } S \sqsubset_{\mathcal{O}^{\text{ref}}} S' \sqsubset_{\mathcal{O}^{\text{ref}}} R \text{ and} \\
&S, S' \text{ are simple in } \mathcal{O}^{\text{full}}\} \ .
\end{aligned}$$

The upward and downward covers for a non-negative integer n are given by

$$\begin{aligned}
\mathsf{UpCover}_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(n) &= \{n, n+1\} \ , \\
\mathsf{DownCover}_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(R) &= \begin{cases} \{n\} & \textit{if } n = 0 \\ \{n, n-1\} & \textit{if } n > 0 \end{cases} \ .
\end{aligned}$$

Since they operate only over the subconcepts of $\mathcal{O}^{\text{full}}$, on their own, the upward and downward covers of concepts are missing some interesting refinements.

Example 2. *Let $N_C = \{A, B, C\}$, $N_R = \{r, s\}$, and $\mathcal{O} = \{A \sqsubseteq B, r \sqsubseteq s\}$. $\mathsf{sub}(\mathcal{O}) = \{\top, \bot, A, B\}$. The upward cover of $C \sqcup A$ is equal to $\mathsf{UpCover}_{\mathcal{O},\mathcal{O}}(C \sqcup A) = \{\top\}$. The potentiality refinement to $C \sqcup B$ will be missed even by iterated application of the upward cover because $C \sqcup B \notin \mathsf{sub}(\mathcal{O})$. Similarly, $\mathsf{UpCover}_{\mathcal{O},\mathcal{O}}(\forall r.A) = \{\top\}$, even if $\forall r.B$ and $\forall s.A$ are reasonable generalizations.*

To also capture these omissions, we define generalization and specialization operators that exploit the recursive structure of the concept being refined to generate more complex refinements. For convenience, we also define these operators for roles.

Definition 3. *Let $\uparrow$ and $\downarrow$ be two functions with domain $\mathcal{L}(N_C, N_R, N_I) \cup \mathcal{L}(N_C) \cup \mathbb{N}_0$. They map every concept to a finite subset of $\mathcal{L}(N_C, N_R, N_I)$, every role to a subset of $\mathcal{L}(N_C)$, and every non-negative integer to a finite subset of $\mathbb{N}_0$. The* abstract refinement operator *is defined recursively by induction on the structure of concepts as follows.*

$$
\begin{aligned}
\zeta_{\uparrow,\downarrow}(A) &= \uparrow(A) \quad , A \in N_C \cup \{\top, \bot\} \ , \\
\zeta_{\uparrow,\downarrow}(\neg C) &= \uparrow(\neg C) \cup \{\neg C' \mid C' \in \zeta_{\downarrow,\uparrow}(C)\} \ , \\
\zeta_{\uparrow,\downarrow}(C \sqcap D) &= \uparrow(C \sqcap D) \cup \{C' \sqcap D \mid C' \in \zeta_{\uparrow,\downarrow}(C)\} \cup \{C \sqcap D' \mid D' \in \zeta_{\uparrow,\downarrow}(D)\} \ , \\
\zeta_{\uparrow,\downarrow}(C \sqcup D) &= \uparrow(C \sqcup D) \cup \{C' \sqcup D \mid C' \in \zeta_{\uparrow,\downarrow}(C)\} \cup \{C \sqcup D' \mid D' \in \zeta_{\uparrow,\downarrow}(D)\} \ , \\
\zeta_{\uparrow,\downarrow}(\forall R.C) &= \uparrow(\forall R.C) \cup \{\forall R'.C \mid R' \in \downarrow(R)\} \cup \{\forall R.C' \mid C' \in \zeta_{\uparrow,\downarrow}(C)\} \ , \\
\zeta_{\uparrow,\downarrow}(\exists R.C) &= \uparrow(\exists R.C) \cup \{\exists R'.C \mid R' \in \uparrow(R)\} \cup \{\exists R.C' \mid C' \in \zeta_{\uparrow,\downarrow}(C)\} \ , \\
&\quad \mathcal{SROIQ} \textit{ concepts:} \\
\zeta_{\uparrow,\downarrow}(\{i\}) &= \uparrow(\{i\}) \ , \\
\zeta_{\uparrow,\downarrow}(\exists R.\mathit{Self}) &= \uparrow(\exists R.\mathit{Self}) \cup \{\exists R'.\mathit{Self} \mid R' \in \uparrow(R)\} \ , \\
\zeta_{\uparrow,\downarrow}(\geq n\ R.C) &= \uparrow(\geq n\ R.C) \cup \{\geq n\ R'.C \mid R' \in \uparrow(R)\} \\
&\quad \cup \{\geq n\ R.C' \mid C' \in \zeta_{\uparrow,\downarrow}(C)\} \cup \{\geq n'R.C \mid n' \in \downarrow(C)\} \ , \\
\zeta_{\uparrow,\downarrow}(\leq n\ R.C) &= \uparrow(\leq n\ R.C) \cup \{\leq n\ R'.C \mid R' \in \downarrow(R)\} \\
&\quad \cup \{\leq n\ R.C' \mid C' \in \zeta_{\downarrow,\uparrow}(C)\} \cup \{\leq n'R.C \mid n' \in \uparrow(C)\} \ , \\
&\quad \mathcal{SROIQ} \textit{ roles:} \\
\zeta_{\uparrow,\downarrow}(R) &= \uparrow(R) \ .
\end{aligned}
$$

From the abstract refinement operator $\zeta_{\uparrow,\downarrow}$, two concrete refinement operators, the generalization operator *and* specialization operator *are respectively defined as*

$$
\gamma_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}} = \zeta_{\mathsf{UpCover}_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}},\mathsf{DownCover}_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}} \quad \textit{and}
$$

$$
\rho_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}} = \zeta_{\mathsf{DownCover}_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}},\mathsf{UpCover}_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}} \ .
$$

Revisiting the case in Example 2 we observe that $\gamma_{\mathcal{O},\mathcal{O}}(C \sqcup A) = \{\top, \top \sqcup A, C \sqcup A, C \sqcup B\}$ does contain $C \sqcup B$ as a possible refinement. Similarly, $\gamma_{\mathcal{O},\mathcal{O}}(\forall r.A) = \{\top, \forall r.A, \forall s.A, \forall r.B\}$ contains $\forall r.B$. We will show now some basic properties of $\gamma_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}$ and $\rho_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}$ that will prove useful in the remainder of this paper.

Lemma 1. *For every pair of $\mathcal{SROIQ}$ ontologies $\mathcal{O}^{\text{ref}}, \mathcal{O}^{\text{full}}$ and every pair of concepts or roles $X, Y \in \mathcal{L}(N_C, N_R, N_I) \cup \mathcal{L}(N_R)$:*

1. ***generalisation:*** *if $X \in \gamma_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(Y)$ then $Y \sqsubseteq_{\mathcal{O}^{\text{ref}}} X$*
   specialisation: *if $X \in \rho_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(Y)$ then $X \sqsubseteq_{\mathcal{O}^{\text{ref}}} Y$*
2. ***generalisation finiteness:*** *$\gamma_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(X)$ is finite*
   specialisation finiteness: *$\rho_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(X)$ is finite*

We define now the *axiom weakening operator* using these generalization and specialization operators.

Definition 4. *Given an axiom ϕ, the set of* weakenings *with respect to the reference ontology $\mathcal{O}^{\text{ref}}$ and full ontology $\mathcal{O}^{\text{full}}$, written $g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(\phi)$ is defined such that*

$$\begin{aligned}
g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(C \sqsubseteq D) &= \{C' \sqsubseteq D \mid C' \in \rho_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(C)\} \cup \{C \sqsubseteq D' \mid D' \in \gamma_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(D)\} \ , \\
g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(C(a)) &= \{C'(a) \mid C' \in \gamma_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(C)\} \ , \\
g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(R(a,b)) &= \{R'(a,b) \mid R' \in \gamma_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(R)\} \cup \{R(a,b), \bot \sqsubseteq \top\} \ , \\
g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(\neg R(a,b)) &= \{\neg R'(a,b) \mid R' \in \rho_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(R)\} \cup \{\neg R(a,b), \bot \sqsubseteq \top\} \ , \\
g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(a = b) &= \{a = b, \bot \sqsubseteq \top\} \ , \quad g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(a \neq b) = \{a \neq b, \bot \sqsubseteq \top\} \ , \\
&\mathcal{SROIQ} \textit{ axioms:} \\
g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(\textit{disjoint}(R,S)) &= \{\textit{disjoint}(R',S) \mid R' \in \rho_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(R)\} \\
&\cup \{\textit{disjoint}(R,S') \mid S' \in \rho_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(S)\} \\
&\cup \{\textit{disjoint}(R,S), \bot \sqsubseteq \top\} \ , \\
g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(S_1 \circ \cdots \circ S_n \sqsubseteq R) &= \{S_1 \circ \cdots \circ S_i' \circ \cdots \circ S_n \sqsubseteq R \mid S_i' \in \rho_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(S_i) \textit{ for } i = 1, \ldots, n\} \\
&\cup \{S_1 \sqsubseteq R' \mid R' \in \gamma_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}} \textit{ and } n = 1 \textit{ and } S_1 \textit{ is simple in } \mathcal{O}^{\text{full}}\} \\
&\cup \{S_1 \circ \cdots \circ S_n \sqsubseteq R, \bot \sqsubseteq \top\} \ .
\end{aligned}$$

The axioms in the set $g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(\phi)$ are indeed weaker than ϕ for every axiom ϕ, in the sense that, given the reference ontology $\mathcal{O}^{\text{ref}}$, ϕ entails them and the opposite in not necessarily true.

Lemma 2. *For every $\mathcal{SROIQ}$ axiom ϕ, if $\phi' \in g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(\phi)$, then $\phi \models_{\mathcal{O}^{\text{ref}}} \phi'$*

Proof. We will handle each type of axiom separately.

- If $\phi = C \sqsubseteq D$, suppose $\phi' = C' \sqsubseteq D'$. From Lemma 1.1 we know that $C' \sqsubseteq_{\mathcal{O}^{\text{ref}}} C$ and $D \sqsubseteq_{\mathcal{O}^{\text{ref}}} D'$. By transitivity of subsumption, we conclude that $C \sqsubseteq D \models_{\mathcal{O}^{\text{ref}}} C' \sqsubseteq D'$.
- If $\phi = C(a)$, suppose $\phi' = C'(a)$. From Lemma 1.1 we know that $C \sqsubseteq_{\mathcal{O}^{\text{ref}}} C'$. Given any model I of $\mathcal{O}^{\text{ref}} \cup \{\phi\}$, $a^I \in C^I$. Since $C^I \subseteq C'^I$ in every model of $\mathcal{O}^{\text{ref}}$, $a^I \in C'^I$. We conclude that $C(a) \models_{\mathcal{O}^{\text{ref}}} C'(a)$.
- If $\phi = R(a,b)$, suppose $\phi' = R'(a,b)$. From Lemma 1.1 we know that $R \sqsubseteq_{\mathcal{O}^{\text{ref}}} R'$. Given any model I of $\mathcal{O}^{\text{ref}} \cup \{\phi\}$, $\langle a^I, b^I \rangle \in R^I$. Since $R^I \subseteq R'^I$ in every model of $\mathcal{O}^{\text{ref}}$, $\langle a^I, b^I \rangle \in R'^I$. We conclude that $R(a,b) \models_{\mathcal{O}^{\text{ref}}} R'(a,b)$.

- If $\phi = \neg R(a, b)$, suppose $\phi' = R'(a, b)$. From Lemma 1.1 we know that $R' \sqsubseteq_{\mathcal{O}^{\mathrm{ref}}} R$. Given any model I of $\mathcal{O}^{\mathrm{ref}} \cup \{\phi\}$, $\langle a^I, b^I \rangle \notin R^I$. Since $R'^I \subseteq R^I$ in every model of $\mathcal{O}^{\mathrm{ref}}$, $\langle a^I, b^I \rangle \notin R'^I$. We conclude that $\neg R(a, b) \models_{\mathcal{O}^{\mathrm{ref}}} \neg R'(a, b)$.
- If $\phi = \mathit{disjoint}(R, S)$, suppose $\phi' = \mathit{disjoint}(R', S')$. From Lemma 1.1 we know that $R' \sqsubseteq_{\mathcal{O}^{\mathrm{ref}}} R$ and $S' \sqsubseteq_{\mathcal{O}^{\mathrm{ref}}} S$. Given any model I of $\mathcal{O}^{\mathrm{ref}} \cup \{\phi\}$, $R^I \cap S^I = \emptyset$. Since $R'^I \subseteq R^I$ and $S'^I \subseteq S^I$ in every model of $\mathcal{O}^{\mathrm{ref}}$, $R'^I \cap S'^I = \emptyset$. We conclude that $\mathit{disjoint}(R, S) \models_{\mathcal{O}^{\mathrm{ref}}} \mathit{disjoint}(R', S')$.
- If $\phi = S_1 \circ \cdots \circ S_n \sqsubseteq R$, suppose $\phi' = S'_1 \circ \cdots \circ S'_n \sqsubseteq R'$. From Lemma 1.1 we know that $R \sqsubseteq_{\mathcal{O}^{\mathrm{ref}}} R'$ and $S'_i \sqsubseteq_{\mathcal{O}^{\mathrm{ref}}} S_i$ for $i = 1, \ldots, n$. Given any model I of $\mathcal{O}^{\mathrm{ref}} \cup \{\phi\}$, $S_1^I \circ \cdots \circ S_n^I \subseteq R^I$. Since $R^I \subseteq R'^I$ and $S_i'^I \subseteq S_i^I$ for $i = 1, \ldots, n$ in every model of $\mathcal{O}^{\mathrm{ref}}$, $S_1'^I \circ \cdots \circ S_n'^I \subseteq R'^I$. We conclude that $S_1 \circ \cdots \circ S_n \sqsubseteq R \models_{\mathcal{O}^{\mathrm{ref}}} S'_1 \circ \cdots \circ S'_n \sqsubseteq R'$.

□

Clearly, replacing an axiom in the full ontology with a weakening can not reduce the number of models of the ontology. However, for the weakening to be useful in practice, we must show additionally that by adding the weakened axioms to the ontology will not violate any of the constraints that ensure the decidability of $\mathcal{SROIQ}$. To do this, we show first that all roles that are simple in $\mathcal{O}^{\mathrm{full}}$ are also simple in the ontology obtained by adding the weakening of any axiom.

Lemma 3. *For every axiom $\phi \in \mathcal{O}^{\mathrm{full}}$ and role R, if $\phi' \in g_{\mathcal{O}^{\mathrm{ref}}, \mathcal{O}^{\mathrm{full}}}(\phi)$ and R simple in $\mathcal{O}^{\mathrm{full}}$, then R is simple in $\mathcal{O}^{\mathrm{full}} \cup \{\phi'\}$.*

Proof. (*Sketch*) Assume, by contradiction, that R is a simple role in $\mathcal{O}^{\mathrm{full}}$ and non-simple in $\mathcal{O}^{\mathrm{full}} \cup \{\phi'\}$. Since R is simple in $\mathcal{O}^{\mathrm{full}}$ it is neither the universal nor the existential role, does not appear as the super role in any complex RIA of $\mathcal{O}^{\mathrm{full}}$, and neither on the right-hand side of a simple RIA in $\mathcal{O}^{\mathrm{full}}$ where the sub role is non-simple. We conclude that ϕ' must be a RIA, that has R as the super role and is either complex, or for which the sub role is non-simple. If ϕ' is a complex RIA then, by definition of the weakening operator, ϕ must be a complex RIA and R must be the super role in ϕ, making it non-simple in $\mathcal{O}^{\mathrm{full}}$, which contradicts our assumption. Similarly, if ϕ' is a simple RIA with a non-simple role as the sub role, the sub role of ϕ must be equal to that of ϕ' because the refinement operators return only roles simple in $\mathcal{O}^{\mathrm{full}}$. Further, since the super role of a RIA is only refined if the sub role is simple, $\phi' = \phi$, which means that R is non-simple in $\mathcal{O}^{\mathrm{full}}$, which contradicts the assumptions. It follows that such a role R does not exist. □

Note that the proof works also for weakening axioms in an ontology $\mathcal{O}$ as long as all non-simple roles in $\mathcal{O}$ are also non-simple in $\mathcal{O}^{\mathrm{full}}$ (or, equivalently, that all simple roles in $\mathcal{O}^{\mathrm{full}}$ are also simple in $\mathcal{O}$). This is an important observation, since it means that repeatedly adding weakened axioms is possible. We will show next that the

Lemma 4. *For every axiom $\phi \in \mathcal{O}^{\mathrm{full}}$, if $\phi' \in g_{\mathcal{O}^{\mathrm{ref}}, \mathcal{O}^{\mathrm{full}}}(\phi)$ and the role hierarchy of $\mathcal{O}^{\mathrm{full}}$ is regular, then the role hierarchy of $\mathcal{O}^{\mathrm{full}} \cup \{\phi'\}$ is also regular.*

Proof. (*Sketch*) Let us first argue that if there exists a preorder $\preceq$ that satisfies the constraints necessary for checking regularity, then there exists on such that $S_1 \preceq S_2$, $S \preceq R$ and $R \not\preceq S$ for all simple roles S, S_1, S_2 and non-simple roles R. Firstly, $S_1 \not\preceq S_2$ and $S \not\preceq R$ can not be required, because absence of a tuple is only required for complex RIAs, where the super role must not be a predecessor of the roles on the left-hand side. Since S_1 and S are simple, they do not appear as the super role in a complex RIA. Similarly, $R \preceq S$ can not be required. Since S is simple and R non-simple, it can not be required directly through an axiom of the form $R \sqsubseteq S$. By induction, it can not be required through transitivity, since $R \preceq T$ and $T \preceq S$ would have to be required. If T is simple, $R \preceq T$ can not be required, and if T is non-simple, $T \preceq S$ can not be required.

Since $\mathcal{O}^{\text{full}}$ has a regular role hierarchy, there exists such a $\preceq$ for $\mathcal{O}^{\text{full}}$. We will show that $\preceq$ is also a witness for regularity of $\mathcal{O}^{\text{full}} \cup \{\phi'\}$. All RIA in $\mathcal{O}^{\text{full}}$ are of one of allowed forms for $\preceq$. It is therefore sufficient to verify that ϕ' has one of the allowed forms. If $\phi' = \phi$ or ϕ' is not a RIA, it does not affect the regularity. Otherwise, if ϕ' is a simple RIA $S \sqsubseteq R$, then by definition of the weakening operator, S is simple in $\mathcal{O}^{\text{full}}$. Given that S is simple, $S \preceq R$ holds for simple and non-simple R by our choice of $\preceq$. If ϕ' is a complex RIA $S'_1 \circ \cdots \circ S'_n \sqsubseteq R$, then ϕ is also a complex RIA $S_1 \circ \cdots \circ S_n \sqsubseteq R$ and R is non-simple in $\mathcal{O}^{\text{full}}$. If $S_i \preceq R$ and $S_i \not\preceq R$, then so will $S'_i \preceq R$ and $R \not\preceq S'_i$, either because $S_i = S'_i$ or because S'_i is simple and R is non-simple. Since $\mathcal{O}^{\text{full}}$ has a regular role hierarchy, the only case in which $R \preceq S_i$ is if $S_i = R$. In this case, $S'_i \preceq R$ and $R \not\preceq S'_i$ will still hold if $S_i \neq R$. If $S_i = R$, either $i = 0$ or $i = n$ which is allowed. The only delicate case is if $\phi = R \circ R \sqsubseteq R$, which will result in either $\phi' = S'_1 \circ R \sqsubseteq R$ or $R \circ S'_2 \sqsubseteq R$, both of which are valid. □

Like for simple roles, also regularity is maintained by repeated addition of weaker axioms. With the help of Lemma 3 and Lemma 4 we will now sketch a proof showing that adding weakened axioms to a $\mathcal{SROIQ}$ ontology will yield another valid $\mathcal{SROIQ}$ ontology.

Lemma 5. *Given that $\mathcal{O}^{\text{ref}}$ and $\mathcal{O}^{\text{full}}$ are valid $\mathcal{SROIQ}$ ontologies. For every axiom $\phi \in \mathcal{O}^{\text{full}}$, if $\phi' \in g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(\phi)$, then $\mathcal{O}^{\text{full}} \cup \{\phi'\}$ is a valid $\mathcal{SROIQ}$ ontology.*

Proof. (*Sketch*) We have established already in Lemma 4, that the regularity of the RBox will be preserved. It is guaranteed by Lemma 3 that all roles that were simple before addition, are still simple afterwards. Therefore, all usages of roles in axioms and concepts that were not touched by the refinement do not pose a problem. The condition static that the upward and downward cover of a role contain only roles that are simple in $\mathcal{O}^{\text{full}}$ (and therefore by Lemma 3 also in $\mathcal{O}^{\text{full}} \cup \{\phi'\}$) forces that every refinement of a role is simple. This restriction to simple roles guarantees that no non-simple role may be used in disjoint role axioms, or the scope of cardinality and self constraints. □

3. Implementing Axiom Weakening for $\mathcal{SROIQ}$

Refinement operator and axiom weakening have previously been implemented for $\mathcal{ALC}$ in [6]. Based on this, we have now extended the implementation to cover the full range of $\mathcal{SROIQ}$ axioms and concepts.[2] The concept refinement and axiom weakening operators for $\mathcal{SROIQ}$

[2]The source code for the implementation is available at https://github.com/rolandbernard/ontologyutils

Algorithm 1 RepairOntologyWeaken($\mathcal{O}$)

$\mathcal{O}^{\text{full}} \leftarrow \mathcal{O}$
$\mathcal{O}^{\text{ref}} \leftarrow$ FindMaximalConsistentSubset($\mathcal{O}$)
while $\mathcal{O}$ is inconsistent **do**
  $\phi_{\text{bad}} \leftarrow$ FindBadAxiom($\mathcal{O}$)
  $\Phi_{\text{weaker}} \leftarrow g_{\mathcal{O}^{\text{ref}},\mathcal{O}^{\text{full}}}(\phi_{\text{bad}})$
  $\phi_{\text{weaker}} \leftarrow$ SelectWeakerAxiom(Φ_{weaker})
  $\mathcal{O} \leftarrow \mathcal{O} \setminus \{\phi_{\text{bad}}\} \cup \{\phi_{\text{weaker}}\}$
end while
Return $\mathcal{O}$

have been implemented as discussed above. Further, we implemented a repair algorithm using the axiom weakening operator based on the procedures already proposed in [6] and [4]. The implementation performs weakening in OWL 2 DL [3] and is implemented in Java using the OWL API [7]. A plug-in for the ontology development tool Protégé has also been implemented, but will not be discussed in detail in this paper.[3] The plug-in allows for manually weakening axioms and executing the automatic repair algorithm.

The automatic repair by weakening is implemented as shown in Algorithm 1. The reference ontology is selected by choosing a maximal consistent subset of the inconsistent ontology. In our implementation used for the evaluation in this paper, the reference ontology was selected by randomly sampling a maximal consistent subset. The procedure FindBadAxiom($\mathcal{O}$) may be implemented in a number of ways. Here we consider an implementation that samples some (or all) of the minimal inconsistent subsets of $\mathcal{O}$ and selects as the bad axiom the one occurring most frequently. Then, SelectWeakerAxiom(Φ_{weaker}) has been chosen such that is selects an axiom uniformly at random form Φ_{weaker}. For all experiments presented in this paper, the FaCT++ reasoner [8] was used to compute all entailment and consistency checks.

4. Weakening makes you strong: evaluation aspects

To experimentally evaluate the proposed axiom weakening operator in the context of its use in automatic repair of ontologies, we need some way to compare the quality of repair. As has already been discussed in [6], the problem of deciding which of two possible repaired ontologies $\mathcal{O}_1$ or $\mathcal{O}_2$ is preferable is not generally well-defined. Similar to what has been proposed in [6] we will base the evaluation of the repairs on the size of the *inferred class hierarchy*. To compare two possible repairs, we use the *inferable information content* (IIC) as defined in [6]. The IIC of an ontology $\mathcal{O}_1$ with respect to a second ontology $\mathcal{O}_2$, written $\mathsf{IIC}(\mathcal{O}_1, \mathcal{O}_2)$, is a number between 0 and 1. Value closer to 0 indicates that $\mathcal{O}_1$ contains more "information" than $\mathcal{O}_2$, while a value towards 1 indicates the opposite. Some weaknesses of this measure when it comes to evaluating repairs, like the fact that only atomic concepts are considered, have already been discussed in [6]. For the case of repairing $\mathcal{SROIQ}$ ontologies this is even more relevant, since the role hierarchy is entirely ignored.

[3] The Protégé plugin is available at https://github.com/rolandbernard/protege-weakening

Abbreviation	Name	Expressivity	Axioms	Concepts	Roles	Subconcepts
admin	Nurse Administrator	$\mathcal{ALCHOIF}$	229	42	29	144
ahso	Animal Health Surveillance Ontology	$\mathcal{ALCRIF}$	166	38	31	104
cdao	Comparative Data Analysis Ontology	$\mathcal{ALCROIQ}$	437	132	68	375
cdpeo	Chronic Disease Patient Education	$\mathcal{ALCHF}$	422	41	31	170
covid19-ibo	Covid19 Impact on Banking Ontology	$\mathcal{ALCH}$	288	160	33	227
ecp	Electronic Care Plan	$\mathcal{ALCRQ}$	127	33	17	99
emo	Enzyme Mechanism Ontology	$\mathcal{ALCHQ}$	368	157	24	255
evi	Evidence Graph Ontology	$\mathcal{ALCRI}$	143	30	38	69
falls	Falls Prevention	$\mathcal{ALCH}$	79	30	20	35
fo	Fern Ontology	$\mathcal{ALCHI}$	59	31	4	46
gbm	Glioblastoma	$\mathcal{ALCIF}$	603	108	28	227
gfvo	Genomic Feature and Variation Ontology	$\mathcal{ALCH}$	332	102	30	170
koro	Knowledge Object Reference Ontology	$\mathcal{ALCHI}$	262	110	19	194
lico	Liver Case Ontology	$\mathcal{ALCHQ}$	366	93	36	230
mamo	Mathematical Modelling Ontology	$\mathcal{ALCR}$	229	107	3	154
mpio	Minimum PDDI Information Ontology	$\mathcal{ALCH}$	38	30	14	45
provo	Provenance Ontology	$\mathcal{ALCRIN}$	170	31	42	128
qudt	Quantities, Units, Dimensions, and Types	$\mathcal{SHOIQ}$	293	74	73	177
trans	Nurse Transitional	$\mathcal{ALCROIF}$	244	44	22	123
triage	Nurse triage	$\mathcal{ALCHF}$	132	33	29	129
vio	Vaccine Investigation Ontology	$\mathcal{ALCRI}$	249	81	44	235
pizza	Pizza Ontology	$\mathcal{SHOIN}$	1131	100	8	376

Table 1
The BioPortal ontologies used for evaluation. The number of axioms, concept names, role names, and subconcepts are taken after preprocessing.

Definition 5. *The* inferred class hierarchy *of an ontology $\mathcal{O}$ is given by*

$$\mathsf{Inf}(\mathcal{O}) = \{A \sqsubseteq B \mid A, B \in N_C \text{ and } \mathcal{O} \models A \sqsubseteq B\} \ .$$

The inferable information content *of an ontology $\mathcal{O}_1$ with respect to another ontology $\mathcal{O}_2$ is given by*

$$\mathsf{IIC}(\mathcal{O}_1, \mathcal{O}_2) = \frac{\mathbf{card}(\mathsf{Inf}(\mathcal{O}_1) \setminus \mathsf{Inf}(\mathcal{O}_2))}{\mathbf{card}(\mathsf{Inf}(\mathcal{O}_1) \setminus \mathsf{Inf}(\mathcal{O}_2)) + \mathbf{card}(\mathsf{Inf}(\mathcal{O}_2) \setminus \mathsf{Inf}(\mathcal{O}_1))} \ ,$$

where $\mathbf{card}(X)$ is the cardinality of the set X.

For the experimental evaluation we have selected ontologies of varying size and expressivity from BioPortal[4] [9]. Additionally, the pizza ontology[5] was included in the testing. Some characteristics of the used ontologies are shown in Table 1. On average, they contain about 289 axioms, 73 concept names, 29 role names, and 168 subconcepts.

Since the ontologies use OWL 2, the axioms and concepts do not map directly to $\mathcal{SROIQ}$. In order to follow the definitions laid out in this paper, the OWL ontologies axioms where normalized to conform with $\mathcal{SROIQ}$. During the preprocessing, we further removed axioms related to data properties and any axiom that caused any OWL 2 DL profile violation, as our weakening does not handle them.

The evaluation precedes by first making the ontologies inconsistent. This was achieved by repeatedly adding axioms to the ontology until they became inconsistent. The newly added

[4]https://bioportal.bioontology.org/
[5]Available from Protégé at https://protege.stanford.edu/ontologies/pizza/pizza.owl

Algorithm 2 RepairOntologyRemove($\mathcal{O}$)

while $\mathcal{O}$ is inconsistent **do**

  $\phi_{\text{bad}} \leftarrow \text{FindBadAxiom}(\mathcal{O})$

  $\mathcal{O} \leftarrow \mathcal{O} \setminus \{\phi_{\text{bad}}\}$

end while

Return $\mathcal{O}$

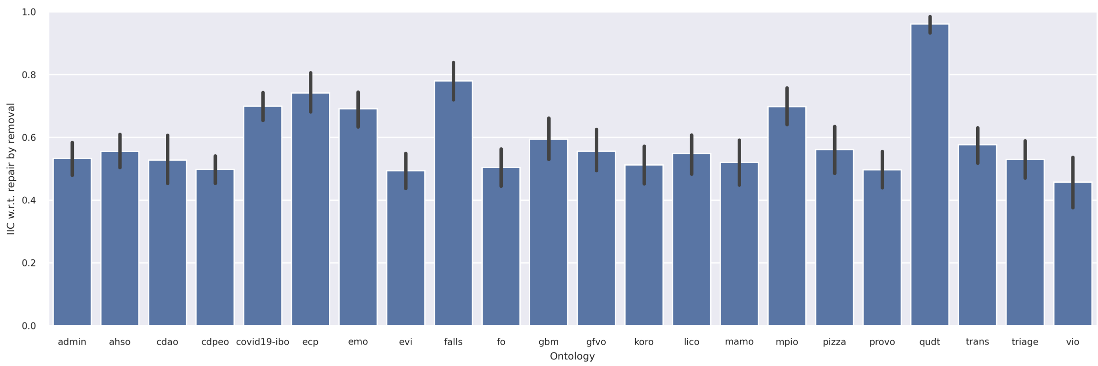

Figure 1: Mean IIC with respect to repair via removal per ontology. The error bars show the 95% confidence interval.

axioms were generated by strengthening randomly selected axioms of the original ontology. It was ensured that the added axioms on their own were not inconsistent. After making the ontology inconsistent, it was repaired once with the repair algorithm using the axiom weakening operator presented in Algorithm 1 and once using Algorithm 2. Additionally, the repair was also performed by selecting a randomly sampled maximal consistent subset. This process, both making the ontology inconsistent and then repairing it, was repeated one hundred times for each ontology, and the IIC was computed between the repair by weakening and the repairs by removal and random maximal consistent subset. The evaluation was performed using a randomly selected maximal consistent subset as the reference ontology and by sampling 16 minimal inconsistent subsets during the selection of bad axioms.

Unfortunately, even though the utilized reasoners are generally very fast to evaluate queries on the selected ontologies, they exhibit undesirable performance in some edge cases. When performance pitfalls are encountered, they make the computations required for weakening unreasonably slow. For this reason a timeout of 5 minutes was placed on the repairs execution and the outputs of these runs where discarded and replaced by new runs. The results of these experiments are listed in Table 2 and shown in Figure 1 and Figure 2.

The results of the evaluation suggest that the repair by weakening is on average about as good or better than the repair by removal of axioms. While this supports the conclusion in [6] that weakening is able to retain more information than removal, the observed advantage was worse than what has been observed in [6]. In contrast, it can be seen that the repair using weakening is not in general better than choosing a random maximal consistent subset. There

Abbreviation	IIC w.r.t. repair by removal	IIC w.r.t. maximal consistent subset
admin	0.53 [0.47; 0.59]	0.39 [0.31; 0.47]
ahso	0.56 [0.50; 0.62]	0.51 [0.44; 0.57]
cdao	0.53 [0.44; 0.61]	0.53 [0.45; 0.61]
cdpeo	0.50 [0.45; 0.55]	0.22 [0.16; 0.28]
covid19-ibo	0.70 [0.65; 0.75]	0.63 [0.57; 0.69]
ecp	0.74 [0.67; 0.81]	0.36 [0.28; 0.44]
emo	0.69 [0.63; 0.75]	0.60 [0.53; 0.66]
evi	0.49 [0.43; 0.55]	0.59 [0.53; 0.66]
falls	0.78 [0.71; 0.85]	0.49 [0.41; 0.58]
fo	0.50 [0.44; 0.57]	0.70 [0.62; 0.76]
gbm	0.59 [0.52; 0.66]	0.52 [0.44; 0.59]
gfvo	0.56 [0.49; 0.62]	0.54 [0.49; 0.60]
koro	0.51 [0.45; 0.57]	0.37 [0.29; 0.45]
lico	0.55 [0.48; 0.62]	0.53 [0.46; 0.60]
mamo	0.52 [0.44; 0.60]	0.68 [0.61; 0.74]
mpio	0.70 [0.63; 0.76]	0.73 [0.67; 0.78]
provo	0.50 [0.43; 0.56]	0.55 [0.49; 0.62]
qudt	0.96 [0.93; 0.99]	0.44 [0.35; 0.55]
trans	0.58 [0.52; 0.64]	0.43 [0.35; 0.52]
triage	0.53 [0.46; 0.60]	0.53 [0.46; 0.60]
vio	0.46 [0.37; 0.55]	0.49 [0.40; 0.57]
pizza	0.56 [0.49; 0.64]	0.61 [0.53; 0.68]
Overall	0.59 [0.57; 0.61]	0.52 [0.50; 0.54]

Table 2
Results of the evaluation. IIC is given as mean and 95% confidence interval in brackets.

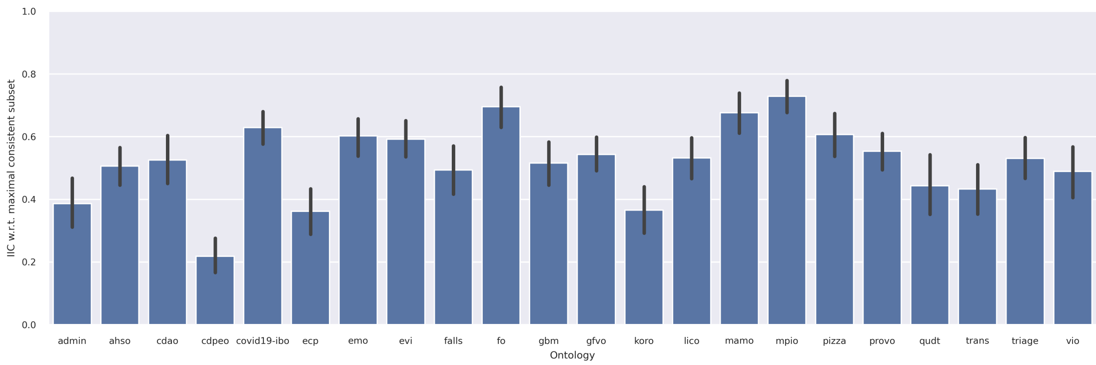

Figure 2: Mean IIC with respect to a random maximal consistent subset per ontology. The error bars show the 95% confidence interval.

are ontologies for which the repairs by weakening are on average significantly worse when comparing using IIC. This is however a somewhat unequal comparison since an alternative repair with weakening could start with a maximal consistent subset and use weakening to add in more information from the remaining axioms. Still, this result suggests that the heuristic used for selecting bad axioms is not reliable for preserving information.

5. Outlook

We have proposed refinement operators and an axiom weakening operator for all aspects of $\mathcal{SROIQ}$ and shown that for repairs of inconsistent ontologies weakening can, in some cases, significantly outperform removal. Further additions to the refinement operators may be studied, e.g., using non-simple roles in the upward and downward covers in certain contexts. Relaxing the allowed weakening for RIAs may also be considered. We have also seen that the repair algorithm likely needs better heuristics to steer the selection of axioms and weakenings. Future work could focus on finding robust measures for comparing the quality of repairs.

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